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Analysis of the Effect of Time Delay on the Integrated GNSS/INS Navigation Systems

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ABSTRACT: The performance of tightly coupled GNSS/INS integration is known to be better than that of loosely coupled GNSS/INS integration. However, if the time synchronization error occurs between the GNSS receiver and INS(Inertial Navigation System), the situation reverses. The performance of loosely coupled GNSS/INS integration and tightly coupled GNSS/INS integration is analyzed and compared due to time synchronization error by computer simulation.

1 INTRODUCTION

GNSS(Global Navigation Satellite System) and INS(Inertial Navigation System) are widely used as standalone navigation systems, respectively. The integration of GNSS and INS not only leads to high accuracy of vehicle's position, velocity, and attitude, but also provides position error bound. To maximize the performance of GNSS/INS integrated systems, time synchronization between GNSS and INS is an important issue.

There has been some research for time synchronization of GNSS/INS measurements for loosely coupled GNSS/INS systems or tightly coupled GNSS/INS systems. However, there are few research to consider both. This paper compares the time delay effect of loosely coupled GNSS/INS systems and tightly coupled GNSS/INS systems. For loosely coupled GNSS/INS systems, we analyze the effect of time delay for the case when only position information is used as Kalman filter measurement. For tightly coupled GNSS/INS systems, we analyze the effect of time delay for the case when only pseudorange information is used as Kalman filter measurement.

To compare the performance of the two integration systems, it is analytically studied how the time delay between GNSS and INS has effect on the Kalman filter innovation for each integration method. Then based on the analysis results, computer simulations are performed to check how each integration method has effect on the navigation performance such as position, velocity, and attitude of the vehicle. The two GNSS/INS integration methods are reviewed briefly in section 2, and the effects of time delay are analyzed in section 3. Computer simulations are performed in section 4 and conclusions are given in section 5.

2 INTEGRATED GNSS/INS NAVIGATION SYSTEMS

Several approaches are possible for the integrated GNSS/INS systems depending on which information is shared between GNSS and INS. Loosely coupled and tightly coupled integrated systems are considered in this paper. The position and velocity calculated from GNSS are used to update the INS filter in the loosely coupled integration, while raw pseudorange is used in the tightly coupled integration for Kalman filter.

2.1 *Loosely coupled GNSS/INS navigation system*

The block diagram of the loosely coupled GNSS/INS integration is shown in Figure 1. The position and velocity is calculated in the GNSS receiver and then used as a measurement in the INS Kalman filter.

As seen in Figure 1 the structure is simple and modularization is possible. The computation time is reduced since the algorithm in the GNSS receiver can be used. The Kalman filter in the loosely coupled GNSS/INS integration has the error model with dimension of 15.

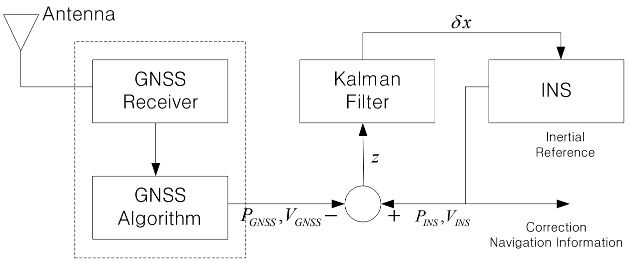


Figure 1. Loosely coupled GNSS/INS integration

$$\dot{x}_{INS} = F_{INS} x_{INS} + w_{LC} \quad , \quad w_{LC} \sim N(0, Q_{LC}) \tag{1}$$

$$x_{INS} = [\delta L \; \delta l \; \delta h \; \delta v^n \; \Phi^n \; B_{accel} \; B_{gyro}]^{\mathrm{T}} \tag{2}$$

The state variable contains the position error with latitude, longitude, and height $(\delta L, \delta l, \delta h)$, and velocity error (δv^n) , attitude error $(\delta\Phi^n)$ expressed in the navigation frame, and bias errors $(B_{accel}$, $B_{gyro})$ of accelerometer and gyroscopes(refer to Knight 1997 for detail error model). The measurement equation of the loosely coupled GNSS/INS integration can be denoted as follows:

$$z = [\mathrm{P}_{INS}] - [\mathrm{P}_{GNSS}] = \mathrm{H}_{LC} x_{INS} + v_{LC} \; , v_{LC} \sim N(0, R_{LC}) \, , \tag{3}$$

where $H_{LC} = \begin{bmatrix} I_{3\times3} & 0_{3\times9} \end{bmatrix}$.

In the case of loosely coupled integration there are some drawbacks. When there are short of visible satellites, navigation information cannot be calculated in the GNSS receiver. Under the large dynamic environment GNSS navigation solution becomes inaccurate, thus the integration performance becomes worse.

2.2 *Tightly coupled GNSS/INS navigation system*

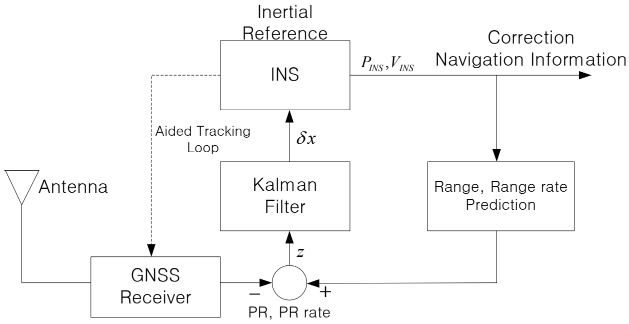


Figure 2. Tightly coupled GNSS/INS integration

The block diagram of the tightly coupled GNSS/INS integration is shown in Figure 2. The GNSS pseudorange and pseudorange rate are used directly in the measurement of Kalman filter and the clock error of the GNSS receiver should be included in the state variable to be estimated. Thus the Kalman filter error model contains position error, velocity error, attitude error, accelerometer bias, gyroscope bias, clock bias and clock drift of GNSS receiver, resulting in 17 state variables as in (4).

$$\begin{bmatrix} \dot{x}_{INS} \\ \dot{x}_{clock} \end{bmatrix} = \begin{bmatrix} F_{INS} & 0_{15\times2} \\ 0_{2\times15} & F_{clock} \end{bmatrix} \begin{bmatrix} x_{INS} \\ x_{clock} \end{bmatrix} + w_{TC} \; , \; w_{TC} \sim N(0, Q_{TC}) \tag{4}$$

$$x_{clock} = \begin{bmatrix} \delta c_{bias} & \delta c_{drift} \end{bmatrix}^{\mathrm{T}} \tag{5}$$

where δc_{bias} , δc_{drift} denotes for clock bias and clock drift of GNSS receiver and F_{INS} , F_{clock} are described in Knight 1997.

The measurement is given as in (6).

$$z = [\rho_{INS}] - [\rho_{GPS}] = \mathrm{H}_{TC} \begin{bmatrix} x_{INS} & x_{clock} \end{bmatrix}^{\mathrm{T}} + v_{TC} \; , v_{TC} \sim N(0, R_{TC}) \tag{6}$$

In the case of GNSS positioning, the former GNSS position is used as the reference of linearization and thus the navigation error increases for large acceleration and angular rate. However, for the tightly coupled GNSS/INS integration, INS information with good dynamic performance is used as the reference of linearization of GNSS measurement, thus the same problem does not happen. Even though the visible GNSS satellites are not enough, the navigation calculation is still possible. So it provides more efficient structure in the usage of information than the loosely coupled integration. The drawback of tightly coupled integration is the complexity of the integration structure and thus computing effort increases as the number of GNSS satellites increases.

3 EFFECTS OF THE TIME DELAY IN THE INTEGRATED GNSS/INS SYSTEM

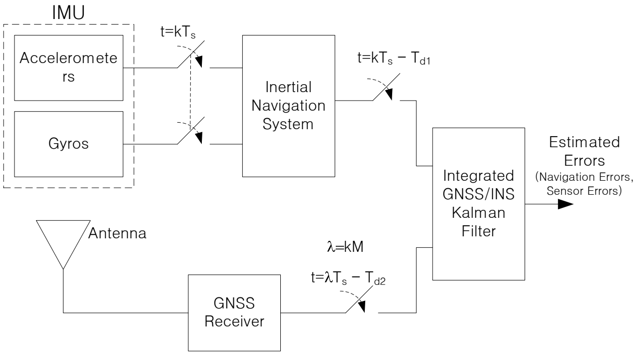


Figure 3. Structure of time synchronization error Td(=Td2 - Td1) in the integrated GNSS/INS navigation system

For the GNSS/INS integration there exist time delay between GNSS receiver and INS since not only sampling time but also the signal processing time is different as in Figure 3. For simplified analysis, the GNSS receiver sampling period is assumed to be a multiple M of the IMU sampling period T_s. T_{d1} and T_{d2} are processing time delays of the inertial navigation system(INS) and the GNSS receiver, respectively and $T_d = T_{d2} - T_{d1}$, i.e., the difference of the INS and GNSS processing time.

3.1 *EKF algorithm*

Consider the error model of INS and EKF equation to analyze the effects of time delay in measurement. The error model can be described as follows:

$$x_{k+1} = \Psi_k x_k + G_k e_k \tag{7}$$

$$z_k = \mathrm{H}_k x_k + w_k \tag{8}$$

Here, Ψ_k is the error state transition matrix, e_k is the process noise, G_k is the process noise gain, H_k is the measurement matrix, and w_k is the measurement noise. The subscript k implies k-th sampling sequence. The process noise e_k and the measurement noise w_k are considered as white noise, uncorrelated with each other and the covariance matrices are $R = \mathrm{E}\{w_k w_k^{\mathrm{T}}\}$ and $Q = \mathrm{E}\{e_k e_k^{\mathrm{T}}\}$, where, $\mathrm{E}\{\ \}$ denotes the expectation operator.

The EKF algorithm for the GNSS/INS integration is given in Table 1.

Table 1. EKF Algorithm for Integration between the GNSS and the INS

Kalman gain update	$\mathrm{K}_k = \mathrm{P}_k^- \mathrm{H}_k^{\mathrm{T}} (\mathrm{H}_k \mathrm{P}_k^- \mathrm{H}_k^{\mathrm{T}} + R)^{-1}$
Difference of the two measurements	$z_k = \hat{p}_{k,INS} - \hat{p}_{k.GNSS}$
Estimation of the state errors	$\hat{x}_k = \begin{bmatrix} \delta\hat{r}_k \\ \delta\hat{\xi}_k \end{bmatrix} = \mathrm{K}_k z_k$
Error correction	$\begin{bmatrix} \hat{r}_k \\ \hat{\xi}_k \end{bmatrix} = \begin{bmatrix} \delta\hat{r}_k^- \\ \delta\hat{\xi}_k^- \end{bmatrix} + \hat{x}_k$
Covariance update	$\mathrm{P}_k = (\mathrm{I} - \mathrm{K}_k \mathrm{H}_k)\mathrm{P}_k^-$
Sensor error compensation	$\hat{u}_k = g(\tilde{u}_k, \hat{\xi}_k)$
Navigation equation update	$\hat{r}_{k+1}^- = f(\hat{r}_k, \hat{u}_k)$
Sensor error update	$\hat{\xi}_{k+1}^- = \hat{\xi}_k$
Covariance prediction	$\mathrm{P}_{k+1}^- = \Psi_k \mathrm{P}_k^- \Psi_k^{\mathrm{T}} + G_k Q G_k^{\mathrm{T}}$

3.2 *Effects of time synchronization error*

Vectors $\hat{r}_k$, $\hat{\xi}_k$ and $\hat{u}_k$ in Table 1 denote the estimated navigation state, the estimated sensor errors, and the IMU measurements, respectively. Effects of time synchronization error between GNSS receiver and INS can be analyzed as follows.

Let the time difference of sampling measurement between GNSS receiver and INS be T_d and suppose that INS sampling has time synchronization error less than 1sec. Then in the case that vehicle position is used in loosely coupled integration, the estimated position in INS is given as in (9), where $p(t)$ means true trajectory.

$$\hat{p}_{k,INS} = p(k\mathrm{T}_S - \mathrm{T}_d) \cong p_k - v_k \mathrm{T}_d + 0.5 a_k \mathrm{T}_d^2 + w_k \tag{9}$$

where $|T_d| \leqslant 1$, v_k is the velocity of the vehicle and a_k is the acceleration.

Suppose that the pseudorange, which is the measurement of tightly coupled integration, has time synchronization error less than 1sec, then pseudorange estimated in INS can be described as (10), where $\rho(t)$ is true pseudorange.

$$\hat{\rho}_{k,INS} = \rho(k\mathrm{T}_S - \mathrm{T}_d) \cong \rho_k - \dot{\rho}_k \mathrm{T}_d + 0.5 \ddot{\rho}_k \mathrm{T}_d^2 + w_k \tag{10}$$

where the pseudorange rate is $\dot{\rho}_k = (v_{k,GNSS} - v_k)l_k$, pseudorange acceleration is $\ddot{\rho}_{k,INS} = (\alpha_{k,GNSS} - a_k)l_k$, l_k is LOS(line of sight) vector between satellite and vehicle, and $v_{k,GNSS}$ and $\alpha_{k,GNSS}$ are velocity and acceleration of satellite, respectively.

Thus the true observation z_k^a of two integration systems are obtained as in (11).

For loosely coupled integration

$$z_k^a = \hat{p}_{k,INS} - \hat{p}_{k,GNSS} \tag{11a}$$
$$\cong p_k - v_k \mathrm{T}_d + a_k \frac{\mathrm{T}_d^2}{2} + w_k - \hat{p}_{k,GNSS}$$

For tightly coupled integration

$$z_k^a = \hat{\rho}_{k,INS} - \hat{\rho}_{k,GNSS} \tag{11b}$$
$$\cong \rho_k - \dot{\rho}_k \mathrm{T}_d + \ddot{\rho}_k \frac{\mathrm{T}_d^2}{2} + w_k - \hat{\rho}_{k,GNSS}$$

Let $\mathrm{H}_k x_k = p_k - \hat{p}_{k,GNSS}$ or $\mathrm{H}_k x_k = \rho_k - \hat{\rho}_{k,GNSS}$, then (11) becomes (12).

$$z_k^a = \mathrm{H}_k x_k + w_k + d_k \tag{12}$$

In (12) the residual error vector of position or the residual error vector of pseudorange is introduced and defined as

$$d_k = -v_k \mathrm{T}_d + a_k \frac{\mathrm{T}_d^2}{2} \text{ or } d_k = -\dot{\rho}_k \mathrm{T}_d + \ddot{\rho}_k \frac{\mathrm{T}_d^2}{2} \tag{13}$$

In Skog & Händel (2011), the errors of the closed-loop system are found to be

$$x_{k+1} = \Psi_k (x_k - \hat{x}_k) G_k e_k \tag{14}$$
$$= \Psi_k x_k - \Psi_k \mathrm{K}_k z_k + G_k e_k$$

Let $\mathrm{K}_{p,k} = \Psi_k \mathrm{K}_k$, then (14) becomes (15).

$$x_{k+1} = (\Psi_k - \mathrm{K}_{p,k}\mathrm{H}_k) x_k - \mathrm{K}_{p,k} w_k + G_k e_k \tag{15}$$

Here, by substituting z_k with z_k^a in (14) and inserting (12) into this, the following difference equation for the error state can be obtained:

$$x_{k+1} = (\Psi_k - \mathrm{K}_{p,k}\mathrm{H}_k) x_k - \mathrm{K}_{p,k} w_k + G_k e_k + \mathrm{K}_{p,k} d_k \tag{16}$$

If we introduce $\Phi_k = \Psi_k - \mathrm{K}_{p,k}\mathrm{H}_k$, then the MSE(mean square error) of the navigation solution can be expressed as in (17).

$$\mathrm{P}_{k+1} = \mathrm{E}\{x_{k+1}(x_{k+1})^{\mathrm{T}}\} \tag{17}$$
$$= \Phi_k \mathrm{P}_k \Phi_k^{\mathrm{T}} + \mathrm{K}_{p,k} R \mathrm{K}_{p,k}^{\mathrm{T}} + G_k Q G_k^{\mathrm{T}}$$
$$+ \mathrm{K}_{p,k} d_k d_k^{\mathrm{T}} \mathrm{K}_{p,k}^{\mathrm{T}} - \Phi_k \bar{x}_k d_k^{\mathrm{T}} \mathrm{K}_{p,k}^{\mathrm{T}} - \mathrm{K}_{p,k} d_k \bar{x}_k^{\mathrm{T}} \Phi_k^{\mathrm{T}}$$

where the mean error $\bar{x}_k = \mathrm{E}\{x_k\}$ can be calculated as in (18).

$$\bar{x}_{k+1} = \Phi_k x_k - \mathrm{K}_{p,k} d_k \tag{18}$$

Notice the residual error vector d_k in (13). For the loosely coupled integration, time delay $\mathrm{T_d}$ has little effect on d_k in the case of small velocity and acceleration, i.e., $d_k \approx 0$. Thus for small velocity d_k can be neglected in the loosely coupled integration and MSE value in (17) becomes

$$\mathrm{P}_{k+1} \cong \Phi_k \mathrm{P}_k \Phi_k^{\mathrm{T}} + \mathrm{K}_{p,k} R \mathrm{K}_{p,k}^{\mathrm{T}} + G_k Q G_k^{\mathrm{T}}.$$

However, for the tightly coupled integration, the residual error vector d_k does not become zero even though the velocity of vehicle is almost zero because of high speed of GPS satellite like 3.9km/sec, i.e.,

$$d_k \approx -(v_{k,GNSS} l_k)\mathrm{T}_d + (\alpha_{k,GNSS} l_k)\frac{\mathrm{T}_d^2}{2}$$

Thus we can notice that time delay in the tightly coupled integration has more effect on the navigation performance than in the loosely coupled integration when velocity and acceleration of vehicle are not large.

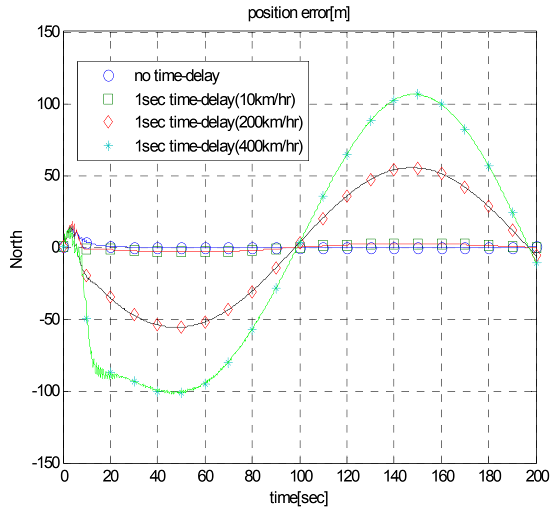
4 SIMULATIONS

In this section computer simulation is performed to verify the analysis result for the effects of time delay. Table 2 shows the specification of INS and GPS used in the simulation. The vehicle trajectory is assumed as circle and run time is 200sec. Vehicle speed is 10km/sec, 200km/sec, and 400km/sec and the time delay is assumed to be 1sec in INS measurement.

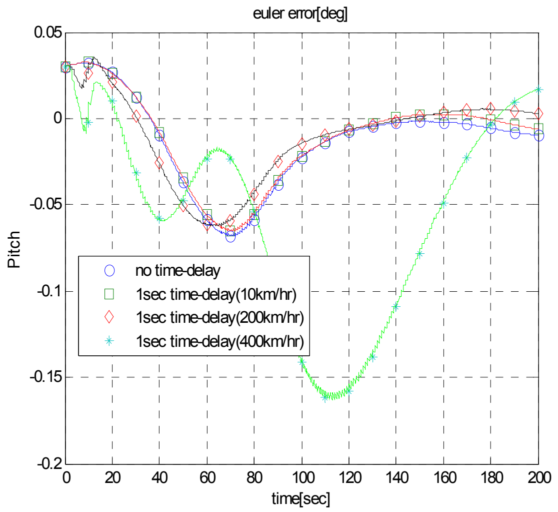
Table 2. Specification of error sources of the INS and the GPS

	Error Sources	1-σ value
INS	initial position error	10m
	initial velocity error	1m/sec
	initial horizontal attitude error	0.03 deg
	initial vertical attitude error	5 deg
	accelerometer bias	500 μg
	gyro bias	3 deg/hr
GPS	clock bias	10m
	clock drift	1m/sec

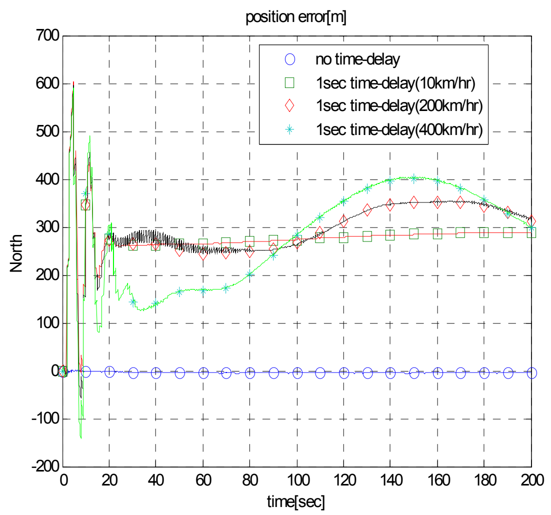
Figure 4 shows the position errors in north direction and pitch errors in the loosely coupled GPS/INS according to the various vehicle speeds. Generally the position error and attitude error become large as the vehicle speed increase. This happens because the residual error vector in (13) becomes large according to the vehicle speed and thus the optimality of Kalman filter gets worse. The oscillation of the position error comes from the circle trajectory.



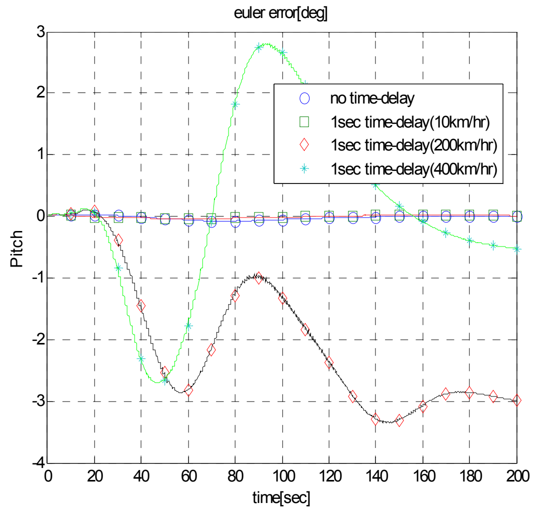
(a) the north position errors



(b) the pitch angle errors
Figure 4. The position errors and attitude errors in the loosely coupled GPS/INS according to the various vehicle speeds (10km/hr, 200km/hr, 400km/hr)



(a) the north position errors



(b) the pitch angle errors
Figure 5. the position errors and attitude errors in the tightly coupled GPS/INS according to the various vehicle speeds(10km/hr, 200km/hr, 400km/hr)

Figure 5 shows the position errors in north direction and pitch errors in the tightly coupled GPS/INS according to the various vehicle speeds.

Notice the position error at first. The position error is large like 300m even though the speed of vehicle is small like 10km/hr. The residual error vector in (13) is small, i.e., $d_k \approx 0$ in loosely coupled integration when vehicle speed is small, while in tightly coupled integration the residual error vector, $d_k \approx -(v_{k,GPS} l_k)T_d + 0.5(\alpha_{k,GPS} l_k)T_d^2$, is large since the satellite speed is large like 3.9km/hr. Figure 5(b) shows that the attitude error becomes large according to vehicle speed and the attitude error is much larger than that in loosely coupled integration since the satellite speed is the dominant term in d_k.

Figure 4 and Figure 5 show that time delay error between GPS and INS causes more effect on tightly coupled integration than on loosely coupled integration

5 CONCLUSIONS

The estimation performance of loosely coupled GNSS/INS integration and tightly coupled GNSS/INS integration is compared due to time synchronization error between GNSS receiver and INS. If the time synchronization error occurs between two sensor measurements, residual error exists. For loosely coupled integration the residual error vector increases according to vehicle speed while for tightly coupled integration the residual error vector increases according to satellite speed as well as vehicle speed. The GPS satellite speed is about 3.9km/sec, which is much larger than the vehicle speed. Thus residual error vector in tightly coupled integration depends mainly on the satellite speed.

Simulations are performed to verify the analysis result for the two GPS/INS integration methods. The simulation shows that time synchronization error has effect more on the tightly coupled integration than the loosely coupled integration

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REFERENCES

Bar-Itzhack, I.Y. & Vitek, Y. 1985. The enigma of false bias detection in a strapdown system during transfer alignment. *Journal of Guidance and Control* 8(2): 175-180.

Bevly, D., Ryu, J. & Gerdes, J. 2006. Integrating INS sensors with GPS measurements for continuous estimation of vehicle sideslip, roll, and tire cornering stiffness. *IEEE Trans. Intell. Transp. Syst.* 7(4): 483-493.

Coelho, E., Carvalhal, P., Ferreira, M., Silva, L., Almeida, H., Santos, C. & Afonso, J. 2006. A Bluetooth-based wireless distributed data acquisition and control system. *in Proc. IEEE Int. Conf. ROBIO*: 543-548.

Ding, W., Wang, J., Li, Y., Mumford, P. & Rizos, C. 2008. Time Synchronization Error and Calibration in Integrated GPS/INS Systems. *ETRI Journal* 30: 59-67.

Fox, J. 2006. Synchronization of input and output measurements using a Kalman filter. *in Proc. 25th Int. Conf. Model., Identification, Control*: 435-440.

Im, N. & Seo, J.-H. 2010. Ship Manoeuvring Performance Experiments Using an Free Running Model Ship. TransNav-International Journal on Marine Navigation and Safety of Sea Transportation 4(1): 29-33.

Knight, D.T. 1997. Rapid Development of Tightly-Coupled GPS/INS System. *IEEE AES System Magazine*: 14-18.

Lee, H.K., Lee, J.G. & Jee, G.I. 2002. Calibration of measurement delay in GPS/SDINS hybrid navigation. *Journal of Guidance, Control, and Dynamics* 25(2): 240-247.

Li, B., Rizos, C., Lee, H.K. & Lee, H.K. 2006. A GPS-slaved time synchronization system for hybrid navigation. *GPS solutions* 10(3): 207-217.

Li, W. & Leung, H. 2004. Simultaneous registration and fusion of multiple dissimilar sensors for cooperative driving. *IEEE Trans. Intell. Transp. Syst.* 5(2): 84-98.

Nilsson, J.O., Skog, I. & Händel, P. 2010. Joint state and measurement time-delay estimation of nonlinear state space systems. *in Proc. Int. Conf. ISSPA*: 324-328. Kuala Lumpur, Malaysia.

Skog, I. & Händel, P. 2011. Time Synchronization Errors in Loosely Coupled GPS-Aided Inertial Navigation Systems. *IEEE Trans. Intell. Transp. Syst.*12(4): 1014-1023.