

## Some estimates on the space scales of vortex pairs emitted from river mouths

V. P. Goncharov<sup>1</sup> and V. I. Pavlov<sup>2</sup>

<sup>1</sup>Institute of Atmospheric Physics, Russian Academy of Sciences, 109017 Moscow, Russia

<sup>2</sup>DMF, UFR de Mathématiques Pures et Appliquées, Université de Lille 1, 59655 Villeneuve d'Ascq Cedex, France

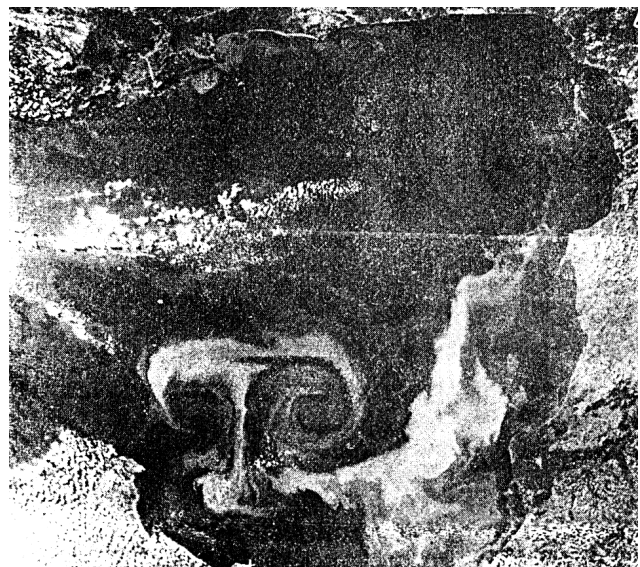
Received: 17 July 1999 – Accepted: 12 November 1999

**Abstract.** Two-dimensional vortex pairs are frequently observed in geophysical conditions, for example, in a shelf zone of the ocean near river mouths. The main aims of the work are to estimate the space scales of such vortex structures, to analyze possible scenarios of vortex pair motion and to give the qualitative classification of their trajectories. We discuss some features of the motion of strong localized vorticity concentrations in a given flow in the presence of boundaries. The analyses are made in the framework of a 2D point vortex model with an open polygonal boundary. Estimations are made for the characteristic parameters of dipole vortex structures emitted from river mouths into the open ocean.

### 1 Introduction

Satellite observations show that the vortex quasi-dipole structures are one of most wide-spread forms of mesoscale two-dimensional motions in the ocean. These structures arise as a result of the ocean response to a localized (maybe, impulsive) action of some kind. According to the analysis of satellite images (Ginsburg and Fedorov, 1984; Fedorov and Ginsburg, 1989), quasi-dipole vortex structures, called mushroom-like currents, are characterized by two closely packed patches of oppositely signed vorticity. Vortex pairs are frequently observed in the shelf zone (see Fig. 1). Evidently, such structures can assure a very effective mechanism for horizontal mixing in an ocean. Because of their self-propelling motion, they can transport captured scalar properties, such as salt, heat and other constituents, through large distances from sources. This fact indicates convincingly that mushroom-like currents can play an important role in exchange between shelf and deep-sea water.

A large body of observation (Ginsburg and Fedorov, 1984, and Refs therein) shows that the formation of vortex pairs is preceded by the appearance of jet currents which can be caused by various natural factors: in coastal zones, by river inflow, ice melting, water exchanging through a strait, etc. It is particularly interesting that in most cases, as inferred from



**Fig. 1.** Vortex pairs observed near the coastal line.

satellite images, in the coastal zone the jet currents ending in vortex pairs are directed approximately normally to the shoreline and their geographic location has an explicitly non-random character; this phenomenon is observed frequently near a river mouth.

The main aim of the paper is to answer some important questions: does a relationship exist between horizontal scales of the observed vortex structures and dynamical characteristics of river mouth? What are the key geometrical parameters and how does the motion of vortices depend on these?

Here we focus on qualitative estimates of vortex dynamics near river mouths providing analytical results of the greatest generality. The numerical results depend frequently on a number of factors of secondary importance, which can distort the over-all picture by introducing details frequently nonexistent in reality. Instead of massaging the computer calculated details of velocity or vorticity distribution, it is more impor-

tant to answer the more practical questions: do formed vortices remain near a source, near a coastal line? Under what external conditions do vortex pairs abandon the domain of the source? Do points of stagnation, where the vortices are not moving, exist? And, if so, what are the key geometrical parameters and how does the motion of vortices depend on them?

## 2 Basic approximations and model

It is natural to consider the simplest approximation in which the vortex structures are considered as quasi-two-dimensional (their horizontal scales  $b$  are much greater than the depth of the shelf zone,  $d$ ), the ocean is considered as inviscid (corresponding Reynolds number  $Re \gg 1$ ).

We can represent the fluid motions by the following parameters: the horizontal,  $v_1$ , and vertical,  $v_2$ , velocities of the fluid, as well as the horizontal length-scale,  $l$ , of the coastal line. The layer thickness,  $d$ , will be regarded as the vertical length-scale. We assume that the Froude number,  $Fr = v_1^2/gl$ , is rather small, and that the Reynolds number,  $Re_d = v_1 d^2/\nu l$ , in contrast, is large ( $g$  is the acceleration due to gravity,  $\nu$  is the kinematic viscosity of the fluid; for this concrete estimation the dominant mechanism of bottom Rayleigh friction is taken into account (Goncharov and Pavlov, 1998, and Refs therein). For example, if the size of a river mouth is on a scale of  $l \sim 10^3 \div 10^4$  m, the depth is of  $d \sim 10^1 \div 3 \cdot 10^2$  m, the characteristic scale of the vortex pair,  $b \sim 10^2$  m, then for a moderate flow velocity  $v_1 \sim 1 \div 10^{-1} \text{ ms}^{-1}$  and typical viscosity  $\nu \sim 10^{-6} \text{ m}^2 \text{ s}^{-1}$ , the characteristic Reynolds number is  $Re_d > 10^6 \gg 1$ . If the inequalities  $Fr \ll 1$ ,  $Re_d \gg 1$ ,  $d \ll l$ ,  $v_2 \ll v_1$  hold true, we can, for our purpose, regard the fluid layer as “thick”, and motions are quasi-two-dimensional. Such estimates of dimensionless numbers are typical for geophysical quasi-2D hydrodynamics.

Thus, we focus on the general picture, and suppose that the vortex pairs are formed near sources (river mouth), characterized by the source potency  $m$  and by the width  $l$ . The circulations of the vortices,  $\pm\kappa$ , are fixed. We consider their motion near boundaries (complex configuration of coast line). We shall not discuss the problem of the formation of vortex pairs here; this problem cannot be resolved in detail with the model of inviscid hydrodynamics discussed below. But, once the vortices are formed and their circulations are given, the motion of such vortex structures can be analyzed at least qualitatively. In the light of observable facts it is not unreasonable to ask what scenarios of motion of generated dipole vortices are possible in the presence of the shoreline and what are their influence on the dynamics of this vortex system.

The qualitative explanation of the features of vortex motion may be following: in the presence of hard boundaries (coastal line), the corresponding boundary conditions for two-dimensional vortices are respected by introducing “images” of the vortices. (The term “image” has to be used with prudence in such a situation, because the boundaries

are curved). If the pair of vortices are sufficiently close to one another, the interaction vortices-“images” can turn out to be weak. The vortices then form a strongly coupled structure and move along open trajectories into the ocean, abandoning the river mouth. If the vortex pair is relatively large (loosely coupled), the interaction vortices-“images” becomes predominant and the vortices cannot abandon a domain near the source and move near the boundaries along closed trajectories, representing a family of super-inserted loops, without escaping from the river mouth.

There exists therefore a relationship between the size of the vortex pair,  $b$ , and the space scale of the coastal line configuration (river mouth),  $l$ . From a physical standpoint, the problem is characterized by the following dimensional parameters:  $b$  and  $l$ , the characteristic vortex structure scale and boundary configuration scale (the characteristic width of the river) respectively, topological number  $\alpha$ , the characteristics of the boundary curvature (see below), the source (river) potency  $m$ , the circulation of vortices,  $\pm\kappa$  (the corresponding non-dimensional problem parameter is  $p = |\kappa|/m$ ).

Simple dimensional analysis shows that the corresponding relationship can be represented by

$$b = l \mathcal{F}_1(p, \alpha, d/l, Re).$$

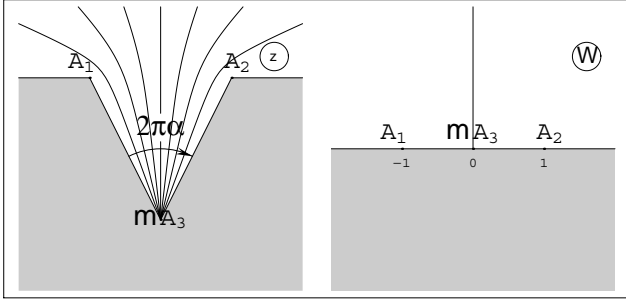
If  $d/l \ll 1$ ,  $Re \gg 1$  (two-dimensional, inviscid model), and the system is not degenerated into the points  $d/l = 0$ ,  $Re = \infty$ , we can rewrite this expression as  $b = l \mathcal{F}(p, \alpha)$ . The structure of  $\mathcal{F}(x, y)$  cannot be obtained only from the above given simple estimates based on the traditional dimensional analysis only. Our goal is thus to find the structure of this function. A detailed discussion of the proposed mathematical model (see below) shows the essentials of the process.

We give below some estimations concerning the motion of *localized vortices* in domains with *polygonal boundaries* only. We hope that such an idealization is useful: the results of the analysis may serve as a theoretical guideline for numerical modeling, on the one hand, and qualitative experimental estimates, on the other.

(i) In the general case, it is impossible to give the analytical expression of the conformal mapping  $w$  for any arbitrary traced contour  $\Gamma$ . Consequently, various approximations are commonly used (Lavrentev and Shabat, 1965; Goncharov and Pavlov, 1998). Some of these essentially consist of the replacement of the real boundary  $\Gamma$  by an open  $n$ -gon which is characterized in the  $z$ -plane by vertices  $A_k$  and corners  $\pi\alpha_k$  ( $0 < \pi\alpha_k \leq 2, k = 1, 2, \dots, n$ ) at the vertices. Then, according to the Schwartz-Christoffel theorem, the desired conformal mapping onto the half-space  $\Im w \geq 0$  is given by

$$\frac{dz}{dw} = c \prod_{k=1}^n (w - a_k)^{\alpha_k - 1}, \quad (1)$$

where  $a_k$  are the images of the vertices, i.e.,  $a_k = w(A_k)$ , and  $\alpha_1 + \alpha_2 + \dots + \alpha_n = n$ . In practice, the problem of the construction of a conformal mapping is reduced to computing for images of the vertices,  $a_k$ , the constant,  $c$ , and the



**Fig. 2.** Sketch illustrating the streamline pattern of flow simulated by the point source  $m$  located in a wedge-shaped river mouth.

constant of integration. Representation (1) has such a degree of arbitrariness that any three points of  $a_k$  can be chosen for the sake of convenience. Then other points and constants are determined unambiguously.

(ii) The models of localized (point) sources (sinks) and point vortices and their superpositions are interesting when the motion of vortices is discussed. The models are among the most basic flow configurations and their study is of fundamental hydrodynamical interest.

There are convincing examples (see papers of Aref, 1979, 1983; Aref et al., 1988; Goldshtik et al., 1991; Meleshko and van Heijst, 1994; Saffman, 1997) of how such elementary flows may serve locally as idealized models of convergent, divergent, and swirling motions prevailing in natural and technological flows and also as structural elements in turbulent motions.

(iii) Note that basic information on the features of point vortex motion near complex boundaries, based on the use of conformal-transformation theory and applying Routh's theorem, may be found, for example, in the books of Saffman (1997) and Milne-Thomson (1968) (see also Refs therein and the works of Lin, 1941, 1943).

For analysis we propose the simplest (minimal) model of two-dimensional flow which occupies a half-plane with rigid boundary simulating the shoreline in the form of an open isosceles triangle  $A_1A_2A_3$ . The sketch illustrating the model of a river mouth is presented in Fig. 2.

For modeling a river mouth, in vertex  $A_2$  of the wedge-shaped excision we will locate the point source which will fulfill the function of a river inflow. Such a model permits not only to investigate the possibility of emitting vortex dipoles from the river mouth into the open ocean but also of studying the character of their motion near the source. By ignoring all non-ideal processes and leaving aside the problems associated with the generation of these dipoles, we will perform the analysis in several possible scenarios initiating similar vortex structures. It is clear that every such scenario must provide the specific asymptotic state in which, after a lapse of time, and at distances well away from shore, the vortex field of the flow will be characterized by dipole structures only.

First we assume that the dipoles have been formed with the help of some mechanism in a river channel and are then

ejected into the river mouth. As will be shown below, if we accept the hypothesis that the generation of vortex dipoles takes place directly in the river mouth, one more scenario is possible. For example, such generation could occur under the influence of viscosity as result of development of the circulation cells adjacent to the shores of the river mouth. By virtue of the Prandtl-Batchelor theorem (for large Reynolds numbers) (Batchelor, 1956) these cells are bound to be filled by a vorticity. This process will obviously continue until the vorticity of the generated cells reaches a level where the vortices could escape from the river mouth at the expense of a self-propelling motion.

We will assume that the point source simulating the river inflow is located in a vertex of the river mouth with angle  $2\pi\alpha$ . If  $U$  is the average flow velocity and  $l = 2a \sin \alpha$  is the characteristic width of the river then the source potency  $m$  can be estimated as  $m = Ul/2\pi\alpha$ .

In order to study this problem it is convenient to make the corresponding conformal mapping

$$z(w) = c \int_0^w dw (w^2 - 1)^{\frac{1}{2} - \alpha} w^{2\alpha - 1}, \quad (2)$$

where  $c = 2a B^{-1}(\alpha, (3/2) - \alpha)$ , and the function  $B(x, y)$  is the beta-function (Abramowitz and Stegun, 1964). Transformation (2) maps the flow domain  $z = x + iy$  onto the half plane  $w = \xi + i\zeta$  so that the shoreline in the form of an open isosceles triangle  $A_1A_2A_3$  ( $A_1A_2 = A_2A_3 = a$ ) is mapped onto the straight line  $\Im w = 0$  (Fig.2).

Restricting our consideration to cases when the evolution is symmetric about the imaginary axis in the  $z$ - or  $w$ -plane, we will follow the dynamics of the right-hand vortex by assuming from the condition of asymptotical behavior of the dipole at infinity, that its intensity  $\kappa < 0$ . Omitting the details of calculations, we give the final result: the dipole dynamics in the domain are governed by the equation

$$\dot{\bar{w}} = \frac{\bar{w}}{\pi c^2} \frac{|w^2 - 1|^{2\alpha - 1}}{|w|^{4\alpha}} \left[ \frac{m + i|\kappa|}{4} - i|\kappa| \frac{w^2}{w^2 - \bar{w}^2} + i \frac{|\kappa|}{2} \frac{2\alpha - 1}{w^2 - 1} \right], \quad (3)$$

Here  $w$  is the complex coordinate of the right-hand vortex (over-bar denotes complex conjugation)

One can also show that the equation (3) can be recast in Hamiltonian form (the discussion on the use of the Hamiltonian approach in the hydrodynamics can be found in Goncharov and Pavlov (1997))

$$\dot{\bar{w}} = -i \frac{2}{|\kappa|} |w'|^2 \frac{\partial H}{\partial w}, \quad \dot{w} = i \frac{2}{|\kappa|} |w'|^2 \frac{\partial H}{\partial \bar{w}}, \quad (4)$$

(where  $w' = dw/dz$ ) with the Hamiltonian of interaction

$$H = \frac{|\kappa|}{4\pi} \left[ i \frac{m}{2} \ln \frac{w}{\bar{w}} + |\kappa| \ln \left| \frac{w + \bar{w}}{w} \right| - |\kappa| \ln \left| \frac{(dw/dz)^2}{w\bar{w}} \right| \right]. \quad (5)$$

The terms of this Hamiltonian describe the interactions:

- (i) source-vortices,
- (ii) vortex-vortex (between the left-hand vortex and the right-hand one), and
- (iii) vortices-boundary (vortices-images).

Such a structure of the Hamiltonian shows that there exists, in principle, the possibility of self-organization of the vortex structure when a stable (or unstable) configuration is formed. In this case the vortices are placed at the points of stagnation (stationary points).

The stationary points ( $\dot{w} = 0$ ,  $\dot{\bar{w}} = 0$ ) of (3) are solutions of the equation:

$$(1 + ip)(u - \bar{u})(u - 1) - i4pu(u - 1) + i2p(2\alpha - 1)(u - \bar{u}) = 0. \quad (6)$$

Here  $u = w^2$  and  $p = |\kappa|/m$  is one of the non-dimensional problem parameters. The solutions of (6) can be expressed in the parametric form

$$u_s = 1 - 2(1 - 2\alpha) \cos \theta \exp i\theta, \quad (7)$$

$$p = \frac{(1 - 2\alpha) \sin \theta \cos \theta}{3(1 - 2\alpha) \cos^2 \theta - 2(1 - \alpha)}, \quad (8)$$

where  $\theta$  is the parameter varying through the range  $-\pi/2 < \theta < 0$ , so that the point  $u_s$  is in the upper half-plane. These relationships give the dependence  $u_s = F[\theta(p)]$  and allows one to find the positions of the stationary points  $u_s$  (or  $z_s$ ) except for the parameter  $p = \kappa/m$ .

### 3 Character of the motion of vortices near boundaries

It follows from (8) that there exist two or three stationary points depending on whether the angle of the river mouth is larger or smaller than  $\pi/2$  ( $\pi/2$  corresponds to the parameter  $\alpha_c = 1/4$ ).

If  $\alpha > \alpha_c$ , the parameter  $p$  has an upper limit and reaches its maximum as a function of  $\theta$

$$p^* = \frac{1}{2} (1 - 2\alpha) / \sqrt{2(1 - \alpha)(4\alpha - 1)} \quad (9)$$

at an interior point on the range  $-\pi/2 < \theta < 0$ . It should be noted that (9) defines the boundary for existence of a solution in the domain of  $\alpha$ ,  $p$  (see Fig. 3).

For each value of  $p$  in  $0 < p < p^*$  equation (8) has two solutions and consequently has two ordinary stationary points of the topological type which are defined by the behavior of the trajectories in the neighborhood of these points. As well as these two ordinary stationary points, there exists one more point  $W = 0$  corresponding to the vanishing of

$$|(dW/dz)^2/W| \sim |W|^{1-4\alpha}$$

at this point.

The topological type of the stationary points determines the possibility for emitting vortex pairs from the river mouth.

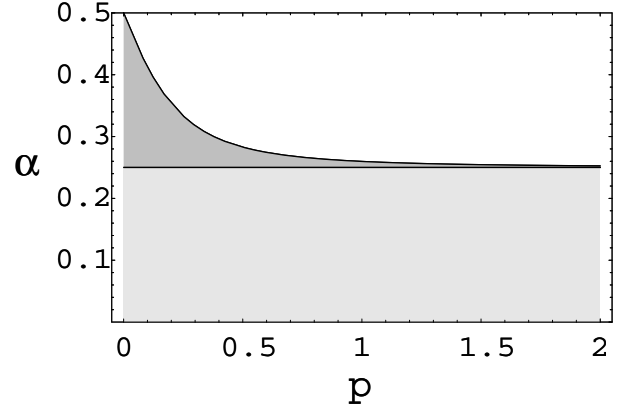


Fig. 3. Domain of solution existence in  $\alpha$ ,  $p$ -plane.

The dimensionless Hamiltonian  $\mathcal{H}$  ( $H = (4\pi)^{-1}m|\kappa|\mathcal{H}$ ) can be expressed in terms of the variable  $u$ . Here

$$\mathcal{H} = \frac{i}{4} \ln \left( \frac{u}{\bar{u}} \right) + p \ln |u - \bar{u}| + p(1 - 2\alpha) \ln |u - 1| + p \left( 2\alpha - \frac{3}{2} \right) \ln |u|, \quad (10)$$

Taylor's expansion (10) into a power series in the neighborhood of the simple stationary points,  $u_s$ , where  $(\partial\mathcal{H}/\partial u)_s = 0$ , shows that the topological type of the stationary points depends on the sign of the value

$$S = \left( \frac{\partial^2 \mathcal{H}}{\partial u \partial \bar{u}} \right)_s^2 - \left| \frac{\partial^2 \mathcal{H}}{\partial u \partial u} \right|_s^2 = \left( \frac{\partial^2 \mathcal{H}}{\partial u \partial \bar{u}} \right)_s^2 \frac{4(1 - 2\alpha) \sin^2 \theta}{|u_s|^2} [2\alpha(\cos^2 \theta + 1) - (1 + \sin^2 \theta)]. \quad (11)$$

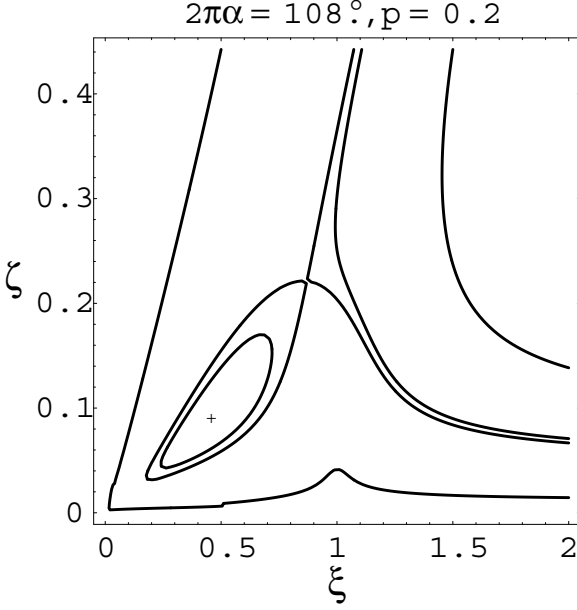
If  $S > 0$ , the stationary point is a center type, if  $S < 0$ , it is a saddle point.

If  $\alpha > \alpha_c$ , as already noted, for each fixed  $p$  in  $0 < p < p^*$  there exist two stationary points one of which is a center type the other is a saddle point. In the case of  $p = p^*$ , these points merge and disappear as might be expected when  $S = 0$ .

There is an additional critical point  $u = 0$ . The nature of this point is determined by the sign of  $\alpha - \alpha_c$ . This result follows from the fact that in the neighborhood of the source  $W = 0$ , when in polar coordinates  $W = \rho \exp(i\varphi)$ , the integral curves are described by the equation

$$\rho^{1-4\alpha} = C |\sin 2\varphi| \exp(\varphi/p) \quad (12)$$

where  $C$  is a integration constant. From (12), one can see whether or not vortices will be emitted from the vertex  $A_2$ , controlled only by the parameter  $\alpha$  and not by the parameter  $p$ . Thus, the vortex emission does not depend on whether a source is available at all. For the analyzed case ( $\alpha > \alpha_c$ ), by virtue of the fact that the phase trajectories have a parabolic



**Fig. 4.** Topologies of trajectories for  $\alpha > 1/4$ .

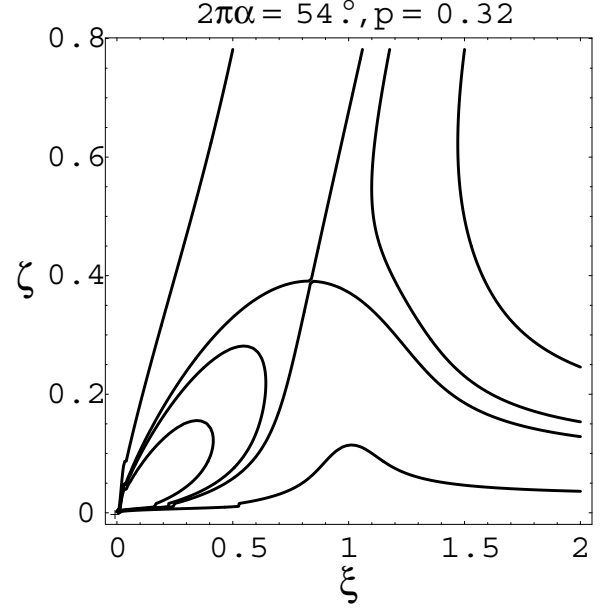
character in the sector  $0 < \varphi < \pi/2$ , the singular point  $w = 0$  is the saddle. Thus, for a wide river mouth ( $\alpha > \alpha_c$ ), the first scenario is impossible since the trajectory's topology in the neighborhood of the point  $A_2$  is of saddle type and hence the dipole is unable to leave this point for any parameter  $p$  (see in Fig. 4).

However, existence of the domain of closed trajectories when  $p < p^*$  indicates that in this case the second scenario turns out to be possible. Note that the domain shrinks to a point as the parameter  $p$  increases. Thus, if vortices increase their vorticity during formation, after reaching the threshold  $p^*$ , they will be found on the open trajectories along which they are able to leave the river mouth.

In the case of a narrow river mouth ( $\alpha \leq \alpha_c$ ) the topology of the trajectories is radically modified. In this case the critical center-type point, corresponding to closed trajectories, merges with the point  $w = 0$  in which the source is positioned, changing the topology in the neighborhood of  $w = 0$ . Being of saddle and center type before merging ( $\alpha \geq \alpha_c$ ), these singular points have indices  $-1$  and  $1$  respectively. Taking into account that there are precisely the same three centers in the remaining quadrants of the  $w$ -plane, after merging ( $\alpha \leq \alpha_c$ ) the index of the point  $w = 0$  becomes equal to  $4 - 1 = 3$ . As a result every quadrant of the  $w$ -plane contains a sector with elliptical trajectories which lie between two sectors with parabolic trajectories as shown in Fig. 5.

Thus, two kinds of trajectories are possible along which vortices emitted from the source can escape from the river mouth.

If vortices composing the dipole are tightly coupled, emitted vortices move along open trajectories, which asymptotically go into straight lines parallel to a bi-separatrix of



**Fig. 5.** Topologies of trajectories for  $\alpha < 1/4$ .

the river mouth angle, so that the dipoles abandon the river mouth. If dipole structures are loosely coupled, emitted vortices move along closed trajectories representing a family of super-inserted loops. Then the dipoles come back into the source without escaping from the river mouth.

#### 4 Estimates of the space scale of emitted vortex pairs

Existence of separatrix open trajectories, passing through the saddle point  $u_s \equiv u^*$ , allows us to obtain both an upper estimate for the size of  $b$  and a lower estimate of velocity  $v$  for dipoles escaping at large distances from the river mouth.

It is only necessary to calculate the rescaled Hamiltonian  $\mathcal{H}$  in the limiting case  $t \rightarrow \infty$  when the oppositely signed vortices propagate uniformly in the regime of vortex pairs with constant velocity  $v$  and mutual distance  $b$ . Because in this regime

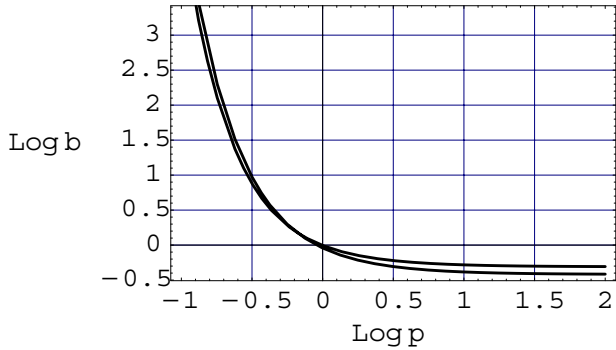
$$W \rightarrow \frac{1}{c} \left( \frac{b}{2} + i vt \right),$$

it is easy to verify directly from (3) that  $v = |\kappa| / (2\pi b)$ . Using this fact, from (10) we find the following estimation

$$\mathcal{H}|_{u^*} \equiv h^* = -\frac{\pi}{2} + p \ln 2 \frac{b}{c}. \quad (13)$$

Whence it follows that characteristic sizes of vortex pairs are given by the relationship

$$b = b(\alpha, p, l) = \frac{l}{2} \sin^{-1} \alpha \quad B^{-1}(\alpha, (3/2) - \alpha) \exp \left[ p^{-1} \left( \frac{\pi}{2} + h^* \right) \right], \quad (14)$$



**Fig. 6.** Upper bound of the dimensionless size  $b$  as a function of the parameter  $p$ .

where the function  $B(x, y)$  is the beta-function (Abramowitz and Stegun, 1964),

$$h^* = (i/4) \ln(u_s/\bar{u}_s) + p \ln|u_s - \bar{u}_s| + p(1 - 2\alpha) \ln|u_s - 1| + p(2\alpha - (3/2)) \ln|u_s|, \quad (15)$$

and the positions of  $u_s = u_s(p)$  are given by (8).

Corresponding plots of  $\log b$  versus  $\log p$  and  $v$  versus  $p$  are presented in Figs. 6, 7. In Figs. 6, 7 the intervortical distance is scaled with the mouth width  $l = 2a \sin \alpha$ , and velocity  $v$  is normalized on  $U/(\pi\alpha)$ , where  $U$  is a average velocity of the river flow.

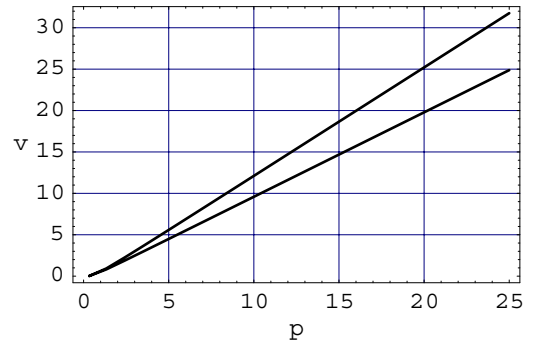
## 5 Conclusion

Characteristics of the motion of strong localized vorticity concentrations in a flow in presence of boundaries have been analyzed.

The main purpose of this work has been the construction of possible scenarios for the motion of vortex structures, their classification and estimations of geophysical parameters for naturally occurring dipole vortex structures emitted from river mouths toward the open ocean. The analyses have been made in the framework of a 2D point-vortex model with a polygonal boundary configuration.

The results of a quasi-qualitative analysis of the dynamics of vortex dipoles are presented. Based upon several possible mechanisms for generating similar vortex structures in a river mouth, two possible scenarios for the motion have been analyzed. In the all the scenarios the river inflow has been simulated by a point source of constant potency.

The first case supposes that the dipoles have been formed in the river channel and are then ejected into the river mouth. Qualitative analysis of the topology of possible trajectories of the vortices in neighborhood of the source shows that processes of this type may be realized only in narrow mouths with half-angle sufficiently small (in the framework of the model, less than  $\pi/4$ ). Two topologically different kinds of trajectories are thus possible for the vortex dipoles outgoing from the source. If the energy of interaction is more than the defined threshold value, the vortices move along open



**Fig. 7.** Lower bound of the dimensionless velocity  $v$  as a functions of the parameter  $p$ .

trajectories and leave the river mouth. Otherwise the vortices move along closed trajectories without escaping from the river mouth.

The second scenario supposes that, directly in the mouth, there is some mechanism for generating the vortex dipoles. For example, the mechanism can be provided by shore friction. However, no matter what mechanism is suggested, its efficiency depends on whether or not the vortices will have sufficient time to form before escaping from the river mouth. A similar possibility is found for a wide river mouth (for the model, with half-angle greater than  $\pi/4$ ). In this case, for each angle of the river mouth there exists a threshold of vortex intensity below which the dipole vortices move along closed trajectories. Under influence of a mechanism of intensification the vortices become increasingly strong and move along spirals until they reach a threshold intensity. After that they go into open trajectories and leave the river mouth.

The estimation of the upper size  $b$  for dipoles escaping at a relatively large distance from the river mouth has the form:  $b \sim l \exp[p^{-1}\mathcal{H}(u_s)]$ , where  $\mathcal{H}(u_s)$  is the Hamiltonian of the system calculated at the stagnation point.

We close by pointing out that a variety of similar dynamical systems exists.

Two-dimensional vortex pairs are observed in many phenomena. For example, the generation of 2D - vortex pairs obtained by pushing fluid down a semi-infinite channel was observed by Brown and Michael (1954). The growth of a double-layer secondary vortex and the formation of near-wedge vortex pairs has been illustrated in the paper of Pullin and Perry (1980). In the experiments of Maxworthy (1977) on the generation of vortex rings by the “puffing” technique (analogy exists between the plane and the axisymmetric cases) it was found that the behavior of the rings is caused by the interaction of formed vortex rings as well as by that of the walls. Sheffield (1977), bypassing the analysis of the process of vortex generation by pushing fluid down a semi-infinite channel by means of an impulsively started piston, calculated the trajectories of an ideal vortex pair near channel openings of different shapes. It was found that the two vortices will not travel back into the channel if their initial po-

sitions lie outside a region adjacent to the wall and bounded by two limiting trajectories. From the results, one can infer that vortices do not collide and break up near the axes of symmetry, only in the case when the distance of the vortex pair from the channel edges is large. The analysis of the phenomenon of vortex pair formation near orifices has been given in the paper by Blondeaux and De Bernardins (1983); attempting to explain the observed experimental results, it was assumed that viscous effects are significant only during the separation process and have negligible influence on the overall flow. In the limit of infinite Reynolds number, the problem becomes one of inviscid flow, large vortex regions being replaced by simple concentrated vortices (see, for example, Saffman, 1979).

*Acknowledgements.* This work was partially supported by the Russian Fundamental Research Foundation under grant No. 00-05-64019.

## References

- Abramovitz, M. and Stegun, I. A. (eds.), Handbook of Mathematical Functions, U. S. Govt. Printing Office, Washington D. C., 1964.
- Aref, H., Motion of three vortices, *Phys. Fluids*, 22, 393, 1979.
- Aref, H., Integrable, chaotic and turbulent vortex motion in two-dimensional flows, *Ann. Rev. Fluid Mech.*, 15, 345, 1983.
- Aref, H., Kadtke, J. B., Zawadzki, I., Campbell, L. J., and Eckhardt, B., Point vortex dynamics: recent results and open problems, *Fluid Dynamics Res.*, 3, 63, 1988.
- Batchelor, G. K., On study laminar flow with closed stream lines at the large Reynolds number, *J. Fluid Mech.*, 1, 177, 1956.
- Blondeaux, P. and De Bernardins, B., On the formation of vortex pairs near orifices, *J. Fluid Mech.*, 135, 111, 1983.
- Brown, C. E. and Michael, W. H., Effect of leading edge separation on the lift of a delta wing, *J. Aero. Sci.*, 21, 690, 1954.
- Fedorov, K. N. and Ginsburg, A. I., Mushroom-like currents (vortex dipoles): One of the most wide-spread forms of non-stationary coherent motions in the ocean, In *Mesoscale/Synoptic Coherent Structures in Geophysical Turbulence*, edited by J. C. Nihoul and B. M. Jamart, Elsevier, Amsterdam, 15, 1989.
- Ginsburg, A. I. and Fedorov, K. N., Mushroom-formed currents in ocean (on the base of analysis of satellite images), *Investigation of the Earth from the Space*, 3, 18, 1984.
- Goldshhtik, M., Hussein, F., and Shtern, V., Symmetry breaking in vortex-source and Jeffery-Hamel flows, *J. Fluid Mech.*, 232, 521, 1991.
- Goncharov, V. P. and Pavlov, V. I., Some remarks on physical foundation of the Hamiltonian description of fluid motions, *Europ. J. Mechanics B/Fluid*, 16, no. 4, 509, 1997.
- Goncharov, V. P. and Pavlov, V. I., Two-dimensional vortex motions of fluid in harbor-like basins at large Reynolds numbers, *Phys. Fluids*, 10, no. 9, 2384, 1998.
- Lavrentev, M. A. and Shabat, B. V., *Methods of theory of complex variable functions*, Nauka, Moscow, 1965.
- Lin, C. C., On the motion of vortices in two dimensions, I. Existence of the Kirchhoff-Routh function, *Proc. Nat. Acad. Sci. USA*, 27, 579, 1941.
- Lin, C. C., On the motion of vortices in two dimensions, Toronto Univ. Press, 1943.
- Maxworthy, T., Some experimental studies of vortex rings, *J. Fluid Mech.*, 81, 465, 1977.
- Meleshko, V. V. and van Heijst, G. J. F., Interacting two-dimensional vortex structures: point vortices, contour kinematics and string properties, *Chaos Solitons Fract.*, 4, 233, 1994.
- Milne-Thomson, L. M., *Theoretical hydrodynamics*, Vth edition, MacMillan, London, 1968.
- Pullin, D. I. and Perry, A. E., Some flow visualization experiments on the starting vortex, *J. Fluid Mech.*, 97, 239, 1980.
- Saffman, P. G., The approach of a vortex pair to a plane surface in inviscid fluid, *J. Fluid Mech.*, 92, 239, 1979.
- Saffman, P. G., *Vortex Dynamics*, Cambridge University Press, Cambridge, 1997.
- Sheffield, J. S., Trajectories of an ideal vortex pair near an orifice, *Phys. Fluids*, 20, 543, 1977.