Spatial dependence of the pairing gap in superfluid nuclei

E. Vigezzi*, F.Barranco†, R. A. Broglia*,**, A. Pastore* and G. Potel*

*INFN, Sezione di Milano and Dipartimento di Fisica, Università di Milano,
Via Celoria 16, 20133 Milano, Italy.
†Departamento de Fisica Aplicada III, Universidad de Sevilla, Escuela Superior de Ingenieros,
Sevilla, 41092 Camino de los Descubrimientos s/n,
Spain.
**The Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17,
2100 Copenhagen Ø, Denmark.

Abstract. The spatial structure of pairing correlations in ^{120}Sn is investigated making use of both the bare nucleon-nucleon potential and the interaction induced by the exchange of collective vibrations, taking into account self-energy effects. The resulting pairing gap is strongly peaked on the nuclear surface.

Keywords: pairing gap, many-body effects, induced interaction
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INTRODUCTION

In this work we report results of a study of the spatial dependence of the pairing gap of the nucleus ^{120}Sn, based on a microscopic approach. We shall first discuss the results obtained making use of the bare v_{14} Argonne nucleon-nucleon interaction, adding afterwards some of the most important medium polarization effects. Because our study aims at determining the basic features of the spatial dependence of the gap, rather than its precise magnitude, use is made of approximations to deal with some of these effects, in particular with self-energy effects, so as to gain in transparency in the presentation of the results. In order to make contact with other, more phenomenological approaches available in the current literature, we shall parametrize our results in terms of a density-dependent, zero-range interaction.

CALCULATIONS WITH THE BARE NUCLEON-NUCLEON FORCE

Contrary to the case of infinite matter, calculations in finite nuclei have seldom used a bare interaction in the pairing channel [1, 2, 3], although this represents the first step in a consistent many-body approach [4]. Here we shall present results obtained solving the pairing gap equation making use of the nucleon-nucleon v_{14} Argonne interaction. We shall start by performing a Hartree-Fock calculation with the two-body interaction SLy4. As expected, the absolute value of the pairing gap depends on the value of the effective mass m_k associated with this particular interaction. Of notice that to deal with

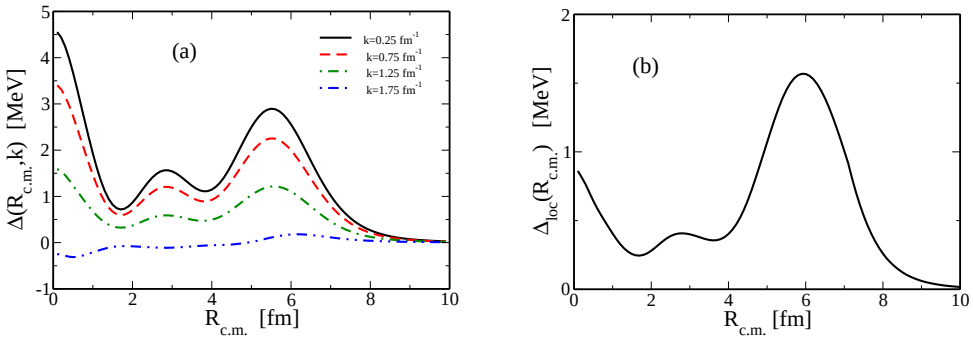

FIGURE 1. (a) Pairing field as a function of the position of the center of mass for different values of the relative momentum k calculated for the Argonne interaction. Going from top to bottom, the curves refer to values of k going from $k = 0.25$ fm^{-1} to $k = 1.75$ fm^{-1}, in steps of 0.5 fm^{-1}. (b) Local pairing field obtained with the semiclassical approximation.

the presence of a hard core in the pairing interaction, one must integrate the gap equation up to about 1 GeV, and the results will be sensitive to the density of single-particle states not only close to Fermi energy, but also at higher energies. On the other hand, the spatial dependence of the pairing gap turns out to be rather independent of the particular properties of m_k.

Instead of considering directly the pairing gap $\Delta(\vec{r},\vec{r}')$, it is better to consider its Fourier transform respect to the relative distance of the two nucleons, $\vec{r}_{12} = \vec{r} - \vec{r}\,'$, (see Fig. 1(a)):

$$\Delta(R_{c.m.},k) = \int dr_{12}\, e^{i\vec{k}\cdot\vec{r}_{12}} \Delta(R_{c.m.},r_{12}), \tag{1}$$

where $R_{c.m.}$ denotes the center of mass of the pair, and we have averaged over the angle between $\vec{R}_{c.m.}$ and $\vec{k}$. In this representation it is simple to separate the effect of the repulsive core, which affects the gap mostly at large values of the relative momentum k, where Δ can become negative. For small values of $R_{c.m.}$, Δ depends strongly on the contributions of the $l = 0$ orbitals. In particular, in ^{120}Sn the $3s_{1/2}$ orbital is close to the Fermi energy and this determines the large value of the gap close to $R_{c.m.} = 0$. The other dominant feature observed in Fig. 1(a) is the peak around $R_{c.m.} = 6$ fm, that is, close to the surface of the nucleus. For any given value of $R_{c.m.}$, the pairing gap decreases as a function of k: this is in keeping with the behaviour of the pairing gap $\Delta(k)$ at a given density in infinite matter [5], and is related to the behaviour of the experimental phase shift in the $^1S_0$ channel,, which decreases monotonously as a function of the relative momentum, until it becomes negative for $k \sim 1.7$ fm^{-1}. It is useful to make a semiclassical approximation, plotting the quantity $\Delta_{loc} \equiv \Delta(R_{c.m.},k_F(R_{c.m.}))$, where $k_F(R_{c.m.})$ denotes the local Fermi momentum. The result is plotted in Fig. 1(b), and clearly shows the surface character of the pairing gap with the Argonne interaction. A qualitatively similar result is obtained applying the local density approximation (LDA) directly to the pairing gap calculated in nuclear matter, shown in Fig. 2. In fact, the pairing gap at the Fermi energy is small for $k_F \sim 1.3$ fm^{-1}, corresponding to saturation density, and is maximum for $k_F \sim 0.9$ fm^{-1}, that is, for densities corresponding to that found at the nuclear surface; this behaviour is directly related to the momentum

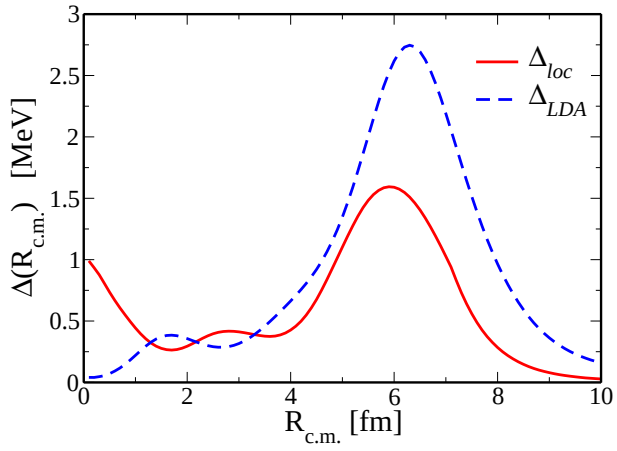

FIGURE 2. The semiclassical pairing gap already shown in Fig. 1(b) is compared to the gap obtained using the simple LDA.

dependence of the experimental phase shifts [6, 7] . However, the LDA produces a too pronounced surface peak.

CALCULATION INCLUDING MANY-BODY POLARIZATION EFFECTS

It is well known that single-particle motion is strongly renormalized by the coupling to collective vibration, which leads to (i) an increase of the level density close to the Fermi energy, and (ii) to a fragmentation of the single-particle strength. These phenomena in turn affect the pairing gap in different ways: while (i) enhances the gap, (ii) suppresses it. Moreover, a contribution to the gap arises from the exchange of collective vibrations between pairs of states in time reversal. The exchange of density and spin vibrations lead respectively to an attractive and to a repulsive pairing interaction [8]. The former is dominant in finite nuclei, where surface vibrations are strongly collective, while the latter is dominant in infinite neutron matter. A consistent treatment of these effects is obtained solving the Nambu-Gorkov equation [4, 9]. Here, we shall limit ourselves to a simple approximation, adding the contribution of the induced interaction to the matrix elements of the bare pairing interaction, multiplying the total matrix element, sum of the Argonne and of the induced interaction, by the average quasiparticle strength Z:

$$v_{Arg+ind} = Z(v_{Arg} + v_{ind}). \tag{2}$$

Using the value $Z = 0.7$, which is close to that obtained in theoretical calculations [10], we obtain a pairing gap at Fermi energy close to that deduced from the experimental odd-even mass difference ($\Delta_F \sim 1.3$ MeV). We then solve the gap equation as was done for the bare interaction. We include the coupling to the phonons of multipolarity up to $\lambda = 5$ and energy up to 30 MeV, calculated in the random phase approximation using the same SLy4 interaction used to calculate the mean field.

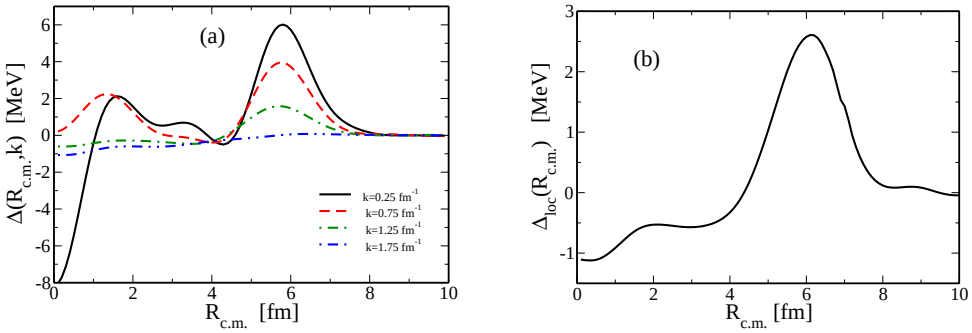

FIGURE 3. (a) Pairing field as a function of the position of the center of mass for different values of the relative momentum k, for the Argonne plus induced interaction $v_{Arg+ind}$. Going from top to bottom, the curves refer to values of k going from $k = 0.25$ fm^{-1} to $k = 1.75$ fm^{-1}, in steps of 0.5 fm^{-1}. (b) Pairing field obtained with the semiclassical approximation.

In Fig. 3(a) and 3(b) we show the pairing gap and its local approximation. It is seen that many-body effects enhance the surface character of the pairing interaction, due to the action of surface collective phonons. At the same time, the interaction within the volume is suppressed by the action of spin modes.

COMPARISON WITH PHENOMENOLOGICAL INTERACTIONS

Pairing correlations in finite nuclei are often parametrized making use of a simple phenomenological zero-range interactions supplemented with a cutoff procedure, as e.g.,

$$v_{pair}(\vec{r},\vec{r}') = v_0 \left(1 - \eta \left(\frac{\rho(r)}{\rho_0}\right)^{\alpha}\right) \delta(\vec{r}-\vec{r}'), \tag{3}$$

where ρ_0 is the saturation density, and include a density dependence (if $\eta \neq 0$). The abnormal density $\Phi(\vec{r},\vec{r}')$is then calculated and the pairing gap obtained as $\Delta(\vec{r},\vec{r}') = -v_{pair}\Phi$. The the spatial dependence of the abnormal density is largely independent of the specific pairing interaction, being dominated by a few orbitals around the Fermi surface [11]. As a consequence, the various choices of the parameters in Eq.(3) lead to pairing gaps with a very different spatial dependence. Recently, large-scale calculations have been performed, studying the dependence of the calculated binding energies on the parameters of the pairing interaction (3) [12, 13, 14]. The r.m.s. deviation between the gaps deduced from the experimental and theoretical odd-even mass differences is of the order of 0.3 MeV [14] and turns out to be rather insensitive to the particular density dependence assumed in Eq.(3), with a weak preference, however, for the surface interaction. It remains to be seen whether other observables, and in particular the cross sections of two-particle transfer reactions represent more sensitive probes [15]. We have determined the parameters α and η in Eq.(3) requiring that they reproduce in the best way the local pairing gap obtained in our microscopic approach, shown in Fig. 3(b). We have adopted a cutoff of 60 MeV above the Fermi energy, fixing the value of v_0 so as to reproduce the scattering length at zero density, as in ref. [16].

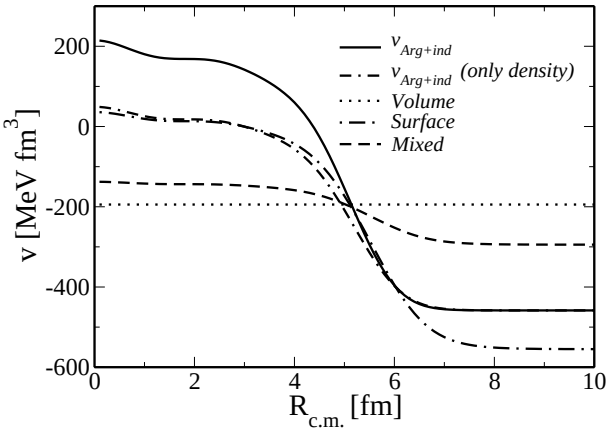

FIGURE 4. Spatial dependence of the contact pairing interaction introduced in this work to simulate the local pairing gap obtained with the bare+induced interaction $v_{Arg+ind}$. We also show the interaction obtained suppressing the coupling to spin modes. These results are compared to the spatial dependence of phenomenological interactions of volume, surface and mixed type.

The resulting interaction is displayed in Fig. 4, and corresponds to the parameters $v_0 = -458.4$ MeV fm^3, $\alpha = 2.0$, $\eta = 1.32$. We compare it to three schematic phenomenological interactions which are often used and have the form of Eq.(3) with $\alpha = 1$: a volume force ($\eta = 0$), a surface force ($\eta = 1$) and a mixed force ($\eta = 0.5$).The value of v_0 in the three cases has been determined imposing that the average value of the pairing field weighted with the nuclear density, be equal to 1.24 MeV using the cut-off described above [17]. It is seen that our microscopic interaction has a pronounced surface character and is repulsive in the interior of the nucleus. The latter feature is associated with the interaction induced by the spin modes: suppressing them, the interaction essentially vanishes in the nuclear volume, as also shown in Fig. 4.

CONCLUSIONS

We have presented a microscopic approach to the calculation of pairing correlations in ^{120}Sn, adding a bare nucleon-nucleon interaction and an induced interaction which takes into account the exchange of RPA vibrational modes, including self-energy effects in a simplified way. The resulting interaction is surface peaked, in keeping with the fact that the bare interaction is mostly attractive for low values of relative momentum, and that the exchange of collective surface modes gives the largest contribution to the induced interaction at the nuclear surface. While this result appears to be well established, to get a more reliable calculation of the magnitude of the gap, one should remove some of the simple approximations we have used to estimate self-energy effects, and go beyond the RPA in the calculation of the properties of nuclear vibrations.

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