Pricing Liquidity Risk with Heterogeneous Investment Horizons[*]

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ABSTRACT

We develop a new asset pricing model with stochastic transaction costs and investors with heterogenous horizons. Short-term investors hold only liquid assets in equilibrium. This generates segmentation effects in the pricing of liquid versus illiquid assets. Specifically, the liquidity (risk) premia of illiquid assets are determined by the heterogeneity in investor horizons and by the correlation between liquid and illiquid assets. We estimate our model for the cross-section of U.S. stocks and find that it fits average returns substantially better than a standard liquidity CAPM. Allowing for heterogenous horizons also leads to much larger estimates for the liquidity premia.

The investment horizon is a key feature distinguishing different categories of investors, with high-frequency traders and long-term investors such as pension funds at the two extremes of the investment horizon spectrum. Most of the literature on horizon effects in portfolio choice and asset pricing builds on the theoretical insight of Merton's (1971) hedging demands and demonstrates that long-horizon decisions can differ substantially from single-period decisions for various model specifications.

Surprisingly, the interaction between investment horizons and liquidity has attracted much less attention. Even in the absence of hedging demands, heterogeneous investment horizons can have important asset pricing effects for the simple reason that different horizons imply different trading frequencies. More specifically, liquidity plays a distinct role for investors with diverse horizons because trading costs only matter when trading actually takes place. The investment horizon then becomes a key element in the asset pricing effects of liquidity.

We explicitly take this standpoint and derive a new liquidity-based asset pricing model featuring risk-averse investors with heterogeneous investment horizons and stochastic transaction costs. Investors with longer investment horizons are clearly less concerned about trading costs, because they do not necessarily trade every period. Our model generates a number of new implications on the pricing of liquidity that are strongly supported empirically when we test them on the cross-section of U.S. stock returns.

Previous theories of liquidity and asset prices have largely ignored heterogeneity in investor horizons, with the exception of the seminal paper of Amihud and Mendelson (1986), who study a setting where risk-neutral investors have heterogenous horizons. Their model generates *clientele* effects: short-term investors hold the liquid assets and long-term investors hold the illiquid assets,

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which leads to a concave relation between transaction costs and expected returns.[1] Besides risk-neutrality, Amihud and Mendelson (1986) assume that transaction costs are constant. However, there is large empirical evidence that liquidity is time-varying. Assuming stochastic transaction costs, Acharya and Pedersen (2005) set out one of the most influential asset pricing models with liquidity risk, where various liquidity risk premiums are generated. However, this model includes homogeneous investors with a one-period horizon and thus implies a linear (as opposed to concave) relation between (expected) transaction costs and expected returns. Our paper bridges these two seminal papers, because our model entails heterogeneous horizons, as in Amihud and Mendelson (1986), with stochastic illiquidity and risk aversion, as in Acharya and Pedersen (2005). This leads to a number of novel and important implications for the impact of both expected liquidity and liquidity risk on asset prices.

Our model setup is easily described. We have multiple assets with i.i.d. dividends and stochastic transaction costs, and many investor types with mean-variance utility over terminal wealth but different investment horizons. We obtain a stationary equilibrium in an overlapping generation setting and we solve for expected returns in closed form.

This theoretical setup implies the existence of an intriguing equilibrium with partial segmentation. Short-term investors optimally choose not to invest in the most illiquid assets, intuitively because their expected returns are not sufficient to cover expected transaction costs. In contrast, long-term investors trade less frequently and can afford to invest in illiquid assets. This clientele

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[1]Hopenhayn and Werner (1996) propose a similar set-up featuring risk-neutral investors with heterogeneity in impatience and endogenously determined liquidity effects.

partition is different from Amihud and Mendelson (1986), because our risk-averse long-horizon investors also buy liquid assets for diversification purposes.

The partial segmentation equilibrium implies different expressions for the expected returns of liquid and segmented assets. For liquid assets, expected returns contain the familiar compensation for expected transaction costs and a mixture of a liquidity premium and standard-CAPM risk premium. The presence of investors with longer investment horizons, however, reduces the importance of liquidity risk relative to a homogeneous investor setting. Furthermore, the effect of expected liquidity is relatively smaller, given that long-horizon investors do not trade every period, and it varies in the cross-section of stocks as a function of the covariance between returns and illiquidity costs. Interestingly, we identify cases in the cross-section of stocks where high liquidity risk may actually lead to a lower premium on expected liquidity because of a greater presence of long-term investors.

The expected returns of segmented assets contain additional terms, both for risk premia and in expected liquidity effects. More specifically, there are *segmentation* and *spillover* risk premia. The segmentation risk premium is positive and is caused by imperfect risk sharing, as only long-term investors hold these illiquid assets. The spillover risk premium can be positive or negative, depending on the correlation between illiquid (segmented) and liquid (non-segmented) asset returns. For example, if a segmented asset is highly correlated with non-segmented assets, the spillover effect is negative and neutralizes the segmentation risk premium, because in this case the segmented asset can be replicated (almost exactly) by a portfolio of non-segmented assets.

The expected liquidity term also contains a segmentation effect, in that expected liquidity matters less for segmented assets that are held only by long-term investors. Along the same lines as

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the risk premium, it also contains an expected liquidity spillover term, with a sign that is a function of the correlation between liquid and illiquid assets. In sum, the presence of these additional effects implies that the total expected liquidity premium can be larger for liquid assets relative to segmented assets. Hence, in contrast to Amihud and Mendelson (1986) and Acharya and Pedersen (2005), the relation between expected returns and expected liquidity in our model is not necessarily strictly increasing.

In summary, our model demonstrates that incorporating heterogeneous investment horizons has a considerable impact on the way liquidity affects asset prices. It changes the relative size of liquidity and market risk premia, leads to cross-sectional differences in liquidity effects, and generates segmentation and spillover effects.

Armed with this array of novel theoretical predictions, we take the model to the data to test its empirical relevance. Specifically, we analyze the cross-section of U.S. stocks over the period 1964 to 2009 and use the illiquidity measure of Amihud (2002) to proxy for liquidity costs, as in Acharya and Pedersen (2005). We estimate our asset pricing model using the Generalized Method of Moments (GMM) and find that a version with two horizons (one month and ten years) generates a remarkable cross-sectional fit of expected stock returns. Specifically, for 25 liquidity-sorted portfolios, the heterogeneous-horizon model generates a cross-sectional R^2 of 82.2% compared to 62.2% for the single-horizon model, with similar improvements when using other portfolio sorting criteria. Our model achieves this substantial increase in explanatory power using the same degrees of freedom and imposing more economic structure on the composition of the risk premium and on the loading of expected returns on expected liquidity. As an upshot of our richer model, the

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empirical estimates can also be used to make inferences about the risk-bearing capacity of investors in each horizon class.

We also estimate a version of our heterogeneous horizon model without liquidity risk, thus incorporating only the effects of expected liquidity and the associated segmentation and spillover effects. As explained above, this model setup deviates from Amihud and Mendelson (1986) in that investors are risk-averse, rather than risk-neutral. Interestingly, the fit of this version of the model is as good as the fit of a model with liquidity risk. For our empirical application to the cross-section of U.S. stocks, what matters is the combination of expected liquidity and partial segmentation. While the cost of the homogenous horizon assumption is about 20% in terms of R^2, in the end the cost of assuming constant transaction costs seems negligible.

The final important implication of the empirical estimates of our model is the more prominent role of the effect of expected liquidity on expected returns compared to the homogeneous horizon case. Averaged across the 25 liquidity-sorted portfolios, the expected liquidity premium generates about 2.40% in annual returns in our model, as compared with 0.36% in the homogeneous-horizon model. The presence of partial segmentation is thus crucial to understand the effect of expected liquidity on asset prices.

The remainder of the paper is organized as follows. Section I reviews the relevant literature. Section II presents the general liquidity asset pricing model that allows for arbitrarily many investment horizons and assets. We describe our estimation methodology in Section III. Section IV illustrates the data and Section V presents our empirical findings. We conclude with a summary of our results in Section VI.

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I. Related Literature

Our paper contributes to the existing literature on liquidity and asset pricing along several dimensions. First, our model is related to theoretical work on portfolio choice and illiquidity (see Amihud, Mendelson, and Pedersen (2005) for an overview). Starting with the work of Constantinides (1986), several researchers have examined multi-period portfolio choice in the presence of transaction costs. In contrast to these papers, we focus on a general equilibrium setting with heterogenous investment horizons in the presence of liquidity risk. We obtain a tractable asset pricing model by simplifying the analysis in some other dimensions. In particular, we assume no intermediate rebalancing for long-term investors.

Second, our empirical results contribute to a rich literature that has empirically studied the asset pricing implications of liquidity and liquidity risk. Amihud (2002) finds that stock returns are increasing in the level of illiquidity both in the cross-section (consistent with Amihud and Mendelson (1986)) and in the time-series. Pástor and Stambaugh (2003) show that the sensitivity of stock returns to aggregate liquidity is priced. Acharya and Pedersen (2005) integrate these effects into a liquidity-adjusted CAPM that performs better empirically than the standard CAPM. The liquidity-adjusted CAPM is such that, in addition to the standard CAPM effects, the expected return on a security increases with the level of illiquidity and is influenced by three different liquidity risk covariances. Several articles build on these seminal papers and document the pricing of liquidity

and liquidity risk in various asset classes.[2] However, none of these papers study the liquidity effects of heterogenous investment horizons.

Third, our paper is also related to empirical research showing the relation between liquidity and investors' holding periods. For example, Chalmers and Kadlec (1998) find evidence that it is not the spread, but the amortized spread that is more relevant as a measure of transaction costs, as it takes into account the length of investors' holding periods. Cremers and Pareek (2009) study how investment horizons of institutional investors affect market efficiency. Cella, Ellul, and Giannetti (2011) demonstrate that investors' short horizons amplify the effects of market-wide negative shocks. All of these articles use turnover data for stocks and investors to capture investment horizons. In contrast, we estimate the degree of heterogeneity in investment horizons by fitting our asset pricing model to the cross-section of U.S. stock returns.

Finally, our modeling approach is somewhat related to recent theories where some investors do not trade every period, although there is no explicit role for transaction costs and illiquidity. For example, Duffie (2010) studies an equilibrium pricing model in a setting where some "inattentive" investors do not trade every period. He uses this setup to study how supply shocks affect price dynamics in a single-asset model. In contrast, besides incorporating transaction costs, our focus is cross-sectional as we study a market with multiple assets in a setting where dividends, transaction costs, and returns are all i.i.d. Similarly, Brennan and Zhang (2012) develop an asset pricing model

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[2]For example, Bekaert, Harvey, and Lundblad (2007) focus on emerging markets, Sadka (2010) studies hedge funds, Franzoni, Nowak, and Phalippou (2011) focus on private equity, Bao, Pan, and Wang (2011) study corporate bonds, and Bongaerts, De Jong, and Driessen (2011) focus on credit default swaps.

where a representative agent has a stochastic horizon.[3] However, liquidity effects are neglected and investors are homogeneous, in that they hold the same assets and those assets are liquidated simultaneously.

II. The Model

In this section, we first lay down the foundations of our liquidity asset pricing model with multiple assets and horizons. We then analyze the main equilibrium implications of the model. Finally, we explore a number of special cases of the model to obtain additional interesting insights.

A. Model Setup and Assumptions

Our liquidity asset pricing model features both stochastic liquidity and heterogenous investment horizons in a setting with multiple assets. We develop a theoretical framework that is also suitable for empirical estimation. Our model is built on the following assumptions:

- There are K assets, with asset i paying each period a dividend $D_{i,t}$.[4] Selling the asset costs $C_{i,t}$. Transaction costs and dividends are *i.i.d.* in order to obtain a stationary equilibrium. There is a fixed supply of each asset, equal to S_i shares, and a risk-free asset with exogenous and constant return R_f.

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[3]Using a similar motivation, Kamara, Korajczyk, Lou, and Sadka (2012) study empirically how the horizon that is used to calculate returns matters for the pricing of various risk factors.

[4]We assume that the proceeds of the dividends at all times are added to the risk-free deposit.

- We model N classes of investors with horizons h_j, where $j = 1,..,N$. It turns out that empirically it is sufficient to take $N = 2$, so we will impose this condition from here onwards to simplify the expressions. We thus have short-term investors with horizon of h_1 periods and long-term investors with horizon h_2. Appendix A solves the model for any N.

- Investors have mean-variance utility over terminal wealth with risk aversion A_j for investor type j.

- We have an overlapping generations (OLG) setup. Each period, a fixed quantity $Q_j > 0$ of type j investors enters the market and invests in some or all of the K assets.

- Investors with horizon h_j only trade when they enter the market and at their terminal date, hence they do not rebalance their portfolio at intermediate dates.

Most assumptions follow Acharya and Pedersen (2005).[5] The key extension is that we allow for heterogenous horizons, while Acharya and Pedersen (2005) only have one-period investors. We make two simplifying assumptions to obtain tractable solutions. First, we assume i.i.d. dividends and transaction costs so as to obtain a stationary equilibrium. In reality transaction costs are relatively persistent over time. In the empirical section of the paper, we show that the i.i.d. assumption does not have a major impact on our empirical results.

The second simplifying assumption is that investors do not rebalance at intermediate dates. This assumption is important mainly for the long-term investors. As long as rebalancing trades are small relative to the total positions, we do not expect that relaxing this assumption would generate

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[5]Acharya and Pedersen (2005) start with investors with exponential utility and normally distributed dividends and costs, which amounts to assuming mean-variance preferences.

very different results. Also note that, in presence of transaction costs, investors only rebalance their portfolio infrequently (see, for example, Constantinides (1986)). In addition, positions in some categories of investment assets, such as private equity, may be hard to rebalance.

B. Equilibrium Expected Returns

In this subsection we describe how we obtain the equilibrium expected returns given our model setup. First, note that, at time t, investors with horizon h_j solve a maximization problem where they choose the quantity of stocks purchased $y_{j,t}$ (a vector with one element for each asset) to maximize utility over their holding period return, taking into account the incurred transaction costs. The utility maximization problem is given by

$$\max_{y_{j,t}} \mathbb{E}\left[W_{j,t+h_j}\right] - \frac{1}{2}A_j \text{Var}\left(W_{j,t+h_j}\right) \tag{1}$$

$$W_{j,t+h_j} = \left(P_{t+h_j} + \sum_{k=1}^{h_j} R_f^{h_j-k} D_{t+k} - C_{t+h_j}\right)' y_{j,t} + R_f^{h_j}\left(e_j - P_t' y_{j,t}\right),$$

where R_f is the gross risk-free rate, $W_{j,t+1}$ is wealth of the h_j investors at time $t+1$, P_{t+1} is the $K \times 1$ vector of prices, and e_j is the endowment of the h_j investors.

In the remainder of the text of the paper, we set $R_f = 1$ to simplify the exposition. Appendix A derives the model for $R_f \geq 1$, which leads to very similar expressions. In the empirical analysis, we obviously estimate the version of the asset pricing model with R_f equal to the historical average of the risk-free rate.

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The optimal portfolio choice may reflect endogenous segmentation, which is the possibility that some classes of investors do not hold some assets in equilibrium because the associated trading costs are too high relative to the expected return over the investment horizon. To this end, we introduce sets B_j ($j = 1, 2$) that are subsets of $\{1, \ldots, K\}$, where K is the number of tradable assets. The set B_j represents the set of assets that investors j will buy in equilibrium. We find that a short-horizon investor (with horizon h_1) will endogenously avoid investing in assets for which the associated transaction costs are too large. The sets B_j thus depend on the level of transaction costs of the assets. Note that, for markets to clear, long-term investors will hold all assets in equilibrium, so that $B_2 = \{1, \ldots, K\}$. In Appendix B, we describe in more detail under which conditions endogenous segmentation arises.

The solution to this utility maximization problem is the usual mean-variance solution, corrected for transaction costs and the possibility of segmentation. As shown in Appendix A, the solution can be written as

$$y_{j,t} = \frac{1}{A_j} \operatorname{diag}(P_t)^{-1} \operatorname{Var}\left(\sum_{k=1}^{h_j} R_{t+k} - c_{t+h_j}\right)^{-1}_{B_j,p}$$
$$\times \left(h_j \mathbb{E}[R_{t+1} - 1] - \mathbb{E}[c_{t+1}]\right), \tag{2}$$

where R_{t+1} denotes the $K \times 1$ vector of gross asset returns, with $R_{i,t+1} = (D_{i,t+1} + P_{i,t+1})/P_{i,t}$, and c_{t+1} the $K \times 1$ vector of percentage costs, with $c_{i,t} = C_{i,t}/P_{i,t}$. For a generic matrix M, the notation M_{B_j} is used to indicate the $|B_j| \times |B_j|$ matrix containing only the rows and columns of M that are in B_j. We write $M_{B_j,p}^{-1}$ for the inverse of M_{B_j} with zeros inserted at the locations where rows and columns of M were removed. With this convention, $\operatorname{Var}\left(\sum_{k=1}^{h_j} R_{t+k} - c_{t+h_j}\right)^{-1}_{B_j,p}$ corresponds to

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the $K \times K$ matrix containing the inverse of the covariance matrix of the set of assets that investors j invest in, with zeros inserted for the rows and columns that were not included (the assets that investors j do not invest in). The optimal demand vector $y_{j,t}$ thus contains zeros for those assets in which investor j does not invest.[6]

With i.i.d. dividends and costs, given a fixed asset supply, a wealth-independent optimal demand, and with the same type of investors entering the market each period, we obtain a stationary equilibrium where the price of each asset $P_{i,t}$ is constant over time. At any point in time, the market clears with new investors buying the supply of stocks minus the amount still held by the investors that entered the market at an earlier point in time,

$$Q_1 y_{1,t} + Q_2 y_{2,t} = S - \sum_{k=1}^{h_1-1} Q_1 y_{1,t-k} - \sum_{k=1}^{h_2-1} Q_2 y_{2,t-k}, \tag{3}$$

where S is the vector with supply of assets (in number of shares of each of the assets). Given the i.i.d. setting, we have constant demand over time, $y_{j,t} = y_{j,t-k}$ for all j and k.

We let $R_t^m = \widetilde{S}_t' R_t / \widetilde{S}_t' \iota$ and $c_t^m = \widetilde{S}_t' c_t / \widetilde{S}_t' \iota$, where $\widetilde{S}_t = \text{diag}(P_t)S$ denotes the dollar supply of assets. Appendix A shows that under the stated assumptions we obtain the following result.

PROPOSITION 1: *A stationary equilibrium exists with the following equilibrium expected returns*

$$\mathbb{E}\left[R_{t+1} - 1\right] = (\gamma_1 h_1 V_1 + \gamma_2 h_2 V_2)^{-1} (\gamma_1 V_1 + \gamma_2 V_2) \mathbb{E}\left[c_{t+1}\right] \tag{4}$$

$$+ (\gamma_1 h_1 V_1 + \gamma_2 h_2 V_2)^{-1} \text{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right),$$

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[6]We compute the long-term covariance matrices using the i.i.d. assumption. Appendix C provides further details.

where

$$V_j = h_j \text{Var}\left(R_{t+1} - c_{t+1}\right) \text{Var}\left(\sum_{k=1}^{h_j} R_{t+k} - c_{t+h_j}\right)^{-1}_{B_{j,p}},$$ (5)

and $\gamma_j = Q_j/(A_j \widetilde{S}' \iota)$.[7] R_f *is set equal to 1 for ease of exposition.*

Proposition 1 shows that the equilibrium expected returns contain two components. The first component is a compensation for the expected transaction costs. The second component is a compensation for market risk and liquidity risk. Note that the loadings on expected costs and return covariances are matrices. This is in contrast to standard linear asset pricing models, where these loadings are scalars and therefore all assets have the same exposure to expected costs and the return covariance.

In the equilibrium equation (4), the parameter γ_j has an interesting interpretation as *risk-bearing capacity*. Specifically, the OLG setup implies that in every period the total number of h_j-investors in the market is equal to $h_j Q_j$. This total number is important because it determines among how many h_j-investors the risky assets can be shared. Their risk aversion A_j is also important, because it determines the size of the position these investors are willing to take in the risky assets. Therefore, we can indeed interpret the quantity

$$h_j \gamma_j = \frac{h_j Q_j}{A_j} \frac{1}{\widetilde{S}' \iota}$$ (6)

as the *risk-bearing capacity* of the h_j-investors (scaled by the total market capitalization).

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[7]The time subscript for supply $\widetilde{S}_t$ is omitted, as supply is constant over time.

C. Interpreting the Equilibrium: Special Cases

We now consider several special cases to gain intuition for the different effects that the general equilibrium model generates. It is important to note that, in the empirical analysis, we estimate the general model in equation (4). Hence, these special cases are only used here to better understand the new implications of our equilibrium model.

We begin with an integration setting where both short-term and long-term investors hold all assets. In this setting, we consider the following special cases:

- the liquidity CAPM of Acharya and Pedersen (2005);

- the expected liquidity effect without liquidity risk;

- the expected liquidity effect with liquidity risk;

- the market and liquidity risk premia with two assets.

We then consider a special case within the endogenous segmentation setting, where the short-term investors do not invest in assets that are very illiquid. Finally, we summarize and discuss the array of novel predictions of our model.

C.1. Liquidity CAPM of Acharya and Pedersen (2005)

If we have only one investor type with a one-period horizon, we obtain a model similar to the liquidity CAPM of Acharya and Pedersen (2005). Specifically, consider the case where $N = 1$ (or $\gamma_2 = 0$), $h_1 = 1$, and $B_1 = \{1, \ldots, K\}$, so that there is just one class of one-period investors. For

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ease of comparison, we write the equilibrium equation in beta form. In this case, the equilibrium expected returns simplify to

$$\mathbb{E}[R_{t+1} - 1] = \mathbb{E}[c_{t+1}] + \frac{\text{Var}\left(R_{t+1}^m - c_{t+1}^m\right)}{\gamma_1} \frac{\text{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right)}{\text{Var}\left(R_{t+1}^m - c_{t+1}^m\right)}, \tag{7}$$

which is an i.i.d. version of the equilibrium relation of Acharya and Pedersen (2005).

C.2. Expected liquidity effect without liquidity risk

We now allow for two distinct investor horizons, but assume constant transaction costs (i.e. $\text{Var}(c_{t+1}) = 0$). In the integration setting ($B_1 = B_2 = \{1, \ldots, K\}$), we obtain a linear asset pricing model with scalar loadings on expected liquidity and risk

$$\mathbb{E}[R_{t+1} - 1] = \frac{\gamma_1 + \gamma_2}{\gamma_1 h_1 + \gamma_2 h_2} \mathbb{E}[c_{t+1}] + \frac{1}{\gamma_1 h_1 + \gamma_2 h_2} \text{Cov}\left(R_{t+1}, R_{t+1}^m\right). \tag{8}$$

We immediately see that the loading on expected liquidity equals $1/h_1$ if $\gamma_2 = 0$ and $1/h_2$ if $\gamma_1 = 0$. As the horizon h_j increases, it follows that the impact of expected liquidity on returns decreases with the investor horizon.

To illustrate the difference with the single-horizon case in equation (7), where the loading on expected liquidity is equal to 1, let us use a simple example with $h_1 = 1$, $h_2 = 2$, $\gamma_1 = 2$, and $\gamma_2 = 1$. In this simple example, the loading on expected liquidity is equal to

$$\frac{\gamma_1 + \gamma_2}{\gamma_1 h_1 + \gamma_2 h_2} = \frac{3}{4}, \tag{9}$$

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which is exactly halfway between the expected liquidity coefficient with only one-period investors ($1/h_1 = 1$) and the loading when there are only two-period investors ($1/h_2 = 1/2$). More generally, we observe that the introduction of long-term investors in the model decreases the impact of expected liquidity on expected returns.

C.3. Expected liquidity effect with liquidity risk

We now extend the previous special case C.2 to a setting with stochastic transaction costs. For simplicity, we take $\mathrm{Var}\,(c_{t+1})$ and $\mathrm{Var}\,(R_{t+1} - c_{t+1})$ to be diagonal matrices (in this example only), we set $h_1 = 1$, and still consider the integration setting ($B_1 = B_2 = \{1, \ldots, K\}$). In this case, we obtain

$$\mathbb{E}\,[R_{i,t+1} - 1] = \frac{\gamma_1 + \gamma_2 V_{2,i}}{\gamma_1 h_1 + \gamma_2 h_2 V_{2,i}} \mathbb{E}\,[c_{i,t+1}] \tag{10}$$
$$+ \frac{1}{\gamma_1 h_1 + \gamma_2 h_2 V_{2,i}} \mathrm{Cov}\,\left(R_{i,t+1} - c_{i,t+1}, R_{t+1}^m - c_{t+1}^m\right),$$

where $V_{2,i}$ denotes the i-th diagonal element of V_2. In this case, we can write $V_{2,i}$ as

$$V_{2,i} = \frac{h_2 \mathrm{Var}\,(R_{i,t+1} - c_{i,t+1})}{(h_2 - 1)\,\mathrm{Var}\,(R_{i,t+1}) + \mathrm{Var}\,(R_{i,t+1} - c_{i,t+1})}. \tag{11}$$

Now consider the following ratio:

$$\frac{\mathrm{Var}\,(R_{i,t+1} - c_{i,t+1})}{\mathrm{Var}\,(R_{i,t+1})}. \tag{12}$$

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This ratio is a good measure of the amount of liquidity risk, as it increases with $\text{Var}(c_{i,t+1})$ and with $\text{Cov}(R_{i,t+1}, c_{i,t+1})$. We can show that the expected liquidity coefficient in (10) decreases with this "liquidity risk" ratio. That is, higher liquidity risk leads to a smaller expected liquidity premium. This result might seem counterintuitive at first sight, but it has a natural interpretation. If an asset has higher liquidity risk, it will be held in equilibrium mostly by long-term investors. Long-term investors care less about liquidity and this leads to the smaller expected liquidity effect.

C.4. Market and liquidity risk premia with two assets

We now focus on interpreting the risk premia that the model generates in equilibrium. In the general model of equation (4), expected returns are determined by a mix of market and liquidity risk premia. This mix becomes especially clear when we consider the two-asset case ($K = 2$), $h_1 = 1$, again in the integration setting. Formally:

PROPOSITION 2: *In the two-asset case ($K = 2$), with two horizons ($N = 2$), $h_1 = 1$, $R_f = 1$, and no segmentation ($B_1 = B_2 = \{1, \ldots, K\}$), the equilibrium expected returns are*

$$\mathbb{E}[R_{t+1} - 1] = (\gamma_1 h_1 V_1 + \gamma_2 h_2 V_2)^{-1} (\gamma_1 V_1 + \gamma_2 V_2) \mathbb{E}[c_{t+1}] \tag{13}$$
$$+ (\gamma_1 \lambda_1 + \gamma_2 \lambda_2) \text{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right) + \gamma_2 \lambda_2 (h_2 - 1) \text{Cov}\left(R_{t+1}, R_{t+1}^m\right),$$

where $\lambda_j = h_j^2 / d_0 d_j$ is a scalar parameter. The definitions of the determinants d_0 and d_j are given by equations (A19) *and* (A21) *in Appendix D.*

In this equilibrium, the total risk premium is a weighted sum of market and liquidity risk premia. Holding everything else constant, we can show that liquidity risk becomes less important

relative to market risk when the long-term investors become less risk averse or more numerous (formally, as γ_2 increases). As γ_2 increases, long-term investors hold a larger fraction of the total supply in equilibrium and these investors care less about liquidity risk compared to short-term investors.

C.5. Segmentation effects

The special cases discussed above show the expected liquidity and risk premia effects when all investors have positive holdings of all assets. Now we show what happens to expected returns when some assets are only held by long-term investors (endogenous segmentation).

To obtain tractable theoretical expressions, we focus on the special case where V_2 equals the identity matrix and set $h_1 = 1$. The simplification $V_2 = I$ is appropriate when the variability of returns is much higher than the variability of transaction costs. As we show later in the empirical section, this is indeed the case in our data and we can thus rely on these theoretical simplified expressions. Of course, our benchmark empirical estimation focuses on the unrestricted equilibrium in equation (4).

Without loss of generality, we order the assets by liquidity, with the most liquid assets first. The returns on the assets that are in B_1 are denoted by R_t^{liq}, and the returns on the assets that are not in B_1 are denoted by R_t^{illiq}. We use this notation also for the costs. Appendix E shows that in this setting we obtain the following proposition.

PROPOSITION 3: *If $N = 2$, $h_1 = 1$, $V_2 = I$, $R_f = 1$, and B_1 contains only those assets that the short-term investors hold, then for these "liquid" assets the expected returns are*

$$\mathbb{E}\left[R_{t+1}^{liq} - 1\right] = \frac{\gamma_1 + \gamma_2}{\gamma_1 h_1 + \gamma_2 h_2}\mathbb{E}\left[c_{t+1}^{liq}\right] + \frac{1}{\gamma_1 h_1 + \gamma_2 h_2}\text{Cov}\left(R_{t+1}^{liq} - c_{t+1}^{liq}, R_{t+1}^{m} - c_{t+1}^{m}\right). \tag{14}$$

The expected returns on "illiquid" assets only held by long-term investors are

$$\mathbb{E}\left[R_{t+1}^{illiq} - 1\right] = \frac{1}{h_2}\mathbb{E}\left[c_{t+1}^{illiq}\right] + \frac{h_2 - h_1}{h_2}\frac{\gamma_1}{\gamma_1 h_1 + \gamma_2 h_2}\beta\mathbb{E}\left[c_{t+1}^{liq}\right]$$
$$+ \frac{1}{\gamma_1 h_1 + \gamma_2 h_2}\text{Cov}\left(R_{t+1}^{illiq} - c_{t+1}^{illiq}, R_{t+1}^{m} - c_{t+1}^{m}\right)$$
$$+ \left(\frac{1}{\gamma_2 h_2} - \frac{1}{\gamma_1 h_1 + \gamma_2 h_2}\right)\text{Cov}\left(R_{t+1}^{illiq} - c_{t+1}^{illiq}, R_{t+1}^{m} - c_{t+1}^{m}\right)$$
$$- \left(\frac{1}{\gamma_2 h_2} - \frac{1}{\gamma_1 h_1 + \gamma_2 h_2}\right)\beta\text{Cov}\left(R_{t+1}^{liq} - c_{t+1}^{liq}, R_{t+1}^{m} - c_{t+1}^{m}\right),$$

where the matrix β denotes the liquidity spillover beta, *defined as*

$$\beta = \text{Cov}\left(R_{t+1}^{illiq} - c_{t+1}^{illiq}, R_{t+1}^{liq} - c_{t+1}^{liq}\right)\text{Var}\left(R_{t+1}^{liq} - c_{t+1}^{liq}\right)^{-1}. \tag{15}$$

First, we note that the equilibrium expected returns for liquid assets are similar to the special cases discussed previously, since these assets are held by both short-term and long-term investors. For the "illiquid" assets, the pricing is more complex. In what follows, we thus discuss separately the different components that make up expected excess returns for illiquid assets.

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We start by analyzing the expected liquidity effect that we can decompose into three parts:

$$\frac{\gamma_1+\gamma_2}{\gamma_1 h_1+\gamma_2 h_2}\mathbb{E}\left[c_{t+1}^{illiq}\right]+\left(\frac{1}{h_2}-\frac{\gamma_1+\gamma_2}{\gamma_1 h_1+\gamma_2 h_2}\right)\mathbb{E}\left[c_{t+1}^{illiq}\right]+\frac{h_2-h_1}{h_2}\frac{\gamma_1}{\gamma_1 h_1+\gamma_2 h_2}\beta\mathbb{E}\left[c_{t+1}^{liq}\right]. \qquad (16)$$

The first component, which we denote *full risk-sharing expected liquidity premium,* is the expected liquidity effect that one would obtain if these assets were held by both short-term and long-term investors. The second term (*segmentation expected liquidity premium*) reflects that, in fact, only long-term investors hold the illiquid assets and this term dampens the effect of expected liquidity since $\frac{1}{h_2}-\frac{\gamma_1+\gamma_2}{\gamma_1 h_1+\gamma_2 h_2}<0$. The third component (*spillover expected liquidity premium*) arises from the exposure (as given by β) of the illiquid assets to the liquid assets. If this exposure is positive, this increases the expected liquidity effect for the illiquid assets since $\frac{h_2-h_1}{h_2}\frac{\gamma_1}{\gamma_1 h_1+\gamma_2 h_2}>0$. In other words, if liquid and illiquid assets are positively correlated, the expected liquidity effect on illiquid assets cannot be much lower than the effect for liquid assets, because long-term investors would take advantage by shorting the illiquid assets and buying the liquid assets.

We now turn to the risk premia, where we have a natural interpretation for each of the various covariance terms in the equilibrium relation for the illiquid assets. The term

$$\frac{1}{\gamma_1 h_1+\gamma_2 h_2}\text{Cov}\left(R_{t+1}^{illiq}-c_{t+1}^{illiq},R_{t+1}^m-c_{t+1}^m\right) \qquad (17)$$

gives the *full risk-sharing risk premium* that would arise if both types of investors would hold the asset. The second term,

$$\left(\frac{1}{\gamma_2 h_2}-\frac{1}{\gamma_1 h_1+\gamma_2 h_2}\right)\text{Cov}\left(R_{t+1}^{illiq}-c_{t+1}^{illiq},R_{t+1}^m-c_{t+1}^m\right), \qquad (18)$$

gives the *segmentation risk premium*, which shows the impact of the lower risk sharing due to long-term investors only holding the illiquid assets. Since $\frac{1}{\gamma_2 h_2} - \frac{1}{\gamma_1 h_1 + \gamma_2 h_2} > 0$, this segmentation premium increases expected returns in case of positive return covariance. The third term,

$$- \left(\frac{1}{\gamma_2 h_2} - \frac{1}{\gamma_1 h_1 + \gamma_2 h_2} \right) \beta \mathrm{Cov} \left(R_{t+1}^{liq} - c_{t+1}^{liq}, R_{t+1}^m - c_{t+1}^m \right), \tag{19}$$

defines a *spillover risk premium*. Along the lines of the discussion above for the expected liquidity effect, this term concerns the relative pricing of the illiquid versus liquid assets. If these two assets are positively correlated (high elements of β), their expected returns cannot be too far apart. This term reduces the effect of segmentation when the elements of β are nonzero. Specifically, if

$$\mathrm{Cov} \left(R_{t+1}^{illiq} - c_{t+1}^{illiq}, R_{t+1}^m - c_{t+1}^m \right) = \beta \mathrm{Cov} \left(R_{t+1}^{liq} - c_{t+1}^{liq}, R_{t+1}^m - c_{t+1}^m \right), \tag{20}$$

the net effect of segmentation is equal to zero.

We can also rewrite the expected returns on segmented assets in Proposition 3 in a more compact form:

$$\mathbb{E} \left[R_{t+1}^{illiq} - 1 \right] = \frac{1}{h_2} \mathbb{E} \left[c_{t+1}^{illiq} \right] + \beta \left(\mathbb{E} \left[R_{t+1}^{liq} \right] - \frac{1}{h_2} \mathbb{E} \left[c_{t+1}^{liq} \right] \right) \tag{21}$$
$$+ \frac{1}{\gamma_2 h_2} \mathrm{Cov} \left(R_{t+1}^{illiq} - c_{t+1}^{illiq} - \beta \left(R_{t+1}^{liq} - c_{t+1}^{liq} \right), R_{t+1}^m - c_{t+1}^m \right),$$

which can provide some additional intuition. In particular, this expression shows in a different way how segmentation matters. The expected returns on segmented assets are driven by the exposure

to net-of-cost returns of the liquid assets, plus an additional effect that comes from the systematic exposure of the residual return on segmented assets, $R_{t+1}^{illiq} - c_{t+1}^{illiq} - \beta(R_{t+1}^{liq} - c_{t+1}^{liq})$.

The total segmentation risk premium, as expressed in equation (21), is in the spirit of the international asset pricing literature (e.g., De Jong and De Roon (2005)), where segmentation also leads to additional effects on expected returns.

To better illustrate how segmentation influences the impact of expected liquidity on expected returns, we consider again the simple example of Section II.C.2, where $h_1 = 1$, $h_2 = 2$, $\gamma_1 = 2$, and $\gamma_2 = 1$. We also impose $\text{Var}(c_{t+1}) = 0$ and $\beta = 0$. In this segmentation setting, we find that the loading on expected liquidity is

$$\frac{\gamma_1 + \gamma_2}{\gamma_1 h_1 + \gamma_2 h_2} = \frac{3}{4} \tag{22}$$

for the liquid assets, and

$$\frac{1}{h_2} = \frac{1}{2} \tag{23}$$

for the illiquid assets. This example shows that the effect of expected liquidity is smaller for the illiquid assets, because these assets are only held by long-term investors in equilibrium. Note that in this case the total expected liquidity component of expected returns for liquid assets ($\frac{3}{4}\mathbb{E}\left[c_{t+1}^{liq}\right]$) is not necessarily smaller than the premium for illiquid assets ($\frac{1}{2}\mathbb{E}\left[c_{t+1}^{illiq}\right]$).

C.6. Summary and Discussion

Our model shows that the asset pricing effects of liquidity are much more complex once we allow for heterogenous horizons and segmentation. In summary, the main theoretical implications are:

(i) the expected liquidity effect is decreasing with investor horizons;

(ii) the expected liquidity effect is decreasing with the amount of liquidity risk;

(iii) for "segmented" assets the expected liquidity effect is dampened because of the exclusive ownership of long-term investors;

(iv) for "segmented" assets the expected liquidity effect also contains a spillover term due to the correlation between segmented and non-segmented assets;

(v) the total risk premium is a mix of a market risk premium and a liquidity risk premium. The liquidity risk premium becomes relatively more important when short-term investors are more numerous or less risk-averse;

(vi) for "segmented" assets there is an additional segmentation risk premium due to limited risk sharing;

(vii) for "segmented" assets there is an additional spillover risk premium due to the correlation between segmented and non-segmented assets.

Note that the sign of the various effects listed above is not always unambiguous. For example, the spillover effects clearly depend on the sign of the correlation between segmented and non-segmented assets. The model thus predicts a more complex relation between liquidity and ex-

pected returns compared to Acharya and Pedersen (2005) and Amihud and Mendelson (1986). For example, one of the most interesting predictions of Amihud and Mendelson (1986) is the concave relationship between expected liquidity and expected returns. In our model, the effect that drives this concave relation is also present (a smaller expected liquidity coefficient for segmented assets, point 3 above). However, there are other segmentation and spillover effects that also play a role. These additional effects are not present in Amihud and Mendelson (1986), because they assume risk-neutral investors. In their model long-term investors only invest in illiquid assets and not in the liquid assets. In contrast, in our model with risk averse agents, long-term investors will diversify and invest in liquid assets as well, leading to spillover and segmentation effects.

We thus observe that the introduction of heterogenous investment horizons into a liquidity asset pricing model has strong implications for the pricing of liquid versus illiquid assets. Specifically, we find various and potentially contrasting effects on the liquidity (risk) premia. It then becomes an empirical question to understand the relevance of these additional effects. We take on this task in the next Sections of the paper.

III. Empirical Methodology

In this section, we explain how our liquidity asset pricing model can be estimated. We also explore the economic mechanism that allows the identification of the parameters. We then discuss alternative approaches for a robust computation of standard errors.

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A. *GMM Estimation*

We use a Generalized Method of Moments (GMM) methodology to estimate the equilibrium condition given by equation (4), but without imposing $R_f = 1$. The key estimated parameters are $\gamma_j = Q_j/(A_j \widetilde{S}' \iota)$, that is, the *risk-bearing capacity* of the different classes of investors. We define the vector of pricing errors of all assets, denoted by $g(\psi, \gamma)$, as

$$g(\psi, \gamma) = \mathbb{E}\left[R_{t+1} - 1\right] - \left(\gamma_1 h_1 V_1 + \gamma_2 h_2 V_2\right)^{-1} \left(\gamma_1 V_1 + \gamma_2 V_2\right) \mathbb{E}\left[c_{t+1}\right] \tag{24}$$
$$- \left(\gamma_1 h_1 V_1 + \gamma_2 h_2 V_2\right)^{-1} \text{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right),$$

where γ is the vector of parameters, and ψ is a vector containing the underlying expectations and covariances that enter the pricing errors. Specifically, ψ contains all expected returns, expected costs, covariances entering the V_j matrices, and the covariances with the market return. In a first step, we estimate all elements of ψ by their sample moments. In a second step, we perform a GMM estimation of γ, using an identity weighting matrix across all assets. We thus minimize the sum of squared pricing errors over γ,

$$\min_{\gamma} \ g(\widehat{\psi}, \gamma)' g(\widehat{\psi}, \gamma). \tag{25}$$

In Appendix F, we derive the asymptotic covariance matrix of this GMM estimator, taking into account the estimation error in all sample moments in ψ, in line with the approach of Shanken (1992).

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B. Identification

To gain insight into the economic mechanism that allows the identification of the parameters, it is useful to illustrate some comparative statics results. Specifically, a change in γ_j means that the horizon h_j investors become either more numerous, or less risk averse, or both. Appendix G shows that the effect of such a change on expected returns is given by

$$\frac{\partial \mathbb{E}\left[R_{t+1} - 1\right]}{\partial \gamma_j} = \left(\gamma_1 h_1 V_1 + \gamma_2 h_2 V_2\right)^{-1} V_j \left(\mathbb{E}\left[c_{t+1}\right] - h_j \mathbb{E}\left[R_{t+1} - 1\right]\right). \tag{26}$$

We observe two contrasting effects of an increase in γ_j. The first effect is an increase in the risk premium due to the impact of expected liquidity. The second effect is the increased amount of risk sharing, which leads to a decrease in the risk premium proportional to the original risk premium. For long-term investors, the latter effect dominates and an increase in γ implies lower expected returns for all assets. For short-term investors, however, the expected costs may exceed the expected return $h_j \mathbb{E}\left[R_{t+1} - 1\right]$ for the more illiquid assets. This is exactly what we observe in the data for some more illiquid stocks. Hence, an increase in γ_1, which corresponds to the short-term investors, may increase the expected return of illiquid assets and decrease the expected return of liquid assets. We also observe that hedging considerations could play a different role for short-term versus long-term investors, because the matrix pre-multiplying the difference between the liquidity cost and the scaled risk premium can reverse the sign of the partial derivatives in equation (26).

In summary, this comparative statics exercise shows that the estimated parameters for short-term versus long-term investors may have opposing effects on equilibrium expected returns for different assets and, as such, can be properly identified.

C. *Bootstrap Standard Errors*

We use a bootstrap test to check the robustness of the asymptotic standard errors. We generate bootstrap samples by re-sampling the data and then carrying out the first step of the estimation procedure to obtain estimates for the different moments that enter the vector of pricing errors.

The test is a bootstrap t-test based on the bootstrap estimate of the standard error. The test does not provide asymptotic refinements, but has the advantage that it does not require direct computation of asymptotically consistent standard errors and thus provides a check on the asymptotic standard errors. Overall, we find that the bootstrap standard errors are close to the asymptotic standard errors.

IV. Data

We largely follow Acharya and Pedersen (2005) in our data selection and construction. We use daily stock return and volume data from CRSP from 1964 until 2009 for all common shares listed on NYSE and AMEX. As our empirical measures of liquidity rely on volume, we do not include Nasdaq since the volume data includes interdealer trades (and only starts in 1982). Overall, we consider a number of stocks ranging from 1056 to 3358, depending on the month. To correct for

survivorship bias, we adjust the returns for stock delisting (see Shumway (1997) and Acharya and Pedersen (2005)).

The relative illiquidity cost is computed as in Acharya and Pedersen (2005). The starting point is the Amihud (2002) illiquidity measure, which is defined as

$$ILLIQ_{i,t} = \frac{1}{Days_{i,t}} \sum_{d=1}^{Days_{i,t}} \frac{|R_{i,t,d}|}{Vol_{i,t,d}} \tag{27}$$

for stock i in month t, where $Days_{i,t}$ denotes the number of observations available for stock i in month t, and $R_{i,t,d}$ and $Vol_{i,t,d}$ denote the trading volume in millions of dollars for stock i on day d in month t, respectively.

We follow Acharya and Pedersen (2005) and define a normalized measure of illiquidity that deals with non-stationarity and is a direct measure of trading costs, consistent with the model specification. The normalized illiquidity measure can be interpreted as the dollar cost per dollar invested and is defined for asset i by

$$c_{i,t} = \min\left\{0.25 + 0.30 ILLIQ_{i,t} P_{t-1}^m, 30.00\right\}, \tag{28}$$

where P_{t-1}^m is equal to the market capitalization of the market portfolio at the end of month $t-1$ divided by the value at the end of July 1962. The product with P_{t-1}^m makes the cost series $c_{i,t}$ relatively stationary and the coefficients 0.30 and 0.25 are chosen as in Acharya and Pedersen (2005) to match approximately the level and variance of $c_{i,t}$ for the size portfolios to those of the effective half spread reported by Chalmers and Kadlec (1998). The value of normalized liquidity $c_{i,t}$ is capped at 30% to make sure the empirical results are not driven by outliers.

We obtain the book-to-market ratio using fiscal year-end balance sheet data from COMPU-STAT in the same manner as Ang and Chen (2002). They follow Fama and French (1993) in defining the book value of a firm as the sum of common stockholders' equity, deferred taxes, and investment credit minus the book value of preferred stocks. The ratio is obtained by dividing the book value by the fiscal year-end market value.

We construct the market portfolio on a monthly basis and only use stocks that have a price on the first trading day of the corresponding month between $5 and $1000. We include only stocks that have at least 15 observations of return and volume during the month. Following Acharya and Pedersen (2005), we use equal weights to compute the return on the market portfolio.

We construct 25 illiquidity portfolios, 25 illiquidity variation portfolios, and 25 book-to-market and size portfolios, as in Acharya and Pedersen (2005). The portfolios are formed on an annual basis. For these portfolios, we require again for the stock price on the first trading day of the corresponding month to be between $5 and $1000. For the illiquidity and illiquidity variation portfolios, we require to have at least 100 observations of the illiquidity measure in the previous year.

Table 1 shows the estimated average costs and average returns across the 25 illiquidity port-folios. The values correspond closely to those found in Table 1 of Acharya and Pedersen (2005). Most importantly, we see that average returns tend to be higher for illiquid assets. Also, the table shows that returns on more illiquid portfolios are more volatile. This finding holds for returns net of costs as well. The returns (net of costs) on more illiquid portfolios tend to co-move more strongly with market returns (also net of costs).

V. Empirical Results

In this section, we take the model to the data. First, we estimate the parameters of the model for the segmented case and compare it with single-horizon models (e.g., Acharya and Pedersen, 2005). We also explore the implications of the estimates for the importance of the different components of expected returns. We then study the robustness of our results to the choice of the investor horizon, to the extent of segmentation, and to pricing different sets of portfolios.

A. Estimation Setup

We estimate the parameters of the equilibrium relation given by equation (4) for the sample period 1964–2009 using the GMM methodology described in Section III.A. We first estimate the model on 25 portfolios of stocks listed on NYSE and AMEX, sorted on illiquidity. In the next subsection, we also estimate the model for 25 illiquidity-variation portfolios and 25 Book/Market-by-Size portfolios.

Our benchmark estimation is based on two classes of investors.[8] The first class (short horizon) has an investment horizon h_1 of one month, the second class (long horizon) has an investment horizon h_2 of 120 months (10 years). The choice of the length of the long horizon can be related to the results of using the methodology of Atkins and Dyl (1997) for our sample.[9] Over the 1964-2009

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[8] Adding a third class of investors does not yield substantial empirical improvement. The corresponding coefficient does not necessarily go to zero, but the R^2 remains essentially unchanged, with little gain in terms of explanatory power.

[9] Atkins and Dyl (1997) find that the mean investor holding period for NYSE stocks during the period 1975–1989 is roughly equal to 4.01 years.

period, we find an average holding period of 5.59 years. The robustness tests later in Section V.C show that the empirical results are virtually unchanged with the long horizon set at five years or longer.

Long-term investors tend to hold more illiquid assets. Consistent with this idea, Table I shows that turnover tends to be much lower and has a smaller standard deviation for the least liquid portfolios. We thus impose a segmentation cutoff, where the one-month investors invest only in the 19 most liquid portfolios. We choose this threshold based on the empirical evidence in Table I. While monthly expected excess returns are larger or similar to expected costs for most portfolios, for the six least liquid portfolios, the costs become roughly 2 to 9 times higher than the monthly average return. As the one-month investors incur the costs each period, these assets can be seen as prohibitively costly.[10]

This simple rule for the one-month investor (hold the asset if the expected monthly return exceeds the expected transaction costs and have a zero position otherwise) would also be the optimal rule with a diagonal covariance matrix of returns, as equation (2) shows.[11] Furthermore, Figure 6 shows that this threshold maximizes the cross-sectional R^2 across all possible cutoffs, including the model without any segmentation.

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[10]A portfolio-level analysis along the lines of Atkins and Dyl (1997) shows that the first 19 portfolios have average holding periods between 2.49 and 7.91 years, while portfolios 20 through 25 have average holding periods between 10.67 and 30.12 years, suggesting that short-term investors are unlikely to trade these illiquid stocks.

[11]To determine endogenously what are the portfolios held by the one-month investors, we can cast the problem as a mean-variance optimization exercise for the one-month investors. However, with this exercise, we run into the often-encountered issue of extreme positions in some portfolios due to close-to-singular covariance matrices.

Having set horizons and the segmentation cutoff, we now estimate the model parameters $\gamma_j = Q_j/(A_j \widetilde{S}' \iota)$ and, in some cases, a constant term in the expected return equation (α). We denote the models with and without a constant term as specifications (SEG+α) and (SEG), respectively. The role of the constant term is basically to provide a specification check, because it should equal zero under the null hypothesis. Recall that we can interpret $h_j\gamma_j$ as the risk-bearing capacity of h_j-investors. The risk-bearing capacity is determined by the risk aversion (A_j) and size (Q_j) of the h_j-investor group. Hence, the interpretation of the estimated parameters can offer interesting insights on the risk aversion or size of the short-term versus long-term investor groups.

We compare our model with a baseline one-period horizon model as in equation (7), with $N = 1$ and $h_1 = 1$. Here, we follow Acharya and Pedersen (2005) and allow for a slope coefficient κ on the expected liquidity term $\mathbb{E}[c_{t+1}]$, although formally the Acharya and Pedersen (2005) model implies a coefficient on expected liquidity equal to one. This coefficient is used by Acharya and Pedersen (2005) to correct for the fact that the typical holding period does not equal the estimation period of one month. We denote these single-horizon specifications as (AP) and (AP+α) if we add the constant term. These single-horizon specifications provide a very useful baseline case to understand the empirical improvement of having multiple horizons and segmentation, because they have the same degrees of freedom of the segmented models. For both categories, there are two estimated parameters and, possibly, a constant. Specifically, the single horizon case contains one horizon parameter and one expected liquidity coefficient, while the multiple horizon case has one parameter for each horizon.

B. Benchmark Estimation Results

Table II shows the results for the illiquidity portfolios. We find that the first specification of the segmented model (SEG), without a constant term, improves the R^2 of the Acharya-Pedersen model by about 20%, from 62% to 82%. Importantly, this improvement is achieved retaining the parsimony of the original model – both models depend on two parameters. The fit is graphically displayed in Figure 2. The graphs indicate that accounting for segmentation and heterogeneous horizons leads to smaller pricing errors in the upper-right end of the plot, i.e., for the more illiquid portfolios (as Table I shows that illiquid portfolios tend to have higher excess returns). Since the more illiquid portfolios are also characterized by segmentation, this is first suggestive evidence that the economic source of the improved fit of our model is obtained by effectively constraining the clientele of the illiquid assets to the long-term investors. Table II also shows that the segmented model still outperforms the AP model when we allow for a constant term α in the asset pricing equation.

We then investigate the sources of this improved fit in more detail and use the empirical estimates to decompose expected returns into an expected liquidity component and risk premium component, according to equation (4). We depict this decomposition for the single-horizon and two-horizon case with segmentation in Figure 3. We notice that in the single horizon (AP) case, the impact of the expected liquidity term is relatively modest. This is because the expected costs increase exponentially when moving from liquid to illiquid portfolios, while the expected returns do not exhibit such an exponentially increasing pattern (see Table I as well as Figure 1). If anything, the expected returns increase with illiquidity at a lower rate for the more illiquid portfolios: the expected return levels off after portfolio 19, but the expected expected liquidity term keeps ris-

ing. The (AP) specification implies a linear relation between expected costs and expected returns, and thus has difficulty fitting the cross-section of liquid versus illiquid portfolios. As a result, the expected liquidity effect is rather small for the (AP) specification (a few basis points per month for most portfolios).

Our model with segmentation reduces the impact of the expected liquidity term on the illiquid portfolios relative to the impact on the liquid portfolios. Hence, our model allows for a much larger overall expected liquidity premium (between 10 and 40 basis points per month) and this improves the fit substantially as shown by Figure 2 and Figure 3. The average expected liquidity premium across portfolios is about 20 basis points per month for the (SEG) specification, compared to an average effect of 3 basis points for the (AP) specification. Since only long-term investors hold the most illiquid assets, the expected liquidity premium is relatively limited for these assets. This explains the drop in the impact of the cost term around portfolios 19 and 20. Figure 3 also shows that the covariance term provides the largest overall contribution to the expected excess returns.

To gain further insight into the impact of segmentation, we make use of Proposition 3 to decompose both the expected liquidity effect and the covariance effect into a full risk-sharing component, a segmentation component, and a spillover component. We show these components in Figure 4. The decomposition indicates clearly how the impact of segmentation on the total expected return builds up. For the expected liquidity premium given in equation (16) (upper panel in Figure 4), the full risk-sharing effect increases sharply for the least liquid portfolios since expected costs increase exponentially when moving to illiquid assets. This effect is mostly canceled out by the negative segmentation effect, which arises because the long-term investors care less about liquidity. There is still a modest liquidity spillover premium. Hence, the liquidity spillover effect drives most of

the expected liquidity effect for the least liquid assets. This is also what causes the drop in the model-implied expected return going from portfolio 19 to 20, as depicted in Figure 3.

For the covariance component of expected returns (lower panel of Figure 4), we observe that the segmentation premium and the spillover risk premium in equations (18) and (19) mostly cancel out because the returns on illiquid portfolios are strongly related to liquid portfolio returns. Hence the risk premia of liquid and illiquid assets are quite similar. This is evidence showing that the effect of segmentation is almost entirely driven by the expected liquidity term.

The estimates in Table II can be used to obtain insight into the structural parameters in the asset pricing model. For example, if we assume for simplicity that risk aversion is constant across investor classes (i.e., $A_1 = A_2$), we can make inferences about the number of investors in each class. More specifically, we examine the ratio $(h_2\gamma_2)/(h_1\gamma_1) = (h_2Q_2)/(h_1Q_1)$.[12] The results for specifications (SEG) and (SEG+α) show that the estimates imply that there are respectively 2.1 and 2.6 times as many long horizon investors as there are short horizon investors.

We show some comparative statics results for each model parameter in Figure 5 (see equation (26) for the analytical expression). The graphs illustrate the impact on the risk premium of an increase in the γ_j, that is, an increase in the quantity of class j investors, a decrease in their risk aversion, or both. The top panel shows the baseline case with one-period homogeneous investors. Here, the larger risk-sharing (with more numerous or less risk averse investors) is all that matters. Looking now at long-term investors in the heterogeneous horizon model (Figure 5, bottom right panel), we see that the effect of an increase in γ_2 on the risk premium is always negative. This is

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[12] As Q_j investors with horizon h_j enter each period, at each point in time the total number of type-j investors equals h_jQ_j. Also note that including $\widetilde{S}'\iota$ in the γ_j does not influence our comparison.

consistent with the theoretical analysis of Section III.B, where we show that for long-term investors the risk sharing effect dominates the liquidity effect (absent hedging considerations). In other words, this finding confirms empirically that long term investors are less concerned about liquidity. For the short term investors (Figure 5, bottom left panel), we see that the effect of γ_1 on expected returns is positive for the most illiquid portfolios and negative for the more liquid portfolios, again in line with our intuition in Section III.B. These comparative statics results show that γ_1 and γ_2 have quite different effects on expected returns, which implies that these parameters are well identified empirically.

C. Robustness Across Horizons and Portfolios

In this subsection, we check the robustness of our empirical findings to different modeling assumptions. We first test the sensitivity of model performance to the choice of the long term investor horizon and we compute the R^2 for $h_2 = 30, 60, 120, 240, 480$ months. The results are given in Figure 6, and show that the explanatory power of the model is relatively insensitive to the choice of horizon. In addition, the coefficients do not vary much across the different choices. The performance is also robust to varying h_1, the short-term investor horizon, as long as it does not grow too large. More specifically, with $h_1 = 6$ months we still obtain a substantial improvement over the single-horizon model.

The second robustness check concerns the assumption of i.i.d. transaction costs, which is required to obtain a tractable solution for the asset pricing model. Empirically, transaction costs are persistent over time. For example, Acharya and Pedersen (2005) estimate an AR(2) model for their monthly measure of transaction costs. For our empirical application, the i.i.d. assumption is not a

major concern for two reasons. First, as shown above, the model generates a good fit even when the short-term investors have a six-month horizon ($h_1 = 6$). The persistence of transaction costs is obviously lower at a semi-annual frequency compared to the monthly frequency in Acharya and Pedersen (2005). Second, and more importantly, we estimate a version of the model without liquidity risk (hence with constant c_{t+1}). The results in Table III show that the model fit is virtually unchanged. This shows that the good fit of the heterogenous-horizon model is not obtained through the liquidity risk channel, but rather via the expected liquidity effect and the associated segmentation and spillover effects. In addition, it follows from the result in Appendix C that without liquidity risk, $V_2 = I$ (assuming $R_f = 1$). The results for the model without liquidity risk thus indicate that the assumption that $V_2 = I$ does not seem to be very restrictive, validating the analysis of Section II.C.

Another robustness test is related to the specific choice of the baseline model. Equation (7) is an i.i.d. version of the Acharya and Pedersen (2005) model, which is a conditional model. To obtain an unconditional version, they take expectations on both sides and apply a standard result regarding the expectation of a conditional covariance. This means that the covariance component in their specification is actually a covariance between *residuals* of $R_{t+1} - c_{t+1}$ and *residuals* of $R_{t+1}^m - c_{t+1}^m$, obtained with an AR(2) model for returns and liquidity. Unreported estimation results show that the conditional model with AR(2) residuals yields very similar results as the unconditional specification of equation (7). Hence, the comparison of the explanatory power between the results of the models in Table II does not depend on the specific version of the one-period single horizon model that is used.

As a final robustness check, we also estimate our model for two different portfolio sorts. As before, the segmentation cutoff is set by comparing the average monthly return to the average transaction costs, with the one-month investors only investing in the portfolios where the monthly return exceeds the costs. Table IV, Panel A, shows even larger improvements in the cross-sectional fit of our model for the σ(illiquidity) portfolios: the R^2 equals 64.1% in the AP model versus 86.5% in the heterogenous horizon model for the case without a constant term. This shows that the model captures well both the pricing of the level of liquidity and liquidity risk. For the B/M-by-size portfolios, the improvement is also very substantial (see Table IV, Panel B): here the cross-sectional R^2 equals 35.0% in the AP model versus 54.4% in the heterogenous horizon model (without constant term).[13] In summary, for any portfolio sorting criteria, our heterogeneous investment horizon model with segmentation provides at least a 20% R^2 improvement in the cross-sectional fit.

VI. Conclusions

Heterogeneous investment horizons can have important asset pricing effects through the distinct role of liquidity. Different horizons imply different trading frequencies and therefore trading costs can have a varying impact for the expected returns of assets held by short-term versus long-term investors.

We present a new liquidity-based asset pricing model with heterogeneous investment horizon investors and stochastic transaction costs. Our model contributes to the literature by effectively

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[13]If we include a constant term in the asset pricing model, the improvement in R^2 is even larger. In this case, the estimate for γ_1 in the heterogenous-horizon model tends to infinity, implying a zero risk premium for the non-segmented portfolios. For these portfolios the returns are best explained by the constant term plus the expected liquidity effect.

bridging the clientele of investors in the seminal Amihud and Mendelson (1986) paper with the risk-averse agents and stochastic illiquidity of the Acharya and Pedersen (2005) model. The increased generality of our model delivers a number of new theoretical insights. It also provides a useful metric to understand the empirical cost of restrictive assumptions, such as horizon homogeneity, in fitting the cross-section of U.S. stock returns.

The most intriguing theoretical result is the existence of an equilibrium with partial segmentation. Short-term investors optimally choose not to invest in the most illiquid assets, intuitively because their expected returns are not sufficient to cover expected transaction costs. In contrast, long-term investors trade less frequently and can afford to invest in illiquid assets. In this equilibrium, the expected returns of segmented assets contain additional terms, both for risk premia and in expected liquidity effects. These additional terms depend partly on the segmented ownership and partly on the correlation between liquid and illiquid assets.

The additional structure imposed by our model delivers a substantial increase in the cross-sectional explanatory power for U.S. stock returns. For a number of portfolio sorting criteria, we find that our heterogeneous horizon model increases the R^2 by at least 20% compared to an homogeneous-horizon liquidity asset pricing model. With the same degrees of freedom, we obtain this large empirical improvement through a suitable characterization of the relation between excess returns and different features of expected liquidity and the liquidity risk premium. This characterization depends crucially on the presence of partial segmentation and agents' risk aversion.

Appendix: Derivations

A. Main result

To derive the main result, we consider N classes of investors, as this shows the generality of the result and shortens the proof. We start by introducing sets B_j ($j = 1, \ldots, N$) that represent the assets that investor j optimally holds in his or her portfolio. In Appendix B we describe the conditions that are required for these optimal portfolios. We let the B_j be subsets of $\{1, \ldots, K\}$, where K is the number of assets. Without loss of generality we assume that for some j it holds that $B_j = \{1, \ldots, K\}$.

Proof of Proposition 1: To derive the equilibrium, we first consider each investor's optimization problem. For the investors with horizon h_j it is given by

$$\max_{y_{j,t}} \mathbb{E}\left[W_{j,t+h_j}\right] - \frac{1}{2}A_j \text{Var}\left(W_{j,t+h_j}\right) \tag{A1}$$

$$W_{j,t+h_j} = \left(P_{t+h_j} + \sum_{k=1}^{h_j} R_f^{h_j-k} D_{t+k} - C_{t+h_j}\right)' y_{j,t} + R_f^{h_j}\left(e_j - P_t' y_{j,t}\right).$$

We first introduce notation that will allow us to derive the equilibrium in the case where investor j holds only assets that are in B_j. For a $K \times K$ matrix M, we denote by M_{B_j} the $|B_j| \times |B_j|$ matrix (with $|\cdot|$ the cardinality of a set) with the rows and columns that are not elements of B_j removed. As it will be used frequently, we also introduce the notation $M_{B_j,p}^{-1}$ for the inverse of M_{B_j} with zeros inserted at the locations where rows and columns of M were removed, so that $M_{B_j,p}^{-1}$ is a $K \times K$ matrix.

For example, let

$$M = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix}$$

and let $B_j = \{1,3\}$. Then

$$M_{B_j} = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix},$$

so that

$$M_{B_j}^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}.$$

We then have

$$M_{B_j,p}^{-1} = \begin{bmatrix} 7 & 0 & -2 \\ 0 & 0 & 0 \\ -3 & 0 & 1 \end{bmatrix}.$$

If we apply this operation to the covariance matrix in the optimization problem of investor j, this yields the solution considering only the assets in B_j padded with zeros, so that it is a $K \times 1$ vector. The benefit is that it makes the solution vectors $y_{j,t}$ $(j = 1, \ldots, N)$ conformable to addition, which allows us to derive the equilibrium.

Thus, given that the optimal portfolio of the investor consists only of assets that are elements of B_j, the solution is

$$y_{j,t} = \frac{1}{A_j} \mathrm{Var} \left(P_{t+h_j} + \sum_{k=1}^{h_j} R_f^{h_j-k} D_{t+k} - C_{t+h_j} \right)_{B_j,p}^{-1}$$

$$\times \left(\mathbb{E} \left[P_{t+h_j} + \sum_{k=1}^{h_j} R_f^{h_j-k} D_{t+k} - C_{t+h_j} \right] - R_f^{h_j} P_t \right). \qquad (\text{A2})$$

Using the i.i.d. assumption for dividends and costs, we obtain a stationary equilibrium with constant prices and i.i.d. returns. It is then straightforward to derive that $y_{j,t}$ can be written as (derivation available on request)

$$y_{j,t} = \frac{1}{A_j} \mathrm{diag} \left(P_t \right)^{-1} \mathrm{Var} \left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j} \right)_{B_j,p}^{-1} \qquad (\text{A3})$$

$$\times \left(\mathbb{E} \left[\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j} \right] - \sum_{k=0}^{h_j-1} R_f^{h_j-k} \right).$$

Similarly, it is also straightforward to show that

$$\mathbb{E} \left[\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} \right] - \sum_{k=0}^{h_j-1} R_f^{h_j-k} = \rho_j \left(\mathbb{E} \left[R_{t+1} \right] - R_f \right), \qquad (\text{A4})$$

where $\rho_j = \sum_{k=1}^{h_j} R_f^{h_j-k}$. Making further use of the i.i.d. assumption by which $\mathbb{E}(c_{t+h_j}) = \mathbb{E}(c_{t+k})$ for all j and k, the allocations can thus be written as

$$y_{j,t} = \frac{1}{A_j} \operatorname{diag}(P_t)^{-1} \operatorname{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j}\right)_{B_j,p}^{-1}$$
$$\times \left(\rho_j \left(\mathbb{E}[R_{t+1}] - R_f\right) - \mathbb{E}[c_{t+1}]\right). \tag{A5}$$

Each period a fixed quantity $Q_j > 0$ of type j investors enters the market. The equilibrium condition at time t is

$$\sum_{j=1}^{N} Q_j y_{j,t} = S - \sum_{j=1}^{N} \sum_{k=1}^{h_j-1} Q_j y_{j,t-k}, \tag{A6}$$

which is equivalent to

$$\sum_{j=1}^{N} \sum_{k=0}^{h_j-1} Q_j y_{j,t-k} = S. \tag{A7}$$

Under the i.i.d. assumption we have $y_{j,t-k} = y_{j,t}$ for all k, so that

$$\sum_{j=1}^{N} h_j Q_j y_{j,t} = S. \tag{A8}$$

Scaling by price we obtain

$$\sum_{j=1}^{N} h_j Q_j \operatorname{diag}(P_t) y_{j,t} = \widetilde{S}_t, \tag{A9}$$

where $\widetilde{S}_t = \text{diag}(P_t)S$. At this point it is useful to introduce the notation $R^m_{t+1} = \widetilde{S}'_t R_{t+1}/\widetilde{S}'_t \iota$, and $c^m_{t+1} = \widetilde{S}'_t c_{t+1}/\widetilde{S}'_t \iota$. We note that in the i.i.d. setting with constant prices, $\widetilde{S}_t$ is constant over time, hence we omit the time subscript and write $\widetilde{S}$ in what follows. This allows us to write

$$\text{Var}\left(R_{t+1} - c_{t+1}\right)\widetilde{S} = \widetilde{S}'\iota\text{Cov}\left(R_{t+1} - c_{t+1}, R^m_{t+1} - c^m_{t+1}\right).$$

Then, multiplying both sides of (A9) by $(1/\widetilde{S}'\iota)\text{Var}\left(R_{t+1} - c_{t+1}\right)$, and filling in the expression for the optimal allocations gives

$$\sum_{j=1}^{N} h_j \frac{Q_j}{A_j\widetilde{S}'\iota}\text{Var}\left(R_{t+1} - c_{t+1}\right)\text{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+1}\right)_{B_j,p}^{-1} \tag{A10}$$

$$\times \left(\rho_j\left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \mathbb{E}\left[c_{t+1}\right]\right) = \text{Cov}\left(R_{t+1} - c_{t+1}, R^m_{t+1} - c^m_{t+1}\right).$$

We define $\gamma_j = Q_j/(A_j\widetilde{S}'\iota)$ and

$$V_j = h_j\text{Var}\left(R_{t+1} - c_{t+1}\right)\text{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j}\right)_{B_j,p}^{-1}. \tag{A11}$$

This allows us to write

$$\sum_{j=1}^{N} \gamma_j V_j\left(\rho_j\left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \mathbb{E}\left[c_{t+1}\right]\right) = \text{Cov}\left(R_{t+1} - c_{t+1}, R^m_{t+1} - c^m_{t+1}\right). \tag{A12}$$

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We can rewrite this equilibrium condition as

$$\mathbb{E}\left[R_{t+1}\right] - R_f = \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j\right)^{-1} \sum_{j=1}^{N} \gamma_j V_j \mathbb{E}\left[c_{t+1}\right] \tag{A13}$$

$$+ \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j\right)^{-1} \mathrm{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right).$$

Q.E.D.

B. *Endogenous Segmentation*

In this Appendix we describe under which conditions endogenous segmentation arises. Consider the usual (non-segmented) mean-variance solution for the short-term investors

$$y_{1,t} = \frac{1}{A_1} \mathrm{diag}\left(P_t\right)^{-1} \mathrm{Var}\left(\sum_{k=1}^{h_1} R_f^{h_1-k} R_{t+k} - c_{t+h_1}\right)^{-1} \tag{A14}$$

$$\times \left(\rho_1 \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \mathbb{E}\left[c_{t+1}\right]\right),$$

Suppose that the costs on some illiquid assets are so high that, in equilibrium, some elements of $y_{1,t}$ are negative. Without loss of generality, order the assets such that $y_{1,t} = (y_{liq,1,t}, y_{illiq,1,t})$ with $y_{liq,1,t}$ having only positive (or non-negative) elements and $y_{illiq,1,t}$ having only negative elements. In this case, these investors do not want to buy the more illiquid assets. Of course, it is still possible that the investor wants to short these illiquid assets, but this is unlikely given the high transaction

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costs. To see this formally, we note that if the optimal position in the illiquid assets were negative (and positive for the liquid assets), the optimal portfolio would be

$$z_{1,t} = \frac{1}{A_1} \operatorname{diag}(P_t)^{-1} \operatorname{Var}\left(\sum_{k=1}^{h_1} R_f^{h_1-k} R_{t+k} - \delta_1 c_{t+h_1}\right)^{-1}$$

$$\times \left(\rho_1 \left(\mathbb{E}[R_{t+1}] - R_f\right) - \delta_1 \mathbb{E}[c_{t+1}]\right), \tag{A15}$$

where δ_1 is a diagonal matrix with elements equal to 1 if the investor is long in the respective asset, and -1 if the investor is short (see Bongaerts, De Jong, and Driessen (2011)). Consider the i-th asset. If $z_{illiq,1,i,t} < 0$, this is indeed the solution to the optimal portfolio rule, but this is unlikely if costs are high for this asset. In turn, if $z_{illiq,1,i,t} > 0$ and the corresponding element of $y_{illiq,1,t}$ is negative, it is optimal for the short-term investors to have a zero position in the illiquid assets. We thus focus here on the case in which costs are high enough so that the short-term investors optimally have a zero position in the illiquid assets. Hence, the set B_1 contains only those assets that are liquid enough for the short-term investors to invest in them.

C. *Computing the long-term covariance matrix*

We use the i.i.d. assumption to rewrite part of the moment conditions as follows

$$\operatorname{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j}\right)^{-1} = \left(\left(\sum_{k=1}^{h_j-1} R_f^{2(h_j-k)}\right)\operatorname{Var}(R_{t+1}) + \operatorname{Var}(R_{t+1} - c_{t+1})\right)^{-1}. \tag{A16}$$

This allows us to compute the covariance terms using only one-period covariances.

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D. Market and liquidity risk premia with two assets

Proof of Proposition 2: We consider the two-asset case ($K = 2$), with two horizons ($N = 2$), $h_1 = 1$, and no segmentation. We start from (A9), multiply both sides by $1/\widetilde{S}_t'\iota$, and use the expression for the allocations to obtain

$$\sum_{j=1}^{N} \frac{Q_j}{A_j\widetilde{S}'\iota} h_j \text{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+1}\right)^{-1} \left(\rho_j \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \mathbb{E}\left[c_{t+1}\right]\right) = \frac{\widetilde{S}}{\widetilde{S}'\iota}. \tag{A17}$$

This yields

$$\mathbb{E}\left[R_{t+1}\right] - R_f = \left(\sum_{j=1}^{N} \gamma_j\rho_j h_j \text{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+1}\right)^{-1}\right)^{-1} \tag{A18}$$

$$\times \left(\sum_{j=1}^{N} \gamma_j\rho_j h_j \text{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+1}\right)^{-1}\right) \mathbb{E}\left[c_{t+1}\right]$$

$$+ \left(\sum_{j=1}^{N} \gamma_j\rho_j h_j \text{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+1}\right)^{-1}\right)^{-1} \frac{\widetilde{S}}{\widetilde{S}'\iota}.$$

Next, we introduce for $j = 1, 2$ the determinants

$$d_j = \det\left(\text{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j}\right)\right), \tag{A19}$$

we note that $K = 2$ implies that the adj$(\cdot)$ operator is additive, and we apply (A16) to write

$$
\text{Var} \left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j} \right)^{-1} = \frac{1}{d_j} \text{adj Var} \left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j} \right) \tag{A20}
$$

$$
= \frac{1}{d_j} \left(\left(\sum_{k=1}^{h_j-1} R_f^{2(h_j-k)} \right) \text{adj Var} \left(R_{t+1} \right) + \text{adj Var} \left(R_{t+1} - c_{t+1} \right) \right).
$$

We now let

$$
d_0 = \det \left(\sum_{j=1}^{N} \gamma_j \rho_j h_j \text{Var} \left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+1} \right)^{-1} \right) \tag{A21}
$$

and

$$
\sigma_j = \sum_{k=1}^{h_j-1} R_f^{2(h_j-k)}. \tag{A22}
$$

Making use of the fact that the adj$(\cdot)$ operator is equal to its own inverse (as $K = 2$), we find

$$
\left(\sum_{j=1}^{N} \gamma_j \rho_j h_j \text{Var} \left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+1} \right)^{-1} \right)^{-1} \tag{A23}
$$

$$
= \frac{1}{d_0} \text{adj} \left(\sum_{j=1}^{N} \gamma_j \frac{\rho_j h_j}{d_j} \left(\sigma_j \text{adj Var} \left(R_{t+1} \right) + \text{adj Var} \left(R_{t+1} - c_{t+1} \right) \right) \right)
$$

$$
= \sum_{j=1}^{N} \gamma_j \lambda_j \left(\sigma_j \text{Var} \left(R_{t+1} \right) + \text{Var} \left(R_{t+1} - c_{t+1} \right) \right),
$$

where $\lambda_j = \rho_j h_j / d_0 d_j$. It now follows from (A10) that

$$\left(\sum_{j=1}^{N} \gamma_j \rho_j h_j \text{Var} \left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+1} \right)_{B_j,P}^{-1} \right)^{-1} \frac{\widetilde{S}}{\widetilde{S}'\iota} \tag{A24}$$

$$= \sum_{j=1}^{N} \gamma_j \lambda_j \left(\sigma_j \text{Cov} \left(R_{t+1}, R_{t+1}^m \right) + \text{Cov} \left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m \right) \right)$$

$$= \left(\sum_{j=1}^{N} \gamma_j \lambda_j \right) \text{Cov} \left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m \right) + \left(\sum_{j=1}^{N} \gamma_j \lambda_j \sigma_j \right) \text{Cov} \left(R_{t+1}, R_{t+1}^m \right).$$

The result now follows by applying (A24) to (A18) with $N = 2$. Q.E.D.

E. Segmentation effects

For this part, we specialize to $N = 2$, and $h_1 = 1$. To derive the result below, we assume that $V_2 = I$, that the h_1-investors invest only in the most liquid assets, and that the h_2-investors invest in all assets.

Proof of Proposition 3: If we sort the assets by liquidity with the most liquid assets first, writing

$$\text{Var} \left(R_{t+1} - c_{t+1} \right) = \begin{bmatrix} V_{liq} & V_{liq,illiq} \\ V_{illiq,liq} & V_{illiq} \end{bmatrix}, \tag{A25}$$

we have

$$V_1 = h_1 \text{Var} \left(R_{t+1} - c_{t+1} \right) \text{Var} \left(\sum_{k=1}^{h_1} R_f^{h_1-k} R_{t+k} - c_{t+h_1} \right)^{-1}_{B_1,p} \tag{A26}$$

$$= \begin{bmatrix} V_{liq} & 0 \\ 0 & V_{illiq} \end{bmatrix} \begin{bmatrix} V_{liq}^{-1} & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} I & 0 \\ V_{illiq,liq}V_{liq}^{-1} & 0 \end{bmatrix}.$$

Using $N = 2$ and $V_2 = I$ in (A13) leads to the equilibrium relation

$$\mathbb{E}\left[R_{t+1} \right] - R_f = (\gamma_1 \rho_1 V_1 + \gamma_2 \rho_2 I)^{-1} (\gamma_1 V_1 + \gamma_2 I) \mathbb{E}\left[c_{t+1} \right]$$

$$+ (\gamma_1 \rho_1 V_1 + \gamma_2 \rho_2 I)^{-1} \text{Cov} \left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m \right).$$

To find the liquidity risk effect, we focus on the factor

$$(\gamma_1 \rho_1 V_1 + \gamma_2 \rho_2 I)^{-1} = \begin{bmatrix} (\gamma_1 \rho_1 + \gamma_2 \rho_2)I & 0 \\ \gamma_1 \rho_1 V_{illiq,liq}V_{liq}^{-1} & \gamma_2 \rho_2 I \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} (\gamma_1 \rho_1 + \gamma_2 \rho_2)^{-1}I & 0 \\ -\gamma_1 \rho_1 (\gamma_2 \rho_2)^{-1} (\gamma_1 \rho_1 + \gamma_2 \rho_2)^{-1} V_{illiq,liq}V_{liq}^{-1} & (\gamma_2 \rho_2)^{-1}I \end{bmatrix}.$$

In what follows, we will use the *liquidity spillover beta*, defined by

$$\beta = V_{illiq,liq}V_{liq}^{-1} \tag{A27}$$

$$= \mathrm{Cov}\left(R_{t+1}^{illiq} - c_{t+1}^{illiq}, R_{t+1}^{liq} - c_{t+1}^{liq}\right)\mathrm{Var}\left(R_{t+1}^{liq} - c_{t+1}^{liq}\right)^{-1}.$$

For the impact of the level of liquidity we write

$$(\gamma_1\rho_1 V_1 + \gamma_2\rho_2 I)^{-1}(\gamma_1 V_1 + \gamma_2 I) \tag{A28}$$

$$= \begin{bmatrix} (\gamma_1\rho_1 + \gamma_2\rho_2)^{-1}I & 0 \\ -\gamma_1\rho_1(\gamma_2\rho_2)^{-1}(\gamma_1\rho_1 + \gamma_2\rho_2)^{-1}\beta & (\gamma_2\rho_2)^{-1}I \end{bmatrix}\begin{bmatrix} (\gamma_1 + \gamma_2)I & 0 \\ \gamma_1\beta & \gamma_2 I \end{bmatrix}$$

$$= \begin{bmatrix} (\gamma_1 + \gamma_2)(\gamma_1\rho_1 + \gamma_2\rho_2)^{-1}I & 0 \\ \left(\gamma_1(\gamma_2\rho_2)^{-1} - \gamma_1\rho_1(\gamma_2\rho_2)^{-1}(\gamma_1 + \gamma_2)(\gamma_1\rho_1 + \gamma_2\rho_2)^{-1}\right)\beta & \rho_2^{-1}I \end{bmatrix}.$$

We rewrite the scalar part of the spillover coefficient using the identity

$$\frac{\gamma_1}{\gamma_2\rho_2} - \frac{\gamma_1\rho_1(\gamma_1 + \gamma_2)}{\gamma_2\rho_2(\gamma_1\rho_1 + \gamma_2\rho_2)} = \frac{\rho_2 - \rho_1}{\rho_2}\frac{\gamma_1}{\gamma_1\rho_1 + \gamma_2\rho_2}. \tag{A29}$$

Combining the results above, we can write the equilibrium relation for the liquid assets as

$$\mathbb{E}\left[R_{t+1}^{liq}\right] - R_f = \frac{\gamma_1 + \gamma_2}{\gamma_1\rho_1 + \gamma_2\rho_2}\mathbb{E}\left[c_{t+1}^{liq}\right] + \frac{1}{\gamma_1\rho_1 + \gamma_2\rho_2}\mathrm{Cov}\left(R_{t+1}^{liq} - c_{t+1}^{liq}, R_{t+1}^{m} - c_{t+1}^{m}\right). \tag{A30}$$

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and the equilibrium relation for the illiquid assets as

$$\mathbb{E}\left[R_{t+1}^{illiq}\right] - R_f = \frac{1}{\rho_2}\mathbb{E}\left[c_{t+1}^{illiq}\right] + \frac{\rho_2 - \rho_1}{\rho_2}\frac{\gamma_1}{\gamma_1\rho_1 + \gamma_2\rho_2}\beta\mathbb{E}\left[c_{t+1}^{liq}\right] \tag{A31}$$
$$+ \frac{1}{\gamma_2\rho_2}\text{Cov}\left(R_{t+1}^{illiq} - c_{t+1}^{illiq}, R_{t+1}^m - c_{t+1}^m\right)$$
$$- \frac{\gamma_1\rho_1}{\gamma_2\rho_2(\gamma_1\rho_1 + \gamma_2\rho_2)}\beta\text{Cov}\left(R_{t+1}^{liq} - c_{t+1}^{liq}, R_{t+1}^m - c_{t+1}^m\right).$$

The desired expressions now follow directly. Q.E.D.

F. Estimation Methodology – Obtaining Standard Errors

We denote the required moments that enter the asset pricing model by the vector ψ. This vector contains expected returns, expected costs, and all required covariances of returns and costs. It is straightforward to derive the asymptotic covariance matrix of the sample estimator of these moments (since covariances can be written as second moments plus products of first moments),

$$\sqrt{T}\left(\widehat{\psi} - \psi\right) \xrightarrow{d} \mathcal{N}\left(0, S_\psi\right). \tag{A32}$$

We can now use the delta method to find the standard errors for $\widehat{\gamma}$.

Consider the GMM minimization problem given by

$$\min_\gamma g(\widehat{\psi}, \gamma)' g(\widehat{\psi}, \gamma), \tag{A33}$$

for which the solution is implicitly given by

$$2G_\gamma(\widehat{\psi}, \gamma)' g(\widehat{\psi}, \gamma) = 0, \tag{A34}$$

where

$$G_\gamma(\psi, \gamma) = \frac{\partial g(\psi, \gamma)}{\partial \gamma}. \tag{A35}$$

Dividing both sides of (A34) by 2 and evaluating at $\widehat{\gamma}$, we may write

$$G_\gamma(\widehat{\psi}, \widehat{\gamma})' g(\widehat{\psi}, \gamma_0) + G_\gamma(\widehat{\psi}, \widehat{\gamma})' (g(\widehat{\psi}, \widehat{\gamma}) - g(\widehat{\psi}, \gamma_0)) = 0. \tag{A36}$$

Next, we expand $g(\widehat{\psi}, \widehat{\gamma})$ around γ_0 to obtain

$$g(\widehat{\psi}, \widehat{\gamma}) - g(\widehat{\psi}, \gamma_0) \approx G_\gamma(\widehat{\psi}, \widehat{\gamma}) (\widehat{\gamma} - \gamma_0). \tag{A37}$$

It follows that

$$G_\gamma(\widehat{\psi}, \widehat{\gamma})' g(\widehat{\psi}, \gamma_0) + G_\gamma(\widehat{\psi}, \widehat{\gamma})' G_\gamma(\widehat{\psi}, \widehat{\gamma}) (\widehat{\gamma} - \gamma_0) = 0. \tag{A38}$$

We now expand $g(\widehat{\psi}, \gamma_0)$ around ψ_0 and use the fact that $g(\psi_0, \gamma_0) = 0$ to find that

$$g(\widehat{\psi}, \gamma_0) \approx G_\psi(\widehat{\psi}, \widehat{\gamma}) (\widehat{\psi} - \psi_0), \tag{A39}$$

where

$$G_\psi(\psi, \gamma) = \frac{\partial g(\psi, \gamma)}{\partial \psi}. \tag{A40}$$

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Hence

$$G_\gamma(\widehat{\psi},\widehat{\gamma})'G_\gamma(\widehat{\psi},\widehat{\gamma})\,(\widehat{\gamma}-\gamma_0) = -G_\gamma(\widehat{\psi},\widehat{\gamma})'G_\psi(\widehat{\psi},\widehat{\gamma})\,(\widehat{\psi}-\psi_0)\,. \tag{A41}$$

Using this result we obtain

$$\sqrt{T}\,(\widehat{\gamma}-\gamma_0) \approx -\left(G_\gamma(\widehat{\psi},\widehat{\gamma})'G_\gamma(\widehat{\psi},\widehat{\gamma})\right)^{-1}G_\gamma(\widehat{\psi},\widehat{\gamma})'G_\psi(\widehat{\psi},\widehat{\gamma})\sqrt{T}\,(\widehat{\psi}-\psi_0)\,. \tag{A42}$$

It follows that

$$\sqrt{T}\,(\widehat{\gamma}-\gamma_0) \xrightarrow{d} \mathcal{N}\left(0,\left(G_\gamma'G_\gamma\right)^{-1}G_\gamma'G_\psi S_\psi G_\psi'G_\gamma\left(G_\gamma'G_\gamma\right)^{-1}\right)\,. \tag{A43}$$

This result allows us to compute standard errors for the γ estimates taking into account the pre-estimation of the various moments ψ. For the final estimation procedure, we restrict the γ_j pertaining to the horizons h_j to be positive by estimating the logs. We use the usual, additional, delta method correction for the computation of the standard errors.

G. Comparative statics

We consider an increase in γ_k, so that the horizon h_k investors become either more numerous, or less risk averse, or both. We find

$$\frac{\partial \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right)}{\partial \gamma_k} = -\left(\sum_{j=1}^{N} \gamma_j \rho_j V_j\right)^{-1} \rho_k V_k \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j\right)^{-1} \sum_{j=1}^{N} \gamma_j V_j \mathbb{E}\left[c_{t+1}\right] \qquad \text{(A44)}$$

$$+ \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j\right)^{-1} V_k \mathbb{E}\left[c_{t+1}\right]$$

$$- \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j\right)^{-1} \rho_k V_k \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j\right)^{-1}$$

$$\times \mathrm{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right).$$

Rearranging gives

$$\frac{\partial \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right)}{\partial \gamma_k} = \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j\right)^{-1} V_k \left(\mathbb{E}\left[c_{t+1}\right] - \rho_k \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right)\right). \qquad \text{(A45)}$$

REFERENCES

Acharya, Viral V., and Lasse Heje Pedersen, 2005, Asset pricing with liquidity risk, *Journal of Financial Economics* 77, 375–410.

Amihud, Yakov, 2002, Illiquidity and stock returns: cross-section and time-series effects, *Journal of Financial Markets* 5, 31–56.

Amihud, Yakov, and Haim Mendelson, 1986, Asset pricing and the bid-ask spread, *Journal of Financial Economics* 17, 223–249.

Amihud, Yakov, Haim Mendelson, and Lasse Heje Pedersen, 2005, Liquidity and asset prices, *Foundations and Trends in Finance* 1, 269–364.

Ang, Andrew, and Joseph Chen, 2002, Asymmetric correlations of equity portfolios, *Journal of Financial Economics* 63, 443–494.

Atkins, Allen B., and Edward A. Dyl, 1997, Transactions costs and holding periods for common stocks, *The Journal of Finance* 52, 309–325.

Bao, Jack, Jun Pan, and Jiang Wang, 2011, The illiquidity of corporate bonds, *The Journal of Finance* 66, 911–946.

Bekaert, Geert, Campbell R. Harvey, and Christian Lundblad, 2007, Liquidity and expected returns: Lessons from emerging markets, *Review of Financial Studies* 20, 1783–1831.

Bongaerts, Dion, Frank De Jong, and Joost Driessen, 2011, Derivative pricing with liquidity risk: Theory and evidence from the credit default swap market, *The Journal of Finance* 66, 203–240.

Brennan, Michael J., and Yuzhao Zhang, 2012, Capital asset pricing with a stochastic horizon, *Working Paper*.

Cella, Cristina, Andrew Ellul, and Mariassunta Giannetti, 2011, Investors' horizons and the amplification of market shocks, *Working Paper*.

Chalmers, John M.R., and Gregory B. Kadlec, 1998, An empirical examination of the amortized spread, *Journal of Financial Economics* 48, 159–188.

Constantinides, George M., 1986, Capital market equilibrium with transaction costs, *The Journal of Political Economy* 94, 842–862.

Cremers, Martijn, and Ankur Pareek, 2009, Institutional investors investment durations and stock return anomalies: Momentum, reversal, accruals, share issuance and R&D increases, *Yale ICF Working Paper*.

De Jong, Frank, and Frans A. De Roon, 2005, Time-varying market integration and expected returns in emerging markets, *Journal of Financial Economics* 78, 583–613.

Duffie, Darrell, 2010, Presidential address: Asset price dynamics with slow-moving capital, *The Journal of Finance* 65, 1237–1267.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Franzoni, Francesco, Eric Nowak, and Ludovic Phalippou, 2011, Private equity performance and liquidity risk, *The Journal of Finance* p. forthcoming.

Hopenhayn, Hugo A., and Ingrid M. Werner, 1996, Information, liquidity, and asset trading in a random matching game, *Journal of Economic Theory* 68, 349–379.

Kamara, Avraham, Robert A. Korajczyk, Xiaoxia Lou, and Ronnie Sadka, 2012, Horizon pricing, *Working Paper*.

Merton, Robert C., 1971, Optimum consumption and portfolio rules in a continuous-time model, *Journal of Economic Theory* 3, 373–413.

Pástor, Luboš, and Robert F. Stambaugh, 2003, Liquidity risk and expected stock returns, *The Journal of Political Economy* 111, 642–685.

Sadka, Ronnie, 2010, Liquidity risk and the cross-section of hedge-fund returns, *Journal of Financial Economics* 98, 54–71.

Shanken, Jay, 1992, On the estimation of beta-pricing models, *Review of Financial Studies* 5, 1–55.

Shumway, Tyler, 1997, The delisting bias in CRSP data, *The Journal of Finance* 52, 327–340.

Table I
Descriptive statistics

This table shows descriptive statistics for the data used to estimate the model. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2009. The average excess return $\mathbb{E}[R_{t+1}] - R_f$, standard deviation of returns $\sigma(R_{t+1})$, standard deviation of returns net of costs $\sigma(R_{t+1} - c_{t+1})$, and covariance between portfolio and market level returns net of costs $\text{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right)$ are computed from the time-series observations.

Portfolio	$\mathbb{E}[R_{t+1}] - R_f$ (%)	$\mathbb{E}[c_{t+1}]$ (%)	$\sigma(R_{t+1})$ (%)	$\sigma(c_{t+1})$ (%)	$\sigma(R_{t+1} - c_{t+1})$ (%)	$\text{Cov}(\ldots,\ldots)$ (·100)	trm (%)	$\sigma(\text{trm})$
1	0.3264	0.2518	4.2022	0.0017	4.2024	0.1767	5.9158	6.2783
2	0.3755	0.2568	4.6114	0.0060	4.6123	0.2135	6.9554	6.2195
3	0.4354	0.2611	4.6451	0.0093	4.6465	0.2212	6.9501	6.0284
4	0.5600	0.2664	4.7600	0.0133	4.7621	0.2286	7.1681	6.8244
5	0.5953	0.2732	5.0470	0.0192	5.0506	0.2442	6.8267	6.0381
6	0.5495	0.2827	4.8503	0.0261	4.8549	0.2371	6.8506	6.4248
7	0.5571	0.2928	4.7960	0.0329	4.8009	0.2377	6.4540	5.5812
8	0.5431	0.3083	4.9547	0.0470	4.9637	0.2441	6.2103	5.4767
9	0.6055	0.3253	4.9567	0.0608	4.9678	0.2473	6.0450	5.0996
10	0.6309	0.3448	4.8916	0.0699	4.9049	0.2442	5.6554	4.8098
11	0.6734	0.3725	5.1438	0.0941	5.1611	0.2572	5.4089	4.2691
12	0.5630	0.4037	4.8935	0.1072	4.9144	0.2445	4.7951	3.7103
13	0.6529	0.4376	4.8893	0.1240	4.9165	0.2463	4.7662	3.6871
14	0.7400	0.4758	5.0515	0.1529	5.0836	0.2535	4.4090	3.0553
15	0.6506	0.5453	5.0894	0.1981	5.1365	0.2565	4.2004	3.0606
16	0.6331	0.6210	5.0289	0.2298	5.0830	0.2524	4.0327	2.5514
17	0.8010	0.7238	5.0751	0.3034	5.1522	0.2531	4.2676	4.4986
18	0.6807	0.8283	5.0265	0.3259	5.1102	0.2504	4.2420	5.3765
19	0.8664	0.9742	5.1872	0.4031	5.2873	0.2563	3.7374	2.6377
20	0.6789	1.2769	5.3153	0.5771	5.4804	0.2594	3.4632	2.3135
21	0.8253	1.5531	5.4042	0.6395	5.5692	0.2597	2.9857	1.7014
22	0.8381	1.9864	5.3899	0.8188	5.5838	0.2556	3.8557	7.9582
23	0.8183	2.8059	5.4890	1.2157	5.9041	0.2685	3.1721	2.6555
24	0.7511	4.4659	5.5545	2.0282	6.3574	0.2732	2.6859	1.9129
25	0.8712	8.0759	6.0417	4.0085	7.8284	0.2780	2.4010	2.1144

Table II
GMM estimation results: Illiquidity portfolios

This table shows the results from estimation of the various specifications of the model. The estimates are based on monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2009. An equal-weighted market portfolio is used. The specifications are special cases of the relation

$$\mathbb{E}[R_{t+1}] - R_f = \alpha + \kappa (\gamma_1 \rho_1 V_1 + \gamma_2 \rho_2 V_2)^{-1} (\gamma_1 V_1 + \gamma_2 V_2) \mathbb{E}[c_{t+1}] \tag{A46}$$
$$+ (\gamma_1 \rho_1 V_1 + \gamma_2 \rho_2 V_2)^{-1} \text{Cov}\left(R_{t+1} - c_{t+1}, R^m_{t+1} - c^m_{t+1}\right),$$

where $\gamma_j = Q_j/(A_j \widetilde{S}' \iota)$, $\rho_j = \sum_{k=1}^{h_j} R_f^{h_j-k}$, and

$$V_j = h_j \text{Var}(R_{t+1} - c_{t+1}) \text{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j}\right)^{-1}_{B_{j,p}}. \tag{A47}$$

We set $h_1 = 1$, and $h_2 = 120$. The parameters are estimated using GMM. For each coefficient the t-value is given in parentheses. The cross-sectional R^2 is reported in the rightmost column. Estimates for the heterogeneous horizon model, where short term investors invest only in the 19 most liquid portfolios, are denoted by SEG. AP indicates that the specification corresponds to a variant of the Acharya and Pedersen (2005) specification (7). Where the value of κ is unreported, it is set to 1.

	γ_1	γ_2	α	κ	R^2
(SEG)	0.2080	0.0036			0.8224
	(0.5922)	(1.9400)			
(SEG+α)	0.0830	0.0018	-0.0050		0.8722
	(0.2763)	(1.1285)	(-0.5358)		
(AP)	0.3973			0.0287	0.6215
	(2.2428)			(0.1672)	
(AP+α)	0.1737		-0.0078	0.0088	0.7660
	(0.8451)		(-0.5432)	(0.0213)	

Table III

GMM estimation results: Illiquidity portfolios, without liquidity risk

This table shows the results from estimation of the various specifications of the model without liquidity risk. The setup is the same as in Table II, but with c_{t+1} taken to be constant and equal to its estimated mean.

	γ_1	γ_2	α	κ	R^2
(SEG)	0.2008	0.0038			0.8243
	(0.3237)	(1.4004)			
(SEG+α)	0.0802	0.0019	-0.0049		0.8725
	(0.1172)	(0.9545)	(-0.2375)		
(AP)	0.3922			0.0425	0.6136
	(2.2360)			(0.2500)	
(AP+α)	0.1675		-0.0081	0.0402	0.7621
	(0.8949)		(-0.5933)	(0.0994)	

Table IV
GMM estimation results: σ(illiquidity) and B/M-by-size portfolios

This table shows the results from estimation of the various specifications of the model for different portfolio types. The setup is the same as in Table II. Panel A shows the results for 25 portfolios sorted on illiquidity variation. For Panel B 25 value-weighted portfolios sorted on book-to-market value and size are used. In both cases the same rule for the segmentation threshold is used as in Table II: the one-month investors only invest in assets for which the monthly average return exceeds the average transaction cost.

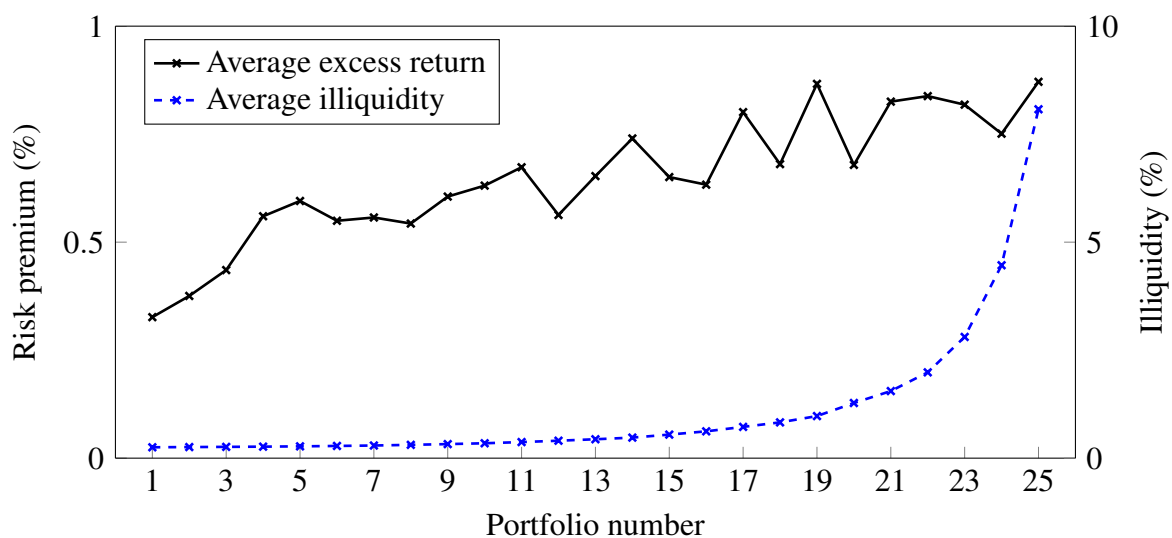|	γ_1	γ_2	α	κ	R^2
Panel A: σ(illiquidity) portfolios					
(SEG)	0.2030	0.0037			0.8650
	(0.5557)	(1.9920)			
(SEG+α)	0.0796	0.0019	-0.0046		0.9078
	(0.2854)	(1.4201)	(-0.7796)		
(AP)	0.3993			0.0278	0.6407
	(2.2445)			(0.1718)	
(AP+α)	0.1755		-0.0076	0.0014	0.7867
	(0.9360)		(-0.6027)	(0.0037Z)	
Panel B: B/M-by-size portfolios					
(SEG)	0.7721	0.0027			0.5442
	(0.3613)	(0.5327)			
(SEG+α)	$1.8 \cdot 10^{14}$	0.0030	0.0018		0.7579
	(0.0000)	(0.9388)	(0.5242)		
(AP)	0.4630			0.0424	0.3498
	(2.0839)			(0.2590)	
(AP+α)	0.8201		0.0025	0.0540	0.3923
	(0.8285)		(0.8093)	(0.5891)	

Figure 1. Expected returns and level of illiquidity. This figure illustrates the average monthly return (left axis) and average transaction costs (right axis) for the 25 US stock portfolios sorted on illiquidity. Portfolio 1 is the most liquid portfolio, while portfolio 25 is the least liquid portfolio.
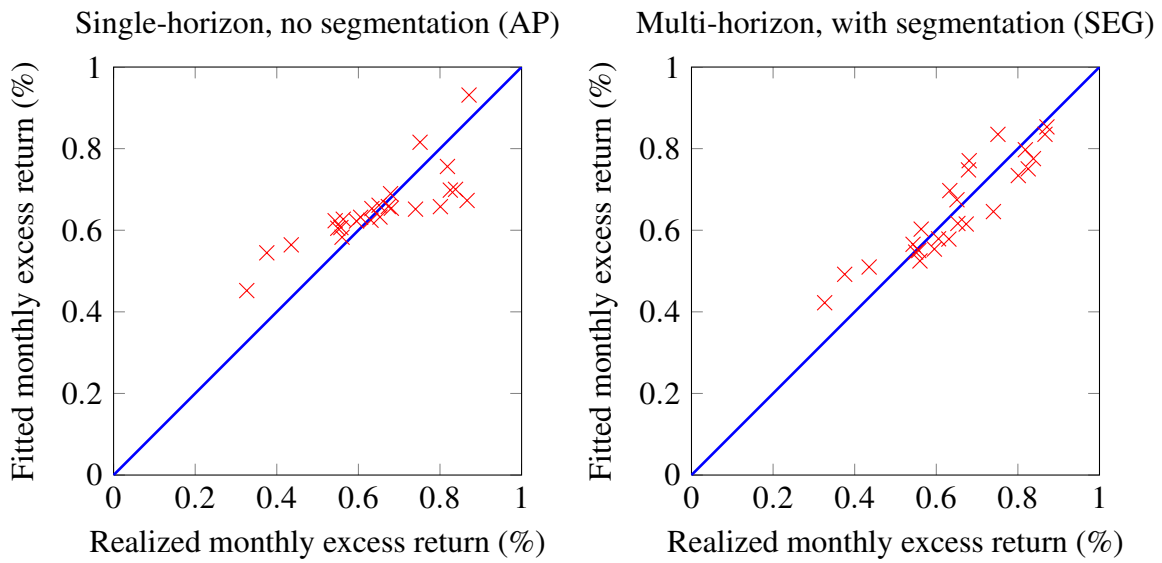
Figure 2. Fitted excess returns vs. realized excess returns. The left panel shows the goodness of fit for the Acharya and Pedersen (2005) specification (AP). The right panel shows the fit for the heterogeneous horizon specification (SEG). The graphs correspond to the estimation results as given in Table II.
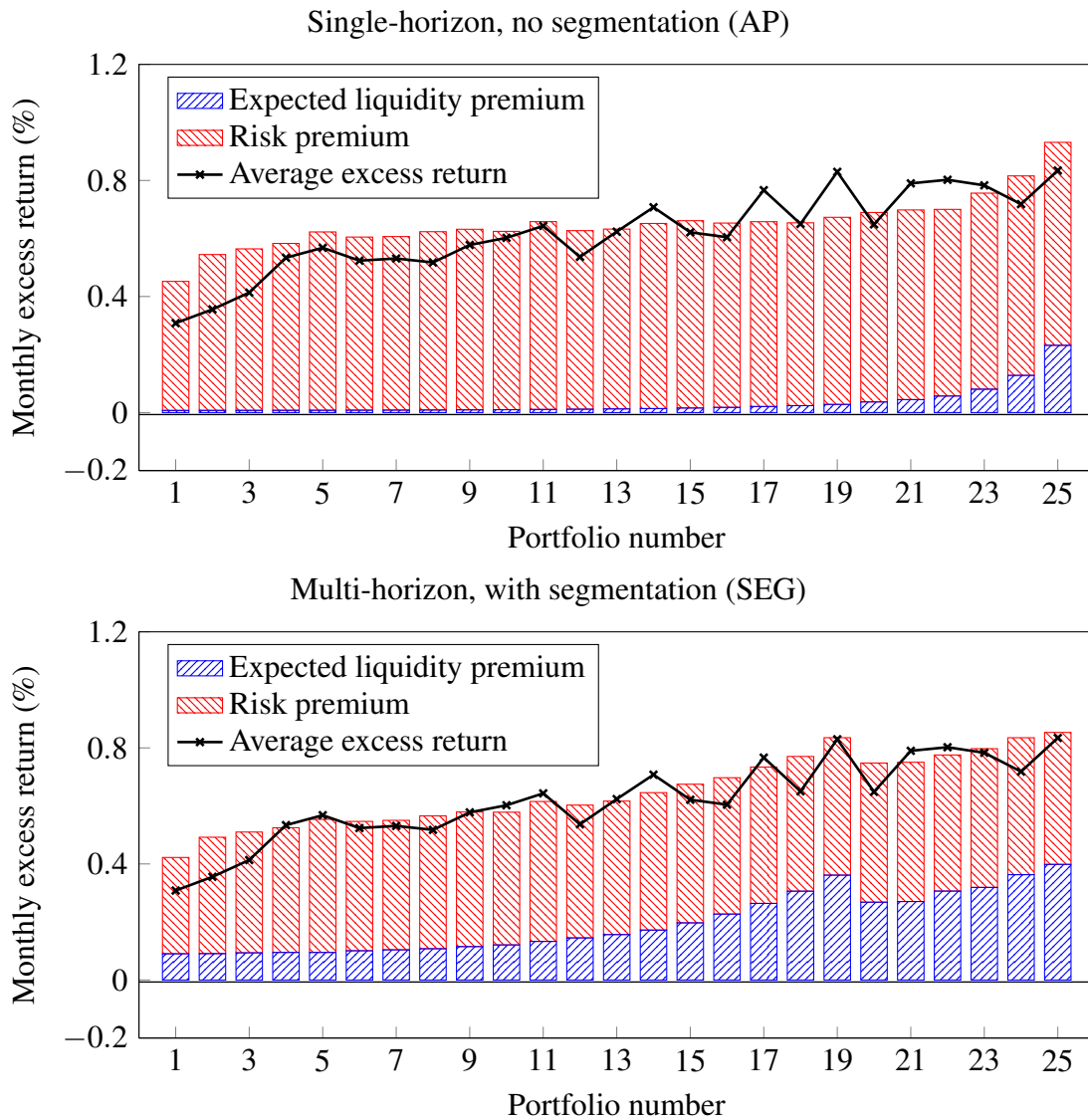
**Figure 3. Decomposition of predicted excess returns in the expected liquidity premium and the risk premium.** In each panel the lower part shows the expected liquidity premium and the upper part the risk premium. The line indicates the actual excess return. The upper panel shows the decomposition for the Acharya and Pedersen (2005) specification (AP). The lower panel shows the decomposition for the heterogeneous horizon specification (SEG). The graphs correspond to the estimation results as given in Table II.
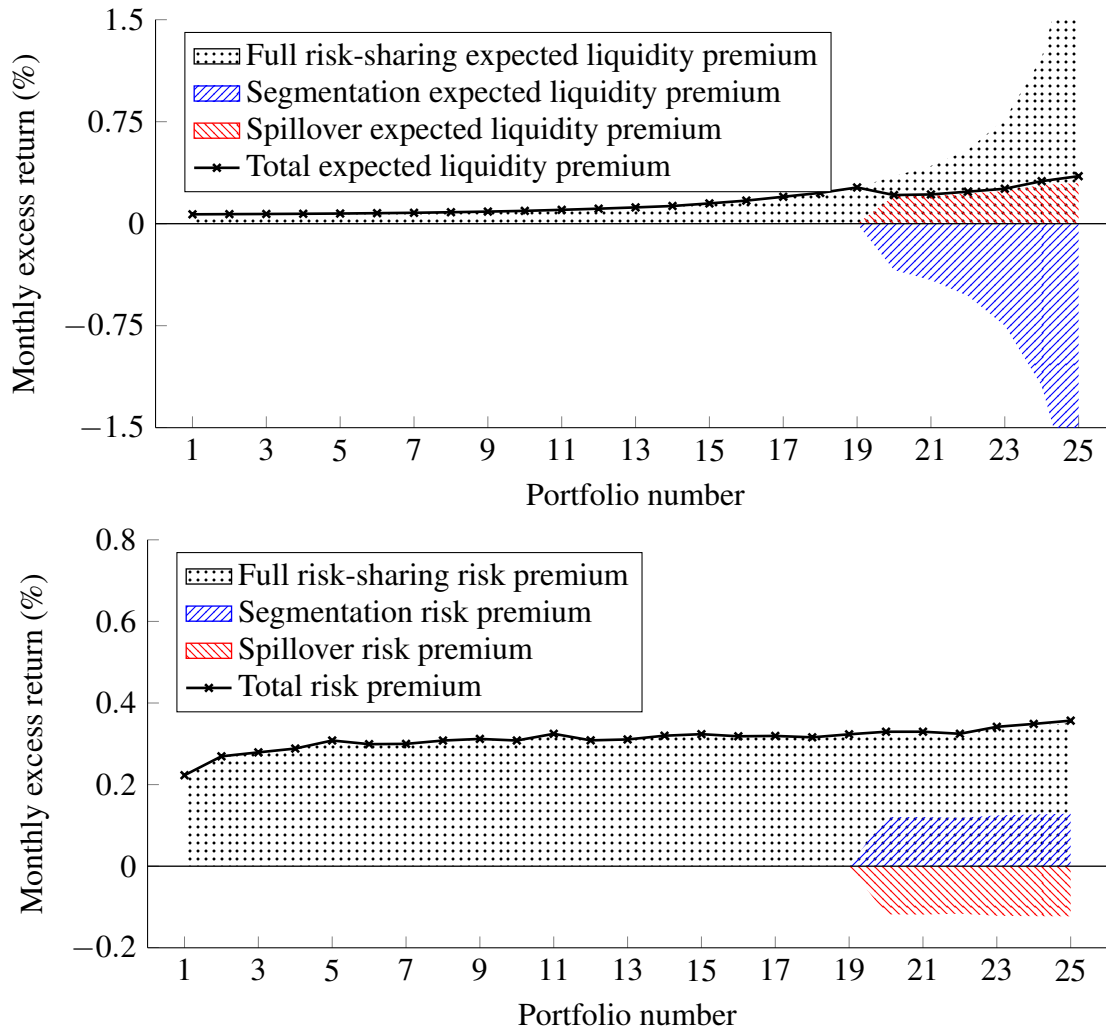
Figure 4. Segmentation effects. The top panel shows the decomposition of the expected liquidity premium into three components: the full risk-sharing component, the segmentation component, and the spillover component. The bottom panel shows a similar decomposition for the risk premium. In all cases the heterogeneous horizon specification (SEG) is used. The coefficient values correspond to the estimation results as given in Table II.
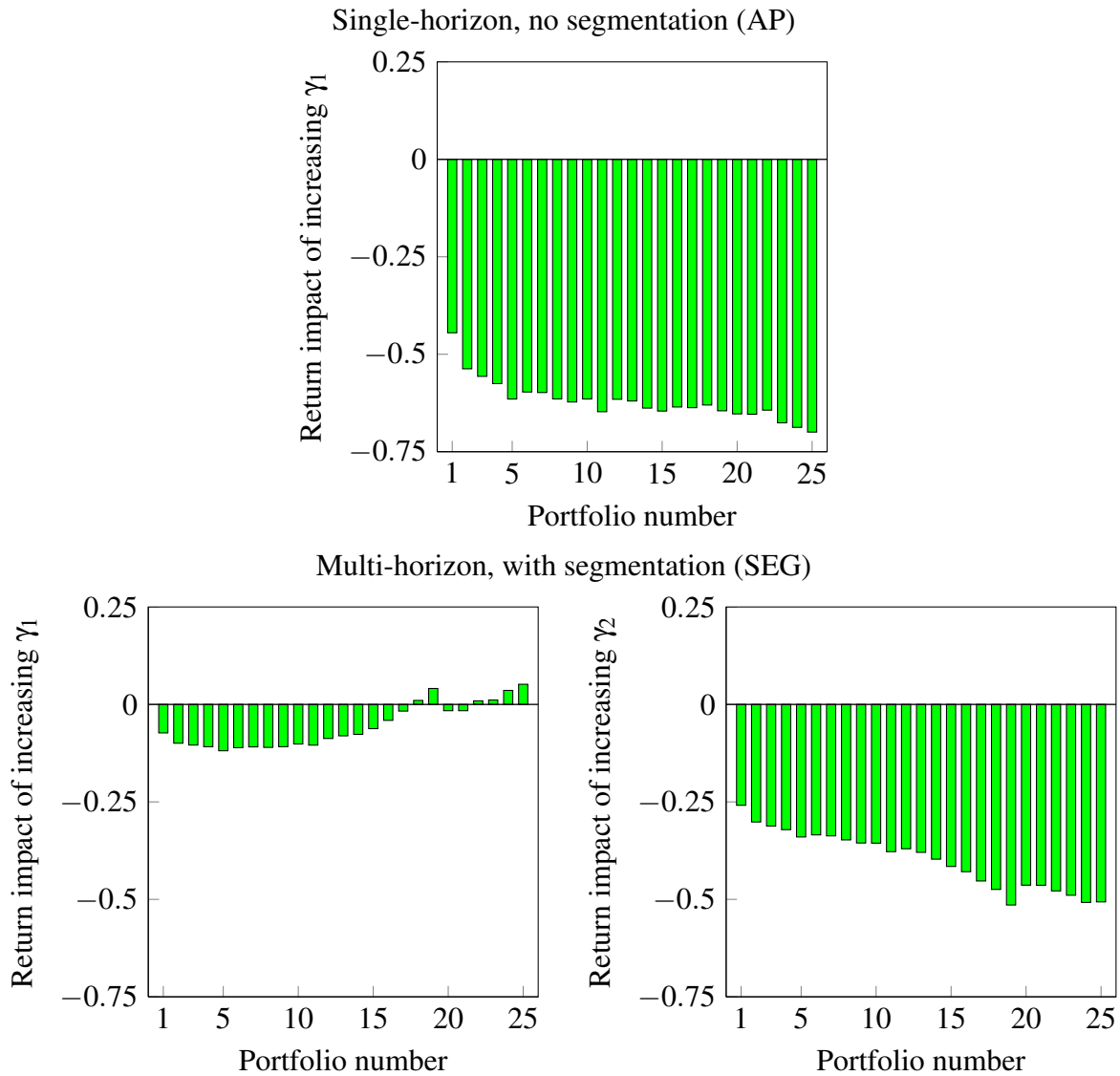
Figure 5. Comparative statics. The comparative statics are computed according to equation (26), and give the sensitivity of the expected return to the parameter γ_j. The top panel shows the comparative statics for the Acharya and Pedersen (2005) specification (AP). The bottom panel shows the comparative statics for the heterogeneous horizon specification (SEG). The graphs correspond to the estimation results as given in Table II.
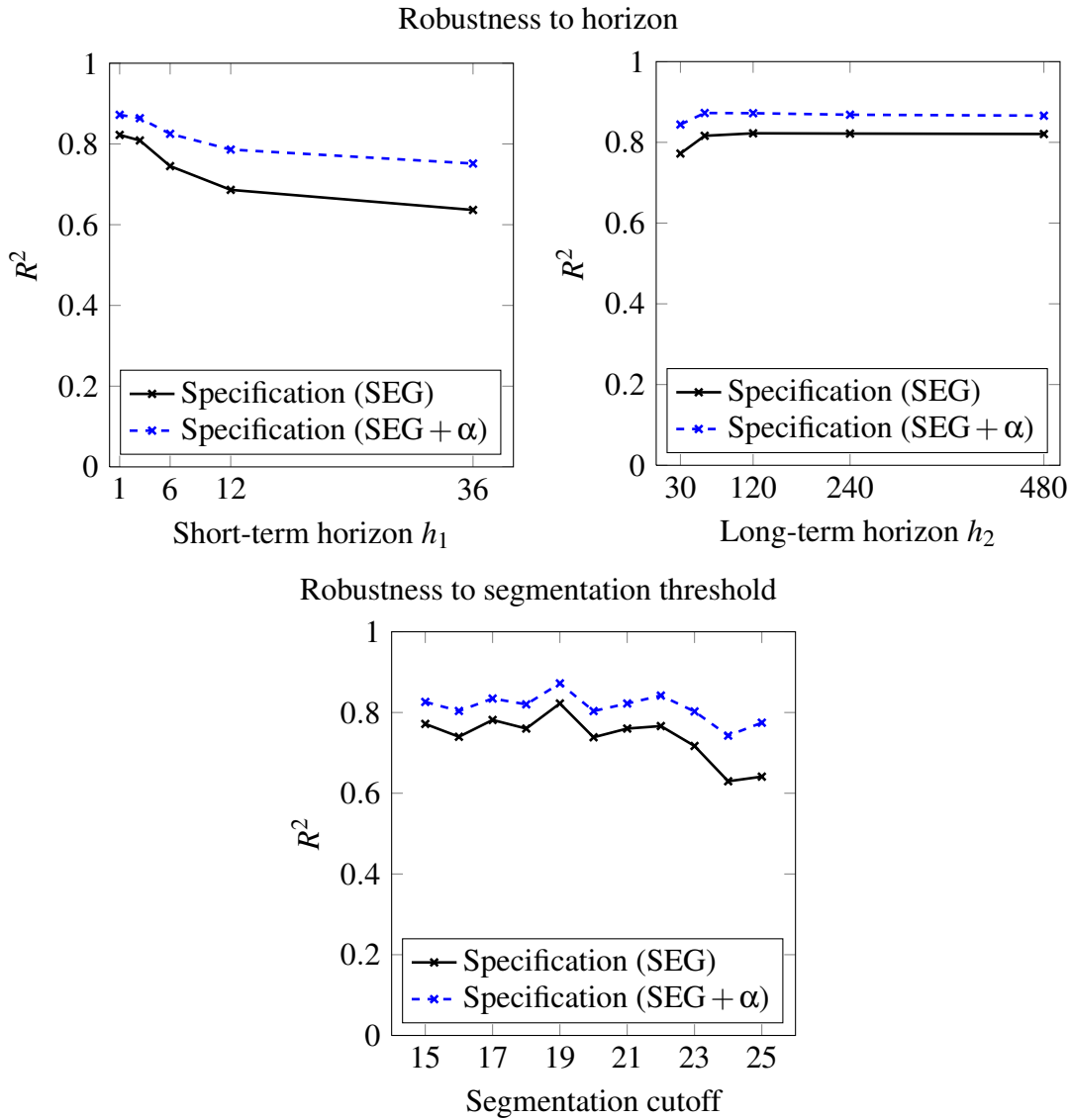
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Figure 6. Robustness to horizons and segmentation. This graph shows the sensitivity of the cross-sectional R^2 to varying the horizons and to varying the segmentation threshold. The data and the specifications are the same as in Table II. Setting $h_1 = 1$, we let $h_2 = 30, 60, 120, 240, 480$. Alternatively, we fix $h_2 = 120$ and let $h_1 = 1, 3, 6, 12, 36$. For the segmentation level we take $h_1 = 1$, $h_2 = 120$ and let the short-term investors invest in the $16, \ldots, 25$ most liquid portfolios. The case of 25 corresponds to integration.