FREEZING AND MELTING OF A BATH MATERIAL ONTO A CYLINDRICAL SOLID ADDITIVE IN AN AGITATED BATH

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Abstract

In melting and assimilation of a cylindrical shaped additive in an agitated hot melt bath during the process of preparation of cast iron and steel of different grades, an unavoidable step of transient conjugated conduction-controlled axisymmetric freezing and melting of the bath material onto the additive immediately after its dunking in bath occurs. Decreasing the time of completion of this step is of great significance for production cost reduction and increasing the productivity of such preparations. Its suitable mathematical model of lump-integral type is developed. Its nondimensional format indicates the dependence of this step upon independent nondimensional parameters- the bath temperature, θ_b the modified Biot number, B_{im} denoting the bath agitation, the property-ratio, B and the heat capacity-ratio, C_r of the melt bath-additive system, the Stefan number, S_t pertaining to the phase-change of the bath material. The model provides the closed-form expressions for both the growth of the frozen layer thickness, ξ onto the additive and the heat penetration depth, η in the additive. Both are functions of these parameters, but when they are transformed to the growth of the frozen layer thickness with respect to the heat capacity ratio per unit Stefan number; and the time per unit property-ratio, B, their expressions become only a function of single parameter, the conduction factor, C_{of} consisting of the parameters, B, B_{im} and θ_b. The closed-form expression for the growth of the maximum thickness of the frozen layer, its time of growth, the time of the freezing and melting; the heat penetration depth are also derived. When the heat penetration depth approaches the central

axis of the cylindrical additive in case of the complete melting of the frozen layer developed $C_{of} \lesssim$ 11/72. It is found that the decreasing C_{of} reduces both the time of this unavoidable step and the growth of the maximum frozen layer thickness and at $C_{of}=0$, the frozen layer does not form leading to zero time for this step. If the bath is kept at the freezing temperature of the bath material, only freezing occurs. To validate the model, it is cast to resemble the freezing and melting of the bath material onto the plate shaped additive. The results are exactly the same as those of the plate.

1. Introduction

Reducing the time of production with decrease in the cost of production without compromise of the quality of the cast iron and steel of a required grade produced is of prime importance for global competitiveness. Here, before following a prescribed route to manufacture these products, their melt of a desired composition is prepared by melting and assimilating solid additives in the hot melt bath after they are immersed in the bath. This process comprises different stages. In the first stage soon after the immersion of the additive in the bath, the contact interface between the surface of the additive and the melt bath attains an equilibrium temperature, T_e, the bath material freezes onto the surface of the additive and the heating of the additive initiates. Then the growth of the frozen layer with its subsequent melting occurs, the interface temperature builds up and the heat penetrates the solid additive. The second stage consists of further heating of the additive to its melting temperature after its emergence at an elevated temperature. The melting and assimilation of the additive in the bath take place in the third stage. These stages are completed in a certain time called production time and regulated by the temperatures of the bath and the additive, the bath condition, the thermo-physical properties of the bath-additive system and the geometry of the additive. It can be accomplished by decreasing the time of completion of the undesirable step of freezing and melting of the bath material onto the additive which always occurs due to the heat conducted to the additive far exceeding the convective heat supplied by the bath during the initial period of this process. The excess amount of conductive heat is compensated by latent heat of fusion released owing to the freezing of the bath material onto the additive. At the later time, the conductive heat to the additive becomes less than the convective heat supplied leading to the melting of the frozen layer by the excess of the convective heat. Consequently, for a specified melt bath-additive system, the convective heat available from the bath acts as a regulating factor to determine the time of completion of the freezing and melting.

In view of these, its high value permits the development of smaller frozen layer and decreases the time of freezing of this layer with its subsequent melting and, in turn, reduces the production time. This can be achieved once the bath is made highly agitated due to its giving high heat transfer

co-efficient and associated higher rate of convective heat transfer.

In the literature, study of such an occurrence which yields negligible thermal resistance of the frozen layer with respect to that of the cylindrical additive and is frequently encountered in manufacturing practices, is seldom reported. Nevertheless, the first stage for the plate [1], cylindrical [2] and spherical [3] shaped solid additives is investigated in case of the thermal resistances of the frozen layer developed on these additives are comparable with those of the additives. In this condition, it was stated that increasing the heat transfer co-efficient of the bath decreases the frozen layer thickness and the time of the first stage for the plate shaped [1] additive. These findings are implicit for the spherical additive [3]. For the cylindrical steel additive-liquid steel bath system [4] the experimental results exhibit that increasing the temperature before immersion of such an additive the maximum thickness of the frozen layer and the time of its growth and melting decrease when the heat supplied from the bath to the additive was by natural convection. Similar behavior is exhibited for increasing the diameter of the additive. In titanium cylindrical additive-liquid steel bath system [5]., numerical results indicate decreasing the time of freezing and melting of the bath material once heat transfer coefficient from the bath increases. For this additive-bath system, [5] comprehensive theoretical study for the growth of the frozen layer with its subsequent melting [6] was undertaken. It states that for a given diameter of the additive at a prescribed heat transfer coefficient from the bath to the additive, the

time of freezing and melting of the bath material reduces as the temperature of the additive before its immersion in the bath increases or diameter of the additive at its particular temperature and heat transfer coefficient decreases. The behavior is found to be similar for increasing heat transfer coefficient or bath temperature. A closed-form expression for the attainment of the instant interface temperature [7] between the cylindrical additive and the developing frozen layer of the bath material onto the additive soon after the immersion of the additive in the bath also appeared.

The present investigation is intended to develop a suitable mathematical model of the Lump-integral form for the freezing and melting of the bath material onto the surface of a cylindrical shaped additive dunked in an agitated bath. The thermal resistance of the frozen layer grown is of negligible thermal resistance with respect to that of the cylindrical additive. The model is non-dimensionalised exhibiting the dependence of this situation upon the independent parameters namely, the Biot number, B_i signifying the bath agitation, the Stefan number, S_t denoting the phase-change of the bath material, the bath temperature, θ_b and the property-ratio B of the melt bath-additive system and gives closed-form solutions for the frozen layer with its melting, its time of completion and the heat penetration thickness. In the solutions, these parameters appear only as a conduction factor, C_{of}. The expressions for the maximum frozen layer thickness, its time of growth, the total time of freezing and melting are obtained. The solution for only freezing of the bath material is also derived. For validation, the

solutions are reduced to those of the freezing and melting of the bath material onto the plate shaped additive giving exactly the same expressions as those of the plate.

2. Mathematical Model

Considering the above facts, this section develops an appropriate mathematical model. Here, a cylindrical shaped solid additive of radius r_a at a uniform temperature, T_{ai} less than its melting temperature, T_{af} is dipped in a hot melt bath of temperature, T_b greater than melting temperature, T_{af} of the additive with the freezing temperature, T_{mf} of the bath material smaller than T_{af} ($T_{mf} < T_{af}$) and in the additive-melt bath system shown in Fig.1, a temperature field $T_b > T_{af} > T_{mf} > T_{ai}$ is established. Immediately after the immersion of the additive in the bath, the bath material

Fig. 1. Schematic of freezing and melting of the bath material onto the cylindrical shaped solid oddditive in an agitated bath

freezes onto the cylindrical additive surface, the contact interface between the bath and the additive arrives at an elevated temperature, T_e, less than the melting temperature of the additive and the temperature gradients are set up both on the additive and bath sides of the interface. With passing of time, the interface temperature, Te rises, the frozen layer grows in thickness and heat penetrates in the solid until the heat conducted to the additive is more than the convective heat given by the bath. Once the rate of heat conduction equals the rate of convective heat supplied the growth of the frozen layer ceases. Beyond this time the convective heat becomes greater than the heat conducted to the additive through the frozen layer resulting in the melting of the frozen layer but the interface temperature, T_e and the heat penetration depth continue to increase. Eventually, the frozen layer completely melts leaving the cylindrical additive at an elevated temperature.

The heating of the additive and the freezing of the bath material around the additive are regulated by transient conjugated axisymmetric heat conduction. The dimensionless integral form of heat conduction equation governing the temperature field in the heated region of the additive can be written as
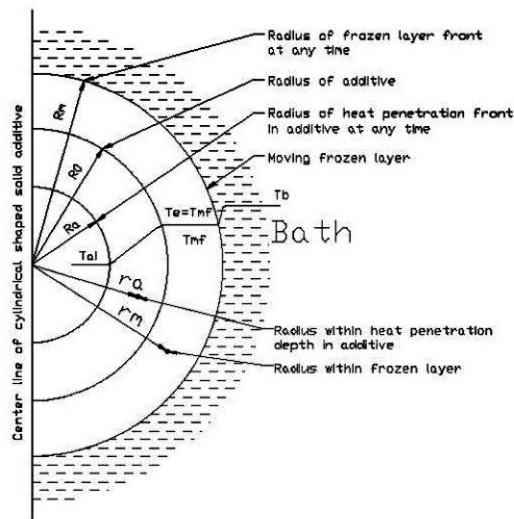
$$\frac{d}{d\tau}\int_{\eta}^{1}\xi_a\theta_a d\xi_a - [\theta_a\xi_a]_{\xi_a=\eta}\frac{d\eta}{d\tau} + [\theta_a\xi_a]_{\xi_a=1}\frac{d1}{d\tau}$$

$$= \frac{1}{B}\left[\xi_a\frac{\partial\theta_a}{\partial\xi_a}\Big|_{\xi_a=1} - \xi_a\frac{\partial\theta_a}{\partial\xi_a}\Big|_{\xi_a=\eta}\right] \qquad (1)$$

Its associated initial and boundary conditions are

$$\theta_a = 0, \ \eta < \xi_a < 1, \eta = 1, \tau = 0 \qquad (2)$$

$$\theta_a = \theta_e < \theta_{af} < 1, \ \xi_a = 1, \tau > 0 \qquad (3)$$

$$\frac{\partial \theta_a}{\partial \xi_a} \qquad \theta_a = 0, \ \xi_a = \eta, \tau > 0 \qquad (4)$$

Since the agitated bath is associated with a large value of heat transfer coefficient, it gives a large amount of convective heat $h(T_b - T_{mf})$ resulting in the requirement of a small amount of latent heat of fusion to compensate the deficient amount of heat due to the difference between the heat conducted to the cylindrical additive and the convective heat supplied by the bath. It is affected once the frozen layer of a very small thickness [1, 8] is grown. Such a situation provides the thermal resistance of the thin frozen layer insignificant with respect to the convective thermal resistance of the melt bath leading to the development of uniform temperature [9, 10, 11] in the entire thickness of the frozen layer. It is equal to the freezing temperature, T_{mf} of the bath material because the freezing front in contact with bath always remains at T_{mf}. Due to this reason, the contact interface temperature, T_e between the cylindrical additive and the frozen layer also assumes the freezing temperature, T_{mf} of the bath material. In view of these, the frozen layer behaves as a lump system that does not absorb or release the sensible heat. An energy balance employed to this lump makes the heat conducted to the additive equal to the sum of the latent heat of fusion evolved due to freezing and the convective heat provided by the bath. Its mathematical expression in nondimensional form becomes

$$\frac{1}{S_t} \frac{d\xi}{d\tau} + B_{im}(\theta_b - 1) = -Q_{mn},$$

$$\xi_m = \xi, \tau > 0 \quad \text{with} \quad BB_{im} = B_i \qquad (5)$$

The initial condition related to this equation is

$$\xi_m = C_r \qquad \tau = 0 \qquad (6)$$

The conjugating conditions at the interface between the frozen layer and the additive are

$$\frac{1}{B} \frac{\partial \theta_a}{\partial \xi_a} = -Q_{mn}, \qquad \xi_a = 1, \xi_m = C_r, \tau > 0 \qquad (7)$$

$$\theta_a = \theta_m = \theta_e = 1, \ \xi_a = 1, \xi_m = C_r, \tau > 0 \qquad (7a)$$

In writing equations (1) to (7a) associated with mathematical model of the present problem, the thermo-physical properties of the materials of the frozen layer and the additive are taken to be uniform but different. Moreover, the surface of the cylindrical additive is assumed to be in perfect contact with the surface of the frozen layer and there is no interface resistance between them. Choice of such assumptions is realistic since in the previous studies of freezing and the melting of the bath material onto the spherical [3] and plate shaped additive [1, 12] with comparative thermal resistances of the frozen layer with respect to the additive yielded accurate results. It may be noted that this nondimensional model indicates that the problem is dependent upon the independent parameters, the bath temperature, θ_b, the modified Biot number, B_{im}, the Stefan number, S_t, the heat capacity-ratio, C_r and the property-ratio, B of the bath-additive system.

3. Solutions

As the model of this problem is mathematically nonlinear due to the appearance of moving phase-change boundary denoted by equation (5) and is

coupled owing to the conjugating conditions at the interface represented by equations (7) and (7a), they do not allow the model to provide an analytical solution employing exact analyses available in the literature. Consequently, semi-analytical methods become important. One of such methods known as the integral method capable of yielding simple and close-form expressions for several heating and phase-change problems in the past [13, 14, 15] is applied. It requires the prescription of the temperature distribution in the heated region of the additive. Here, the distribution is assumed to be cubic,

$$\theta_a = \theta_e \left[1 - \frac{1 - \xi_a}{1 - \eta} \right]^3 \tag{8}$$

It satisfies the boundary conditions, equations (3) and (4) which also reduce the integral equation (1) to

$$\frac{d}{d\tau} \int_\eta^1 \xi_a \theta_a d\xi_a = \frac{1}{B} \left[\xi_a \frac{\partial \theta_a}{\partial \xi_a} \Big|_{\xi_a=1} \right] \tag{9}$$

Application of Eq.(8) to Eq.(9) leads to

$$\frac{d}{d\tau} \left[\theta_e \left\{ \frac{1 - \eta}{4} - \frac{(1 - \eta)^2}{20} \right\} \right] = \frac{3\theta_e}{B(1 - \eta)} \tag{10}$$

whereas the conjugating condition, Eq.(7) gives

$$\frac{3\theta_e}{B(1 - \eta)} = -Q_{mn} \tag{11}$$

Using Eq.(11), Eq.(5) takes the form

$$\frac{1}{S_t} \frac{d\xi}{d\tau} + B_{im}(\theta_b - 1) = \frac{3\theta_e}{B(1 - \eta)} \tag{12}$$

Since the interface temperature, θ_e in view of the description appeared earlier is one ($\theta_e = 1$), its application converts Eqs.(10) and (12) respectively, to

$$\frac{d}{d\tau} \left[\frac{1 - \eta}{4} - \frac{(1 - \eta)^2}{20} \right] = \frac{3}{B(1 - \eta)} \tag{13}$$

and

$$\frac{1}{S_t} \frac{d\xi}{d\tau} + B_{im}(\theta_b - 1) = \frac{3}{B(1 - \eta)} \tag{14}$$

Although these two equations are coupled due to the presence of η in them, their close examination indicates that they provide closed form expressions in terms of η. Here, Eq.(13) is rearranged to

$$\left[-\frac{1 - \eta}{4} + \frac{(1 - \eta)^2}{10} \right] \frac{d\eta}{d\tau} = \frac{3}{B} \tag{15}$$

giving readily the closed-form solution

$$\frac{\tau}{B} = \frac{(1 - \eta)^2}{24} - \frac{(1 - \eta)^3}{90} \tag{16}$$

It satisfies the initial condition, Eq.(2) and states that the heat penetration depth (1- η) in cylindrical additive is an inverse function of time, τ. To solve Eq.(14), it is combined with Eq.(13) leading to

$$\frac{1}{S_t} \frac{d\xi}{d\tau} + B_{im}(\theta_b - 1) = \frac{d}{d\tau} \left[\frac{1 - \eta}{4} - \frac{(1 - \eta)^2}{20} \right] \tag{17}$$

Its rearranged form becomes

$$\frac{d}{d\tau} \left[\frac{1 - \eta}{4} - \frac{(1 - \eta)^2}{20} - \frac{\xi}{S_t} \right] = B_{im}(\theta_b - 1) \tag{18}$$

Its solution becomes

$$\frac{1 - \eta}{4} - \frac{(1 - \eta)^2}{20} - \frac{\xi - C_r}{S_t} = B_{im}(\theta_b - 1)\tau \tag{19}$$

Note that it fulfils the initial condition, Eq.(6). Substitution of Eq.(16) changes it in terms of only heat penetration depth.

$$\xi^* = \frac{\xi - C_r}{S_t} = -BB_{im}(\theta_b - 1) \left[\frac{(1 - \eta)^2}{24} - \frac{(1 - \eta)^3}{90} \right] + \left[\frac{1 - \eta}{4} - \frac{(1 - \eta)^2}{20} \right] \tag{20}$$

Eqs.(16) and (20) represent, respectively, the closed-form solutions for the heat penetration depth in the cylindrical additive and the growth of the frozen layer around the surface of the additive.

3.1 Maximum Thickness of the Frozen Layer and its Growth Time

To determine the growth of the maximum frozen layer thickness and its time of occurence, $\frac{d\xi^*}{d\tau}$ is made zero. As Eq.(20) is a function of η only, $\frac{d\xi^*}{d\tau}$ is replaced by $\frac{d\xi^*}{d\eta} \cdot \frac{d\eta}{d\tau}$ and is then made zero. Using Eq.(16)

$$\frac{d\eta}{d\tau} = B\left[-\frac{1}{12}(1-\eta) + \frac{1}{30}(1-\eta)^2 \right]^{-1} \quad (21)$$

whereas Eq.(20) leads to

$$\frac{d\xi^*}{d\eta} = -BB_{im}(\theta_b - 1)\left[-\frac{(1-\eta)}{12} + \frac{(1-\eta)^2}{30} \right] + \left[-\frac{1}{4} + \frac{(1-\eta)}{10} \right] \quad (22)$$

In view of the above application of Eq.(21) and (22) to

$$\frac{d\xi^*}{d\tau} = \frac{d\xi^*}{d\eta} \cdot \frac{d\eta}{d\tau} = 0 \quad (23)$$

provides

$$-BB_{im}(\theta_b - 1)\left[-\frac{(1-\eta)}{12} + \frac{(1-\eta)^2}{30} \right] + \left[-\frac{1}{4} + \frac{(1-\eta)}{10} \right] = 0 \quad (24)$$

It results in the condition

$$BB_{im}(\theta_b - 1) = \frac{3}{(1-\eta)} \quad (25)$$

for the maximum growth of the frozen layer thickness as it satisfies the requirement of $\frac{d^2\xi^*}{d\tau^2} < 0$ for the maximum growth. It is

transformed to

$$1 - \eta = 3C_{of} \quad (26)$$

where,

$$C_{of} = \frac{1}{BB_{im}(\theta_b - 1)} = \frac{1}{B_i(\theta_b - 1)} = \frac{K_a(T_{mf} - T_{ai})}{hR_0(T_b - T_{mf})} \quad (27)$$

It is known as conduction factor and signifies the ratio of the heat conduction to the cylindrical additive, $K_a(T_{mf} - T_{ai})/r_a$ caused by the temperature difference of the freezing temperature of the bath material and the initial temperature of the additive and the convective heat, $h(T_b - T_{mf})$ supplied by the bath. It lies between 0 to ∞ ($0 \le C_{of} \le \infty$). Zero represents the preheated additive at the freezing temperature of the bath material resulting in no conductive heat transfer to the additive and there is no formation of the frozen layer whereas ∞ is indicative of the bath at the freezing temperature of the bath material leading to non availability of the convective heat from the bath. Owing to this situation, only freezing occurs onto the surface of the additive immersed in the bath at its initial temperature, T_{ai}. Using Eq.(26) in Eq.(20) yields

$$\xi^* = -\frac{1}{C_{of}}\left[\frac{(1-\eta)^2}{24} - \frac{(1-\eta)^3}{90} \right] + \left[\frac{1-\eta}{4} - \frac{(1-\eta)^2}{20} \right] \quad (28)$$

whereas application of Eq.(26) to Eq.(20) yields the maximum thickness of frozen layer

$$\xi^*_{max} = 3C_{of}\left[\frac{1}{8} - \frac{1}{20}C_{of} \right] \quad (29)$$

Time of this maximum growth is readily obtained once Eq.(26) is employed in

Eq.(16)

$$\frac{\tau_{max}}{B} = 3C_{of}^2 \left[\frac{1}{8} - \frac{1}{10}C_{of} \right] \qquad (30)$$

3.2 Total Time Of Freezing And Melting Of The Bath Material, τ_t :

At this time τ_t , the frozen layer grown melts completely giving $\xi^* = 0$, and the heat penetrates the additive through the thickness $1 - \eta_t$. . Applying them to Eq.(28) yields

$$0 = -\frac{1}{C_{of}} \left[\frac{(1-\eta_t)^2}{24} - \frac{(1-\eta_t)^3}{90} \right] + \left[\frac{(1-\eta_t)}{4} - \frac{(1-\eta_t)^2}{20} \right]$$

It is cast in the form

$$4(1-\eta_t)^2 - (18C_{of} + 15)(1-\eta_t) + 90C_{of} = 0 \qquad (31)$$

which is a quadratic equation in $(1-\eta_t)$. Its solution can be written as

$$(1-\eta_t) = \frac{(18C_{of} + 15) \pm \sqrt{(18C_{of} + 15)^2 - 1440C_{of}}}{8} \qquad (32)$$

As $(1-\eta)$ remains below one $\{(1-\eta_t) \le 1\}$ for the heat not penetrating beyond the central axis of the cylindrical additive, the root from the Eq.(32) for any value of C_{of} can be selected such that $(1-\eta_t) \le 1$. When it is employed in Eq.(16), the total time of freezing and melting, τ_t is obtained. To determine the range of values of C_{of} with respect to these conditions of $\xi^* = 0$ and $(1-\eta_t) \le 1$ Eq.(28) is again considered. Here, for $\xi^* = 0$ it gives

$$C_{of} = \left\{ \frac{(1-\eta_t)}{24} - \frac{(1-\eta_t)^2}{90} \right\} \Big/ \left\{ \frac{1}{4} - \frac{(1-\eta_t)}{20} \right\} \qquad (33)$$

To satisfy the condition of $(1-\eta_t) \le 1$, it becomes $C_{of} \le (11/90)/(4/5)$ which leads to

$$C_{of} \le \frac{11}{72} \qquad (34)$$

It is noted that $C_{of} \le \frac{11}{72}$ signifies the value of C_{of} for which the heat penetration depth reaches the central axis, $(1-\eta_t) = 1$ of the cylindrical additive and the frozen layer grew completely melts $(\xi^* = 0)$. For this condition, the maximum growth of the frozen layer can readily be obtained from Eq.(29) once Eq.(34) is employed.

$$\xi_{max}^* \le \frac{11x169}{24x1440} \cong \frac{1}{20} \qquad (35)$$

3.3 Limiting case: Freezing without subsequent melting

When the bath temperature is maintained at the freezing temperature of the bath material, no convective heat is available from the bath owing to which the heat conducted to the additive is met only by the latent heat of fusion released due to freezing of the bath material. It results in freezing the bath material only and makes the conduction factor, C_{of} infinity $(C_{of} \to \infty)$. Using it in Eq.(28) provides the growth of the frozen layer thickness

$$\xi^* = \frac{1-\eta}{4} - \frac{(1-\eta)^2}{20} \qquad (36)$$

when the heat penetration approaches the central axis of the cylindrical additive, $(1-\eta) \le 1$.

This equation gives the development of the frozen layer thickness

$$\xi^* \le \frac{1}{5} \qquad (37)$$

It is noted that for this condition of , $(1-\eta) \le 1$

the growth of the maximum frozen layer thickness when the freezing and melting of the bath material takes place due to availability of the convective heat from the bath remains always less than Eq.(37)

4. Validation

To confirm the efficacy of the model, the present problem is converted to that of the freezing of the bath material onto the cylindrical [7] and plate [12] shaped additives immediately after the immersion of these additives in the bath. In such a situation a thin frozen layer of the bath material onto the additive grows and heat penetrates the additive simulating the present problem. Here, the heat penetration depth obtained in Eq.(16) is exactly the same as that appeared in [7]. For comparing with the plate shaped additive the heated annular portion of the cylindrical additive due to the assumption of its radius-ratio, $\eta \rightarrow 1$ and the heat penetration thickness, $1 - \eta = t_h \rightarrow 0$ resembles a plate and Eq.(16) pertaining to this penetration depth becomes

$$\frac{\tau}{B} = \frac{t_h^2}{24} - \frac{t_h^3}{90} \tag{38}$$

Applying the order of magnitude analysis [11] of various terms with respect to each other gives $0(t_h^3) << 0(t_h^2)$ and Eq.(38) reduces to

$$\frac{\tau}{B} = \frac{t_h^2}{24} \tag{39}$$

It truly represents the heat penetration thickness in the plate. Note that it is the same expression reported in the previous study for plate additive [12].

5. Results and Discussion

A Lump-inetgral nondimensional form of mathematical model for transient conjugating axisymmentric freezing and melting of the bath material around the surface of the cylindrical additive immediately after its immersion in the agitated bath is evolved. It exhibits that this process is controlled by independent parameters namely, the bath temperature, θ_b the modified Biot number, B_{im}, the property-ratio B and heat capacity-ratio, C_r of the melt-bath additive system and the Stefan number, S_t. The modified Biot number represents agitation of the bath which increases the convective heat available from the bath to the additive. The bath temperature, θ_b signifies the thermal potential of the bath. Its increasing value aids to increase this heat transfer. The Stefan number which is the ratio of the sensible heat and the latent heat of fusion of the phase-changing bath material is a phase-change parameter. Its high value is indicative of the material of low latent heat of fusion leading to growth of large thickness of frozen layer for the same convective heat supplied from the bath. The property-ratio, B denotes the thermal force of the bath. Its lower value increases this force to transfer more heat to the additive whereas the large heat capacity-ratio, C_r permits the storage of less heat in the heated region of the additive with respect to that in the frozen layer. In the closed-form solution, B, B_{im} and θ_b appear together as a single parameter, the conduction factor, C_{of}. It varies from zero to infinity ($0 \le C_{of} \le \infty$). $C_{of}=0$ states that there is no conductive heat transfer to the additive owing to which the

freezing does not take place. Consequently, the first step of this freezing and melting get vanished. Such a condition is arrived once the additive is preheated to the freezing temperature of the bath material before its immersion or the bath is made highly agitated. $C_{of} = \infty$ implies nonavailabilty of the convective heat from the bath. It is achieved by maintaining the bath at the freezing temperature of the bath material. Here, the conductive heat transferred to the additive after its immersion in the bath is provided by only latent heat of fusion evolved owing to freezing of the bath material. As a result, the frozen layer continues to grow without its subsequent melting. These facts allow to select C_{of} close to zero for the growth of a smaller frozen layer thickness in order that much less time needed for the freezing of this layer along with its melting. The values of independent parameters stated above for different cylindrical additive-bath systems employed in the manufacture of the cast iron and steel are also presented in Table1.

5.1 Effect of the conduction factor, C_{of}

The closed-form solution for the frozen layer thickness converted to the frozen layer thickness with respect to the heat capacity-ratio per unit Stefan number $[\xi^* = (\xi - C_r)/S_t]$ with the time taken per unit property-ratio, τ/B becomes only a function of the conductor factor, C_{of} Eq.(28). The behavior of the growth of such a frozen layer thickness and the heat penetration with time, τ/B is exhibited in Fig.2 for different values of C_{of}. The Fig.2 denotes that for each of C_{of}, the freezing with melting takes a parabolic shape. The height of its apex represents the growth of the maximum frozen layer thickness whereas the left of the apex gives the time of growth of the frozen layer thickness and its right corresponds to the time of melting of this frozen layer. Moreover, the frozen layer builds up with much faster rate than that of its melting. For $C_{of} = 11/72$, this built up takes 25.3% of the total time of the freezing and melting whereas the melting of the frozen layer is completed in 74.7% of this total time. These imply that the time ratio of the growth of the frozen layer and the melting of this layer is approximately 1/3:3/4. It is found that this

Table1: Thermo-physical properties of typical Bath- Solid system [3, 9, 11, 16] and their dimensionless parameters.

Bath Material							Solid Additive						Non-Dimensional parameters				
	K_m	ρ_m	C_{pm}	$L_m \times 10^{-3}$	T_{mf}	T_b		K_a	ρ_a	C_{pa}	T_{ai}	R_0	θ_b	B	$*B_i$	C_{of}	S_t
	W/mk	Kg/m³	J/kgk	J/ kg	⁰C	⁰C		W/mk	Kg/m³	J/kgk	⁰C	m					
Cast- Iron 4%C [11]	51.9	7304	417	275.7	1433	1550	Ni [9]	90	8906	449.5	35	0.025	1.084	0.438	2.22	5.38	2.17
Hot-Metal [16]	35	6850	670	275.7	1150	1500	Ferro-Magnese [16]	7.5	7200	700	35	0.025	1.314	4.25	26.66	0.12	2.79
Slag [3]	1.063	2890	920	544	1500	1600	DRI [3]	2.13	2600	820	35	0.025	1.068	0.622	93.9	0.16	2.54

Based on heat transfer co-efficient, h= 8000 $Wm^{-2}k^{-1}$ and R_0= 0.025m.
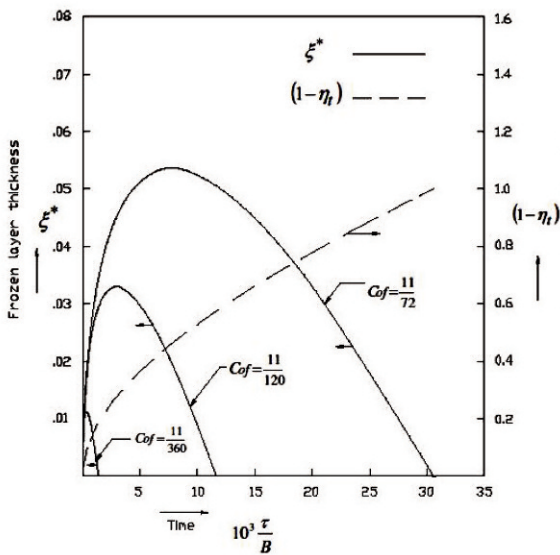
Fig. 2. Effect of conduction factor, C_{of} on the time of development of the frozen layer thickness, ξ' of the bath material onto the cylindrical shaped additive alongwith total time of freezing and melting and the heat penetration depth, $(1-\eta)$ in the additive

ratio remains almost unaltered for all the values of C_{of} ($0 < C_{of} \leq 11/72$) taken. Similar result was obtained in an earlier investigation pertaining to the same process of the freezing and melting onto the plate shaped additive [1] with the cubic temperature profile taken in the heated portion of the plate. Decreasing C_{of} provides smaller parabola and reduces both the total time of the freezing and melting and the time for the growth of the maximum frozen layer thickness, Eq.(30). Also, the height of the apex of the parabola signifying the maximum thickness of the frozen layer gets shortened, Eq.(29), with the apex moving towards the zero time. These predictions can be corroborated from the following facts. For a specified initial thermal condition of the additive, a certain amount of heat is conducted to, Eq.(27), the additive whereas at a prescribed bath

temperature, reduction in C_{of} increases the convective heat supplied by the bath resulting in increase in the Biot number and the agitation of the bath. Since the heat conducted to the additive is met by the convective heat available from the bath and the latent heat of the fusion released due to the freezing of the bath material onto the additive, the availability of the increased convective heat due to reduced C_{of} needs smaller amount of latent heat of fusion to match the above required conductive heat. Consequently, a smaller thickness of the frozen layer is built up, Fig.2. With respect to time, the behavior of the heat penetration depth, η is nonlinear and it lies on the same plot for all C_{of} ($0 < C_{of} \leq 11/72$), Fig.2. Decreasing C_{of} decreases η at the time of the completion of the freezing and melting of bath material.

Fig.3 displays the behavior of the total time τ_t/B of the freezing and melting of the bath material, the time, τ_{max}/B of the
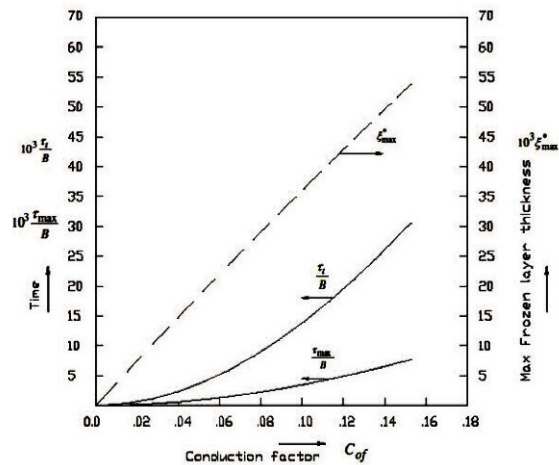

Fig. 3. Variation of maximum frozen layer thickness ξ_{max}, its growth time τ_{max}/B and total time of freezing with its subsequent melting τ_t/B with conduction factor, C_{of}

growth of the frozen layer to its maximum thickness and the maximum frozen layer thickness, with the conduction factor $C_{of} \leq 11/72$. It is observed that these times increase with a faster rate as C_{of} increases beyond its zero value whereas the growth of the values of the maximum frozen layer thickness shows almost linear feature although in reality it follows the nonlinearity behavior, Eq.(29).

6. Conclusion

Lump-integral formulation in nondimensional form for the transient axisymmetric freezing and melting of the bath material onto the cylindrical shaped solid additive developed provides the independent parameters controlling this occurrence and gives closed-form solutions for the time variant growth of the frozen layer along with its melting and the heat penetration thickness. With the skillful arrangements of these parameters, the solutions become dependent upon the conduction factor, C_{of}. It is predicted that the reduction in C_{of} decreases both the growth of the maximum thickness of the frozen layer and the total time of freezing and melting. This occurrence almost vanishes with no growth of the frozen layer when the bath is made extremely agitated, $C_{of} \rightarrow 0$. For the radius-ratio of the heated annulus formed in the cylindrical additive in the above occurrence is of order of unity, the expressions of the present solutions become those of plate shaped additive validating the present analysis.

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Appendix - Nomenclature

R_0 radius of the cylindrical shaped solid additive, m

B property ratio, $(K_m C_m / K_a C_a)$

B_i Biot number, $(h R_0 / K_a)$

B_{im} Modified Biot number, $(h R_0 / K_a)*(K_a C_a / K_m C_m)$

C_a heat capacity of the cylindrical additive, $Jm^{-3}K^{-1}$

C_m heat capacity of the frozen layer, $Jm^{-3}K^{-1}$

C_{of} conduction factor, $1/B_i (\Theta b - 1)$

R_a radius of the heat penetration front in the additive at any time, m

R_m radius of the frozen layer front onto the additive at any time, m

h heat transfer coefficient, $Wm^{-2}K^{-1}$

K_a thermal conductivity of the additive, $Wm^{-1}K^{-1}$

K_m thermal conductivity of the frozen layer, $Wm^{-1}K^{-1}$

L_m latent heat of fusion of the frozen layer, JKg^{-1}

Q_m heat transfer from the frozen layer to the additive, Wm^{-2}

Q_{mn} non-dimensional heat transfer from the frozen layer to the additive, $(Q_m R_0 / K_a T_{mf})/B$

S_t Stefan number, $C_m(T_{mf} - T_{ai})/L_m \rho_m$

t time, s

t_{mc} time for freezing and its subsequent melting, s

T temperature, K

T_a temperature of the additive, K

T_{ai} initial temperature of the additive, K

T_{af} freezing or melting temperature of the additive, K

T_b bulk temperature of the bath material, K

T_e instant equilibrium temperature at the interface between the additive and the frozen layer, K

T_{em} instant equilibrium temperature at the interface between the additive and the frozen layer when the frozen layer completely melted, K

T_{mf} freezing or melting temperature of the frozen layer, K

r_a radius within the heat penetration region in the additive, m

r_m radius within the frozen region, m

α_a thermal diffusivity of the additive, $m^2 s^{-1}$

α_m thermal diffusivity of the frozen layer, $m^2 s^{-1}$

ξ non-dimensional radius of the frozen layer front at any time, $(C_m R_m / C_a R_0)$

ξ_a non-dimensional radius in the heat penetration region in the additive, (r_a / R_0)

ξ_m non-dimensional radius within the frozen layer region, $(C_m r_m / C_a R_0)$

η non-dimensional radius of the heat penetration front in the additive at any time, (R_a / R_0)

ρ_m density of the frozen layer, Kgm^{-3}

θ non-dimensional temperature, $(T - T_{ai} / T_{mf} - T_{ai})$

θ_a non-dimensional temperature of the additive at any time, $(T_a - T_{ai} / T_{mf} - T_{ai})$

θ_{af} non-dimensional freezing or melting temperature of the additive, $(T_{af} - T_{ai} / T_{mf} - T_{ai})$

θ_b non-dimensional bulk temperature of the bath material, $(T_b - T_{ai} / T_{mf} - T_{ai})$

θ_e non-dimensional instant equilibrium temperature at the interface between the additive and the frozen layer, $(T_e - T_{ai} / T_{mf} - T_{ai})$

τ non-dimensional time, $(K_m C_m / C_a^2 R_0^2)t$

τ_n non-dimensional time for heating of the additive without freezing of the bath material, $(\alpha_a t / R_0^2)$