RELATIONSHIP BETWEEN PRICE INDICES

- Weights used to update and comparability; monthly basis

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Abstract

The paper starts from the premise: why the statistical agencies do not select the quantitative vector of reference q out of the Lowe formula as being the monthly quantitative vector q^0 *that belongs to the transactions of the month 0*

Key words: *transaction, indices, goods, price, quantity*

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It is necessary to discuss a serious practical issue of the fix basket indices theory. So far, it has been assumed that the quantitative $q = (q_1, q_2, \ldots, q_n)$ that appears in the definition of the Lowe index $P_{Lo}(p^0, p^1, q)$, is either the quantitative vector of the base period q^0, or the quantitative vector of the current period q^1, or an average of the two quantitative vectors. As a matter of fact, in terms belonging to the real practices of the statistical offices, the quantitative vector q is usually considered as being a yearly quantitative, which refers to a base year, designed as b, previous to the period 0 basic for premiums. A typical price index will have the formula $P_{Lo}(p^0, p^t, q^b)$, where p^0 is a price vector from the month that is the base period for prices, the month 0, p^t is the vector for the month that is the current period for prices, designed as the month t, and q^b is a quantitative vector of the reference basket that refers to the base year b that is the same or previous to the month 0. To note that this Lowe index $P_{Lo}(p^0, p^t, q^b)$ in not a real Laspeyres index (as the quantitative annual index qb is not equal to the monthly quantitative vector q^0, altogether).

Most of the economies are subject to seasonal fluctuations, so that the choice of the quantitative vector for the month 0 as being the quantitative vector of reference for all the months of the year would not be representative for the transactions being achieved all over the year;

The quantitative or households expenses monthly averages are usually collected by the statistical agencies by using an inquiry on the households expenses, based on a relatively small sample. As a result, the out coming weights may often show very large sampling errors – therefore, the current practice goes for calculating the average of these monthly expenses or quantitative weights for a whole year (or, in some cases, for several years), in the attempt to reduce these sampling errors.

One can argue that the utilization of annual weights for an index formula is just a method to settle the seasonality matter.

At this point, there is another issue to notice, i.e., the matter of using the annual weights corresponding to a, probably, more remote year, in the context of a monthly consumption price index: if there are systematic tendencies (but divergent) of the goods prices and the households are multiplying their acquisitions of goods which relative prices decrease reducing meantime the goods acquisitions which relative prices increase, then the utilization of quantitative weights within a more remote year would tend to lead to a bigger deviation ("bias") for this Lowe index comparatively with the one arising from the utilization of weights from a more recent year, as shown below. This observation suggests that the statistical agencies should pay efforts in order to obtain permanently weights as recent as possible.

It is useful to explain the way the annual quantitative vector q^b can be obtained starting from the monthly expenses for each commodity, during the chosen base year b. In this respect, we shall note the expenses for the month m of the population reference from the base year b relating to the commodity I, as $v^{b,m}{}_i$, while the corresponding price and quantity as $p^{b,m}{}_i$ respectively $q^{b,m}{}_i$. Of course, the value, the price and the quantity for each commodity are connected through the following equations:

$v^{b,m}{}_i = p^{b,m}{}_i \, q^{b,m}{}_i$, unde $i = 1, \ldots, n$ iar $m = 1, \ldots, 12$ (1)

For each commodity i, the yearly total qbi can be obtained by deflating the prices for the monthly values and by adding the prices of the base year b, as follows:

$$q_i^b = \sum_{m=1}^{12} \frac{v_i^{b,m}}{p_i^{b,m}} = \sum_{m=1}^{12} q_i^{b,m} \qquad (2)$$

where the equation (1) used to derive the second equation (2). In practice, the above equations would be evaluated by using the aggregated expenses for the commodities closely related while the price $p^{b,m}_i$ will be the price of the month m for this group of elementary goods i from the year b related to the first month of the year b.

For certain purposes, it is useful to have annual prices by type of commodity in order to connect them with the annual quantities defined by the equation (2). In accordance with the conventions used by the National Accounts, we can obtain a reasonable price p^{bi} which we associate with the annual quantity q^{bi} dividing the value of the total consumption of goods I from the year b by la q^{bi}. Thus, we have:

$$p_i^b \equiv \sum_{m=1}^{12} v_i^{b,m}/q_i^b = \frac{\sum_{m=1}^{12} v_i^{b,m}}{\sum_{m=1}^{12} v_i^{b,m}/p_i^{b,m}} \quad , i=1, \ldots , n$$

using (2)

$$= \left[\sum_{m=1}^{12} s_i^{b,m} (p_i^{b,m})^{-1} \right]^{-1} \tag{3}$$

where the quota from the annual expenses for the commodity I from the month m of the base year is:

$$s_i^{b,m} \equiv \frac{v_i^{b,m}}{\sum_{k=1}^{12} v_i^{b,k}}; \quad i=1, \ldots, n \tag{4}$$

Hence, is results that the annual price from the base year for the commodity i, p^{bi} , is the harmonic average of the monthly prices for the commodity i from the base year, weighted according to the monthly expenses $p^{b,1}_i$, $p^{b,2}_i$, . . . , $p^{b,12}_i$.

By using the annual prices of the commodity for the base year defined by the equation (3), we can define a vector of these prices as follows: $p^b = [\, p^b_1, ..., p^b_n]$. Applying this definition, the Lowe index $P_{Lo}(p^0, p^t, q^b)$ can be expressed as a ration of two Laspeyres indices, where the prices vector pb plays the role of the prices from the base period for each of the two Laspeyres indices:

$$P_{Lo}(p^0,p^t,q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \frac{\sum_{i=1}^{n} p_i^t q_i^b \Big/ \sum_{i=1}^{n} p_i^b q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b \Big/ \sum_{i=1}^{n} p_i^b q_i^b}$$

$$= \frac{\sum_{i=1}^{n} s_i^b (p_i^t/p_i^b)}{\sum_{i=1}^{n} s_i^b (p_i^0/p_i^b)}$$

$$= P_L(p^b,p^t,q^b)/P_L(p^b,p^0,q^b) \qquad (5)$$

where the formula of the Laspeyres index PL has been defined by the equation (4).

Consequently, the above equation shows the fact that the Lowe monthly price index that compares the prices from the month 0 with those from the month t by using the quantities of the base year b as well as the weights $P_{Lo}(p^0, p^t, q^b)$, equals the Laspeyres index that compares the prices of the month t with those from the year b, $P_L(p^b, p^t, q^b)$, divided by the Laspeyres index which compares the prices from the month 0 with those from the year b, $P_L(p_b, p_0, q_b)$. To note that the Laspeyres index from the denominator can be calculated if the expenses with the respective commodity from the base year, s^b_i , are known, along with the prices ratios that compare the prices of the commodity i from the month t, p^t_i, with the corresponding annual average prices from the base year b, p^b_i . The Laspeyres index at the denominator can be calculated if the expenses quotas with the respective commodity from the base year, s^b_i, are known along with the prices ratios that compare the prices of the commodity i from the month 0, p^0_i, with the corresponding annual average prices from the base year, p^b_i.

There is another suitable formula for evaluating the Lowe index, PLo(p0, pt,qb), that is using the formula of the hybrid weights (4). In this context, the formula becomes:

$$P_{Lo}(p^0,p^t,q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \frac{\sum_{i=1}^{n} (p_i^t/p_i^0) p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0}\right) s_i^{0b} \qquad (6)$$

Where the hybrid weights s^{0b}_i that are using the prices of the month 0 an dthe quantities from the year b are defined by :

$$s_i^{0b} \equiv \frac{p_i^0 q_i^b}{\sum_{j=1}^{n} p_j^b q_j^b}; \quad i=1, \ldots, n$$
$$= \frac{p_i^b q_i^b (p_i^0/p_i^b)}{\sum_{j=1}^{n} \left[p_j^b q_j^b (p_j^0/p_j^b) \right]} \tag{7}$$

The second equation from (7) shows how to multiply the expenses $p^b_i q^b_i$ from the base year by the prices indices of the commodities p^0_i/p^b_i, in order to calculate the hybrid quotas. Here we have the additional formula for the Lowe index, $P_{Lo}(p^0, p^t, q^b)$. To note that the Laspeyres decomposing of the Lowe index defined by the third term of the equation (5) involves the long term price ratios, p^t_i/p^b_i, that compares the prices from the month t, p^t_i , with the prices from the base year (possibly remote) p^b_i, as well as the fact that the decomposition with hybrid quotas of the Lowe index defined by the third term of the equation (6) involves the long term monthly prices ratios, p^t_i/p^0_i, that compares the prices from the month t, p^t_i , with the prices from the base month p^0_i.

Both formulas are not satisfying in practice because of the sampling wearing out: each month, a substantial group of goods disappears from the market. Thus, it is useful to have a formula for updating the price index from the previous month by using the month-to-month price ratios only. In other [illegible] viable [illegible] h t+1, [illegible] month

$$P_{Lo}(p^0, p^{t+1}, q^b) \equiv \frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \left[\frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} \right] \left[\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b} \right] = P_{Lo}(p^0, p^{t+1}, q^b) \left[\frac{\sum_{i=1}^{n} \left(\frac{p_i^{t+1}}{p_i^t} \right) p_i^t q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b} \right]$$
$$= P_{Lo}(p^0, p^{t+1}, q^b) \left[\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b} \right] = P_{Lo}(p^0, p^{t+1}, q^b) \left[\sum_{i=1}^{n} \left(\frac{p_i^{t+1}}{p_i^t} \right) s_i^{tb} \right] \tag{8}$$

where the hybrid weights $s^{tb}i$ are defined by:

$$s_i^{tb} \equiv \frac{p_i^t q_i^b}{\sum_{j=1}^{n} p_j^t q_j^b}; \quad i=1, \ldots, n \tag{9}$$

The required updating factor, from the month t to the month t+1 is the chaining index $\sum_{i=1}^{n} s_i^{tb}(p_i^{t+1}/p_i^t)$

which uses the hybrid weights s^{tb}_i corresponding to the month t and to the base year b.

The Lowe index $P_{Lo}(p^0, p^t, q^b)$ can be considered as an approximation of the common Laspeyres index, $P_L(p^0, p^t, q^0)$, that compares the prices from the base month 0, p^0, with those from the month t, p^t, by using the quantitative vectors from the month 0, q^0, as weights.

Consequently, there is a relatively simple formula, which connects these two indices. In order to explain this formula, first we have to provide some definitions. We shall define thus the price ratio of order I between the months 0 and *t* as:

$$r_i \equiv p_i^t/p_i^0; \quad i=1, \ldots, n \tag{10}$$

The common Laspeyres price index, between the months 0 and t, can be defined with the help of the price ratios as follows:

$$P_L(p^0, p^t, q^0) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} = \frac{\sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0}\right) p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} = \sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0}\right) s_i^0 = \sum_{i=1}^{n} s_i^0 r_i \equiv r^* \tag{11}$$

where the expenses quotas from the month 0 s^0_i are defined as follows:

$$s_i^0 \equiv \frac{p_i^0 q_i^0}{\sum_{j=1}^{n} p_j^0 q_j^0}; \quad i=1, \ldots, n$$

(12)

We define then the relative quantity t_i as the ratio between the quantity of goods I used in the base year b, q^b_i, and the quantity used in the month 0, q^0_i, as follows:

$$t_i = \frac{q_i^b}{q_i^0}, \ i = 1,\ldots,n \qquad (13)$$

The Laspeyres quantitative index $Q_L(q_0, q_b, p_0)$, which compares the quantities from the year b, q_b, with the corresponding quantities from the month 0, q_0, by using the prices from the month 0, p_0, the weights being defined as a weighted average of the quantitative ratios t_i, shows as follows:

$$Q_L(q^0, q^b, p^0) \equiv \frac{\sum_{i=1}^{n} p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^0} = \frac{\sum_{i=1}^{n} \left(\frac{q_i^b}{q_i^0}\right) p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} = \sum_{i=1}^{n} \left(\frac{q_i^b}{q_i^0}\right) s_i^0$$

$$= \sum_{i=1}^{n} s_i^0 t_i$$ using the definition (13)

$$\equiv t^*$$

(14)

The relation between the Lowe index (p_0, p_t, q_b) that uses the quantities from the year b as weights in order to compare the prices from the month t with those from the month 0, and the corresponding Laspeyres index $P_L(p_0, p_t, q_0)$ that uses the quantities from the month 0 as weights, is the following

$$P_{Lo}(p^0, p^t, q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$

$$= P_L(p^0, p^t, q^0) + \frac{\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0}{Q_L(q^0, q^b, p^0)} \qquad (15a)$$

The outcome shows that the Lowe price index that uses the quantities from the year b as weights, $P_{Lo}(p_0, p_t, q_b)$, equals to the usual Laspeyres index that uses the quantities from the month 0 as weights, $P_L(p_0, p_t, q_0)$, plus a covariance term

$$\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0 \qquad (15b)$$

Between the price ratios $r_i = p^t_i/p^0_i$ and the quantitative ones $t_i = q^b_i/q^0_i$, divided by the quantitative Laspeyres index $Q_L(q_0, q_b, p_0)$ between the month 0 and the base year b.

The formula (15) shows that the Lowe price index will coincide with the Laspeyres price index if the covariance between the prices ratios of the months 0 şi t, namely $r_i = p^t_i/p^0_i$ and the relative quantities between the month 0 and the year b, namely $t^i = q^b_i/q^0_i$ is zero. To note that this covariance will be zero in the following three situations:

- If the prices from the month t are proportional with the prices from the month 0, so that all the ratios $r_i = r^*$;
- If the quantities from the base year b are proportional with the quantities from the month 0 so that all the ratios $t_i = t^*$;
- If the distribution of the relative prices r_i is independent of the distribution from the relative quantities t_i.

The first two conditions can be accomplished more difficulty but the third one is possible, at least approximately if the consumers do not change systematically their purchasing preferences as response to the relative prices changes.

If this covariance from the formula (15) is negative, then the Lowe index will be smaller then the Laspeyres index. Finally, if the covariance is positive, then the Lowe index will be bigger then the Laspeyres index. Although the sign and the size of the covariance term

$$\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0 \qquad (15c)$$

represent in fact an empirical issue, it is possible to make some reasonable considerations on them. If the base year b is previous to the reference month 0 and there are tendencies for long term prices, probably this covariance will be positive and, thus, the Lowe index will be bigger than the corresponding Laspeyres price index, namely:

$$P_{Lo}(p^0, p^t, q^b) > P_L(p^0, p^t, q^0) \tag{16}$$

In order to see the reason for which the covariance may be positive, we shall assume that there an increasing tendency on long-term basis for the price of the commodity I, so that:

ri - r* = (p^t_i/p^0_i) – r* is positive.

When there are normal responses of substitution from the consumers' side, q^t_i /q^0_i out of which an average quantitative modification of cast type has been deducted, it may be negative or, reciprocally, q^0_i / q^t_i minus an average quantitative modification of a reciprocal type, it may be positive. Nevertheless, the increasing tendency of the long-term basis prices maintained until the base year b and, then, t_i - t* = (q^b_i/q^0_i) – t* may be also positive. Therefore, the covariance will be positive under these conditions. In fact, to the extent the base year b is more remote from the base month 0, the residual difference t_i – t* as well as the positive covariance will be bigger. Similarly, to he extent the current month t is more remote from the base month 0, the residual difference r_i – r* as well as the positive covariance will be bigger. Thus, assuming that there long-term are tendencies for prices and normal responses to substitution of the consumers, the Lowe index will be normally higher than the corresponding Laspeyres index.

The Paasche index between the months 0 and t is defined as follows:

$$P_P(p^0, p^t, q^t) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^t}{\sum_{i=1}^{n} p_i^0 q_i^t} \tag{17}$$

We can reasonably define an index that measures the price modification between the months 0 and t as being a kind of symmetrical average between the Paasche index $P_P(p_0, p_t, q_t)$, defined by the formula (17), and the corresponding Laspeyres index $P_L(p_0, p_t, q_0)$, defined by the formula (11). The relation between the Paasche and Laspeyres indices can be written

$$P_P(p^0, p^t, q^t) = P_L(p^0, p^t, q^0) + \frac{\sum_{i=1}^{n} (r_i - r^*)(u_i - u^*) s_i^0}{Q_L(q^0, q^t, p^0)} \tag{18}$$

where the relative prices $r_i = p^t_i / p^0_i$ are defined by the equation (10), their weighted average r* by the equation (11), and u_i, u* and Q_L are defined as follows:

$$u_i \equiv q_i^t / q_i^0; \quad i = 1, \ldots, n \tag{19}$$

$$u^* \equiv \sum_{i=1}^{n} s_i^0 u_i = Q_L(q^0, q^t, p^0) \tag{20}$$

where the expenses quotas from the month 0, s^0_i, are defined through the identity (12). It is resulting that u* equals to the quantitative Laspeyres index between the months 0 and t. This means that the Paasche price index that uses the quantities from the month t as weights, $P_P(p^0, p^t, q^t)$, equals to the usual Laspeyres index for the quantities from the month 0 as weights, $P_L(p^0, p^t, q^0)$, plus a covariance term.

$$\sum_{i=1}^{n} (r_i - r^*)(u_i - u^*) s_i^0$$

Between the relative prices $r_i = p^t_i / p^0_i$ and the quantitative ratios $u_i = q^t_i / q^0_i$, divided by the quantitative index $Q_L(q^0, q^t, p^0)$ between the months 0 and t.

Although the sign and the size of the covariance term:

$\sum_{i=1}^{n} (r_i - r^*)(u_i - u^*) s_i^0$, are also constituting an empirical issue, it is possible to make some reasonable considerations on them as well.

If there are long-term tendencies for the prices and the consumers respond in a normal way to the modification of the prices for the acquisitions they are making, then it is prfobable that this covariance is negative and, consequently, the Paasche index will be smaller then the corresponding Laspeyres index, namely

$$P_P(p^0, p^t, q^t) < P_L(p^0, p^t, q^0) \tag{21}$$

In order to see whether this covariance can be negative, we shall assume that there is an increasing tendency on a long-term basis for the price of the commodity i so that $r_i - r^* = (p^t_i / p^0_i) - r^*$ is positive. At normal responses of the consumers as regards the substitution, q^t_i / q^0_i minus a quantitative modification of cast type, will be probably negative. As a result, $u_i - u^* = (q^t_i / q^0_i) - u^*$ will be also negative. Under these conditions, the covariance will be negative. In fact, to the extent the base month 0 is more remote from the current month t, the residual value $u_i - u^*$ will be bigger, as well as the negative covariance. Similarly, to the extent the current month t is more remote from the base month 0, the residual value $r_i - r^*$, will be bigger, as well as the positive covariance. Thus, assuming that there are long-term tendencies for prices and normal responses of the consumers as regards the substitution, the Laspeyres index will be higher than the corresponding Paasche index, the divergence between them tending to increase to the extent that the month t remotes from the month 0.

As a resume of the previous three paragraphs, one can state that – assuming that there are long-term tendencies for the prices and normal responses of the consumers as regards the substitution, the Lowe price index between the month 0 and t will exceed the corresponding Laspeyres index, which, at its turn, will exceed the corresponding Paasche index, namely:

$$P_{Lo}(p^0,p^t,q^b) > P_L(p^0,p^t,q^0) > P_P(p^0,p^t,q^t) \qquad (22)$$

Thus, if the price index watched over a long-term period is an average of the Laspeyres and Paasche indices, it can be observed that the Laspeyres index will have an increasing deviation as against this index, while the Paasche index will have a decreasing deviation. In addition, if the base year b is previous to the month 0 of reference for prices, then the Lowe index will have also an increasing deviation as against the Laspeyres index and, hence, as against the index being watched on a long-term basis.

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