Theoret.Appl.Mech. TEOPM7, Vol.41, No.2, pp. 141–157, Belgrade 2014 *

S-WAVE PROPAGATING IN AN ANISOTROPIC INHOMOGENEOUS ELASTIC MEDIUM UNDER THE INFLUENCE OF GRAVITY, INITIAL STRESS, ELECTRIC AND MAGNETIC FIELD

Rajneesh Kakar **Shikha Kakar**

*doi:10.2298/TAM1402141K Math. Subj. Class.: 74J05; 74J99; 74B10; 74E10; 74F15.

According to: *Tib Journal Abbreviations (C) Mathematical Reviews*, the abbreviation TEOPM7 stands for TEORIJSKA I PRIMENJENA MEHANIKA.

Theoret. Appl. Mech., Vol.41, No.2, pp. 141–157, Belgrade 2014

S-wave propagating in an anisotropic inhomogeneous elastic medium under the influence of gravity, initial stress, electric and magnetic field

Rajneesh Kakar* **Shikha Kakar†**

Abstract

The purpose of this paper is to study the effect of gravity, initial stress, non-homogeneity, electric and magnetic field on the propagation of shear waves in an anisotropic incompressible medium. Various graphs are plotted to show the effect of direction of propagation, the anisotropy, magnetic field, electric field, non-homogeneity of the medium and the initial stress on shear waves. The dispersion equations for shear waves are obtained and discussed for different cases. In fact, in the absence of various material parameters, these equations are in agreement with the classical results for isotropic medium.

Keywords: incompressible, initial-stress, anisotropic, shear-wave, magnetic field, electric field.

1 Introduction

Most of the elastic materials behave as incompressible medium such as rock and the variation of longitudinal wave velocity in these mediums is very high. That is why the study of propagation of S-wave (shear wave) in such materials is useful in various branches of science such as geophysics and earthquake science. Gupta *et al.* [1-2] discussed the effect of stress, anisotropy and non-homogeneity on the propagation of S-waves also they have not taken gravity

*Principal, DIPS Polytechnic College, Hoshiarpur, 146001, India
†HOD, Department of Electronics, SBBSIET, Padhiana Jalandhar, 144001, India

and magnetic field. Gehlot *et al.* [4] studied influence of initial stress, non-homogeneity and magnetic field on S-wave propagation but they have not shown the effect of electric field on shear wave propagation, further they have shown that shear wave velocity depends on density parameter which is wrongly computed. In our study we observed that shear wave rarely depends on density parameter under practical limit. We have studied the shear waves in an anisotropic incompressible medium subjected to magnetic field, electric field, non-homogeneity and initial stress. It is observed that the shear wave phase velocity also depends on the direction of angle of incidence of the shear wave.

2 Governing equations

The governing equations of linear, isotropic and homogenous electro-magneto-elastic solid with hydrostatic initial stress are

a. The displacement-strain relation:

$$e_{ij} = \frac{1}{2}(u_{i.j} + u_{j,i}), \tag{1}$$

b. The small rotation-displacement relation:

$$\omega_{ij} = \frac{1}{2}(u_{i.j} - u_{j,i}). \tag{2}$$

Here u_i are the components of the displacement vector.

c. Maxwell's equations:

$$\nabla \cdot \mathbf{E} = 0, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \mu_e \varepsilon_e \frac{\partial \mathbf{E}}{\partial t}, \tag{3}$$

where $\mathbf{E}$, $\mathbf{B}$, μ_e and ε_e are electric field, magnetic field, permeability and permittivity of the medium.

d. The components of electric and magnetic field:

$$\mathbf{E}\,(0, 0, E) = \mathbf{E}_0 + \mathbf{e}, \quad \mathbf{H}\,(0, 0, H) = \mathbf{H}_0 + \mathbf{h}, \tag{4}$$

where $\mathbf{h}$ is the perturbed magnetic field over $\mathbf{H}_0$ and $\mathbf{e}$ is the perturbed electric field over $\mathbf{E}_0$.

e. Maxwell stress components:

$$T_{ij} = \mu_e \left[H_i e_i + H_j e_j - (H_k e_k)\,\delta_{ij}\right] + \varepsilon_e [E_i e_i + E_j e_j - (E_k e_k)\,\delta_{ij}] \quad \text{(with } i, j, k = 1, 2, 3\text{)}, \tag{5}$$

where δ_{ij} is the Kronecker delta symbol.

Using Eq.(5), we get

$$T_{22} = \mu_e H_0^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \varepsilon_e E_0^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \quad \text{and} \quad T_{12} = 0. \tag{6}$$

The dynamical equations of motion for the propagation of wave have been derived by Biot [3] and in two dimensions these are given by

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} - P\frac{\partial \omega}{\partial y} - \rho g \frac{\partial v}{\partial x} + B_x = \rho \frac{\partial^2 u}{\partial t^2}, \tag{7}$$

$$\frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} - P\frac{\partial \omega}{\partial x} + \rho g \frac{\partial u}{\partial x} + B_y = \rho \frac{\partial^2 v}{\partial t^2}, \tag{8}$$

where s_{11}, s_{22} and s_{12} are incremental thermal stress components. The first two are principal stress components along x- and y-axes, respectively and last one is shear stress component in the x-y plane, ρ is the density of the medium, g is acceleration due to gravity and u, v are the displacement components along x and y directions respectively, B is body force and its components along x and y axis are B_x and $B_y$respectively. ω is the rotational component i.e. $\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ and $P = s_{22} - s_{11}$.

We consider a non-homogeneous anisotropic prestressed elastic medium, under constant primary magnetic field H_0 and electric field E_0 parallel to x-axis. Therefore the body forces along x and y axis are given by

$$B_x = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 v}{\partial x \partial y}\right) + \varepsilon_e E_0^2 \left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 v}{\partial x \partial y}\right), \tag{9}$$

$$B_y = \mu_e H_0^2 \left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y}\right) + \varepsilon_e E_0^2 \left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y}\right), \tag{10}$$

where μ_e and ε_e are permeability and permittivity of the medium.

Following Biot [3] the stress-strain relations are

$$s_{11} = 2\mu e_{xx}, \tag{11}$$

$$s_{22} = s + 2\mu e_{yy}, \tag{12}$$

$$s_{12} = 2\eta e_{xy}, \tag{13}$$

with

$$e_{xx} = \frac{\partial u}{\partial x}, \qquad e_{yy} = \frac{\partial v}{\partial x}, \qquad e_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{14}$$

where $s = \frac{s_{11}+s_{22}}{2}$. Here e_{xx} and e_{yy} are the principal strain components and e_{xy} is the shear strain component, η and μ are the rigidities of the medium.

3 Formulation of the problem

We consider an unbounded incompressible anisotropic medium under initial stresses s_{11} and s_{22} along the x, y directions, respectively. We further consider a non-homogeneous anisotropic prestressed elastic medium, under constant primary magnetic field H_0 and electric field E_0 parallel to x-axis. When the medium is slightly disturbed, the incremental stresses are developed, and the equations of motion in the incremental state from equations (3-10) become

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} - P\frac{\partial \omega}{\partial y} - \rho g\frac{\partial v}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{15}$$

$$\begin{aligned}\frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} - P\frac{\partial \omega}{\partial x} + \rho g\frac{\partial u}{\partial x} + \mu_e H_0^2\left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y}\right)\\ + \varepsilon_e E_0^2\left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y}\right) = \rho\frac{\partial^2 v}{\partial t^2},\end{aligned} \tag{16}$$

where μ_e and ε_e are permeability and permittivity of the medium.

The incompressibility condition $e_{xx} + e_{yy} = 0$ is satisfied by

$$u = -\frac{\partial \varphi}{\partial y}, \; v = \frac{\partial \varphi}{\partial x}. \tag{17}$$

Substituting relations (11-14) and (17) into equations (15) and (16), we get

$$\begin{aligned}\frac{\partial s}{\partial x} - 2\mu\frac{\partial^2 \varphi}{\partial x^2 \partial y} + \frac{\partial}{\partial y}\left[\eta\left(\frac{\partial^2 \varphi}{\partial x^2} - \frac{\partial^2 \varphi}{\partial y^2}\right)\right] -\\ \frac{P}{2}\left(\frac{\partial^3 \varphi}{\partial x^2 \partial y} + \frac{\partial^3 \varphi}{\partial y^3}\right) - \rho g\frac{\partial^2 \varphi}{\partial x^2} = -\rho\left(\frac{\partial^3 \varphi}{\partial t^2 \partial y}\right),\end{aligned} \tag{18}$$

$$
\begin{aligned}
&\frac{\partial s}{\partial y} + \eta \left(\frac{\partial^3 \varphi}{\partial x^3} - \frac{\partial^3 \varphi}{\partial x \partial y^3} \right) + \frac{\partial}{\partial y} \left(2\mu \frac{\partial^2 \varphi}{\partial x \partial y} \right) \\
&\quad - \frac{P}{2} \left(\frac{\partial^3 \varphi}{\partial x^3} + \frac{\partial^3 \varphi}{\partial x \partial y^2} \right) - \rho g \frac{\partial^2 \varphi}{\partial x \partial y} + \mu_e H_0^2 \ \left(\frac{\partial^3 \varphi}{\partial x^3} + \frac{\partial^3 \varphi}{\partial x \partial y^2} \right) \\
&\qquad + \varepsilon_e E_0^2 \left(\frac{\partial^3 \varphi}{\partial x^3} + \frac{\partial^3 \varphi}{\partial x \partial y^2} \right) = \rho \left(\frac{\partial^3 \varphi}{\partial t^2 \partial y} \right).
\end{aligned} \tag{19}
$$

Assuming non-homogeneities as

$$
\eta = \eta_0 \left(1 + ay\right), \qquad \mu = \mu_0 (1 + by), \qquad \rho = \rho_0 (1 + cy) \tag{20}
$$

and substituting from equation (20) into equations (18) and (19), we get

$$
\begin{aligned}
&\frac{\partial^2 s}{\partial x \partial y} - \left[2\mu_0 b - 2a\eta_0\right] \frac{\partial^3 \varphi}{\partial x^2 \partial y} - 2a\eta_0 \frac{\partial^3 \varphi}{\partial y^3} \\
&\quad - \left[2\mu_0 \left(1 + by\right) - \eta_0 \left(1 + ay\right) + \frac{P}{2} \right] \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} \\
&\quad - \left[\eta_0 \left(1 + ay\right) + \frac{P}{2} \right] \frac{\partial^4 \varphi}{\partial y^4} - \rho_0 g c \frac{\partial^2 \varphi}{\partial x^2} \\
&\quad = -\rho_0 \left(1 + cy\right) \frac{\partial^4 \varphi}{\partial t^2 \partial y^2} - \rho_0 c \frac{\partial^3 \varphi}{\partial t^2 \partial y},
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
&\frac{\partial^2 s}{\partial x \partial y} + \left[\eta_0 \left(1 + ay\right) - \frac{P}{2} + \mu_e H_0^2 + \varepsilon_e E_0^2 \right] \frac{\partial^4 \varphi}{\partial x^4} \\
&\quad + \left[2\mu_0 b - \rho_0 g (1 + cy)\right] \frac{\partial^3 \varphi}{\partial x^2 \partial y} + \Big[2\mu_0 \left(1 + by\right) \\
&\quad - \eta_0 \left(1 + ay\right) - \frac{P}{2} + \mu_e H_0^2 + \varepsilon_e E_0^2 l \Big] \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} \\
&\quad = \rho_0 \left(1 + cy\right) \frac{\partial^4 \varphi}{\partial t^2 \partial x^2}.
\end{aligned} \tag{22}
$$

Eliminating "s" from equations (21) and (22), we get

$$\begin{aligned}&\left[\eta_0\ (1+ay)\ -\frac{P}{2}\ +\mu_e\, H_0^2+\varepsilon_e E_0^2\right]\ \frac{\partial^4\varphi}{\partial x^4}+\rho_0 gc\frac{\partial^2\varphi}{\partial x^2}\\&+\left[4\mu_0\ (1+by)-2\eta_0\ (1+ay)\ -\frac{P}{2}\ +\ \mu_0\, H_0^2+\varepsilon_e E_0^2\right]\ \frac{\partial^4\varphi}{\partial x^2\,\partial y^2}\\&+\left[\eta_0\ (1+ay)+\frac{P}{2}\right]\ \frac{\partial^4\varphi}{\partial y^4}+2a\,\eta_0\,\frac{\partial^3\varphi}{\partial y^3}-\ [2a\eta_0-4\mu_0 b]\ \frac{\partial^3\varphi}{\partial x^2\partial y}\\&=\rho_0\ (1+cy)\ \left[\frac{\partial^4\varphi}{\partial t^2\,\partial y^2}+\frac{\partial^4\varphi}{\partial t^2\,\partial x^2}\right]+\rho_0\, c\ \frac{\partial^3\varphi}{\partial t^2\,\partial y}.\end{aligned}\tag{23}$$

4 Solution of the problem

We consider the solution of equation (21) as

$$\varphi(x,y,t)=\varphi_1\exp\left[ik\ (x\ \cos\ \theta\ +y\ \sin\ \theta\ -c_1 t)\right],\tag{24}$$

where θ is incident angle of the shear wave with the x-axis, and c_1 and k are the shear wave velocity and wave number, respectively.

Using equation (22) in equation (21) and equating real and imaginary parts separately, we get

$$\begin{aligned}\frac{c_1^2}{\beta^2}=\frac{1}{1+cy}\Bigg\{&\left[(1+ay)-\frac{P}{2\eta_0}+\frac{\mu_e\, H_0^2+\varepsilon_e E_0^2}{\eta_0}\right]\cos^4\ \theta\\&+\left[4\,\frac{\mu_0}{\eta_0}\,(1+by)-2\,(1+ay)+\frac{\mu_e H_0^2+\varepsilon_e E_0^2}{\eta_0}\right]\cos^2\ \theta\sin^2\theta\\&+\left[(1+ay)+\frac{P}{2\eta_0}\right]\sin^4\theta-\frac{cg}{k^2\beta^2}\cos^2\theta\Bigg\},\end{aligned}\tag{25}$$

$$\frac{c_1^2}{\beta^2}=\left[4\,\frac{\mu_0}{\eta_0}\ \left(\frac{b}{c}\right)-\frac{2a}{c}\right]\ \cos^2\ \theta+2\,\frac{a}{c}\ \sin^2\ \theta.\tag{26}$$

4.1 Analysis of real part of equation (25)

Case I

In this case η is homogeneous ($a\to 0$) i.e., rigidity along vertical direction is constant

$$\left(\frac{c_1}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\left[1 - \frac{P}{2\eta_0} + \frac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\eta_0}\right]\cos^4\theta + \left[4\frac{\mu_0}{\eta_0}(1+by) - 2 + \frac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\eta_0}\right]\cos^2\theta\sin^2\theta + \left[1 + \frac{P}{2\eta_0}\right]\sin^4\theta - \frac{cg}{k^2\beta^2}\cos^2\theta\right\}. \tag{27}$$

The velocity along x-direction ($cos\theta = 1$, $sin\theta = 0$, $c_1 = c_{11}$) as

$$\left(\frac{c_{11}}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\left[1 - \frac{P}{2\eta_0} + \frac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\eta_0} - \frac{cg}{k^2}\right]\right\}. \tag{28}$$

Similarly the velocity of propagation along y-direction ($cos\theta = 0$, $sin\theta = 1$, $c_1 = c_{22}$), is obtained as

$$\left(\frac{c_{22}}{\beta}\right)^2 = \frac{1}{1+cy}\left[1 + \frac{P}{2\eta_0}\right]. \tag{29}$$

Case II

In this case μ is homogeneous (b $\to$ g) i.e., rigidity along horizontal direction is constant

$$\left(\frac{c_1}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\left[(1+ay) - \frac{P}{2\eta_0} + \frac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\eta_0}\right]\cos^4\theta + \left[4\frac{\mu_0}{\eta_0} - 2(1+ay) + \frac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\eta_0}\right]\cos^2\theta\sin^2\theta + \left[(1+ay) + \frac{P}{2\eta_0}\right]\sin^4\theta - \frac{cg}{k^2\beta^2}\cos^2\theta\right\}. \tag{30}$$

The velocity along x-direction ($cos\theta = 1,\ sin\theta = 0,\ c_1 = c_{11}$) is given by

$$\left(\frac{c_{11}}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\left[(1+ay) - \frac{P}{2\eta_0} + \frac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\eta_0} - \frac{cg}{k^2}\right]\right\}. \tag{31}$$

The velocity of propagation along y-direction ($cos\theta = 0, sin\theta \text{g} 1, c_1 = c_{22}$), is given by

$$\left(\frac{c_{22}}{\beta}\right)^2 = \frac{1}{1+cy}\left[(1+ay) + \frac{P}{2\eta_0}\right]. \tag{32}$$

For $P > 0$, the velocity along y-direction may increase considerably at a distance from free surface and the wave becomes dispersive.

Case III

In this case, we take η, μ and ρ to be homogeneous (a $\to$ g, b $\to$ g, c $\to$ g)

$$\left(\frac{c_1}{\beta}\right)^2 = \left\{\left[1 - \frac{P}{2\eta_0} + \frac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\eta_0}\right] \cos^4 \theta + \left[4\frac{\mu_0}{\eta_0} - 2 + \frac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\eta_0}\right] \cos^2 \theta \sin^2 \theta + \left[1 + \frac{P}{2\eta_0}\right] \sin^4 \theta\right\}. \tag{33}$$

In the absence of initial stress the velocity equation becomes

$$\left(\frac{c_1}{\beta}\right)^2 = \frac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\eta_0} \cos^4 \theta + 1 + \Big[4\left(\frac{\mu_0}{\eta_0} - 1\right) + \frac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\eta_0}\Big] \Big[4\frac{\mu_0}{\eta_0} - 2 + \frac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\eta_0}\Big] \cos^2 \theta \sin^2 \theta. \tag{34}$$

In x-direction ($cos\theta = 1,\ sin\theta = 0,\ c_1 = c_{11}$), the velocity is given by

$$\left(\frac{c_{11}}{\beta}\right)^2 = \left[1 + \frac{\mu_e H_0^2 + \varepsilon_e E_0^2}{\eta_0}\right]. \tag{35}$$

In y-direction ($cos\theta = 0,\ sin\theta = 1,\ c_1 = c_{22}$) , the velocity reads

$$\left(\frac{c_{22}}{\beta}\right)^2 = 1. \tag{36}$$

4.2 Analysis of imaginary part of equation (26)

In absence of the initial stress P in equation (26), three cases are given below

Case I

In this case η is homogeneous ($a \to$ g) i.e., rigidity along vertical direction is constant

$$\left(\frac{c_1}{\beta}\right)^2 = \left[4\ \frac{\mu_0}{\eta_0}\ \frac{b}{c}\right]\ \cos^2\theta. \tag{37}$$

This allows that velocity of shear wave is always damped. The velocity of wave along x-direction ($cos\theta = 1,\ sin\theta = 0,\ c_1 = c_{11}$) is obtained as

$$\left(\frac{c_{11}}{\beta}\right)^2 = \left[4\ \frac{\mu_0}{\eta_0}\ \frac{b}{c}\right]. \tag{38}$$

This shows that actual velocity in x-direction is damped by $\left[4\ \frac{\mu_0}{\eta_0}\ \frac{b}{c}\right]$, and no damping takes place along y-direction.

Case II

In this case μ is homogeneous (b $\to$ g), i.e., rigidity along horizontal direction is constant.

$$\left(\frac{c_1}{\beta}\right)^2 = \left[-\frac{2a}{c}\right]\ \cos^2\theta\ + 2\,\frac{a}{c}\ \sin^2\theta. \tag{39}$$

The velocity of wave along x-direction ($cos\theta = 1,\ sin\theta = 0,\ c_1 = c_{11}$) is

$$\left(\frac{c_{11}}{\beta}\right)^2 = \left[-\frac{2a}{c}\right]. \tag{40}$$

The existence of negative sign shows that damping does not take place along x-direction for ($b \to$ g). The velocity along y-direction is given by

$$\left(\frac{c_{22}}{\beta}\right)^2 = \frac{2a}{c} \tag{41}$$

indicating that a damping of magnitude $\frac{2a}{c}$ takes place along y-direction.

Case III

In this case μ and η are homogeneous (a $\to$ 0, b $\to$ 0) but density is linearly varying with depth and

$$\left(\frac{c_1}{\beta}\right)^2 = 0 \tag{42}$$

i.e. no damping takes place.

5 Numerical analysis

We introduce the following non-dimensional parameters to analyze shear waves in an anisotropic incompressible medium subjected to magnetic field, electric field, non-homogeneity and initial stress.

$$a' = \frac{a}{b}, \quad b' = by, \quad c' = \frac{c}{b}, \quad c_1' = \frac{c_1}{b};$$
$$\mu' = \frac{\mu}{\eta_0}, \quad P' = \frac{P}{2\eta_0}, \quad H' = \frac{H}{2\eta_0}, \quad E' = \frac{E}{2\eta_0}.$$

Using above parameters in the equation (25), we obtain

$$c_1'^2 = \frac{1}{1+\bar{c}\bar{b}}\Big\{\left[\left(1+a'b'\right) - P' + H' + E'\right]\cos^4\theta + \left[4N'\left(1+b'\right) - 2\left(1+a'b'\right) + H' + E'\right]\cos^2\theta\sin^2\theta + \left[\left(1+a'b'\right) + P'\right]\sin^4\theta - \bar{c}\bar{g}\cos^2\theta\Big\} \tag{43}$$

Various graphs are plotted with the help of MATLAB. The effect of a non-homogeneity, anisotropy, electric field, magnetic field, direction and initially stressed on shear wave velocity c'(dimensionless) with respect to nondimensional depth b' is shown in figures [1-7].

Figure 1 shows the variation of nondimensional shear wave velocity c_1' at an angle $\theta = 30^0$ with nondimensional depth b' at various values of magnetic field parameter $H' = 0.5, 0.6, 0.7, 0.8, 0.9$ by taking $g' = 0.2$ (gravity parameter),$N' = 0.5$ (anisotropy parameter) ,$a' = 4$(non-homogeneity parameter), $P' = 0.5$ (stress parameter), $E' = 50$(electric parameter) and $c' = 0.8$ (density parameter). It is seen from the graphs, with the increase in magnetic parameter the phase velocity of the shear waves decreases.

Figure 2 shows the variation of nondimensional shear wave velocity c_1' at an angle $\theta = 30^0$ with nondimensional depth b' at various values of electric field parameter $E' = 50, 60, 70, 80, 90$ by taking $g' = 0.2$,$N' = 0.5$,$a' = 4$, $P' = 0.5$, $H' = 0.5$ and $c' = 0.8$. It is seen from the graphs, with the increase in electric parameter the phase velocity of the shear waves decreases.

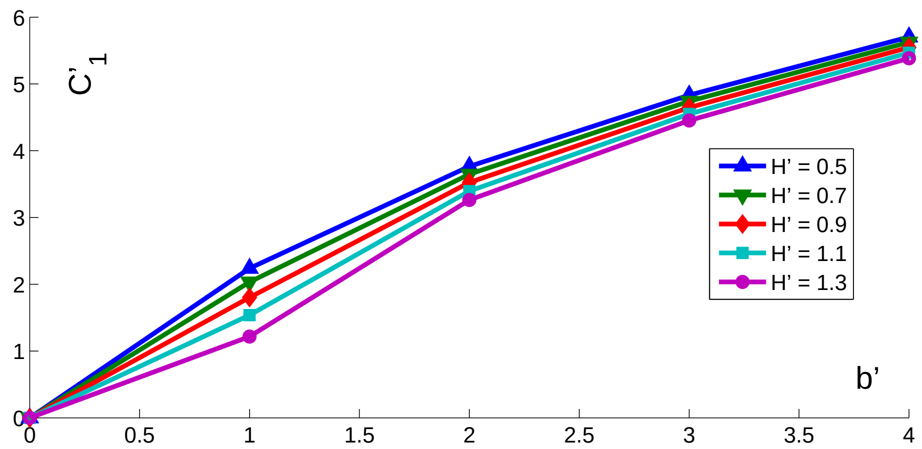

Figure 1: Variation of shear wave velocity c_1 at an angle $\theta = 30^0$ with depth $b$at various values of magnetic field parameter $H' = 0.5, 0.6, 0.7, 0.8, 0.9$ by taking$g' = 0.2$,$N' = 0.5$,$a' = 4$, $P' = 0.5$, $E' = 50$ and $c' = 0.8$

Figure 3 represents the variation of nondimensional shear wave velocity c_1' at an angle $\theta = 30^0$ with nondimensional depth b' at various values of stress parameter $P' = 0.5, 0.9, 1.3, 1.7, 2.1$ by taking $g' = 0.2$,$N' = 0.5$,$a' = 4$, $E' = 50$, $H' = 0.5$ and $c' = 0.8$. It is observed that with the increase in stress parameter the phase velocity also increases.

Figure 4 represents the variation of nondimensional shear wave velocity c_1' at an angle $\theta = 30^0$ with nondimensional depth b' at various values of anisotropy parameter $N' = 0.5, 1, 1.5, 2, 2.5$ by taking $g' = 0.2$, $P' = 0.5a' = 4$, $E' = 50$, $H' = 0.5$ and $c' = 0.8$. The anisotropy of the medium plays an important role in the variation of shear velocity. It is observed that with the increase in anisotropy parameter the phase velocity also increases remarkably.

Figure 5 represents the variation of nondimensional shear wave velocity c_1' at an angle $\theta = 30^0$ with nondimensional depth b' at various values of non-homogeneity parameter $a' = 4, 5, 6, 7, 8$ by taking$g' = 0.2$,$N' = 0.5$,$P' = 0.5$, $E' = 50$, $H' = 0.5$ and $c' = 0.8$. It is observed that with the increase in non-homogeneity parameter the phase velocity also increases remarkably.

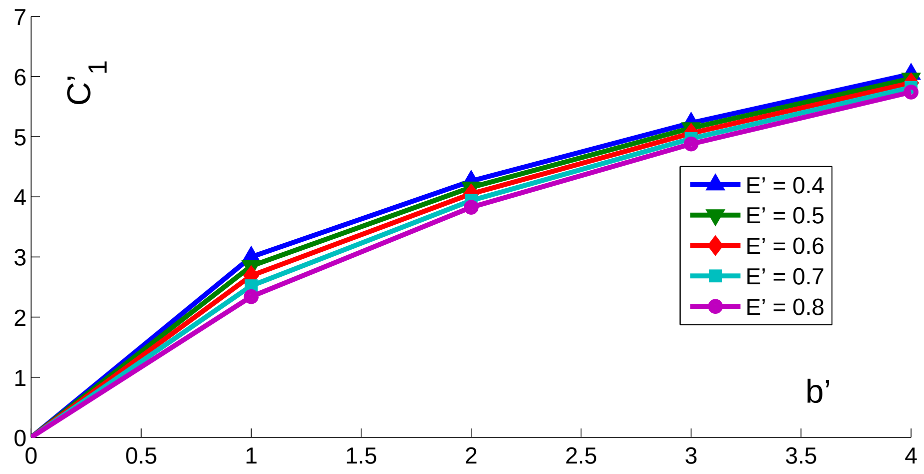

Figure 2: Variation of shear wave velocity c_1 at an angle $\theta = 30^0$ with depth $b$at various values of electric field parameter$E' = 50, 60, 70, 80, 90$ by taking$g' = 0.2$,$N' = 0.5$,$a' = 4$, $P' = 0.5$, $H' = 0.5$ and $c' = 0.8$

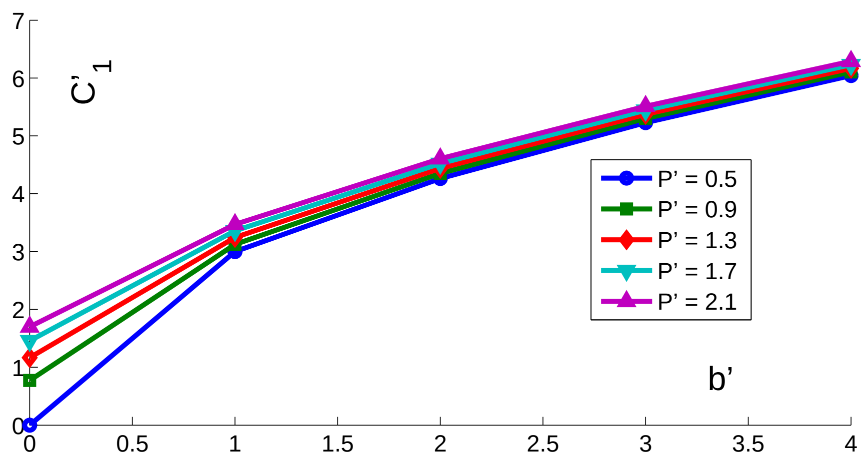

Figure 3: Variation of shear wave velocity c_1 at an angle $\theta = 30^0$ with depth $b$at various values of stress parameter $P' = 0.5, 0.9, 1.3, 1.7, 2.1$ by taking$g' = 0.2$,$N' = 0.5$,$a' = 4$, $E' = 50$, $H' = 0.5$ and $c' = 0.8$

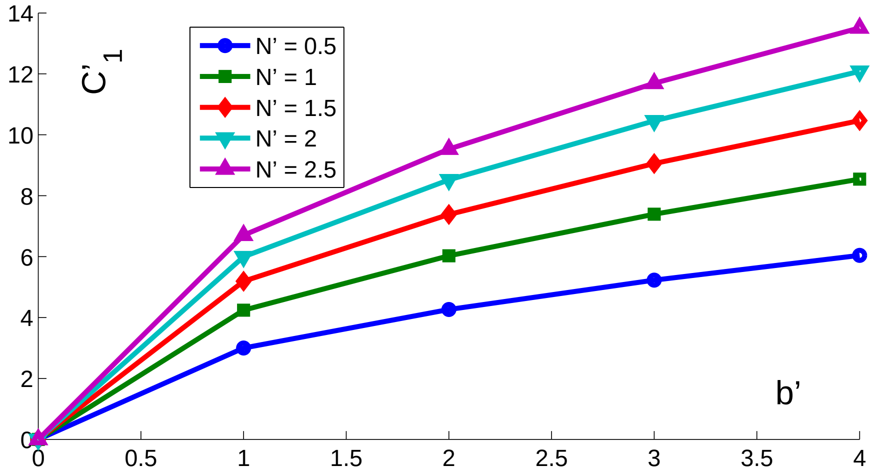

Figure 4: Variation of shear wave velocity c_1 at an angle $\theta = 30^0$ with depth $b$at various values of anisotropy parameter $N' = 0.5, 1, 1.5, 2, 2.5$ by taking$g' = 0.2$, $P' = 0.5a' = 4$, $E' = 50$, $H' = 0.5$ and $c' = 0.8$

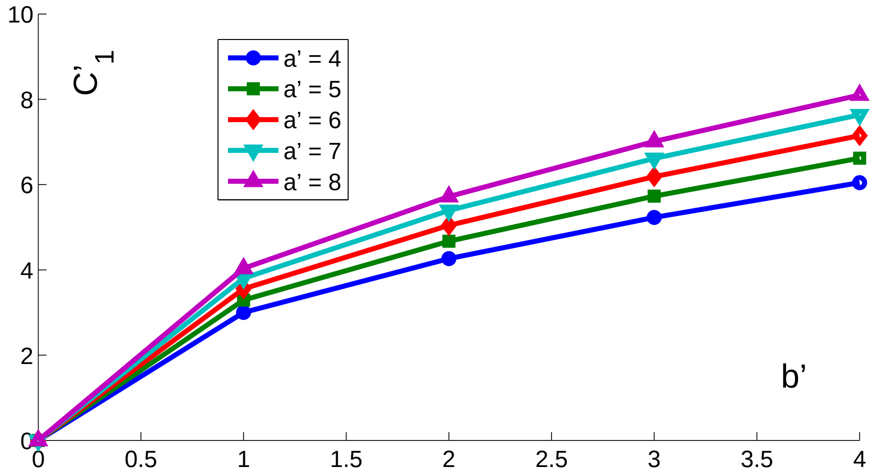

Figure 5: Variation of shear wave velocity c_1 at an angle $\theta = 30^0$ with depth $b$at various values of non-homogeneity parameter $a' = 4, 5, 6, 7, 8$ by taking$g' = 0.2$,$N' = 0.5$,$P' = 0.5$, $E' = 50$, $H' = 0.5$ and $c' = 0.8$

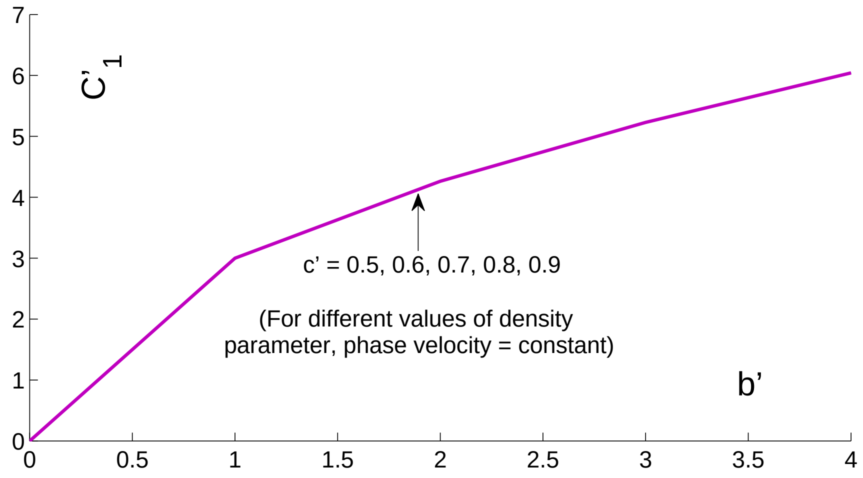

Figure 6: Variation of shear wave velocity c_1 at an angle $\theta = 30^0$ with depth $b$at various values of density parameter $c' = 0.5, 0.6, 0.7, 0.8, 0.9$ by taking$g' = 0.2$,$N' = 0.5$,$P' = 0.5$, $E' = 50$, $H' = 0.5$ and $a' = 4$

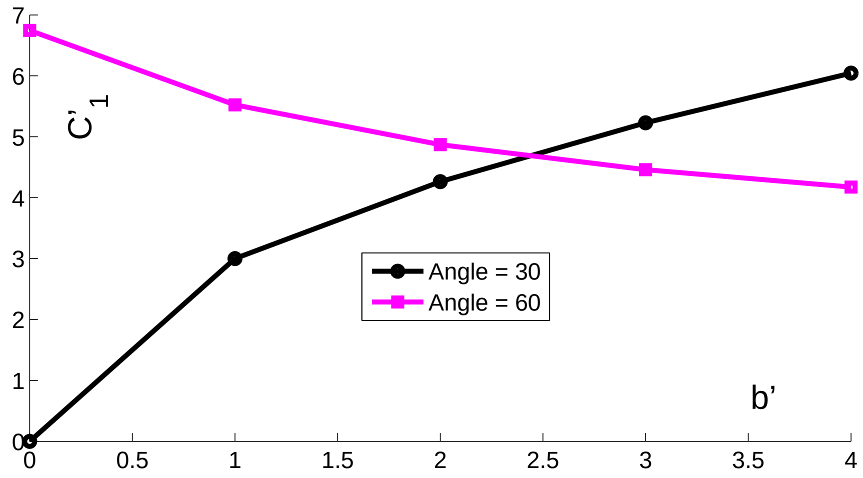

Figure 7: Variation of shear wave velocity c_1 at an angle $\theta = 30^0$ and $\theta = 60^0$with depth $b$at$c' = 0.8$,$g' = 0.2$,$N' = 0.5$,$P' = 0.5$, $E' = 50$, $H' = 0.5$ and $a' = 4$

Figure 6 shows the variation of nondimensional shear wave velocity c_1' at an angle $\theta = 30^0$ with nondimensional depth b' at various values of density parameter $c' = 0.5, 0.6, 0.7, 0.8, 0.9$ by taking $g' = 0.2$,$N' = 0.5$,$P' = 0.5$, $E' = 50$, $H' = 0.5$ and $a' = 4$. It is interesting to note that density parameter has very rare effect on shear wave velocity.

Figure 7 shows the variation of nondimensional shear wave velocity c_1' at an angle $\theta = 30^0$ and $\theta = 60^0$ with nondimensional depth b' at $c' = 0.8$,$g' = 0.2$,$N' = 0.5$,$P' = 0.5$, $E' = 50$, $H' = 0.5$ and $a' = 4$. Shear wave velocity increases as the depth increases. Shear wave velocity is also dependent on direction of propagation. With the increase in angle of incidence the velocity decreases in the beginning and it takes a minimum value before increasing.

6 Conclusion

The anisotropy, magnetic field, electric field, nonhomogeneity of the medium, the direction of propagation, the initial stress and the depth have considerable effect in the velocity of propagation of shear wave. It is found that the variation in parameters associated with anisotropy and non-homogeneity of the medium directly affects the velocity of the wave. This problem attracts the attention of geologists and seismologists.

Acknowledgments

The authors are thankful to unknown reviewers for their valuable comments.

References

[1] Gupta S., Kundu S., Verma AK, Verma AK (2010). *Propagation of S-waves in a non-homogeneous anisotropic incompressible and initially stressed medium*, International Journal of Engineering, Science and Technology, 2(2), pp. 31-42.

[2] Gupta S., Chattopadhyay A., Kumari P (2007). *Propagation of Shear Wave in Anisotropic Medium*, Applied Mathematical Sciences, 1, 2699 – 2706.

[3] Biot MA (1965). *MECHANICS OF INCREMENTAL DEFORMATIONS*, John Wiley and Sons Inc, New York.

[4] Gehlot D., Mehta V., Gehlot KS (2011). *Effect of initial stress and magnetic field on shear wave propagation*, Journal of International Academy of Physical Sciences, 15(1), pp. 71-82.

[5] Landau L.D., Lifshitz E.M. (1960). *ELASTODYNAMICS OF CONTINUOUS MEDIA* (English Translation), Pergamon Press, London, New York.

Submitted in November 2013, revised in March 2014.

Prostiranje S-talasa u nehomogenoj anizotropnoj elastičnoj sredini pod uticajem gravitacije, početnog napona, electričnog i magnetnog polja

Svrha ovog rada je da se prouči uticaj gravitacije, početnog napona, nehomogenosti, električnog i magnetnog polja na prostiranje smičućih talasa u anizotropnoj nestišljivoj sredini. Nacrtani su razni dijagrami da pokažu uticaj pravca prostiranja, anizotropije, magnetnog polja, električnog polja, nehomogenosti sredine i početnog napona na smičuće talase. Jednačine disperzije za smičuće talase su dobijene i razmatrane u različitim slučajevima. Ustvari, u odsustvu raznih materijalnih parametara, te jednačine su u saglasnosti sa klasičnim rezultatima za izotropnu sredinu.

doi:10.2298/TAM1402141K Math. Subj. Class.: 74J05; 74J99; 74B10; 74E10; 74F15.