

Experimental evidence of the compatibility of the cumulative electromagnetic energy release data, with the hierarchical models for the catastrophic fracturing process

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Received: 28 November 2010 – Revised: 28 January 2011 – Accepted: 3 March 2011 – Published: 6 June 2011

Abstract. In this paper, we performed experiments of uniaxial compression of granite samples and recorded time series of electromagnetic pulses during the evolution of the catastrophic fracturing process. The cumulative energy release of the electromagnetic emission (EME) up to the critical point at the moment of rupture was then calculated. It was shown, that the validity of the proposed hierarchy models for the catastrophic fracturing process of composite materials, in analogy to critical phenomena, can be experimentally established not only via acoustic emission data, but via electromagnetic emission data as well. The above conclusion could be a useful tool for the improvement of the earthquake prediction method, based on precursory electromagnetic signals.

1 Introduction

The discrete nature of the fracturing process in various composite materials, such as rocks that are subjected to continuous loading, is of particular interest. As the material is under load, stress concentration develops around cracks and other defects and micro-fractures start to occur locally. The stress is then redistributed and transferred to other neighbouring interacting elements, by analogy to percolation phenomena. Hence, the catastrophic fracture propagates in the bulk of the material, in an accelerating mode, until its total failure.

Newman et al. (1995) proposed a statistical failure model based on the discrete scale hierarchy in the fracturing process. This model manifests self-similarity (multi-scaling)

features and it applies from the laboratory scale up to the earthquake rupture large scale.

Also a power-law distribution of failures is found prior to total failure and a log-periodic distribution of failure appear.

According to the statistical hierarchy model of discontinuous damage, Moura et al. (2005, 2006) consider that “the fracturing process of stressed materials is analogous to a critical phenomenon at a second order transition and the moment of rupture is similar to a critical point”. In the vicinity of the critical point of rupture, the variations in free energy reflected in energy release, can be characterised by power-law accompanying log-periodic oscillations related to complex critical exponent.

Moura et al. (2005, 2006) consider that during the fracturing process of the stressed material, the cumulative energy release is reflected by the acoustic emission. That’s why they were concentrating on recording the bursts of acoustic emission during their laboratory experiments. Their experimental results fitted with the curve they suggested after the extrapolation of a mathematical relationship based on the theory of critical phenomena.

The question now arises whether these theoretical models can be verified not only for the acoustic activity, but for the electromagnetic activity which is emitted during the catastrophic deformation of materials in the laboratory as well (Hadjicontis et al., 2004).

In this paper, we show that our data – the electromagnetic emission from a granite sample undergoing catastrophic failure – are compatible to the proposed models.



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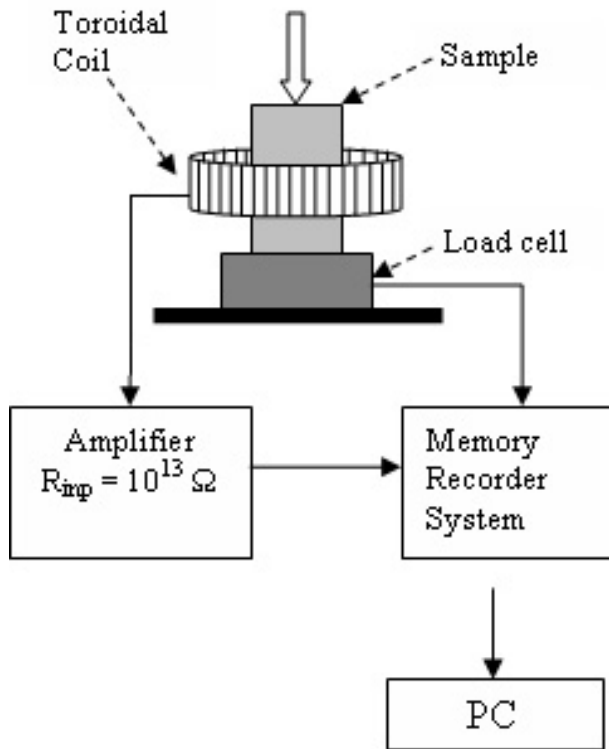


Fig. 1. Block diagram of the experimental setup.

2 Experimental set-up

Instrumentation that was particularly developed was used for the detection and monitoring of the time series of electromagnetic pulses emitted during the uniaxial deformation of rock samples and other crystalline materials. The system includes a mechanical and an electronic part. The mechanical part is the hand-operated loading machine and the electronic part consists of:

1. a toroidal coil used as the sensor for the detection of the magnetic component of the electromagnetic signals emitted from the deformed sample. Using this toroidal coil, we can eliminate the induced external noise,
2. a low-signal amplification system with very high input resistance,
3. a load cell for measuring the mechanical load.

The analog signals are fed to the memory digital recorder. The block diagram of the system is shown in Fig. 1.

The toroidal sensor consists of a ferrite core with a coil wire distributed along it, as is shown in Fig. 1. The sample, mounted in the centre of the coil, produces polarization currents when being plastically deformed. The density J_P of these currents is described by

$$J_P = \frac{\partial P}{\partial t} \quad (1)$$

where P is the electrical polarization. The polarization is part of the electrical displacement

$$D = \varepsilon_0 E + P \quad (2)$$

where E is the electric field. In non-conductive materials, the displacement currents produce a magnetic field H determined by the Maxwell equation:

$$\nabla \times H = \frac{\partial D}{\partial t} \quad (3)$$

Therefore, the polarization currents contribute to the magnetic field since they are a radical part of the displacement currents. This magnetic field excites an alternative magnetic field in the core ring because of the electromagnetic induction, which is the curl of the solenoid electric field E excited in the wire coil

$$\nabla \times E = -\mu_0 \mu \frac{\partial H}{\partial t} \quad (4)$$

where μ is the magnetic permeability of the toroidal's core. The excited electric field E produces a conductivity current in coil wire

$$J = \sigma E \quad (5)$$

where σ is the electrical conductivity of the wire. In this way, the current J produced in the coil wire is a direct response of the EME signals produced by the plastic deformation of the sample. These currents compose the analog signals that are being amplified and fed to the memory of a digital recorder.

3 Results and discussion

Figure 2 represents an example of a square amplitudes' time series of electromagnetic pulses. This time series represents the power $P(t)$ of the emitted electromagnetic signals corresponding to the evolution of the fracturing process as the loading increases gradually up to the total failure. Consequently, the cumulative electromagnetic energy released $E(t)$ can be obtained by the integration of the power's time series

$$E(t) = \int_0^t P(u) du \quad (6)$$

Figure 3 depicts the integral of the square amplitudes of the above time series, versus time, which is analogous to the cumulative electromagnetic energy release, in arbitrary units.

The multi-step character, which appears in the experimental curve of the released electromagnetic energy in granite (Fig. 3), reveals the discrete nature of the fracturing process. A careful consideration, of the two small figures in the main Fig. 3, also reveals fine discrete scale hierarchy and self similarity in accordance with the theoretical hierarchical failure model of Newman et al. (1995). If such an

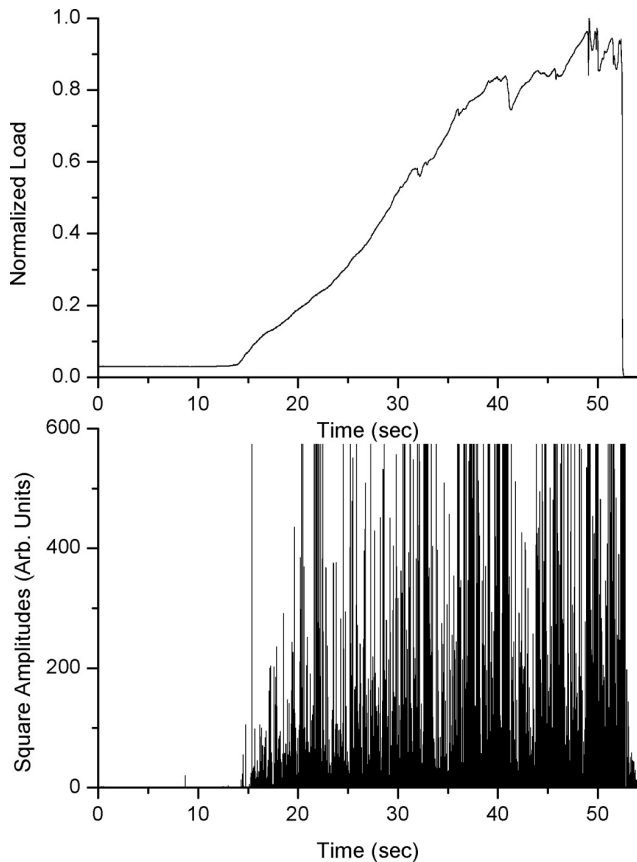


Fig. 2. Upper: mechanical load versus time. Lower: the square amplitudes time series of electromagnetic emission pulses during the evolution of fracturing process in granite.

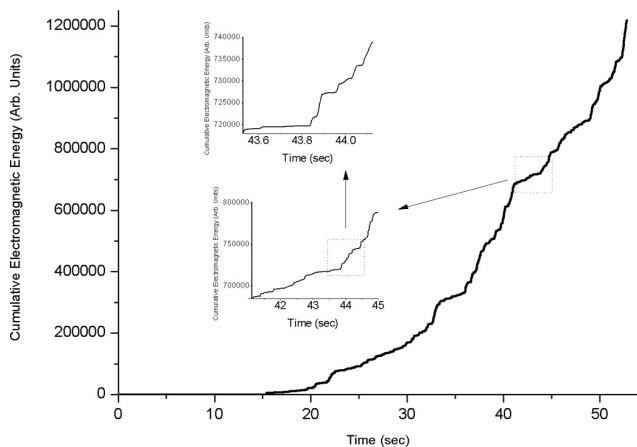


Fig. 3. The cumulative electromagnetic energy release versus time, up to the moment of rupture, in granite under continuous loading.

assumption is valid, the cumulative energy release will exhibit a log-periodic behaviour, a characteristic feature of critical phenomena. Therefore, the obtained experimental data will verify the equivalent laws that describe the behaviour

of a critical phenomenon, which abides by the hierarchical model, near its critical region.

According to Moura et al. (2005, 2006), if we define the dimensionless time as

$$x = \frac{t_c - t}{t_c} \tag{7}$$

and the dimensionless energy release as

$$f(x) = \frac{E(x)}{E_c} \tag{8}$$

where t_c is the critical time of rupture and E_c the total energy released during the whole fracturing phenomena, then in the critical region the dimensionless energy release can be described by the equation

$$f(x) = 1 + a_1 x^\alpha \cos(\omega \ln x + \varphi) \tag{9}$$

which could be accounted for as the real part of $1 + a_1 x^\alpha + a_2 e^{i\varphi} x^{\alpha+i\omega}$ where a_1, a_2 and φ are unknown control parameters and α and ω are the real and imaginary part of the complex critical exponent $z = \alpha + i\omega$.

Outside the critical region, the extrapolated form of the equation is

$$f^*(x) = \cos(cx^\alpha \sin g(x) e^{h(x)}) e^{cx^\alpha \cos(g(x)) e^{h(x)}} \tag{10}$$

where $g(x) = \rho \sin(\omega \ln x - \varphi)$ and $h(x) = \rho \cos(\omega \ln x + \varphi)$. The above equation is used for the fitting of the experimental data, as discussed later.

As explained in detail by Moura et al. (2005, 2006), the parameters α, ω and ρ are in correspondence with the sample's heterogeneity, disorder and induced damage. In particular the quantity α is a structural parameter, which depicts the sample's heterogeneity. Smaller values of α correspond to a lower disorder of the material, therefore, it is expected to be $\alpha \rightarrow 0^+$ for pure crystals. The parameter ω is similar to a damage variable which reflects the transportation of stress in cracks or defects and how these interact. For instance, when ω values are higher than about π , log-periodic oscillations can occur far from the critical point. Concerning the parameter ρ , its role is the control of the amplitude of the log-periodicity correction. Parameter ρ must abide by the restriction of

$$\rho < \frac{\alpha}{\sqrt{\alpha^2 + \omega^2}} \tag{11}$$

in order to have a positive value for the energy release and, therefore, a physical meaning of the mathematical calculation.

Using the equation $f^*(x)$, we performed a fitting in our data of cumulative electromagnetic energy release (Fig. 4), in analogy with the work of Moura et al. (2005, 2006). The results of the fitting were satisfactory and the corresponding parameters values were: $\alpha = 1.067, \omega = 1.440, \rho = 0.445$, which are in good agreement with the physical conditions

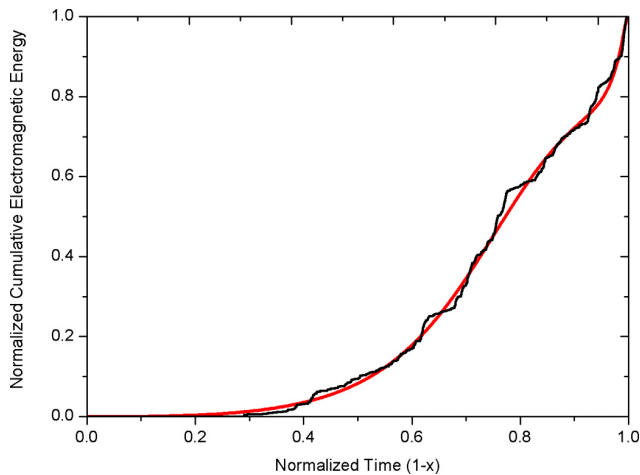


Fig. 4. Cumulative electromagnetic energy release. The experimental data (black line) are fitted with a red curve.

set by the above authors. Note that α and ω are the real and imaginary parts, respectively, of the complex critical exponent, which characterises log-periodic oscillations and ρ controls the amplitudes of the log-periodicity, as has already been mentioned.

4 Conclusions

In this work, we wanted to see if the cumulative energy released by the emitted electromagnetic signals during granite deformation is in accordance with the hierarchical model proposed by Newman et al. (1995) and respectively described by the same power law for the energy released by the acoustic emissions, as proposed by Moura et al. (2005, 2006).

Our experimental results on electromagnetic emission, as they are shown in Fig. 3, indeed seem to be in accordance with the final computed results of the hierarchy model presented by Newman et al. (1995).

The validity of the proposed model by Moura et al. (2005, 2006), in analogy to critical phenomena, can be experimentally established not only via acoustic emission data, but via electromagnetic emission data as well.

Edited by: K. Eftaxias

Reviewed by: P. Koktavý and another anonymous referee

References

- Hadjicontis, V., Mavromatou, C., and Ninos, D.: Stress induced polarization currents and electromagnetic emission from rocks and ionic crystals, accompanying their deformation, *Nat. Hazards Earth Syst. Sci.*, 4, 633–639, doi:10.5194/nhess-4-633-2004, 2004.
- Moura, A., Lei, X., and Nishisawa, O.: Prediction scheme for the catastrophic failure of highly loaded brittle materials or rocks, *J. Mech. Phys. Solids*, 53, 2435–2455, 2005.
- Moura, A., Lei, X., and Nishisawa, O.: Self-similarity in rock cracking and related complex critical components, *J. Mech. Phys. Solids*, 54, 2544–2553, 2006.
- Newman, W., Turcotte, D., and Gabrielov, V. A.: Log-periodic behaviour of hierarchical failure model with applications to precursory seismic activation, *Phys. Rev. E*, 52, 4824–4835, 1995.