Research Article

On the Role of Diffusion Behaviors in Stability Criterion for p-Laplace Dynamical Equations with Infinite Delay and Partial Fuzzy Parameters under Dirichlet Boundary Value

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By the way of Lyapunov-Krasovskii functional approach and some variational methods in the Sobolev space $W_0^{1,p}(\Omega)$, a global asymptotical stability criterion for p-Laplace partial differential equations with partial fuzzy parameters is derived under Dirichlet boundary condition, which gives a positive answer to an open problem proposed in some related literatures. Different from many previous related literatures, the nonlinear p-Laplace diffusion item plays its role in the new criterion though the nonlinear p-Laplace presents great difficulties. Moreover, numerical examples illustrate that our new stability criterion can judge what the previous criteria cannot do.

1. Introduction

Very recently, time-delay p-Laplace ($p > 1$) dynamical equations have attracted rapidly growing interest, because the nonlinear Laplace diffusion dynamical equations admit many physics and engineering background [1–6], such as Cohen-Grossberg neural networks and recurrent neural networks. In real world, diffusion phenomena cannot be unavoidable. Particularly, 2-Laplace ($p = 2$) is called the linear Laplace, and the diffusion phenomenon is always simulated by linear Laplace diffusion for simplicity ([7–10] and their references therein). However, diffusion behavior is so complicated that the nonlinear reaction-diffusion models were considered in many other recent literatures [1–6, 11–13]. Even the nonlinear p-Laplace diffusion ($p > 1$) is considered in simulating some diffusion behaviors [1–6]. But the previous related literature lost sight of the role of the nonlinear diffusion in their stability criteria. As pointed out in [2], the problem how the p-Laplace diffusion item plays a role in stability criteria remains open and challenging. And such a situation motivates our present

study. Besides, fuzzy logic theory has been shown to be an appealing and efficient approach to dealing with the analysis and synthesis problems for complex nonlinear systems. Among various kinds of fuzzy methods, Takagi-Sugeno (T-S) fuzzy models provide a successful method to describe certain complex nonlinear systems using some local linear subsystems [14–16]. Motivated by some ideas and methods of [17–21], we obtain a global asymptotical stability criterion for fuzzy T-S p-Laplace partial differential equations with Dirichlet boundary value by the way of Lyapunov-Krasovskii functional approach and some variational methods in the Sobolev space $W_0^{1,p}(\Omega)$. And in the obtained criterion, the nonlinear p-Laplace diffusion item plays a positive role.

2. Model Description and Preliminaries

Let us consider a class of fuzzy Takagi-Sugeno (T-S) p-Laplace partial differential equations described as follows.

Fuzzy Rule j.
IF $\omega_1(t)$ is μ_{j1} and $\cdots\omega_s(t)$ is μ_{js}, **THEN**

$$\frac{\partial v}{\partial t} = \nabla \cdot \left(\Gamma(t, x, v) \circ \nabla_p v \right) - A(v)$$
$$\times \left\{ B(v) - \left[C_j f(v) + D_j g(v(t - \tau(t), x)) \right] \right\},$$
$$v(\theta, x) = \phi(\theta, x), \quad (\theta, x) \in (-\infty, 0] \times \Omega,$$
$$v(t, x) = 0 \in R^n, \quad (t, x) \in R \times \partial\Omega,$$

(1)

where Ω is an arbitrary open bounded subset in R^n. $\omega_k(t)$ $(k = 1, 2, \ldots, s)$ is the premise variable, μ_{jk} $(j = 1, 2, \ldots, r; k = 1, 2, \ldots, s)$ is the fuzzy set that is characterized by membership function. And r is the number of the **IF-THEN** rules; s is the number of the premise variables. Fuzzy partial differential system (1) admits its many physics and engineering background, including the famous Cohen-Grossberg neural networks [1–6]. Consider $v(t, x) = (v_1(t, x), v_2(t, x), \ldots, v_n(t, x))^T \in R^n$, where $v_i(t, x)$ is the state variable of the ith neuron and the jth neuron at time t and in space variable x. Matrix $\Gamma(t, x, v) = (\Gamma_{ij}(t, x, v))_{n \times m}$ with each $\Gamma_{ij}(t, x, v) \geq 0$, and $\Gamma_{ij}(t, x, v)$ is diffusion operator. $\Gamma(t, x, v) \circ \nabla_p v = (\Gamma_{ik}(t, x, v)|\nabla v_i|^{p-2}(\partial v_i/\partial x_k))_{n \times m}$ denotes the Hadamard product of matrix $\Gamma(t, x, v)$ and $\nabla_p v$ (see [1–6] for details). Matrix $A(v) = \text{diag}(a_1(v_1), a_2(v_2), \ldots, a_n(v_n))$ and vector $B(v) = (b_1(v_1), b_2(v_2), \ldots, b_n(v_n))^T$, where $a_i(v_i)$ and $b_i(v_i)$ represent an amplification function at time t and an appropriately behaved function at time t. $C_j = (c_{ik}^{(j)})_{n \times n}$ and $D_j = (d_{ik}^{(j)})_{n \times n}$ are connection matrices. The time-varying delays are $\tau(t) \in [0, +\infty)$. $f(v) = (f_1(v_1), f_2(v_2), \ldots, f_n(v_n))^T$ and $g(v(t - \tau(t))) = (g_1(v_1(t - \tau(t))), g_2(v_2(t - \tau(t))), \ldots, g_n(v_n(t - \tau(t))))^T$ are the activation functions of the neurons. And the second and third equations of (1) represent the initial condition and the Dirichlet boundary condition, respectively.

With the help of a standard fuzzy inference method, (1) can be inferred as follows:

$$\frac{\partial v}{\partial t} = \nabla \cdot \left(\Gamma(t, x, v) \circ \nabla_p v \right)$$
$$- A(v) \left\{ B(v) - \sum_{j=1}^r h_j(\omega(t)) \right.$$
$$\times \left[C_j f(v) + D_j g \right.$$
$$\left. \times (v(t - \tau(t), x)) \right] \Bigg\},$$
$$v(\theta, x) = \phi(\theta, x), \quad (\theta, x) \in (-\infty, 0] \times \Omega,$$
$$v(t, x) = 0 \in R^n, \quad (t, x) \in R \times \partial\Omega,$$

(2)

where $\omega(t) = [\omega_1(t), \omega_2(t), \ldots, \omega_s(t)]$, $h_j(\omega(t)) = (w_j(\omega(t)))/(\sum_{k=1}^r w_k(\omega(t)))$, $w_j(\omega(t)) : R^s \to [0, 1]$ $(j = 1, 2, \ldots, r)$

is the membership function of the system with respect to the fuzzy rule j. h_j can be regarded as the normalized weight of each **IF-THEN** rule, satisfying $h_j(\omega(t)) \geq 0$ and $\sum_{j=1}^r h_j(\omega(t)) = 1$.

For convenience's sake, we need to introduce some standard notations.

(i) Denote $|C| = (|c_{ij}|)_{n \times n}$ for any matrix $C = (c_{ij})_{n \times n}$.

(ii) Denote $|v(t, x)| = (|v_1(t, x)|, |v_2(t, x)|, \ldots, |v_n(t, x)|)^T$ for any $v(t, x) = (v_1(t, x), v_2(t, x), \ldots, v_n(t, x))^T$.

(iii) $u \geq v$ if $u_i \geq v_i$ for all $i = 1, 2, \ldots, n$, where $u = (u_1, u_2, \ldots, u_n)^T$, $v = (v_1, v_2, \ldots, v_n)^T \in R^n$.

In addition, we also introduce the following standard notations similarly as [2, (iii)–(X)]:

$$Q = (q_{ij})_{n \times n} > 0 (< 0), \quad Q = (q_{ij})_{n \times n} \geq 0 (\leq 0),$$
$$Q_1 \geq Q_2 (Q_1 \leq Q_2),$$

(3)

$$Q_1 > Q_2 (Q_1 < Q_2), \text{ and the identity matrix } I.$$

Throughout this paper, we assume the following.

(H1) There exist positive definite matrices $\underline{A} = \text{diag}(\underline{a}_1, \underline{a}_2, \ldots, \underline{a}_n)$ and $\overline{A} = \text{diag}(\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n)$ such that

$$0 < \underline{a}_i \leq a_i(r) \leq \overline{a}_i, \quad r \in R, \ i = 1, 2, \ldots, n.$$

(4)

(H2) There exists a positive definite matrix $B = \text{diag}(b_1, b_2, \ldots, b_n)$ such that $b_i(0) = 0$ and

$$\frac{b_i(r)}{r^{p-1}} \geq b_i, \quad 0 \neq r \in R, \ i = 1, 2, \ldots, n.$$

(5)

(H3) There exist positive definite matrices $F = \text{diag}(F_1, F_2, \ldots, F_n)$ and $G = \text{diag}(G_1, G_2, \ldots, G_n)$ such that

$$|f_i(r)| \leq F_i |r|^{p-1}, \quad |g_i(r)| \leq G_i |r|^{p-1}, \quad r \in R,$$
$$i = 1, 2, \ldots, n.$$

(6)

From (H1)–(H3), we know that $f(0) = g(0) = 0 \in R^n$, and $v = 0$ is an equilibrium of (2).

Lemma 1. *For any $a, b \in (0, +\infty)$, One has*

$$a^{p-1}b \leq \frac{p-1}{p}a^p + \frac{b^p}{p}.$$

(7)

3. Main Result

Before giving the main result of this paper, we have to present the following Lemma via some variational methods in the Sobolev space $W_0^{1,p}(\Omega)$, which is the completion of $C_0^\infty(\Omega)$ with respect to the norm $\|\eta\| = (\int_\Omega |\nabla\eta(x)|^p dx)^{1/p}$. Denote by λ_1 the first eigenvalue of $-\Delta_p$ in Sobolev space $W_0^{1,p}(\Omega)$, where $\lambda_1 = \min\{\int_\Omega |\nabla\eta(x)|^p dx : \eta(x) \in W_0^{1,p}(\Omega), \int_\Omega |\eta(x)|^p dx = 1\}$ (see [22–24] for details).

Lemma 2. *Let* $Q = \mathrm{diag}(q_1, q_2, \ldots, q_n)$ *be a positive definite matrix, and let* v *be a solution of the fuzzy system* (2). *Then One has*

$$\int_\Omega v^T Q\left(\nabla \cdot \left(\Gamma(t, x, v) \circ \nabla_p v\right)\right) dx \leqslant -\lambda_1 \underline{q} \Gamma \|v\|_p^p, \quad (8)$$

where $\Gamma = \min_{i,k}(\inf_{t,x,v} \Gamma_{ik}(t, x, v))$, $\|v\|_p^p = \sum_{i=1}^n \int_\Omega |v_i|^p dx$, *and* $\underline{q}$ *is a positive scalar, satisfying* $Q > \underline{q}I$.

Proof. Since v is a solution of system (2), it follows by Gauss formula and the Dirichlet boundary condition that

$$\int_\Omega v^T Q\left(\nabla \cdot \left(\Gamma(t, x, v) \circ \nabla_p v\right)\right) dx$$

$$= \int_\Omega v^T Q \left(\sum_{k=1}^m \frac{\partial}{\partial x_k}\left(\Gamma_{1k}|\nabla v_1|^{p-2}\frac{\partial v_1}{\partial x_k}\right), \ldots, \right.$$

$$\left. \sum_{k=1}^m \frac{\partial}{\partial x_k}\left(\Gamma_{nk}|\nabla v_n|^{p-2}\frac{\partial v_n}{\partial x_k}\right)\right)^T dx \quad (9)$$

$$= -\sum_{k=1}^m \sum_{j=1}^n \int_\Omega q_j \Gamma_{jk}|\nabla v_j|^{p-2}\left(\frac{\partial v_j}{\partial x_k}\right)^2 dx$$

$$\leqslant -\lambda_1 \underline{q}\Gamma\|v\|_p^p. \qquad \square$$

Remark 3. Lemma 2 actually generalizes the conclusion of [7, Lemma 2.1] and [25, Lemma 2.4] from Hilbert space $H_0^1(\Omega)$ to Banach space $W_0^{1,p}(\Omega)$. Particularly in the case of $\Omega = (0, T) \subset R^1$ or $W_0^{1,p}(0, T)$, the first eigenvalue $\lambda_1 = ((2/T)\int_0^{(p-1)^{1/p}}(dt/(1 - (t^p/(p-1)))^{1/p}))^p$ (see, e.g., [22]).

Theorem 4. *Suppose that* $p = m_1/m_2 > 1$, *where* m_1 *is an even number while* m_2 *is an odd number. If, in addition, there exist a positive definite matrix* $Q = \mathrm{diag}(q_1, q_2, \ldots, q_n)$ *and two positive scalars* $\underline{q}, \bar{q}$ *such that the following inequalities hold:*

$$[\lambda_1\Gamma + \lambda_{\min}(\underline{A}B)]\underline{q}$$

$$> \left[n\left(\lambda_{\max}\overline{A}\right)\left(\lambda_{\max}F\right)\right.$$

$$\times \sum_{j=1}^r |c_j| + \left(\frac{n}{p} + \frac{n(p-1)}{(1-\tau)p}\right) \quad (10)$$

$$\times \left(\lambda_{\max}\overline{A}\right)\left(\lambda_{\max}G\right)\sum_{j=1}^r |d_j| \bigg]\bar{q},$$

$$q_i > \underline{q}, \quad i \in \mathcal{N}, \quad (11)$$

$$q_i < \bar{q}, \quad i \in \mathcal{N}, \quad (12)$$

then the null solution of (2) *is globally asymptotically stable, where* $\mathcal{N} = \{1, 2, \ldots, n\}$, $\tau'(t) \leqslant \tau < 1$, $|c_j| = \max_{i,k}|c_{ik}^{(j)}|$ *for*

the matrix $C_j = (c_{ik}^{(j)})_{n\times n}$, *and* $|d_j| = \max_{i,k}|d_{ik}^{(j)}|$ *for the matrix* $D_j = (d_{ik}^{(j)})_{n\times n}$.

Proof. Define the Lyapunov-Krasovskii functional as follows:

$$V(t) = V_1(t) + \frac{1}{1-\tau}V_2(t), \quad (13)$$

where

$$V_1(t) = \int_\Omega v^T Q v\, dx,$$

$$V_2(t) = 2n\bar{q}\left(\lambda_{\max}\overline{A}\right)\left(\lambda_{\max}G\right)\frac{p-1}{p} \quad (14)$$

$$\times \sum_{j=1}^r \sum_{k=1}^n |d_j| \int_{t-\tau(t)}^t \int_\Omega |v_k(s, x)|^p\, ds\, dx.$$

Evaluating the time derivation of $V_1(t)$ along the trajectory of the fuzzy system (2), we can get by Lemma 2

$$V_1'(t) \leqslant -2\lambda_1 \underline{q}\Gamma\|v\|_p^p$$

$$-2\int_\Omega v^T Q A(v) B(v)\, dx$$

$$+ 2\sum_{j=1}^r h_j(\omega(t)) \int_\Omega v^T Q\left[A(v)C_j f(v)\right.$$

$$\left. + A(v)D_j g(v_\tau)\right] dx$$

$$\leqslant -2\lambda_1\underline{q}\Gamma\|v\|_p^p - 2\int_\Omega v^T Q A(v) B(v)\, dx$$

$$+ 2\sum_{j=1}^r \int_\Omega |v^T|Q\left[\overline{A}|C_j||f(v)| + \overline{A}|D_j||g(v_\tau)|\right] dx,$$

$$(15)$$

where we denote $v_\tau = v(t - \tau(t), x)$ for convenience.

Then we can get by (H1), (H2) and the restrictive conditions on the parameter p

$$\int_\Omega v^T Q A(v) B(v)\, dx$$

$$\geqslant \underline{q}\int_\Omega |v^T|\underline{A}B\left(|v_1|^{p-1}, |v_2|^{p-1}, \ldots, |v_n|^{p-1}\right)^T dx \quad (16)$$

$$\geqslant \underline{q}\lambda_{\min}(\underline{A}B)\|v\|_p^p.$$

It follows by (H3) and Lemma 1 that

$$\int_\Omega |v^T|Q\overline{A}|C_j||f(v)|\, dx$$

$$\leqslant \int_\Omega |v^T Q|\overline{A}|C_j|F\left(|v_1|^{p-1}, |v_2|^{p-1}, \ldots, |v_n|^{p-1}\right)^T dx$$

$$= \bar{q}\sum_{k=1}^n \sum_{i=1}^n \int_\Omega |v_i|\bar{a}_i\left|c_{ik}^{(j)}\right|F_k|v_k|^{p-1}\, dx$$

$$
\leqslant \bar{q} \sum_{k=1}^{n} \sum_{i=1}^{n} \int_{\Omega} \bar{a}_i \left| c_{ik}^{(j)} \right| F_k \left(\frac{p-1}{p} |v_k|^p + \frac{|v_i|^p}{p} \right) dx
$$

$$
= \bar{q} \frac{p-1}{p} \sum_{k=1}^{n} \sum_{i=1}^{n} \int_{\Omega} \bar{a}_i \left| c_{ik}^{(j)} \right| F_k |v_k|^p \, dx
$$

$$
+ \bar{q} \frac{1}{p} \sum_{k=1}^{n} \sum_{i=1}^{n} \int_{\Omega} \bar{a}_i \left| c_{ik}^{(j)} \right| F_k |v_i|^p \, dx
$$

$$
\leqslant \bar{q} \left| c_j \right| \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} F \right)
$$

$$
\times \left(\frac{p-1}{p} \sum_{k=1}^{n} \sum_{i=1}^{n} \int_{\Omega} |v_k|^p dx + \frac{1}{p} \sum_{k=1}^{n} \sum_{i=1}^{n} \int_{\Omega} |v_i|^p \, dx \right)
$$

$$
= n\bar{q} \left| c_j \right| \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} F \right)
$$

$$
\times \left(\frac{p-1}{p} \sum_{k=1}^{n} \int_{\Omega} |v_k|^p dx + \frac{1}{p} \sum_{i=1}^{n} \int_{\Omega} |v_i|^p \, dx \right)
$$

$$
= n\bar{q} \left| c_j \right| \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} F \right) \|v\|_P^p.
$$

$$(17)$$

Similarly,

$$
\int_{\Omega} \left| v^T \right| Q\overline{A} \left| D_j \right| \left| g \left(v_\tau \right) \right| dx
$$

$$
\leqslant \bar{q} \sum_{k=1}^{n} \sum_{i=1}^{n} \int_{\Omega} \bar{a}_i \left| d_{ik}^{(j)} \right| G_k
$$

$$
\times \left(\frac{p-1}{p} |v_k (t - \tau(t), x)|^p + \frac{|v_i(t,x)|^p}{p} \right) dx \qquad (18)
$$

$$
\leqslant n\bar{q} \left| d_j \right| \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} G \right)
$$

$$
\times \left(\frac{p-1}{p} \sum_{k=1}^{n} \int_{\Omega} |v_k (t - \tau(t), x)|^p dx + \frac{1}{p} \|v\|_P^p \right).
$$

Combining (15)–(18) results in

$$
V_1'(t) \leqslant -2 \left(\lambda_1 \Gamma \underline{q} + \underline{q} \lambda_{\min} \left(\underline{A}B \right) - n\bar{q} \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} F \right) \right.
$$

$$
\times \sum_{j=1}^{r} \left| c_j \right| - \frac{1}{p} n\bar{q} \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} G \right) \sum_{j=1}^{r} \left| d_j \right| \right)
$$

$$
\times \|v\|_p^p + 2n\bar{q} \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} G \right) \frac{p-1}{p}
$$

$$
\times \sum_{j=1}^{r} \sum_{k=1}^{n} \left| d_j \right| \int_{\Omega} |v_k (t - \tau(t), x)|^p \, dx.
$$

$$(19)$$

On the other hand,

$$
V_2'(t) = 2n\bar{q} \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} G \right) \frac{p-1}{p} \|v\|_P^p \sum_{j=1}^{r} \left| d_j \right|
$$

$$
- \left(1 - \tau'(t) \right) 2n\bar{q} \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} G \right) \frac{p-1}{p}
$$

$$
\times \sum_{j=1}^{r} \sum_{k=1}^{n} \left| d_j \right| \int_{\Omega} |v_k (t - \tau(t), x)|^p \, dx
$$

$$(20)$$

$$
\leqslant 2n\bar{q} \left| d_j \right| \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} G \right) \frac{p-1}{p} \|v\|_P^p \sum_{j=1}^{r} \left| d_j \right|
$$

$$
- (1 - \tau) 2n\bar{q} \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} G \right) \frac{p-1}{p}
$$

$$
\times \sum_{j=1}^{r} \sum_{k=1}^{n} \left| d_j \right| \int_{\Omega} |v_k (t - \tau(t), x)|^p \, dx.
$$

So we conclude from (10) that

$$
V'(t) \leqslant -2 \left(\lambda_1 \underline{q} \Gamma + \underline{q} \lambda_{\min} \left(\underline{A}B \right) - n\bar{q} \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} F \right) \right.
$$

$$
\times \sum_{j=1}^{r} \left| c_j \right| - \frac{1}{p} n\bar{q} \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} G \right) \sum_{j=1}^{r} \left| d_j \right|
$$

$$
- \frac{1}{1 - \tau} n\bar{q} \left(\lambda_{\max} \overline{A} \right) \left(\lambda_{\max} G \right) \frac{p-1}{p} \sum_{j=1}^{r} \left| d_j \right| \right)
$$

$$
\times \|v\|_P^p \leqslant 0.
$$

$$(21)$$

It follows by the standard Lyapunov functional theory that the null solution of the fuzzy system (2) is globally asymptotically stable. □

Remark 5. In many previous related literatures (see, e.g., [1–6]), the nonlinear p-Laplace ($p > 2$) diffusion terms were omitted in the deductions, which results in that their stability criteria do not contain the diffusion terms. In other words, the diffusion terms do not affect their results. In addition, when $p = 2$, 2-Laplace is the linear Laplace, and there are many papers (see, e.g., [25–29]) in which the Laplace diffusion item plays its role in their stability criteria, for the linear Laplace PDEs can be considered in the special Hilbert space $H^1(\Omega)$ that can be orthogonally decomposed into the direct sum of infinitely many eigenfunction spaces. However, the nonlinear p-Laplace ($p > 1$, $p \neq 2$) brings great difficulties, for the nonlinear p-Laplace PDEs should be considered in the frame of Sobolev space $W^{1,p}(\Omega)$ that is only a reflexive Banach space. Indeed, owing to the great difficulties, the authors only provide in [4] the stability criterion in which the nonlinear p-Laplace items play roles in the case of $1 < p < 2$. However, in this paper, the nonlinear p-Laplace diffusion terms play a positive role in our Theorem 4 for the case of $p > 2$ or

$p > 1$, which also gives a positive answer to the open problem proposed in [2] to some extent. Besides, we will provide a numerical example where our Theorem 4 works whereas [2, Corollary 15] do not (see, Example 2).

Remark 6. Particularly when $p = 2$, Theorem 4 provides a global asymptotical stability criterion for the familiar reaction-diffusion fuzzy CGNNs with infinite delay. Even in this particular case, the result is also good thanks to the infinite allowable upper bounds of time delays.

4. Numerical Example

Example 1. Consider the p-Laplace fuzzy T-S dynamic equations as follows.

Fuzzy Rule 1.
    IF $\omega_1(t)$ is $1/e^{-2\omega_1(t)}$, **THEN**

$$\frac{\partial v}{\partial t} = \nabla \cdot \left(\Gamma(t, x, v) \circ \nabla_p v\right)$$

$$- A(v)\left\{B(v) - \left[C_1 f(v) + D_1 g(v(t - \tau(t), x))\right]\right\},$$

$$v(\theta, x) = \phi(\theta, x), \quad (\theta, x) \in (-\infty, 0] \times \Omega,$$

$$v(t, x) = 0 \in R^2, \quad (t, x) \in R \times \partial\Omega.$$

$$(22)$$

Fuzzy Rule 2.
    IF $\omega_2(t)$ is $1 - (1/e^{-2\omega_1(t)})$, **THEN**

$$\frac{\partial v}{\partial t} = \nabla \cdot \left(\Gamma(t, x, v) \circ \nabla_p v\right) - A(v)$$

$$\times \left\{B(v) - \left[C_2 f(v) + D_2 g(v(t - \tau(t), x))\right]\right\},$$

$$v(\theta, x) = \phi(\theta, x), \quad (\theta, x) \in (-\infty, 0] \times \Omega,$$

$$v(t, x) = 0 \in R^2, \quad (t, x) \in R \times \partial\Omega,$$

$$(23)$$

where $v(t, x) = (v_1(t, x), v_2(t, x))^T$, $\Omega = (0, \pi)$, $p = 4/3$, and then the first eigenvalue $\lambda_1 = ((2/\pi)\int_0^{(p-1)^{1/p}}(dt/(1 - (t^p/(p-1))))^{1/P}))^p = 0.7915$ (see Remark 3). The initial value function is presented as follows:

$$\phi(s, x) = \begin{pmatrix} 0.25\left(1 - \cos\left(5\pi x^2\right)\right)\sin^{398}\left(x^2 - 0.25\right)e^{-100s} \\ 0.2\cos^2\left(4\pi x^2\right)\sin^{508}\left(x^2 - 0.55\right)e^{-100s} \end{pmatrix}.$$

$$(24)$$

    Let $\tau(t) = t/3$, and then $\tau'(t) \leq 1/3 = \tau < 1$. Besides, $A(v) = \text{diag}(0.7 + 0.2\sin^2(v_1), 0.6 + 0.28\sin^2(v_2))$, $B(v) = (2v_1^{1/3}, 1.9v_2^{1/3})^T$, and $f(v) = g(v) = (0.1v_1^{1/3}, 0.2v_2^{1/3})^T$, and correspondingly we assume

$$\underline{A} = \begin{pmatrix} 0.7 & 0 \\ 0 & 0.6 \end{pmatrix}, \qquad \overline{A} = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.88 \end{pmatrix},$$

$$B = \begin{pmatrix} 2 & 0 \\ 0 & 1.9 \end{pmatrix}, \qquad F = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.2 \end{pmatrix} = G.$$

$$(25)$$

In addition, $r = 2$, and

$$C_1 = \begin{pmatrix} -0.2 & 0.3 \\ 0.28 & 0.3 \end{pmatrix} = D_1, \qquad C_2 = \begin{pmatrix} 0.29 & 0.18 \\ 0.2 & -0.1 \end{pmatrix} = D_2,$$

$$\Gamma(t, x, v) = \begin{pmatrix} 0.008 & 0.007 \\ 0.006 & 0.006 \end{pmatrix}.$$

$$(26)$$

Hence, $\Gamma = 0.006$, $|c_1| = |d_1| = 0.3$, $|c_2| = |d_2| = 0.29$, $\lambda_{\min}(\underline{AB}) = 1.1400$, $\lambda_{\max}\overline{A} = 0.9$, and $\lambda_{\max}F = \lambda_{\max}G = 0.2$. Then we can use MATLAB LMI toolbox to solve the inequalities (10)–(12), and obtain $t_{\min} = -0.5569 < 0$ which implies feasible. Further, extracting the datum shows $\underline{q} = 19.7994$, $\overline{q} = 31.1620$, $q_i = 24.3210$, $i = 1, 2$. Thereby, we can conclude from Theorem 4 that the null solution for this fuzzy dynamic equation is globally asymptotically stable (see Figures 1 and 2).

Example 2. To compare our Theorem 4 with [2, Corollary 15], we may consider the following p-Laplace dynamic equations.

$$\frac{\partial v}{\partial t} = \nabla \cdot \left(\Gamma(t, x, v) \circ \nabla_3 v\right)$$

$$- A(v)\left\{B(v) - \left[Cf(v) + Dg(v(t - 30, x))\right]\right\},$$

$$v(\theta, x) = \phi(\theta, x), \quad (\theta, x) \in (-\infty, 0] \times \Omega,$$

$$v(t, x) = 0 \in R^2, \quad (t, x) \in R \times \partial\Omega,$$

$$(27)$$

where $v(t, x) = (v_1(t, x), v_2(t, x))^T$, $\Omega = (0, \pi/2)$, $p = 3$, and then the first eigenvalue $\lambda_1 = 7.3104$ (see Remark 5). Let $\tau(t) \equiv 30$, and then $\tau'(t) \equiv 0 < 1$. In addition, $r = 1$ is the number of the **IF-THEN**. Assume

$$\underline{A} = \begin{pmatrix} 0.7 & 0 \\ 0 & 0.6 \end{pmatrix}, \qquad \overline{A} = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.88 \end{pmatrix},$$

$$B = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.1 \end{pmatrix}, \qquad F = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.2 \end{pmatrix} = G,$$

$$C = \begin{pmatrix} -0.2 & 0.3 \\ 0.3 & 0.3 \end{pmatrix} = D, \qquad \Gamma(t, x, v) = \begin{pmatrix} 0.08 & 0.07 \\ 0.06 & 0.06 \end{pmatrix}.$$

$$(28)$$

    Below we shall employ [2, Corollary 15] and our Theorem 4 to judge the stability of System (27), respectively.
    In [2, Corollary 15], letting $\lambda = 0.001$, we use MATLAB LMI toolbox to solve the LMI conditions [2, (37)] and obtain $t_{\min} = 7.1357 \times 10^{-13} > 0$, which implies that these LMI constraints are not strictly feasible. Again, let $\lambda = 1$ and $\lambda = 10$ in [2, Corollary 15], respectively. And then $t_{\min} = 2.1479 \times 10^{-20} > 0$ and $t_{\min} = 2.1026 \times 10^{-16} > 0$, respectively. Hence, we cannot judge the stability of System (27) from [2, Corollary 15]. However, the LMIs of [2, Corollary 15] are only the sufficient condition for the stability, not necessary for stability. Indeed, we use MATLAB LMI toolbox to solve (10)–(12) in Theorem 4 and obtain $t_{\min} = -0.0794 < 0$, which implies being feasible. Further, extracting the datum results in that $\underline{q} = 0.9803$, $\overline{q} = 1.8381$, and $q_i = 1.3671$,
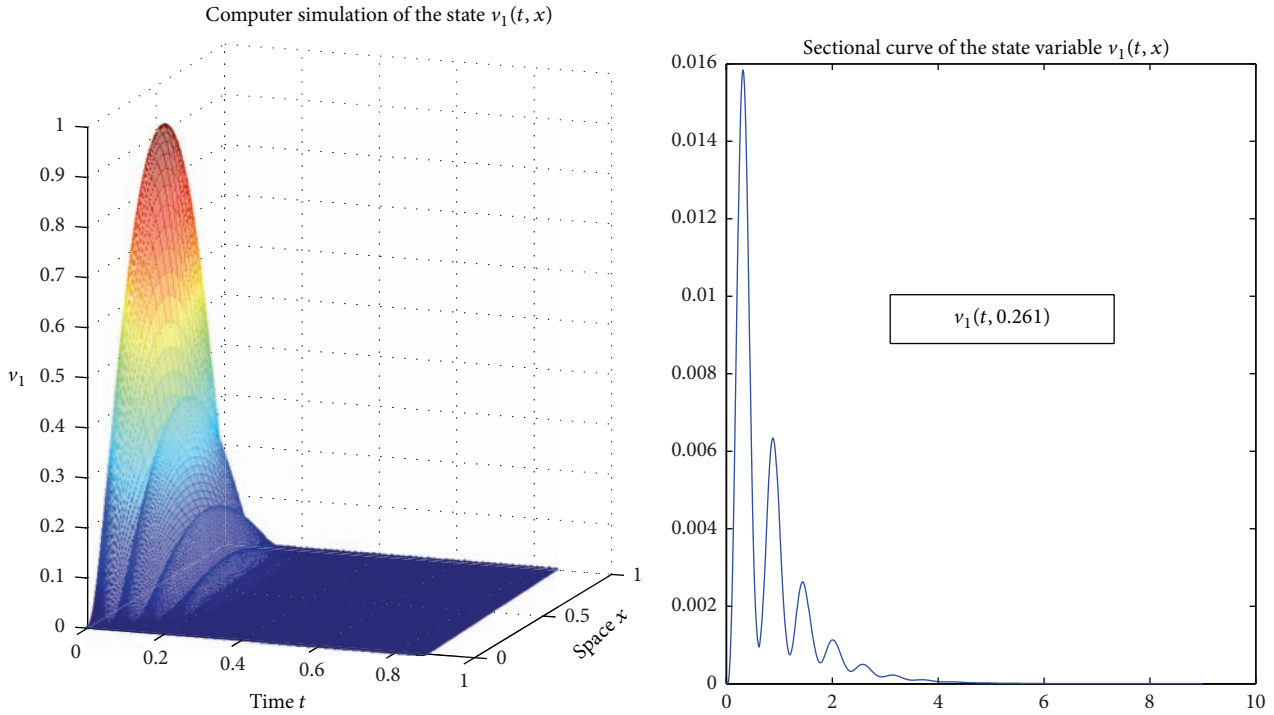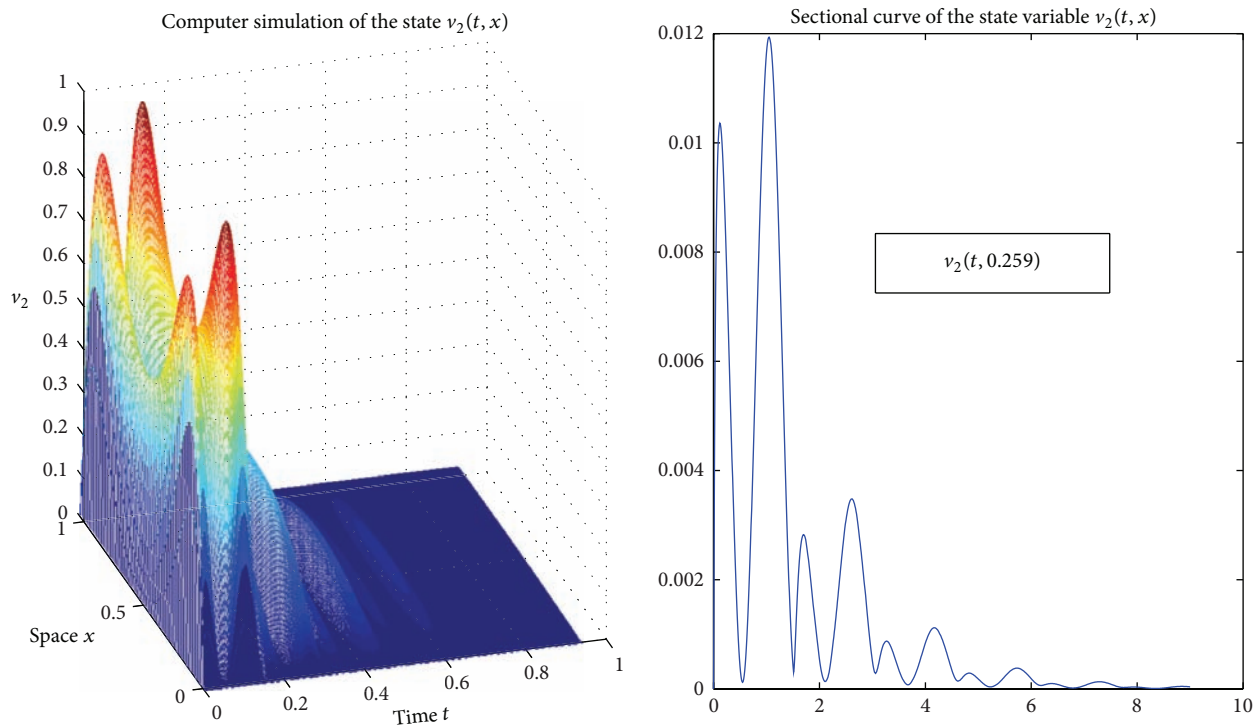
FIGURE 1: The state variable $v_1(t, x)$.

TABLE 1: Comparisons between [2, Corollary 15] and Theorem 4.

	[2, Corollary 15] $\lambda = 0.001$	$\lambda = 1$	$\lambda = 10$	Theorem 4
$t_{\min}$	>0	>0	>0	<0
Feasibility	Unfeasible	Unfeasible	Unfeasible	Feasible
Stability	Unknown	Unknown	Unknown	Global asymptotical stability

for $i = 1, 2$. Thereby, we can conclude from Theorem 4 that the null solution for this fuzzy dynamic equation is globally asymptotically stable.

There are some interesting comparisons between [2, Corollary 15] and our Theorem 4 (see Table 1).

Remark 7. Since we consider the role of the nonlinear p-Laplace diffusion item in the stability criterion, Example 2 illustrates that our Theorem 4 can judge what [2, Corollary 15] cannot do.

Conclusions

In this paper, the global asymptotical stability criterion of the nonlinear p-Laplace fuzzy T-S dynamical equations with infinite delay was derived by the way of Lyapunov-Krasovskii functional approach and some variational methods in the Sobolev space $W_0^{1,p}(\Omega)$. The p-Laplace diffusion item plays its role in our stability criterion while the stability criteria obtained in many previous literatures did not contain the diffusion terms. In fact, when $p = 2$, 2-Laplace is the linear

Laplace, and there are many papers (see, e.g., [25–29]) in which the Laplace diffusion item plays its role in their stability criteria, for the linear Laplace PDEs can be considered in the special Hilbert space $H^1(\Omega)$ that can be orthogonally decomposed into the direct sum of infinitely many eigenfunction spaces. However, the nonlinear p-Laplace ($p > 1$, $p \neq 2$) brings great difficulties, for the nonlinear p-Laplace PDEs should be considered in the frame of Sobolev space $W^{1,p}(\Omega)$ that is only a reflexive Banach space. Indeed, owing to the great difficulties, the authors only provide in [4] the stability criterion in which the nonlinear p-Laplace items play roles in the case of $1 < p < 2$. Now in this paper, we present the stability criterion in which the nonlinear p-Laplace items play roles in the case of $p > 2$ or $p > 1$. Moreover, numerical example shows the effectiveness of the proposed methods. Since the non-linear p-Laplace dynamical equations have many physics and engineering background, including the famous Cohen-Grossberg neural networks, a further profound study is very interesting in mathematical theories, methods, and even practice. Up to now, we do not know how the p-Laplace ($p > 1$) diffusion

Figure 2: The state variable $v_2(t,x)$.

item plays a role in p-stability criteria. This problem remains open and challenging.

Acknowledgments

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