The Effect of Modeling Qualities, Tones and Gages in Ceramic Supply Chains' Master Planning

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Ceramic production processes are characterized by providing quantities of the same finished goods that differ in qualities, tones and gages. This aspect becomes a problem for ceramic supply chains (SCs) that should promise and serve customer orders with homogeneous quantities of the same finished good. In this paper a mathematical programming model for the centralized master planning of ceramic SC is proposed. Inputs to the master plan include demand forecasts in terms of customer order classes based on their order size and splitting percentages of a lot into homogeneous sub-lots. Then, the master plan defines the size and loading of lots to production lines and their distribution with the aim of maximizing the number of customer orders fulfilled with homogeneous quantities in the most efficient manner for the SC. Finally, the effect of modeling qualities, tones and gages in master planning is assessed.
Keywords: *Ceramic Supply Chains, Mathematical Programming Model, Qualities, Tones, Gages, Lack of Homogeneity in the Product*

1 Introduction

Lack of Homogeneity in the Product (LHP) appears in those productive processes which include raw materials that directly originate from nature and/or production processes with operations which confer heterogeneity to the characteristics of the outputs obtained, even when the inputs used are homogeneous. LHP appears in certain industries like ceramics, textile, wood, marble, tanned hides and leather goods, and it becomes a problem when the customer needs to be served with homogeneous units of one same product [1]. These companies are obliged to include a classification stage [2] whose localization in the production process depends on each industry. This is true to the extent that the various homogeneous quantities available of one same product are known only after finalizing each classification stage, and not beforehand. The classification criterion used differs from one industry to another [1]. For instance, in the furniture sector, color and grain sorting of furniture parts is an important manufacturing step where color uniformity has an impact on the value of final products [3]. In the horticulture sector, important criteria for sorting and grading fresh fruit are size, weight, ripeness, damages, color, shape and firmness [4].

LHP in ceramic supply chains (SCs) implies the existence of units of the same finished good (FG) in the same lot that differ in the aspect (quality), tone (color) and gage (thickness) [1,5] that should not be mixed to serve the same customer order. The usual consideration of three qualities, two tones and three gages causes the existence of thirteen different subtypes of the same model (FG). This fact increases the volume of information and makes the system management more complex. Additionally, the customers from this type of companies tend to request quantities of different FGs in one same order, and they also require that the units of one same FG in the order are homogeneous. This is because ceramic pavings and coverings must normally be placed and presented together, so their appearance needs to be homogeneous.

However, the real homogeneous quantities of each subtype in a FG lot will not be known until their production was finished. Not to know the homogeneous quantities available of the same FG to be promised to customers

proves to be a problem when customers' orders have to be committed, reserved and served from homogeneous units available derived from the planned production. Furthermore, not to accomplish with this homogeneity requirement can lead to returns, product and company image deterioration, decreasing customer satisfaction and even lost of customers.
The order promising process (OPP) plays a crucial role in customer requirements satisfaction [6] and, therefore, in properly managing the special LHP characteristics. The OPP refers to the set of business activities that are triggered to provide a response to customer order requests [7]. This process requires information about available-to-promise (ATP) quantities, i. e. the stocks on hand or projected inflows of items stocked at the customer order decoupling point (already in transit or planned by the master plan) that has not yet been allocated to specific orders and thus can be promised to customers in the future. Because one of the main inputs to the OPP is the master plan, the objective of this paper is to define a master plan that anticipate LHP features and can provide this process with reliable information about future available homogeneous quantities.
The paper is structured as follows. Section 2 describes the problem under consideration. Section 3 presents the mixed integer linear programming model proposed for the centralized master planning of ceramic SCs that explicitly takes into account LHP. Section 4 reports the methodology followed for the model validation. In this section a comparison between results obtained from master planning in ceramic SC with and without LHP is made. Finally, Section 5 states the conclusions derived from the obtained results and future research lines.

2 Problem Characteristics

In this paper, we consider the master planning problem for replenishment, production, and distribution in ceramic tiles SCs with LHP. The characteristics of the problem under study are the same as in [8] but with relevant differences introduced by the LHP consideration. In the following paragraphs the main features of the problem addressed are describe, highlighting those novel aspects introduced by LHP consideration.
These ceramic SCs are assumed to be multi-item, multi-supplier, multi-facility, multi-type and multi-level distribution centers. For the problem under consideration, it is assumed that the possibilities of flow between the nodes of the various stages (arcs), as well as the parts, components, raw materials (RMs), and FGs that might circulate through them, have been considered beforehand. The existence of several production plants situated in various geographical locations is also assumed. These production plants are supplied with various RMs provided by different suppliers with a limited supply capacity. This represents the total capacity of the supplier assigned to the SC under study because it is assumed that RM suppliers may supply production plants belonging to other SCs. Each production plant has one or several production lines (processors in parallel) with a limited capacity. Different FGs can be processed by each production line. There are FGs with high added values that are manufactured only in production plants; others may be partly subcontracted, while some may be totally subcontracted to external suppliers (normally products with a low added value). FGs are grouped into product families for production and commercial reasons. A product family is defined as a group of FGs of identical use (flooring or coverings), format (size), grout (white or red), and whose preparation on production lines is similar. This is done to minimize setup times and costs. Changeovers from one product family to the next incur setup costs owing to the time spent in changing, for instance, moulds. Lines may not be standardized, in which case each product family can be processed according to specific facilities with the appropriate technical features. Therefore, not all production lines are capable of processing all the product families, although the product families that may be processed on each line are known. Given the important setup times between product families on production lines, production

within a minimum number of consecutive time periods should be carried out whenever a production line is set up for a specific product family (minimum run length). Item setups among the products belonging to the same product family also exist. Because of technological factors involved in the production process itself, when a certain product is manufactured on a specific line, it should be produced in an equal or greater amount than the minimum lot size. This is partly because a certain defects occur during the production process, and only a percentage of the manufactured items may be sold as first quality FG. Furthermore, in this paper, it is assumed that for first quality quantities of the same FG different tones and gages can appear in the same lot. That is, the LHP real characteristic is taking into account in the master plan.

In the majority of the production planning models developed at the tactical level, the capacities at each stage are aggregated and setup changes are not explicitly considered. However, if at this level the setup times involve an important consumption capacity and have been completely ignored this may lead to an overestimation of the real capacity availability which, in turn, may lead to unfeasible events during the subsequent disaggregation of tactical plans. Considerable savings may also be achieved through optimum lot-sizing decisions. However, accounting for setup times at the tactical level would mean simultaneously including decisions about the allocation and lot sizing of production. This problem is known as the capacitated lot-sizing and loading problem (CLSLP) [9]. Given the lengthy setup times involved in the manufacturing of ceramic floorings and coverings, these setup times need to be considered at the tactical level. This work also aims to solve this problem within the CLSLP framework

The distribution of several FGs (multi-item) from production plants to end customers is carried out in various stages (multi-level) by different types of distribution centers (multi-type), such as central warehouses, logistic centers and shops. Neither manufactured nor subcontracted FGs can be stored in manufacturing plants. So they are sent to the first distribution level, which is made up of a number of central warehouses with a limited storage capacity. Outgoing FGs from central warehouses are designed to not only cover the demand of certain end customers (for instance, independent distributors that do not belong to the firm, construction firms, etc.), but to also supply logistics centers. Logistics centers, unlike warehouses, do not have the required storage capacity and only supply FGs to shops that have been previously assigned to them. Finally, shops, which do not have storage capacity, attend to end customers' demands. Although this type of SC attempts to achieve a maximum customer service level, backorders are permitted in both central warehouses and shops. However, backorders quantities are limited to a certain demand percentage to ensure the accomplishment of an objective customer service level defined by the SC. This is a usual situation in the ceramic tile sector, given its limited production flexibility owing to setup costs and times.

In short, the characteristics of the problem under study are the same as in [8] but with relevant differences introduced by the LHP consideration summarized in the following. As in [8] the master plan considers the CLSLP to reflect the fact that production lots of the same product processed in different production lines present a high probability of not being homogeneous. Furthermore, the splitting of each lot into homogeneous sub-lots of the same FG is also incorporated to reflect the LHP characteristics: different tones and gages for the first quality items. The sizing of lots is made in such a way that an integer number of customer order classes can be served from homogeneous quantities of each sub-lot. To this end, different customer order classes are defined according to their size.

At the master plan level, demand forecasts are usually expressed in aggregate manner without taking into account customer classes. Customer classes definition (also known as customer segmentation) has been traditionally used in the field of the so called "allocation planning". The allocation planning follows a push strategy (based on forecasts), as the master plan, but it is carried out after the master plan and before the OPP. The allocation planning has been used for SC operating under a supply constrained mode where not all customer demand can be fulfilled and should answer on-line to customer requirements based on the first-come-first-served policy. Yet in shortage situations where demand is higher than ATP quantities, single-order processing entails the risk of promising scarce availabilities to the wrong customers; e.g., to less important customers or to customers with smaller profit margins. Allocation planning promises to be a way to improve real-time single-order processing by reserving shares of the ATP, the so-called "quotas" or "allocated ATP", for important customers in the mid-term and by afterward promising orders in relation to these allocated quotas in the short term [10]. In doing so, a classification is defined that is used to segment and prioritize customer orders. The defined classes could be either flat or they could form a hierarchy [11]. Examples of different customer classes' definition can be found in [10], [12], [13], [14].

Therefore, the consideration of customer classes for sizing lots and defining demand forecasts jointly with the splitting of lots into homogeneous sub-lots constitute the most relevant aspects that differentiate the model for master plan proposed in this paper from that proposed by Alemany et al. [8] and other models for SC master plan. The next section describes the mixed integer programming model proposed to solve the described problem.

3 Modeling Lack of Homogeneity in the Product in Ceramic Supply Chains through Master Planning

The following mixed integer linear programming model (MP-CSC-LHP) is proposed to solve the master planning problem described above. The model MP-RDSINC proposed by Alemany et al. [8] is considered as the starting point to formulate the present model but properly modified in order to reflect the LHP characteristics cited previously. Tables 1 to 4, respectively, describe the indices, sets of indices, model parameters and decision variables of the MP-CSC-LHP, respectively. Those model elements that differ from the MP-RDSINC are written in *italics*.

Table 1. Indices

i	Finished goods (*i= 1, ..., I*)	*q*	Logistics centers (*q= 1, ..., Q*)
f	Product families (*f= 1, ..., F*)	*w*	Shops (*w= 1, ..., W*)
c	Raw materials and components (*c= 1, ..., C*)	*r*	Suppliers of raw materials and components (*r= 1,... ,R*)
p	Production plants (*p= 1, ..., P*)	*k*	*Customer order classes (k= 1, ..., K)*
a	Warehouses (*a= 1, ..., A*)	*t*	Periods of time (*t= 1, ..., T*)

Table 2. Set of Indices

Il(l)	Set of FGs that can be manufactured on manufacturing line *l*
Fl(l)	Set of product families that can be manufactured on manufacturing line *l*
If(f)	Set of FGs that belong to product family *f*
Ip(p)	Set of FGs that can be produced in production plant *p*
Ia(a)	Set of FGs that can be stored in warehouse *a*
Ic(c)	Set of FGs of that RM *c* form part
Iq(q)	Set of FGs that can be sent to logistic center *q*
Iw(w)	Set of FGs that can be sent to shop *w*
Lf(f)	Set of manufacturing lines that may produce product family *f*
Lp(p)	Set of manufacturing lines that belong to production plant *p*
Pa(a)	Set of production plants that can send FGs to warehouse *a*
Aq(q)	Set of warehouses that can supply logistic center *q*

Rc(c)	Set of suppliers that can supply RM *c*
Rp(p)	Set of suppliers of RMs that can supply production plant *p*
Cr(r)	Set of RMs that can be supplied by supplier *r*
Qa(a)	Set of logistics centers that can be supplied by warehouse *a*
Wq(q)	Set of shops that can be supplied by logistic center *q*
Qw(w)	Set of logistics centers capable of supplying shop *w*
Ap(p)	Set of warehouses that can be supplied by production plant *p*

Table 3. Model Parameters

ca_{crt}	Capacity (units) of supplying RM *c* of supplier *r* in period *t*
$costtp_{crp}$	Purchase and transport cost of one unit of RM *c* from supplier *r* to production plant *p*
caf_{lpt}	Production capacity available (time) of production line *l* at plant *p* during time period *t*
cm_i	Loss ratio of FG *i* (percentage of faulty m2 obtained of the production process)
cq_i	Percentage of m^2 that can be sold of product *i* as first quality
$costp_{ilp}$	Cost of producing one m^2 of FG *i* on production line *l* of production plant *p*
$costsetupf_{flp}$	Setup costs for product family *f* on production line *l* of production plant *p*
$costsetup_{ilp}$	Setup costs for FG *i* on production line *l* of production plant *p*
$tfab_{ilp}$	Time to process one m^2 of FG *i* on production line *l* of production plant *p*
$tsetup_{flp}$	Setup time for product family *f* on production line *l* of production plant *p*
$tsetupi_{ilp}$	Setup time for article *i* on production line *l* of production plant *p*
lmi_{ilp}	Minimum lot size (m^2) of FG *i* on production line *l* of production plant *p*
tmf_{flp}	Minimum run length (expressed as multiples of the time period used) of product family *f* on production line *l* of production plant *p*
v_{ic}	Units of RM *c* needed to produce one m^2 of FG *i*
ssc_{cp}	Safety stock of RM *c* in production plant *p*
ssa_{ia}	Safety stock (m^2) of FG *i* at warehouse *a*
$capal_a$	Storage capacity (m^2) in warehouse *a*
$costtak_{ipak}$	*Unitary transport cost of FG i from production plant p to warehouse for customer order class k*
$costtclk_{iaqk}$	*Unitary transport cost of FG i from warehouse a to logistic centre q for customer order class k*
$costinak_{iak}$	*Unitary holding cost of FG i of customer order class k in the warehouse a in a period*
$costdifak_{iak}$	*Unitary backorder cost of FG i for customer order class k in warehouse a in a period*
pak_{iak}	*Sales value of FG i in warehouse a for customer order class k*
$\alpha 1_k$	*Maximum backorder quantity permitted by customer order class k in a period in warehouses expressed as a percentage of the demand of that period*
$costtwk_{iqwk}$	*Unitary transport cost of FG i from logistics centre q to shop w for customer order class k*
$costdifwk_{iwk}$	*Unitary backorder cost of FG i of customer order class k in a time period at shop w*
pwk_{iwk}	*Sales price of FG i in shop w for customer order class k*
$\alpha 2_k$	*Maximum backorder quantity permitted in a period by customer order class k in shops expressed as a percentage of the demand of that period*
M1, M2	*Very large integers*

$ordq_{ik}$	*Average size of the order of FG i of customer order class k*
dw_{iwkt}	*Forecast of demand of FG i at the warehouse a of customer order class k in period t*
da_{iakt}	*Forecast of demand of FG i in shop w of customer order class k in period t*
$\beta 1_{ilp}$	*Percentage of a batch of FG i produced on the line l of the plant p at any period which can be considered as the first homogeneous sub- batch of product i*
$\beta 2_{ilp}$	*Percentage of a batch of FG i produced on the line l of the plant p at any period which can be considered as the second homogeneous sub- batch of product i*
$\beta 3_{ilp}$	*Percentage of a batch of FG i produced on the line l of the plant p at any period that can be considered as the third homogeneous sub- batch of product i*

Table 4. Decision Variables

CTP_{crpt}	Amount of RM *c* to be purchased and transported from supplier *r* to production plant *p* in period *t*
INC_{cpt}	Inventory of the RM *c* at plant *p* at the end of period *t*
MPF_{flpt}	Amount of product family *f* manufactured on production line *l* of production plant *p* in period *t*
MP_{ilpt}	Amount of FG *i* manufactured on production line *l* of production plant *p* in period *t*
X_{ilpt}	Binary variable with a value of 1 if FG *i* is manufactured on production line *l* of production plant *p* in time period *t*, and with a value of 0 otherwise
Y_{flpt}	Binary variable with a value of 1 if product family *f* is manufactured on production line *l* of production plant *p* in time period *t*, and with a value 0 otherwise
ZI_{ilpt}	Binary variable with a value of 1 if a setup takes place of product *i* on production line *l* of production plant *p* in time period *t*, and with a value of 0 otherwise
ZF_{flpt}	Binary variable with a value of 1 if a setup takes place of product family *f* on production line *l* of production plant *p* in time period *t*, and with a value of 0 otherwise
$CTAK_{ipakt}$	*Amount of FG i to be transported from production plant p to warehouse a for customer order class k in time period t*
$INVNAK_{iakt}$	*Inventory of FG i in warehouse a for customer order class k in period t*
$VENAK_{iakt}$	*Amount of FG i sold in warehouse a to customer order class k during period t*
$DIFAK_{iakt}$	*Backorder quantity of FG i of customer order class k in warehouse a during period t*
$CTCLK_{iaqkt}$	*Amount of FG i of customer order class k transported from warehouse a to logistics centre q in period t*
$CTTWK_{iqwkt}$	*Amount of FG i of customer order class k transported from logistics centre q to shop w in period t*
$VENWK_{iwkt}$	*Amount of FG i of customer order class k sold in shop w during period t*
$DIFWK_{iwkt}$	*Backorder quantity of FG i of customer order class k in shop w during time period t*
NKL_{ilpkt}	*Number of orders of FG i from customer order class k which can be served from the lot of the FG i to be produced on line l of the plant p in period t*
NKL1ilpkt	*Number of orders of FG i from customer order class k which can be served from the first homogeneous sub-lot of the FG i to be produced on line l of the*

	plant p in period t
NKL2ilpkt	*Number of orders of FG i from customer order class k which can be served from the second homogeneous sub-lot of the FG i to be produced on line l of the plant p in period t*
NKL3ilpkt	*Number of orders of FG i from customer order class k which can be served from the third homogenous sub-lot of the FG i to be produced on line l of the plant p in period t*
NKPipkt	*Number of orders of FG i from customer order class k which can be served from lots of the article i to be produced on all lines of the plant p in period t*

Objective Function:

$$
\begin{aligned}
Máx \sum_t \sum_i \sum_k \Big\{ \sum_a pak_{iak} * VENAK_{iakt} + \sum_w pwk_{iwk} * VENWK_{iwkt} \Big\} - \\
- \sum_t \sum_p \sum_{r \in Rp(p)} \sum_{c \in Cr(r)} costtp_{crp} * CTP_{crpt} - \sum_t \sum_p \sum_{l \in Lp(p)} \sum_{i \in Il(l)} costp_{ilp} * MP_{ilpt} - \\
- \sum_t \sum_p \sum_{l \in Lp(p)} \sum_{f \in Fl(l)} costsetup_{flp} * ZF_{flpt} - \sum_t \sum_p \sum_{l \in Lp(p)} \sum_{i \in Il(l)} costsetupi_{ilp} * ZI_{ilpt} - \\
- \sum_t \sum_a \sum_{p \in Pa(a)} \sum_{i \in Ip(p)} \sum_k costtak_{ipak} * CTAK_{ipakt} - \sum_t \sum_a \sum_{i \in Ia(a)} \sum_k costinak_{iak} * INVNAK_{iakt} - \\
- \sum_t \sum_a \sum_{i \in Ia(a)} \sum_k costdifak_{iak} * DIFAK_{iakt} - \sum_t \sum_a \sum_{q \in Qa(a)} \sum_{i \in Iq(q)} \sum_k costtclk_{iaqk} * CTCLK_{iaqkt} - \\
- \sum_t \sum_q \sum_{w \in Wq(q)} \sum_{i \in Iw(w)} costtwk_{iqwk} * CTTWK_{iqwkt} - \sum_t \sum_w \sum_{i \in Iw(w)} \sum_k costdifwk_{iwk} * DIFWK_{iwkt}
\end{aligned}
\quad (1)
$$

Constraints:

$$INC_{cpt} = INC_{cpt\;1} + \sum_{r \in Rc(c)} CTP_{crpt} - \sum_{i \in Ic(c)} \Big(v_{ic} * \sum_{l \in Lp(p)} MP_{ilpt} \Big) \quad \forall c, p, t \quad (2)$$

$$INC_{cpt} \geq ssc_{cp} \quad \forall c, p, t \quad (3)$$

$$\sum_p CTP_{crpt} \leq ca_{crt} \quad \forall c, p, t \quad (4)$$

$$\sum_{f \in Fl(l)} tsetupf_{flp} * ZF_{flpt} + \sum_{i \in Il(l)} \big(tsetupi_{ilp} * ZI_{ilpt} + tfab_{ilp} * MP_{ilpt} \big) \leq caf_{lpt} \quad \forall p, l \in Lp(p), t \quad (5)$$

$$MPF_{flpt} = \sum_{i \in If(f)} MP_{ilpt} \quad \forall p, l \in Lp(p), f \in Fl(l), t \quad (6)$$

$$MP_{ilpt} \geq lmi_{ilp} * X_{ilpt} \quad \forall p, l \in Lp(p), i \in Il(l), t \quad (7)$$

$$MP_{ilpt} \leq M1 * X_{ilpt} \quad \forall p, l \in Lp(p), i \in Il(l), t \quad (8)$$

$$MPF_{flpt} \leq M2 * Y_{flpt} \quad \forall p, l \in Lp(p), f \in Fl(l), t \quad (9)$$

$$ZI_{ilpt} \geq X_{ilpt} - X_{ilpt\;1} \quad \forall p, l \in Lp(p), i \in Il(l), t \tag{10}$$

$$\sum_i ZI_{ilpt} \geq \sum_i X_{ilpt} - 1 \quad \forall p, l \in Lp(p), t \tag{11}$$

$$ZF_{flpt} \geq Y_{flpt} - Y_{flpt\;1} \quad \forall p, l \in Lp(p), f \in Fl(l), t \tag{12}$$

$$\sum_f ZF_{flpt} \geq \sum_f Y_{flpt} - 1 \quad \forall p, l \in Lp(p), t \tag{13}$$

$$\sum_{t=t'}^{t'+tmf_{flp}\;1} ZF_{flpt} \leq 1 \quad \forall p, l \in Lp(p), f \in Fl(l), t' = 1, \ldots., T \quad tmf_{flp} + 1 \tag{14}$$

$$\left(1 - cm_i\right) * cq_i * \beta 1_{ilp} * MP_{ilpt} = \sum_k NKL1_{ilpkt} * ordq_{ik} \quad \forall p, \forall l \in Lp(p), i \in Ip(p), t \tag{15}$$

$$\left(1 - cm_i\right) * cq_i * \beta 2_{ilp} * MP_{ilpt} = \sum_k NKL2_{ilpkt} * ordq_{ik} \quad \forall p, \forall l \in Lp(p), i \in Ip(p), t \tag{16}$$

$$\left(1 - cm_i\right) * cq_i * \beta 3_{ilp} * MP_{ilpt} = \sum_k NKL3_{ilpkt} * ordq_{ik} \quad \forall p, \forall l \in Lp(p), i \in Ip(p), t \tag{17}$$

$$NKL_{ilpkt} = NKL1_{ilpkt} + NKL2_{ilpkt} + NKL3_{ilpkt} \quad \forall p, i \in Ip(p), \forall l \in Lp(p), \forall k, \forall t \tag{18}$$

$$NKP_{ipkt} = \sum_{l \in Lp(p)} NKL_{ilpkt} \quad \forall p, i \in Ip(p), \forall k, \forall t \tag{19}$$

$$NKP_{ipkt} * ordq_{ik} = \sum_{a \in Ap(p)} CTAK_{ipakt} \quad \forall p, i \in Ip(p), \forall k, \forall t \tag{20}$$

$$INVNAK_{iakt} = INVNAK_{iakt-1} + \sum_{p \in Pa(a)} CTAK_{ipakt} - VEANK_{iakt} - \sum_{q \in Qa(a)} CTCLK_{iaqkt} \; \forall i \in Ia(a), a, k, t \tag{21}$$

$$VENAK_{iakt} + DIFAK_{iakt} - DIFAK_{iakt\;1} = da_{iakt} \quad \forall i \in Ia(a), a, k, t \tag{22}$$

$$DIFAK_{iakt} \leq \alpha l_k \, da_{iakt} \qquad \forall i \in Ia(a), a, k, t \tag{23}$$

$$\sum_k INVNAK_{iakt} \geq ssa_{ia} \quad \forall a, i \in Ia(a), t \tag{24}$$

$$\sum_{i \in Ia(a)} \sum_k INVNAK_{iakt} \leq capal_a \quad \forall a, t \tag{25}$$

$$\sum_{a \in Aq(q)} CTCLK_{iaqkt} = \sum_{w \in Wq(q)} CTTWK_{iqwkt} \quad \forall q, i \in Iq(q), k, t \tag{26}$$

$$\sum_{q \in Qw(w)} CTTWK_{iqwkt} = VENWK_{iwkt} \quad \forall w, i \in Iw(w), k, t \tag{27}$$

$$VENWK_{iwkt} + DIFWK_{iwkt} - DIFWK_{iwkt-1} = dw_{iwkt} \qquad \forall i \in I(w), w, k, t \tag{28}$$

$$DIFWK_{iwkt} \leq \alpha 2_k \, dw_{iwkt} \qquad \forall i \in I(w), w, k, t \tag{29}$$

$$\begin{gathered} MPF_{flpt}, MP_{ilpt}, CTP_{crpt}, INC_{cpt}, CTAK_{ipakt}, INVNAK_{iakt}, CTCLK_{iaqkt}, CTTWK_{iqwkt} \geq 0 \\ VENAK_{iakt}, VENWK_{iwkt}, DIFAK_{iakt}, DIFWK_{iwkt} \geq 0 \\ NKL_{ilpkt}, NKP_{ipkt}, NKL1_{ilpkt}, NKL2_{ilpkt}, NKL3_{ilpkt} \geq 0 \text{ y enteras} \\ \text{and} \quad X_{ilpt}, Y_{flpt}, ZF_{flpt}, ZI_{ilpt} \in \{0, 1\} \\ \forall f \in F, \forall i \in I, \forall c \in C, \forall l \in L, \forall p \in P, \forall a \in A, \forall q \in Q, \forall w \in W, \forall r \in R, \forall k \in K, \forall t \in T \end{gathered} \tag{30}$$

For being concise, in this section only the MP-CSC-LHP functions that differ from the MP-RDSINC are described. For more details, the reader is referred to [8]. The objective function (1) expresses the gross margin maximization over the time periods that have been computed by subtracting total costs from total revenues. In this model, selling prices and other costs including the backlog costs can be defined for each customer class allowing reflect their relative priority.

Constraints (2) to (14) coincide with those of the MP-RDSINC and make reference to suppliers and productive limitations related to capacity and setup. Constraints (15)-(17) reflect the splitting of a specific lot into three homogeneous sub-lots of first quality ($\beta 1_{ilp} + \beta 2_{ilp} + \beta 3_{ilp} = 1$). The number of sub-lots considered in each lot can be easily adapted to other number different from three. Through these constraints the sizing of lots is decided based on the number of orders from different customer order classes that can be served from each homogeneous sub-lot.

Customer order classes are defined based on the customer order size (i.e, the m^2 ordered). Constraint (18) calculates for each time period, customer class and FG the total number of orders of a specific customer class that can be served from a certain lot by summing up the corresponding number of orders served by each homogeneous sub-lot of this lot. Constraint (19) derives the number of each customer order class that is possible to serve from the planned production of a specific plant. Through constraints (15-19), the production is adjusted not to the aggregate demand forecast as traditionally, but to different customer orders classes.

Furthermore, in contrast to the MP-RDSINC, the distributed, stocked and sold quantities downstream the production plants are expressed in terms of the customer class whose demand will be satisfied through them, being possible to discriminate the importance of each order class. Constraint (20) calculates the quantity of each FG to be transported from each production plant to each warehouse for each customer class based on the order number of each customer class that is satisfied by each production plant and the mean order size. Constraint (21) represents the inventory balance equation at warehouses for each finished good, customer class and time period. As backorders are permitted in both central warehouses and shops, sales may not coincide with the demand for a given time period. Backorder quantities in warehouses for each customer class are calculated using constraint (22). Constraint (23) limits these backorder quantities per customer class in each period in terms of a percentage of the demand of each time period. Constraint (24) forces to maintain a total inventory quantity higher or equal to the safety stock in warehouses. Constraint (25) is the limitation in the warehouses' capacity that is assumed to be shared by all the FG and customer order classes.

Constraint (26) represents the inflows and outflows of FGs and customer order classes through each logistic center. Because it is not possible to maintain inventory in shops, constraint (27) ensures that the total input quantity of a FG for a specific customer class from warehouses to shops coincides with the quantity sold in shops. As backorders are permitted in both central warehouses and shops, sales may not coincide with the demand for a given time period. Constraints (28) and (29) are similar to constraints (22) and (23), respectively, but referred to shops instead of warehouses. The model also contemplates non-negativity constraints and the definition of variables (30).

4 Model Validation: Assessing the Impact of LHP Modeling

The MP-CSC-LHP model has been implemented in MPL (V4.11) and solved with CPLEX 6.6.0. With the aim of comparing the performance of the model with and without LHP modeling, the input data for validation has been mainly extracted for the paper of Alemany et al. [8] that do not consider LHP: MP-RDSINC.

However, some additional parameters have been necessary for the model considering LHP (MP-CSC-LHP). These parameters have been defined maintaining the coherence of the data used by the two models. With this input data the MP-CSC-LHP and the MP-RDSINC have been solved. Results show that MP-RDSINC obtains a greater gross margin than the MP-CSC-LHP mainly due to the lower production costs of the former. This is due to the fact that the MP-RDSINC should produce a lower quantity than the MP-CSC-LHP for satisfying the aggregate demand (Table 5).

This result can lead to the wrong conclusion that the MP-RDSINC outperforms the MP-CSC-LHP. This is not true because the MP-RDSINC does not take into account the homogeneity requirement for customer orders. Due to the fact that MP-RDSINC considers all the units of the same lot homogeneous and considers the demand forecasts in an aggregate manner, this model can allow serve the same customer order with quantities of the same FG manufactured in the different lots, thus not guaranteeing the homogeneity in orders.

To obtain results from both models that were really comparable, the lots obtained by the MP-RDSINC model solution (value of decision variable MP_{ilpt}) was transferred as an input data (mp_{ilpt}) to the MP-CSC-LHP computing the new gross margin obtained (MP-RDSINC-LHP). Because the sizing of lots derived from MP-RDSINC were made without considering the customer order size, it may occur that it was impossible to serve an integer number of different customer order classes from the lots of MP-RDSINC leaving some units of lots without being possible to assign them to a specific customer class and, therefore, obtaining unfeasible solutions. To avoid obtaining unfeasible solutions, the constraints (15-17) have been relax from "=" to "$\geq$" . These new constraints (31-33) allow homogeneous sub-lots defined by the MP-RDSINC (MP_{ilpt}, now mp_{ilpt}) being equal or greater than the sum of an integer number of customer order classes. The difference between the left and the right hand side of the constraints cause the appearance of fragmented stocks (rests) that cannot be assigned to any customer because of the impossibility of accumulating them due to their heterogeneity.

$$(1-cm_i)*cq_i*\beta 1_{ilp}*mp_{ilpt} \geq \sum_k NKL1_{ilpkt}*ordq_{ik} \quad \forall p, \forall l \in Lp(p), i \in Ip(p), t \quad (31)$$

$$(1-cm_i)*cq_i*\beta 2_{ilp}*mp_{ilpt} \geq \sum_k NKL2_{ilpkt}*ordq_{ik} \quad \forall p, \forall l \in Lp(p), i \in Ip(p), t \quad (32)$$

$$(1-cm_i)*cq_i*\beta 3_{ilp}*mp_{ilpt} \geq \sum_k NKL3_{ilpkt}*ordq_{ik} \quad \forall p, \forall l \in Lp(p), i \in Ip(p), t \quad (33)$$

A comparison of the results obtained for the different models are shown in Table 5.

Table 5. Comparison of results from MP-RDSINC, MP-CSC-LHP and MP-RDSINC-LHP

	MP-RDSINC	MP-CSC-LHP	MP-RDSINC-LHP
Incomes	1.008.539,55	1.008.539,55	1.003.116,65
Supply costs	208.465,58	216.835,92	208.465,58
Production costs	381.918,37	397.034,01	381.918,37
Inventory costs	9.313,91	11.397,90	9.387,50
Setup costs	7.584,24	9.676,45	7.584,24
Transport costs	42.642,71	42.775,75	42.269,60
Backorder costs	0	0	94.500,00
Total costs	649.924,81	677.720,03	744.125,29
Gross margin	358.614,74	330.819,52	258.991,36

As expected, the new value of the gross margin for the MP-RDSINC-LHP was lower than the MP-CSC-LHP because a lower number of customer orders were able to be served with homogeneous quantities by the lots initially defined by the MP- RDSINC (see backorder costs for MP-RDSINC-FHP). It can also be observed an increment of the inventory holding costs of the MP-RDSINC-LHP with respect to those of the MP-RDSINC, due to the fact that the rests of lots that cannot be used to complete a customer order are maintained in inventory. On the other hand, transport costs of the MP-RDSINC model are lower than those of the MP-RDSINC-LHP because a lower number of customer orders are served and, therefore, a lower quantity needs to be transported from the warehouses to shops.

5 Conclusions

Poor LHP management may have very negative effects on SC's competitiveness: (a) LHP leads to fragmented stocks, which can rapidly become obsolete for products with a short life cycle as they cannot be accumulated to be used in the same order given their heterogeneity; (b) uncertainty in the homogeneous quantities available of FGs entails having to produce more than is necessary, thus increasing stocks; and (c) the customer service level may prove deficient, even with high stock volumes, if the order-promising system is not supplied with reliable information about the real and future homogeneous quantities available of a product.

When faced with this situation, there are two clearly different ways to act: technology and management. Research into the technological field focuses mainly on automating the classification process of the FG into different homogeneous subtypes because, to date, eliminating the heterogeneity of the input material or that caused by the production process itself appears to be unachievable. From the management viewpoint, LHP introduces a new requirement in customer's orders that should be served not only on time and with the right quantity, as usual, but also with the adequate homogeneity degree. The OPP plays a crucial role in customer requirements satisfaction and, therefore, in properly managing the special LHP characteristics. But in turn, one of the main inputs to this process is the master plan. Therefore, in this paper a mathematical programming model to solve the master planning problem for replenishment, production, and distribution in ceramic tiles SCs with LHP has been proposed. The result is a master plan that anticipates LHP features in sizing lots and distributing produced quantities along SC and, additionally, provides the OPP with reliable information about future homogeneous quantities available.

The MP-RDSINC model proposed by Alemany et al. [8] has been considered as the starting point to formulate the present model but properly modified in order to reflect the LHP characteristics. Traditionally, the master plan defines the quantities that should be available per product and time period for achieving the aggregate demand forecasts, without specifying the productive resources.

In LHP environments the productive process and/or the input materials originates units of the same FG that are not homogeneous regarding some attribute required by the customer. For these cases, it is recommendable to define the master plan in such a way that the future available homogeneous quantities in the production lots can be anticipated as much as possible. To achieve this objective, it could be necessary to define the master plan with a higher level of detail.
Along these lines, because in ceramic sector lots of the same FG manufactured in different production lines and time periods present a high probability of not being homogeneous, it has been necessary to define the quantities to be produced not only for each FG and time period but also specifying the productive resource (production line). This aspect has led to solve the CLSLP. Another novel aspect has been the consideration of splitting one lot into different homogeneous sub-lots. Finally, to model the homogeneity requirement of customer orders, the sizing of lots is made in such a way that an integer number of customer order classes can be served from homogeneous quantities of each sub-lot. To this end, different customer order classes have been defined according to their size and the demand forecasts are expressed in terms of these customer order classes and not in terms of aggregate demand as usual.
The impact of modeling qualities, tones and gages has been assessed by comparing results obtained from the model with LHP (MP-CSC-LHP) and without LHP (MP-RDSINC). Results show that profits and customer service level is higher when considering LHP because lots are sizing to serve an integer number of customer order classes. This aspect also leads to a reduction of the rests of stocks of the same FG along the SC that cannot be assigned to any customer because they cannot be joined due to the lack of homogeneity. Additionally, the obtained information at the master plan level about the homogeneous sub-lots of each FG can be used to calculate the homogeneous ATP quantities, improving the OPP.
Future work will be focused on the following research lines. The first one implies the consideration of uncertainty in the splitting of lots into homogeneous sub-lots as well as in the demand forecasts based on customer classes. The second one implies an analysis of the LHP under an information system's perspective because LHP implies the existence of several references of the same FG. Therefore, this aspect jointly with other ones should be taking into account when designing and building information systems that can provide the right information to the proposed model under a decision-making perspective [15]. Finally, the LHP modeling and its inherent uncertainty increases the complexity of the problem, converting LHP productive systems in large-scale complex systems [16]. As a consequence, another research line will be the development of sustainable decision support systems to help decision-makers in such complex situations [17].

Acknowledgments

This research has been carried out in the framework of the project funded by the Spanish Ministry of Economy and Competitiveness (Ref. DPI2011-23597) and the Polytechnic University of Valencia (Ref. PAID-06-11/1840) entitled "Methods and models for operations planning and order management in supply chains characterized by uncertainty in production due to the lack of product uniformity" (PLANGES-FHP)

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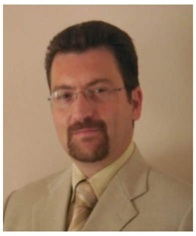

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