Journal of Engineering Science and Technology
Vol. 6, No. 3 (2011) 300 - 310

HARMONIC LOAD FLOW FOR RADIAL DISTRIBUTION SYSTEMS

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Abstract

Radial distribution systems (RDS) require special load flow methods to solve power flow equations owing to their high *R*/*X* ratio. Increasing use of power electronic devices and effect of magnetic saturation cause harmonics in RDS. This paper proposes a novel algorithm to compute the power flow solution of a RDS accounting for all the harmonic components. It uses a recursive solution technique. The proposed method uses a novel dynamic data structure reported in the paper. The proposed method is tested upon a 33-bus RDS and the results are reported.

Keywords: Radial distribution systems, Magnetic saturation, Harmonics, Dynamic data structure.

1. Introduction

Analysis of distribution systems using power flow is important in the field of power systems. Distribution systems are predominantly characterized by their high *R*/*X* ratio and radial topology. Matrix based iterative methods do not lend themselves for radial distribution systems owing to these characteristics. Numerous algorithms have been developed using simple recursive equations [1-3]. Rapid industrialization has led to increasing use of power electronic devices in transmission and distribution systems. Modern industrial and domestic consumers use an ever-increasing number of devices that primarily employ power electronics based power-conditioners. Use of AC machines employing magnetic circuits in the saturated region also introduces harmonics in electric power systems. Numerous power flow methods have been reported in literature that is meant to handle harmonics [4]. These methods seldom address radial distribution systems.

Nomenclatures

$P_{ij(k)}$	k^{th} harmonic component of real power flowing in the line at the sending end
$PL_{ij(k)}$	Real power losses in the transmission line for the k^{th} order of harmonics
$Q_{ij(k)}$	k^{th} harmonic component of reactive power flowing in the line at the sending end
$QL_{ij(k)}$	Reactive power losses in the transmission line for the k^{th} order of harmonics
r_{ij}	Resistance offered to the k^{th} harmonic power flow between buses *i* and *j*
$V_{i(k)}$	Sending end bus voltage for k^{th} order of harmonics
$V_{j(k)}$	Receiving end bus voltage for k^{th} order of harmonics
$x_{ij(k)}$	Reactance offered to the k^{th} harmonic power flow between buses *i* and *j*

Greek Symbols

$\angle\delta_{i(k)}$	Angle of sending end bus voltage for k^{th} order of harmonics
$\angle\delta_{j(k)}$	Angle of receiving end bus voltage for k^{th} order of harmonics
$\varepsilon(k)$	acceptable tolerance value for the power mismatch of k^{th} harmonic

Load flow calculation in harmonic polluted radial system with distributed generation has been carried out using abstract data types with complex parameters [5]. A multiple-frequency three-phase load-flow with two sub models including the fundamental power flow (FPF) and harmonic frequency power-flow (HPF) model has been developed and the standard Fourier analysis was used to deal with the harmonic loads to get injection currents [6]. Fuzzy number based methodology for harmonic load-flow calculation including uncertainties has been applied for interconnected system [7].

From the above, one may see the need for an efficient algorithm that reliably solves the power flow equations for radial distribution systems characterized by high *R*/*X* ratio, radial topology and harmonic loads. Large distribution systems employ supervisory control and data acquisition (SCADA) systems for efficient management. SCADA systems employ power flow solutions methods to ascertain the state of the distribution system. Distribution Load Flow (DLF) also forms an integral part of algorithms that assess the cost and benefit of transformer taps changes, change in static var settings, reconfiguration of the system for various purposes ranging from load balancing, loadability enhancement, distribution loss minimization, and voltage profile improvement amongst several others. Existing methods of distribution load flow utilize look up tables and/or load tables and/or switch tables for the purpose of system representation. Reconfiguration and other actions render the look up table based representation schemes ineffective.

Artificial Neural Network (ANN) approach has been applied for harmonic load flow analysis of a distribution system [8]. This paper proposes a dynamic data structure (DDS) that helps to store information of a branch of a RDS to determine the bus voltages and angle of a particular load pattern for a given order of harmonics. This DDS is adaptation of the DDS reported in [9]. These DDS are handled as a linked list representing the entire radial distribution system. A pseudo code that generates the DDS for a RDS is presented. A function is then developed that computes the voltages from the farthermost end up to the head of the branch. This function is called recursively to find out voltages at all branches of the RDS from the farthermost branch up to the branches emanating from the main substation of the RDS. A pseudo code for this recursive function is also presented. The resulting DLF algorithm is computationally efficient and can deal with the topology changes quickly. The proposed method allows modeling of loads of any type, any number of harmonic components and radial system of any configuration with respect to buses and branches.

2. RDS Representation Using Dynamic Data Structure

Consider a radial distribution system shown in Fig. 1. This system represents typical RDS. It has several branches and buses. The dynamic data structure must represent the RDS or an integral part of the RDS. The proposed DDS represents an integral part of the RDS, namely a generic branch. This section presents the details of the proposed novel DDS that is used to store details of a branch.

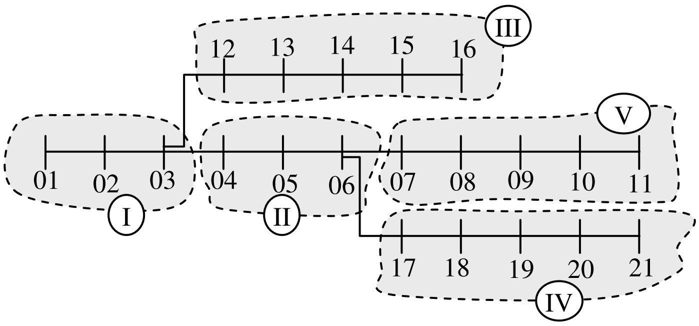

Fig. 1. A Simple Radial Network with Several Branches.

The following attributes were envisioned while developing the proposed DDS.

(a) The data structure must be dynamic in nature so that it may be created and altered in the execution stage.
(b) DDS required for holding the information of a branch must be compact and addressable from any function.
(c) It must be flexible to accommodate any number of buses and harmonic loads within a branch.
(d) It must be flexible to address as many data structures of branches that emanate from the end of a branch.

The proposed data structure of a branch needs to hold the details of:

(a) Parent bus to which the branch is connected,
(b) Number of buses in the branch.
(c) An array to store IDs of all the buses in the order of their location from the head of the branch,
(d) Details of line resistance and reactance for various harmonic components with in a branch,
(e) Number of branches emanating from the last bus of this branch,

(f) A list of pointers pointing to the data structures that store information of emanating branches, and
(g) An array to store IDs of buses at the head of emanating branches.

The definition of the data structure is given below.

```
Typedef       struct {
Integer       parent-bus-id;
Integer       number-of-buses-in-branch, array-of-bus-ids;
Float         resistance and reactance for all harmonic
              components of lines from each bus to a bus
              towards  head bus;
Integer       number-of-branches, array-of-head-bus-id-of-
              emanating-branches;
void pointers-to-structure-of-emanating-branches;
} branch;
```

The pictorial representation of the proposed DDS is shown in Fig. 2. Starting from the main substation, branches are sequentially stored in the data structure using a recursive function that is outlined in the following pseudo code.

```
void create_structure(first bus of the branch, address
(pointer to) of the data structure, parent bus ID number from
which this branch emanates)
{
temp = temporary array to store the IDs of buses in this
branch;
number-of-buses-in-branch = 1
temp[first location] = bus-ID = head bus ID
while ( number of branches emanating from bus-ID = 2)
     {
     increase number-of-buses count by 1;
     store this bus-ID in temp array;
     bus Id changes = next connected bus ID;
     }
Dynamically define array-of-bus-IDs to hold bus IDs of buses in
     the branch;
Store the content of temp in array-of-bus-IDs;
Dynamically define array-of-r-x to hold resistance and reactance
     for various harmonics of lines between each bus and another
     leading to the head bus in the branch;
Fill these arrays with appropriate values based upon the loads
     encountered;
Determine number-of-branches emanating from the last bus this
     branch -> nbr;
If nbr > 0
   {
   dynamically define array-of-head-bus-id-of-emanating-branches
and fillup;
   dynamically define pointers-to-structure-of-emanating-
branches;
   allocate space for nbr data structure and store their addresses
in above;
   for each of the nbr emanating branches -> call function
     create_structure();
   }
}
```

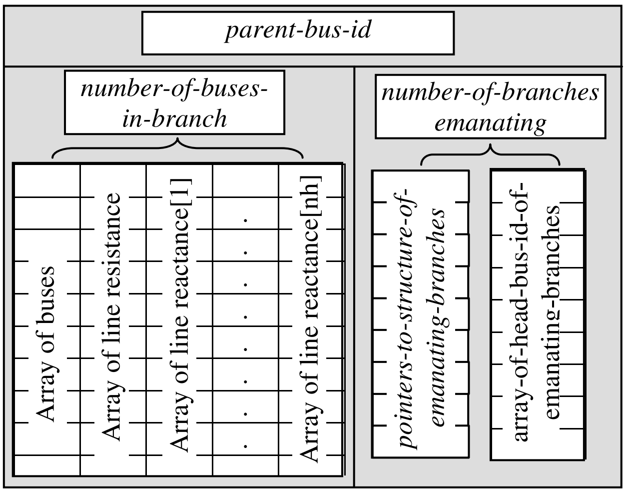

Fig. 2. Proposed DDS with nh Harmonics.

The pseudo code presented above creates a structure for a branch. Starting at the main substation bus, the pseudo code forms the first branch by sequentially considering buses until it finds one that has several branches emanating from it. Referring to Fig. 1 that presents a sample radial system, it builds the first branch from bus 1 up to bus 3 where two branches emanate. The data structure created for branch labeled I, starts with bus 01 and ends with bus 03 comprising three buses. It has two branches emanating from it namely II and III. This structure stores pointers that point towards the structures storing details of branches II and III. The pseudo code calls itself twice, once each for branches II and III. This process proceeds until data structure for all the branches are built. A pictorial representation of the data structure storing details of branch II is shown in Fig. 3. The DDS proposed in this paper is highly flexible and is convenient to use when the RDS is reconfigured under the umbrella of SCADA. This build up of DDS to represent the entire RDS is convenient for the solving the power flow equations for the entire radial network using a recursive function outlined in the next section.

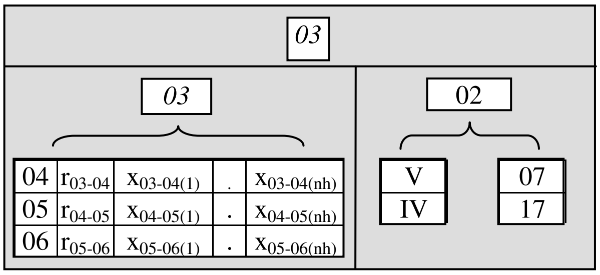

Fig. 3. Example of Dynamic Data Structure-Branch II.

3. Proposed Harmonic Distribution Load Flow

Distribution power flow is presented in this section. It uses the dynamic data structure proposed in Section 2. First the line model of a generic line is presented with modelling for harmonics. Then a recursive algorithm is presented. This is followed by the flowchart of the proposed harmonic distribution load flow.

3.1. Line model and voltage equation considering harmonics

In this section, a simple circuit model of a transmission line considering k^{th} order harmonic component and associated recursive voltage equations is presented. It is assumed that the three-phase RDS is balanced and can be represented by an equivalent single-phase system. The transmission line to ground capacitance elements at the distribution voltage level are small and thus neglected. A simple circuit model of a transmission line is shown in Fig. 4.

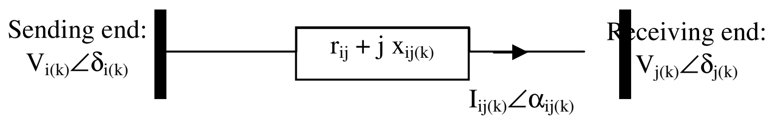

Fig. 4. Simple Equivalent Circuit of Transmission Line Considering Harmonics.

The values of $V_{i(k)}\angle\delta_{i(k)}$ and $V_{j(k)}\angle\delta_{j(k)}$ represent the sending and receiving end voltages. The values $r_{ij} + j\, x_{ij(k)}$ represent resistance and reactance offered to the k^{th} harmonic power flow between buses i and j. Several methods of solving the power flow equations of distribution systems have been proposed [1-3]. It may be observed that these methods use recursive equations as below in several forms considering either sending or receiving end power (Fig. 4 for the variables used):

(a) Equation derived considering sending end powers of k^{th} harmonic:

$$V_{j(k)}^2 = V_{i(k)}^2 - 2\left(r_{ij}P_{ij(k)} + x_{ij(k)}Q_{ij(k)}\right) + \frac{\left(r_{ij}^2 + x_{ij(k)}^2\right)\left(P_{ij(k)}^2 + Q_{ij(k)}^2\right)}{V_{i(k)}^2} \quad (1)$$

where $P_{ij(k)}$ and $Q_{ij(k)}$ refer to the k^{th} harmonic component power flowing in the line at the sending end.

(b) Equation derived considering receiving end powers of k^{th} harmonic:

$$V_{j(k)}^2 = -\left(r_{ij}P_{ij(k)} + x_{ij(k)}Q_{ij(k)} - \frac{V_{i(k)}^2}{2}\right) - \sqrt{\left(r_{ij}P_{ij(k)} + x_{ij(k)}Q_{ij(k)} - \frac{V_{i(k)}^2}{2}\right)^2 - \left(r_{ij}^2 + x_{ij(k)}^2\right)\left(P_{ij(k)}^2 + Q_{ij(k)}^2\right)} \quad (2)$$

where $P_{ij(k)}$ and $Q_{ij(k)}$ refer to the k^{th} harmonic component of power flowing in the line at the receiving end. These equations are not amenable for matrix computation as in the case of conventional methods of solving power flow equations for transmission grids like NR method or Fast De-coupled Load Flow method.

3.2. Line model and voltage equation considering harmonics

The proposed recursive algorithm to compute the voltage solution of the RDS is presented in this section. The routine starts by computing voltage from the farthermost bus of the first branch. If the branch has other branches emanating, the pseudo code recursively calls itself to compute the state of buses in these branches and the power they draw from the farthermost bus. This recursive call is continued until the branches from where other branches do not emanate. Then, the pseudo code computes the voltage at farthermost bus using (2) with the knowledge of its load. Then using the expressions below, it computes the power loss in the transmission line connecting this bus and next bus in the direction leading to the first bus of the branch.

$$\left.\begin{aligned} PL_{ij(k)} &= r_{ij}\frac{P_{ij(k)}^2 + Q_{ij(k)}^2}{V_{j(k)}^2} \\ QL_{ij(k)} &= x_{ij(k)}\frac{P_{ij(k)}^2 + Q_{ij(k)}^2}{V_{j(k)}^2} \end{aligned}\right\} \tag{3}$$

where $PL_{ij(k)}$ and $QL_{ij(k)}$ are the real and reactive power losses in the transmission line model shown in Fig. 4 and $P_{ij(k)}$ and $Q_{ij(k)}$ refer to the power flowing in the line at the receiving end corresponding to the k^{th} harmonic component. With load at bus j and transmission power loss in the line between buses i and j known, the pseudo code computes the load at bus i. This process continues until the voltage is computed until the first bus. The phase angle of the voltage phasor at the j^{th} bus is computed by the following expression:

$$\delta_{j(k)} = \delta_{i(k)} - \cos^{-1}\left(\sqrt{1 - \left(\frac{P_{ij(k)}x_{ij(k)} - Q_{ij(k)}r_{ij}}{V_{i(k)}V_{j(k)}}\right)^2}\right) \tag{4}$$

where $P_{ij(k)}$ and $Q_{ij(k)}$ refer to the power flowing in the line at the receiving end corresponding to the k^{th} component. During each computation, it repeats the solution for each harmonic component.

In this work convergence is checked by ascertaining whether the sum of powers flowing out in the lines connected to each bus equals, or nearly equals within a tolerable limit, the net power injected in to that bus by the connected generations and loads. Mathematically, convergence criterion is represented as

$$PG_{i(k)} - PD_{i(k)} - \left[\sum_j \left\{V_{i(k)}V_{j(k)}Y_{ij(k)}\cos\left(\delta_{i(k)} - \delta_{j(k)} - \theta_{ij(k)}\right)\right\}\right] \le \varepsilon(k) \tag{5}$$

$$QG_{i(k)} - QD_{i(k)} - \left[\sum_j \left\{V_{i(k)}V_{j(k)}Y_{ij(k)}\sin\left(\delta_{i(k)} - \delta_{j(k)} - \theta_{ij(k)}\right)\right\}\right] \le \varepsilon(k) \tag{6}$$

where $\varepsilon(k)$ is an acceptable tolerance value k^{th} harmonic.

3.3. Overall algorithm

This section presents the details of the proposed method that uses the novel data structure and the proposed recursive algorithm. The overall algorithm is presented in Fig. 5. The algorithm reads in the data at first. It then creates one DDS each to represent all the branches of the RDS. The down stream harmonics are considered to generate appropriate reactance values of all the lines. The DDS and pseudo code required are presented in Section 2. Subsequently, the algorithm calls the function *calculate load / generation ()* recursively to compute the voltage solution at all the buses in the system. The algorithm checks for convergence by checking whether the inequalities (5) and (6) are satisfied. The function *calculate load / generation ()* is repeatedly called until the inequalities (5) and (6) are satisfied. In the implementation of the overall algorithm, transformers are modelled using conventional π-model. The effect of shunt compensating element at any bus is considered. π-model of a transformer is used for all computations.

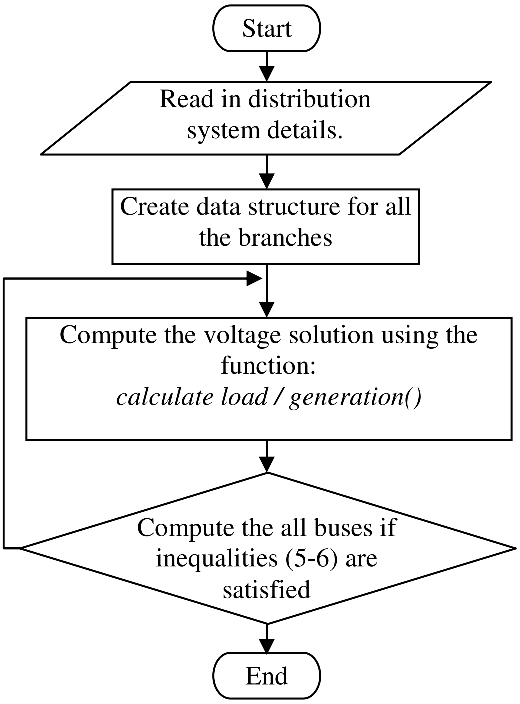

Fig. 5. Flowchart for the Overall Algorithm.

4. Results

To test the effectiveness of the proposed algorithm 33-bus is considered. The single line diagram of the 33-bus RDS is presented in Fig. A-1 (*Appendix A*) Three harmonic components namely 3, 5 and 7 are considered for the purpose of simulation. The tolerance was chosen to be 0.001 p.u. It took seven iterations for the proposed algorithm to converge in for the 33-bus RDS. The convergence depends upon the base load condition and the chosen tolerance. The proposed method is expected to give the power flow solution even for large systems with minimum number of iterations. Table A-1 (*Appendix A*) presents the results of the proposed method with voltage solution for various harmonics. Table A.2 presents comparison of base case solution with solution from ladder iterative technique and from BPN [8]. Figure A-2 (*Appendix A*) presents a graph that shows total system loss variation with respect to harmonic components.

5. Conclusions

Inclusion of power electronic devices and saturation of magnetic circuits introduce several load side harmonics. This paper reports a new distribution system load flow algorithm that uses recursive voltage equations considering harmonic load components. This paper proposes a new dynamic data structure for modular representation of a radial distribution system with loads having harmonic component. The DDS are stored and retrieved using a linked list. A recursive load flow algorithm is then proposed that uses the dynamic data structure to solve the power flow equations efficiently considering harmonic components. Pseudo codes for generation of dynamic data structure are included. Results of the tests on a 33-bus RDS with harmonic components are presented that demonstrates the applicability of the method.

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Appendix A

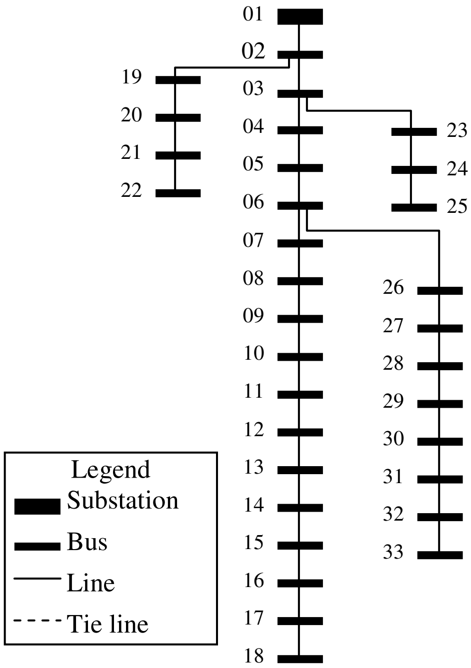

Fig. A-1 33-Bus Radial Distribution System.

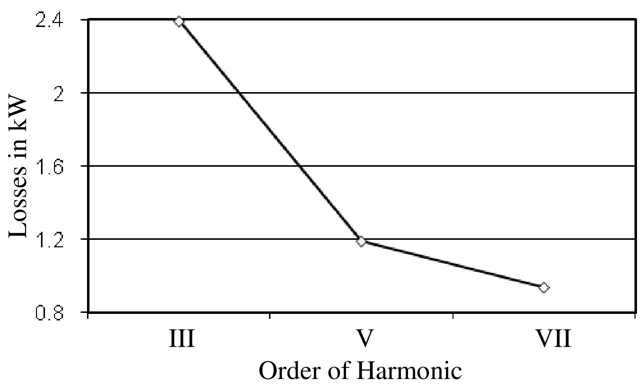

Fig. A-2 Graph of Variation in Losses with Harmonics.

Table A-1 Results of 33-Bus System with Fundamental and Three Harmonic Components.

S. No.	From Bus	To Bus	R (Ω)	Fundamental				III Harmonic				V Harmonic				VII Harmonic			
				X (Ω)	P (kW)	Q (kvar)	V (p.u.)	X (Ω)	P (kW)	Q (kvar)	V (p.u.)	X (Ω)	P (kW)	Q (kvar)	V (p.u.)	X (Ω)	P (kW)	Q (kvar)	V (p.u.)
1	1	2	0.0922	0.047	100	60	1	0.141	1	0.6	0.1000	0.235	0.5	0.3	0.0750	0.329	0.25	0.15	0.0500
2	2	3	0.493	0.2511	90	40	0.997	0.7533	0.9	0.4	0.0995	1.2555	0.45	0.2	0.0745	1.7577	0.225	0.1	0.0494
3	3	4	0.366	0.1864	120	80	0.9829	0.5592	1.2	0.8	0.0972	0.932	0.6	0.4	0.0723	1.3048	0.3	0.2	0.0467
4	4	5	0.3811	0.1941	60	30	0.9754	0.5823	0.6	0.3	0.0960	0.9705	0.3	0.15	0.0711	1.3587	0.15	0.075	0.0450
5	5	6	0.819	0.707	60	20	0.9679	2.121	0.6	0.2	0.0947	3.535	0.3	0.1	0.0698	4.949	0.15	0.05	0.0433
6	6	7	0.1872	0.6188	200	100	0.9495	1.8564	2	1	0.0911	3.094	1	0.5	0.0658	4.3316	0.5	0.25	0.0376
7	7	8	1.7114	1.2351	200	100	0.946	3.7053	2	1	0.0901	6.1755	1	0.5	0.0646	8.6457	0.5	0.25	0.0356
8	8	9	1.03	0.74	60	20	0.9323	2.22	0.6	0.2	0.0877	3.7	0.3	0.1	0.0621	5.18	0.15	0.05	0.0320
9	9	10	1.044	0.74	60	20	0.926	2.22	0.6	0.2	0.0866	3.7	0.3	0.1	0.0609	5.18	0.15	0.05	0.0302
10	10	11	0.1966	0.065	45	30	0.9201	0.195	0.45	0.3	0.0856	0.325	0.225	0.15	0.0598	0.455	0.1125	0.075	0.0286
11	11	12	0.3744	0.1238	60	35	0.9192	0.3714	0.6	0.35	0.0855	0.619	0.3	0.175	0.0597	0.8666	0.15	0.0875	0.0285
12	12	13	1.468	1.155	60	35	0.9177	3.465	0.6	0.35	0.0852	5.775	0.3	0.175	0.0595	8.085	0.15	0.0875	0.0282
13	13	14	0.5416	0.7129	120	80	0.9115	2.1387	1.2	0.8	0.0841	3.5645	0.6	0.4	0.0583	4.9903	0.3	0.2	0.0263
14	14	15	0.591	0.526	60	10	0.9092	1.578	0.6	0.1	0.0836	2.63	0.3	0.05	0.0577	3.682	0.15	0.025	0.0254
15	15	16	0.7463	0.545	60	20	0.9078	1.635	0.6	0.2	0.0834	2.725	0.3	0.1	0.0574	3.815	0.15	0.05	0.0249
16	16	17	1.289	1.721	60	20	0.9064	5.163	0.6	0.2	0.0831	8.605	0.3	0.1	0.0572	12.047	0.15	0.05	0.0245
17	17	18	0.732	0.574	90	40	0.9043	1.722	0.9	0.4	0.0827	2.87	0.45	0.2	0.0567	4.018	0.225	0.1	0.0236
18	2	19	0.164	0.1565	90	40	0.9037	0.4695	0.9	0.4	0.0826	0.7825	0.45	0.2	0.0565	1.0955	0.225	0.1	0.0234
19	19	20	1.5042	1.3554	90	40	0.9965	4.0662	0.9	0.4	0.0994	6.777	0.45	0.2	0.0745	9.4878	0.225	0.1	0.0494
20	20	21	0.4095	0.4784	90	40	0.9929	1.4352	0.9	0.4	0.0989	2.392	0.45	0.2	0.0739	3.3488	0.225	0.1	0.0488
21	21	22	0.7089	0.9373	90	40	0.9922	2.8119	0.9	0.4	0.0988	4.6865	0.45	0.2	0.0738	6.5611	0.225	0.1	0.0487
22	3	23	0.4512	0.3083	90	50	0.9916	0.9249	0.9	0.5	0.0986	1.5415	0.45	0.25	0.0737	2.1581	0.225	0.125	0.0486
23	23	24	0.898	0.7091	420	200	0.9793	2.1273	4.2	2	0.0967	3.5455	2.1	1	0.0718	4.9637	1.05	0.5	0.0461
24	24	25	0.896	0.7011	420	200	0.9726	2.1033	4.2	2	0.0956	3.5055	2.1	1	0.0708	4.9077	1.05	0.5	0.0451
25	6	26	0.203	0.1034	60	25	0.9693	0.3102	0.6	0.25	0.0951	0.517	0.3	0.125	0.0703	0.7238	0.15	0.0625	0.0445
26	26	27	0.2842	0.1447	60	25	0.9475	0.4341	0.6	0.25	0.0907	0.7235	0.3	0.125	0.0654	1.0129	0.15	0.0625	0.0371
27	27	28	1.059	0.9337	60	20	0.945	2.8011	0.6	0.2	0.0902	4.6685	0.3	0.1	0.0649	6.5359	0.15	0.05	0.0365
28	28	29	0.8042	0.7006	120	70	0.9335	2.1018	1.2	0.7	0.0877	3.503	0.6	0.35	0.0621	4.9042	0.3	0.175	0.0325
29	29	30	0.5075	0.2585	200	600	0.9253	0.7755	2	6	0.0859	1.2925	1	3	0.0601	1.8095	0.5	1.5	0.0297
30	30	31	0.9744	0.963	150	70	0.9217	2.889	1.5	0.7	0.0852	4.815	0.75	0.35	0.0593	6.741	0.375	0.175	0.0287
31	31	32	0.3105	0.3619	210	100	0.9176	1.0857	2.1	1	0.0844	1.8095	1.05	0.5	0.0585	2.5333	0.525	0.25	0.0273
32	32	33	0.341	0.5302	60	40	0.9167	1.5906	0.6	0.4	0.0842	2.651	0.3	0.2	0.0582	3.7114	0.15	0.1	0.0270
				Losses: 210.9983 kW				Losses: 2.3947 kW				Losses: 1.189 kW				Losses: 0.9358 kW			

Table A-2 Comparison of Load Flow Results for the Base Case with the Proposed Method.

Bus No.	Test Input		Solution from BPN		Expected Solution (Solution from Ladder iterative technique)		solution from the proposed method with DDS	
	P in KW	Q in KVAr	Voltage Mag (pu)	Angle in rads	Voltage Mag (pu)	Angle in rads	Voltage Mag (pu)	Angle in rads
1	0.00	0.00	1.00	0.00	1.00	0.00	1.000	0.000
2	1.98	1.58	0.99	0.00	0.99	0.00	0.990	0.000
3	1.88	1.38	0.96	0.01	0.96	0.01	0.960	0.010
4	2.18	1.78	0.95	0.01	0.95	0.01	0.955	0.010
5	1.58	1.28	0.93	0.02	0.93	0.02	0.930	0.020
6	1.58	1.18	0.89	0.01	0.88	0.02	0.881	0.020
7	2.98	1.98	0.87	0.00	0.87	0.01	0.870	0.010
8	2.98	1.98	0.83	0.01	0.83	0.01	0.830	0.010
9	1.58	1.18	0.81	0.01	0.81	0.01	0.812	0.010
10	1.58	1.18	0.79	0.01	0.79	0.01	0.790	0.011
11	1.43	1.28	0.79	0.01	0.78	0.01	0.780	0.010
12	1.58	1.33	0.78	0.01	0.78	0.01	0.780	0.010
13	1.58	1.33	0.76	0.01	0.76	0.01	0.763	0.010
14	2.18	1.78	0.76	0.00	0.75	0.01	0.750	0.010
15	1.58	1.08	0.75	0.01	0.74	0.01	0.741	0.010
16	1.58	1.18	0.75	0.01	0.74	0.01	0.742	0.010
17	1.58	1.18	0.74	0.00	0.73	0.01	0.730	0.011
18	1.88	1.38	0.74	0.00	0.73	0.01	0.730	0.010
19	1.88	1.38	0.99	0.00	0.99	0.00	0.990	0.000
20	1.88	1.38	0.98	0.00	0.98	0.00	0.980	0.000
21	1.88	1.38	0.98	0.00	0.98	0.00	0.980	0.00
22	1.88	1.38	0.98	0.00	0.98	0.00	0.980	0.000
23	1.88	1.48	0.96	0.01	0.96	0.01	0.960	0.010
24	5.18	2.98	0.95	0.00	0.95	0.01	0.950	0.010
25	5.18	2.98	0.95	0.00	0.94	0.01	0.940	0.010
26	1.58	1.23	0.88	0.02	0.88	0.02	0.880	0.020
27	1.58	1.23	0.88	0.02	0.87	0.02	0.870	0.020
28	1.58	1.18	0.86	0.02	0.85	0.02	0.850	0.020
29	2.18	1.68	0.84	0.02	0.84	0.02	0.840	0.020
30	2.98	6.98	0.83	0.03	0.83	0.03	0.830	0.030
31	2.48	1.68	0.82	0.03	0.82	0.03	0.820	0.031
32	3.08	1.98	0.82	0.02	0.82	0.03	0.820	0.030