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P AND CP VIOLATION IN B PHYSICS

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ABSTRACT

While the Kobayashi–Maskawa single phase origin of CP violation passed its first crucial precision test in $B \to J/\psi K_S$, the chirality of weak b-quark couplings has not yet been carefully tested. We discuss recent proposals for studying the chiral and CP-violating structure of these couplings in radiative and in hadronic B decays.

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1 Introduction

It has often been stated that CP violation is among the least tested and poorly understood properties of the Standard Model (SM) of electroweak and strong interactions. CP asymmetries measured in meson decays are accounted for by arbitrary complex Yukawa couplings, yielding a single phase in the Cabibbo–Kobayashi–Maskawa (CKM) matrix [1, 2]. The Kobayashi–Maskawa (KM) model for CP violation was suggested thirty years ago [2] to explain CP non-conservation in K decays. Recently it passed in a remarkable way its first crucial [3] precise [4] test in B decays when a large CP asymmetry was measured in $B^0 \to J/\psi K_S$ [5]. This opens a new era, in which the model is expected to be scrutinized through a variety of other B and B_s decay asymmetries. Impressive progress has already been made in search for asymmetries in several hadronic B decays, including $B^0(t) \to \pi^+\pi^-$ [6], $B^{0,\pm} \to K\pi$ and $B^\pm \to DK^\pm$ [7]. Current measurements are approaching the level of tightening bounds on the CP-violating phase γ [8]. These and forthcoming measurements of B_s decays [9] will enable a cross-check of the KM model. Two complementary tools for studying hadronic decays, an isospin [10] and flavor SU(3)-based approach [11] and a QCD-factorization approach [12], can be checked and refined when confronted by data. This is expected to establish the KM hypothesis at a high precision, up to a point where deviations will hopefully be observed from this simple picture.

The CP transformation consists of a product of parity (P) and charge-conjugation (C). In the SM, parity is violated in a maximal way by assuming that left-handed chiral fermion fields transform as doublets under the SU(2) gauge group, while right-handed fermions transform as singlets. This left–right asymmetry, which is introduced by hand in order to account for the observed low energy phenomenology, may ultimately disappear at high energies in a left–right symmetric theory [13]. This theory would hopefully explain the origin and pattern of fermion masses, mixing and CP-violating phases, which in the current model are represented by arbitrary complex Yukawa couplings. In such a theory, parity and CP violation would have a common origin in a new type of interaction responsible for fermion masses. If parity violation and the quark mass hierarchy have a common origin, then it may be anticipated that right-handed couplings grow with quark masses and are larger for b quarks than for s quarks, in contrast to the measured left-handed weak couplings. In this case the very small b couplings [14], $|V_{cb}| = 0.04$, $|V_{ub}| = 0.003$, would be sensitive to right-handed interactions. In order for right-handed b couplings to be observable in the near future, the left–right symmetry scale would have to be around a TeV or several TeV [15], not far above the electroweak scale. In the absence of good experimental tests for the chirality of b quark couplings [16], such a scenario cannot be ruled out and adequate tests are desirable. This report will focus on several such tests.

I will discuss photonic and hadronic B decays, the study of which serves two purposes:

1. Examining the chiral structure of b-quark couplings in P-odd observables.

2. Testing CKM phases in CP-violating observables.

Since the second topic has recently been the subject of frequent discussions [17], I will pay more attention to the first aspect, discussing it in Sections 2 and 3. The second topic will be addressed in Section 4 while Section 5 will conclude.

2 The photon polarization in radiative B decays

In radiative b quark decays, $b \to s\gamma$, the s quark, which couples to a W in the loop, is left-chiral. Therefore, the photon is predominantly left-handed. The effective Hamiltonian for $b \to s\gamma$ contains a dominant dipole-type operator given by [18]

$$\mathcal{H}_{\rm rad} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left(C_{7L}\mathcal{O}_{7L} + C_{7R}\mathcal{O}_{7R} \right) \ , \tag{1}$$

$$\mathcal{O}_{7L,R} = \frac{e}{16\pi^2} m_b \bar{s}\sigma_{\mu\nu} \frac{1 \pm \gamma_5}{2} b F^{\mu\nu} \ , \tag{2}$$

where the Wilson coefficients C_{7L} and C_{7R} describe the amplitudes for left- and right-handed photons, respectively. In the SM one has $C_{7R}/C_{7L} = m_s/m_b$. Defining the photon polarization in $b \to s\gamma$,

$$\lambda_\gamma \equiv \frac{|C_{7R}|^2 - |C_{7L}|^2}{|C_{7R}|^2 + |C_{7L}|^2} \ , \tag{3}$$

the SM predicts

$$\lambda_\gamma = -1 \ , \tag{4}$$

where $\mathcal{O}(m_s^2/m_b^2)$ corrections are expected to be at the per cent level. Also four-quark operators, $O_{1,2}$ with (V–A)(V–A) structure, contribute to a dominantly left-handed photon, whereas contributions from penguin operators, O_{3-6}, with a different chiral structure, involve much smaller Wilson coefficients and can be safely neglected.

As we will now argue [19], the prediction (4) for the photon polarization holds also in the presence of hadronic effects in exclusive radiative decays in which the final hadronic system carries well-defined spin and parity. Let us consider the decay of a $\bar{B}$ meson into a strangeness $+1$ state X_s with spin-parity J^P, and let us denote hadronic and photonic states with helicities ± 1 by $X_s^{R,L}$ and $\gamma_{R,L}$, respectively. One clearly has

$$\langle X_s^L \gamma_L | \mathcal{O}_{7R} | \bar{B} \rangle = \langle X_s^R \gamma_R | \mathcal{O}_{7L} | \bar{B} \rangle = 0 \ , \tag{5}$$

while parity and rotational invariance of the strong interactions imply

$$\langle X_s^R \gamma_R | \mathcal{O}_{7R} | \bar{B} \rangle = (-1)^{J-1} P \langle X_s^L \gamma_L | \mathcal{O}_{7L} | \bar{B} \rangle \ . \tag{6}$$

Defining weak radiative amplitudes for left- and right-polarized photons,

$$c_i \equiv \langle X_s^i \gamma_i | \mathcal{H}_{\rm rad} | \bar{B} \rangle \quad , \qquad i = L, R \quad , \tag{7}$$

one obtains

$$\frac{c_L}{c_R} = (-1)^{J-1} P \frac{C_{7L}}{C_{7R}} \quad \Rightarrow \quad \frac{|c_R|^2 - |c_L|^2}{|c_R|^2 + |c_L|^2} = \frac{|C_{7R}|^2 - |C_{7L}|^2}{|C_{7R}|^2 + |C_{7L}|^2} \quad . \tag{8}$$

Thus, both in inclusive radiative B decays and in exclusive decays to J^P states, there is a prediction for the photon polarization in terms of ratio of Wilson coefficients. The prediction (4) holds in the SM at the per cent level. It is amusing to note that, while the current calculation of the measured inclusive rate has not yet achieved a level of 10% in spite of great efforts [18], the more precisely predicted photon chirality has not yet been put to an experimental test.

The photon polarization prediction is very sensitive to new physics effects. In several extensions of the SM the photon in $b \to s\gamma$ acquires an appreciable right-handed component due to chirality flip along a heavy fermion line in the electroweak loop. Two well-known examples of such extensions are the left–right-symmetric model and the unconstrained Minimal Supersymmetric Standard Model. In the first model chirality flip along the t-quark line in the loop involves W_L–W_R mixing [20], while in the second scheme a chirality flip along the gluino line in the loop involves left–right squark mixing [21]. In both types of models it was found that, in certain allowed regions of the parameter space, the photons emitted in $b \to s\gamma$ can be largely right polarized without affecting substantially the SM prediction for the inclusive radiative decay rate. This situation calls for an independent measurement of the photon polarization.

Several ways were proposed for measuring photon helicity effects in radiative B decays. We will briefly describe three early suggestions. The first two proposals are sensitive to interference between left and right polarization amplitudes; they can be used to forbid a large interference at present B factories. This may exclude certain parameters in left–right-symmetric and SUSY models. The third method, which measures the photon polarization directly, requires an extremely high luminosity e^+e^- Z factory. Finally, we will focus our attention on a recent proposal to measure the photon polarization, which is feasible at currently operating B factories.

2.1 CP asymmetry in $B^0(t) \to X^{CP}_{s(d)}\gamma$

Consider the time-dependent rate of [22] $B^0(t) \to X^{CP}_{s(d)}\gamma$, where $X^{CP}_s = K^{*0} \to K_S\pi^0$ or $X^{CP}_d = \rho^0 \to \pi^+\pi^-$. The time-dependent CP asymmetry follows from an interference between B^0 and $\bar{B}^0$ decay amplitudes into a common state of definite photon polarization, and is proportional to C_{7R}/C_{7L}. For instance, in the SM the asymmetry in $B^0(t) \to f,\ f = K^{*0}\gamma \to (K_S\pi^0)\gamma$, is given by

$$\mathcal{A}(t) \equiv \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)} = \frac{2C_{7L}C_{7R}}{C_{7L}^2 + C_{7R}^2} \sin 2\beta \sin(\Delta m t)\ . \qquad (9)$$

The ratio C_{7R}/C_{7L} is expected to be a few per cent in the SM, whereas it may be much larger in extensions of the SM [22].

2.2 Angular distribution in $\bar{B} \to \bar{K}^*\gamma \to \bar{K}\pi e^+e^-$

Consider the decay distribution in this process as a function of the angle ϕ between the $\bar{K}\pi$ and e^+e^- planes, where the photon can be virtual [23] or real, converting in the beam pipe to an electron–positron pair [24]. The e^+e^- plane acts as a polarizer, the distribution in ϕ is isotropic for purely circular polarization, and the angular distribution is sensitive to interference between left and right polarization. One finds

$$\frac{d\sigma}{d\phi} \propto 1 + \xi \frac{C_{7L}C_{7R}}{C_{7L}^2 + C_{7R}^2} \cos(2\phi + \delta)\ , \qquad (10)$$

where the parameters ξ and δ are calculable and involve hadronic physics.

2.3 Forward–backward asymmetry in $\Lambda_b \to \Lambda\gamma \to p\pi\gamma$

The forward–backward asymmetry of the proton with respect to the Λ_b in the Λ rest frame is proportional to the photon polarization [25]. Using polarized Λ_b's from extremely high luminosity e^+e^- Z factories, one can also measure the forward–backward asymmetry of the Λ momentum with respect to the Λ_b boost axis [26]. This asymmetry is proportional to the product of the Λ_b and photon polarizations.

2.4 Angular distribution in $B \to K_1(1400)\gamma \to K\pi\pi\gamma$

Radiative decays into an excited tensor meson resonance state, $K_2^*(1430)$, identified through its $K\pi$ decay mode, were observed by the CLEO [27] and Belle [28] collaborations. The Belle collaboration also observed a $K\pi\pi$ resonant structure above an invariant mass of 1.2 GeV, consistent with a large mixture of $K_1(1400) \to K^*\pi$. We will show that a detailed angular analysis of $K\pi\pi\gamma$ events can be used to study the photon polarization.

First, we point out [29] that, in order to measure the photon polarization λ_γ in radiative B decays through the recoil hadron distribution, one requires that the hadrons consist of at least three particles. This necessary condition follows from the simple fact that λ_γ is parity-odd. Consequently, the hadronic quantity multiplying λ_γ in the decay distribution must be P-odd. The pseudoscalar quantity, which contains the smallest number of hadron momenta, is a triple product that requires three hadrons recoiling against the photon. The idea is then to measure an expectation value $\langle \vec{p}_\gamma \cdot (\vec{p}_1 \times \vec{p}_2)\rangle$, where $\vec{p}_1$ and $\vec{p}_2$ are momenta of two of the hadrons in the centre-of-mass frame of the recoil hadrons. Since the triple product is also time-reversal-odd, a non-zero expectation value requires a phase due to final state interactions. The final state interaction phase is calculable in the special case that the decay occurs through two isospin-related intermediate K^* resonance states. This is the case that we will discuss now [19, 29].

Consider the decays $B^+ \to K_1^+(1400)\gamma$ and $B^0 \to K_1^0(1400)\gamma$, where K_1^+ and K_1^0 are observed through

$$K_1^+(1400) \to \left\{ \begin{array}{c} K^{*+}\pi^0 \\ K^{*0}\pi^+ \end{array} \right\} \to K^0\pi^+\pi^0, \;\; K_1^0(1400) \to \left\{ \begin{array}{c} K^{*+}\pi^- \\ K^{*0}\pi^0 \end{array} \right\} \to K^+\pi^-\pi^0 \;, \quad (11)$$

with a branching ratio $\mathcal{B}(K_1 \to K^*\pi) = 0.94 \pm 0.06$ [14]. In both charged and neutral K_1 decays, two Breit–Wigner amplitudes interfere through intermediate K^{*+} and K^{*0}. Decays to ρK, with $\mathcal{B}(K_1 \to \rho K) = 0.03 \pm 0.03$ [14], will be neglected at this point. The two K^* amplitudes are related by isospin; therefore phases other than the two Breit–Wigner phases cancel. The decay $K_1 \to K^*\pi$ is dominated by an S wave and involves a small D wave amplitude, where the D/S ratio of rates is $|A_D/A_S|^2 = 0.04 \pm 0.01$ [14]. Using Lorentz invariance, it is straightforward to write down the decay amplitude for $B \to (K\pi\pi)_{K_1}\gamma$, and to calculate the decay distribution in the K_1 rest frame [19, 29],

$$\frac{d\Gamma}{ds_{13}ds_{23}d\cos\theta} \propto |\vec{J}|^2(1+\cos^2\theta) + \lambda_\gamma 2\mathrm{Im}\left(\hat{n}\cdot(\vec{J}\times\vec{J}^*)\right)\cos\theta \;. \quad (12)$$

The vector $\vec{J}$ is an antisymmetric function of the two pion momenta $p_1,\ p_2$ and of the K momentum p_3. It involves a Breit–Wigner function $B(s) = \left(s - m^2_{K^*} - im_{K^*}\Gamma_{K^*}\right)^{-1}$ of $s_{i3} = (p_i + p_3)^2$ $(i = 1, 2)$. The angle θ lies between the normal to the decay plane $\hat{n} \equiv (\vec{p}_1 \times \vec{p}_2)/|\vec{p}_1 \times \vec{p}_2|$ and $-\vec{p}_\gamma$, all measured in the K_1 rest frame. A useful definition of the normal is in terms of the slow and fast pion momenta, $(\vec{p}_{\rm slow} \times \vec{p}_{\rm fast})/|\vec{p}_{\rm slow} \times \vec{p}_{\rm fast}|$. The angle between this normal and $-\vec{p}_\gamma$ will be denoted by $\tilde{\theta}$.

The decay distribution exhibits an up–down asymmetry of the photon momentum with respect to the K_1 decay plane. The asymmetry is proportional to the photon polarization. When integrating over the entire Dalitz plot one finds

$$\mathcal{A}_{\rm up-down} \equiv \frac{\int_0^{\pi/2}\frac{d\Gamma}{d\cos\tilde{\theta}}d\cos\tilde{\theta} - \int_{\pi/2}^{\pi}\frac{d\Gamma}{d\cos\tilde{\theta}}d\cos\tilde{\theta}}{\int_0^{\pi}\frac{d\Gamma}{d\cos\tilde{\theta}}d\cos\tilde{\theta}} = (0.33 \pm 0.05)\lambda_\gamma \;. \quad (13)$$

The uncertainty follows from experimental errors in the ρK amplitude and in the $K^*\pi$ D-wave amplitude. In the SM, where $\lambda_\gamma \approx -1$, the asymmetry is $(33 \pm 5)\%$ and the polarization signature is unambiguous: *in B^- and $\bar{B}^0$ decays the photon prefers to be emitted in the hemisphere of $\vec{p}_{\rm slow} \times \vec{p}_{\rm fast}$, while in B^+ and B^0 it is more likely to be emitted in the opposite hemisphere.*

The $K\pi\pi$ invariant mass region above 1.2 GeV contains in addition to $K_1(1400)$ other K resonances: an axial-vector $K_1(1270)$, a vector $K^*(1410)$, a tensor $K_2^*(1430)$ and higher resonance states. In order to avoid interference between $K_1(1400)$ and $K_1(1270)$, one would have to study the region $m(K\pi\pi) = 1400$–1550 MeV. The $K^*(1410)$ state leads to no up–down asymmetry, while the $K_2^*(1430)$ involves a small asymmetry [29]. These two resonances dilute the overall up–down asymmetry in this mass range compared to the asymmetry from $K_1(1400)$; however, they do not affect the sign of the asymmetry. Thus, a first crude measurement of merely the sign of the photon polarization can be made using an integrated sample of $K\pi\pi\gamma$ events. In order to improve the efficiency, one would have to isolate events from $K_1(1400)$, where the asymmetry is largest. A procedure for achieving this goal by studying decay angular distributions is described in [19].

Assuming then that $B \to K_1(1400)\gamma \to K\pi\pi\gamma$ events involving one neutral pion [30] can be effectively isolated, one may check the feasibility of measuring the photon polarization at currently operating B factories. A 3σ measurement of a 33% up–down asymmetry requires about 80 reconstructed $B \to K_1(1400)\gamma \to K\pi\pi\gamma$ events, including charged and neutral B and $\bar{B}$ decays. Fewer events may be needed using the full $\tilde{\theta}$ dependence. Assuming $\mathcal{B}(B \to K_1\gamma) = 0.7 \times 10^{-5}$ [31], including known K_1 and K^* branching ratios to the relevant charge states, and allowing another order of magnitude for experimental efficiencies, resolution and background, one finds that this number of reconstructed events can be obtained from a total of 2×10^8 $B\bar{B}$ pairs, including charged and neutrals. This number has already been produced at e^+e^- colliders. In order to measure λ_γ to about 15%, which is the level of precision of the theoretical result (13), one would need about 10^9 $B\bar{B}$ pairs.

3 Chirality tests in hadronic B decays

As mentioned in the introduction, the chirality of weak b quark couplings has not yet been put to a test in hadronic B decays. In semileptonic decays $B \to (D^* \to D\pi)e\bar{\nu}$, angular decay distributions are sensitive to a W_L–W_R mixing amplitude, in which the $b \to c$ coupling is $V + A$ and the leptonic current is $V - A$. These measurements [32] set upper bounds on W_L–W_R mixing; however, it cannot distinguish between W_L and W_R exchange, in which quark and lepton currents have equal chiralities [33]. Such a distinction requires a parity-odd observable, which cannot be constructed from two-body hadronic D^* decays.

In the present section we will discuss a method for testing $V - A$ in b-quark couplings by studying hadronic B decays involving a vector meson and an axial-vector meson, $B \to D^* a_1$, in which the axial vector meson decays to three pseudoscalars. The idea is similar to that used to measure the photon or the K_1 chirality in $B \to K_1\gamma$. In order to calculate helicity amplitudes in $B \to D^* a_1$, we will assume factorization and heavy quark symmetry. Predictions following from this assumption have recently been tested experimentally in B decays to two vector mesons, $B \to D^*\rho$. Let us therefore start with a discussion of this process.

3.1 Helicity amplitudes in $\bar{B}^0 \to D^{*+}\rho^-$

The decays $\bar{B}^0 \to D^{*+}\ (\to D^0\pi^+)\ \rho^-\ (\to \pi^-\pi^0)$, in which each of the two vector mesons decays to two spinless particles whose momenta are measured, can be used to study the vector meson polarization [34]. Using an angular momentum decomposition, the decay amplitude can be written in terms of three helicity amplitudes, $H_0,\ H_+,\ H_-$, corresponding to the three polarization states of the vector mesons,

$$A = \frac{3}{2\sqrt{2\pi}}\left[H_0 \cos\theta_1 \cos\theta_2 + \frac{1}{2}(H_+ e^{i\phi} + H_- e^{-i\phi}) \sin\theta_1 \sin\theta_2\right] \quad . \tag{14}$$

Here θ_1 and θ_2 are the angles between each of the two vector mesons momenta in the B rest frame and the momenta of the corresponding daughter particles in the decaying vector mesons rest frame; ϕ is the angle between the D^* and ρ decay planes. In the above we use a convention in which the normalized decay angular distribution is given by $|A|^2$,

$$\frac{1}{\Gamma}\frac{d^3\Gamma}{d\cos\theta_1 \cos\theta_2 d\phi} = |A|^2 \quad \Rightarrow \quad |H_0|^2 + |H_+|^2 + |H_-|^2 = 1 \quad . \tag{15}$$

The decay distribution is symmetric under $(H_0,\ H_+,\ H_-) \to (H_0^*,\ H_-^*,\ H_+^*)$, implying that rates into left and right polarizations, $|H_-|^2$ and $H_+|^2$ respectively, are indistinguishable. Namely, one cannot distinguish in this process between left- and right-polarized vector mesons. As mentioned before, this follows from the lack of a parity-odd observable when each of the vector mesons decays into two spinless particles. Thus, while the rate into longitudinally polarized state $|H_0|^2$ can be measured, only the magnitude of $|H_+|^2 - |H_-|^2$ is measurable, but not its sign.

The following values were reported very recently by the CLEO collaboration [35]:

$$\begin{aligned}
|H_0| &= 0.941 \pm 0.009 \pm 0.006 \quad , \\
|H_+| \text{ or } |H_-| &= 0.107 \pm 0.031 \pm 0.011 \quad , \\
|H_-| \text{ or } |H_+| &= 0.322 \pm 0.025 \pm 0.016 \quad .
\end{aligned} \tag{16}$$

In their report the collaboration quotes a value for $|H_+|$, which is smaller than $|H_-|$, assuming that the D^* predominantly carries the chirality of the c quark, as would follow from a $\bar{c}\gamma_\mu(1-\gamma_5)b$ coupling. It is important to check this assumption experimentally.

The three helicity amplitudes $H_{0,\pm}$ can be calculated using factorization and heavy quark symmetry [36]. In this approximation, the three normalized amplitudes can be written in terms of meson masse. For a $\bar{c}\gamma_\mu(1-\gamma_5)b$ current one finds [37]

$$H_0 = \left(1 + \frac{4y}{y+1}\epsilon^2\right)^{-\frac{1}{2}} \quad , \qquad H_\pm = \left(1 \mp \sqrt{\frac{y-1}{y+1}}\right)\epsilon\left(1 + \frac{4y}{y+1}\epsilon^2\right)^{-\frac{1}{2}} \quad , \tag{17}$$

where $y \equiv (m_B^2 + m_{D^*}^2 - m_\rho^2)/2m_B m_{D^*} = 1.476$, $\epsilon \equiv m_\rho/(m_B - m_{D^*}) = 0.236$. Thus, the values

$$|H_0| = 0.940 \ , \qquad |H_+| = 0.125 \ , \qquad |H_-| = 0.318 \ \ , \tag{18}$$

are obtained, in good agreement with (16). One expects deviations from factorization in the amplitudes $H_\pm$ which are subleading in $1/m_b$. The above predictions of the Standard Model apply to $\bar{B}^0$ decays, while in B^0 decays the values of $|H_+|^2$ and $|H_-|^2$ are interchanged. In the case of a $\bar{c}\gamma_\mu(1+\gamma_5)b$ current, the roles of $|H_+|$ and $|H_-|$ are interchanged. Thus, while the measured value of $|H_0|$ is in excellent agreement with the SM prediction using factorization and heavy quark symmetry, measurements of $|H_\pm|$ cannot distinguish between $V-A$ and $V+A$ currents. The present experimental precision allows also for an admixture of the two chiralities in the $b \to c$ coupling.

3.2 Chirality test in $\bar{B}^0 \to D^{*+}a_1^-$

A large sample of 18000 ± 1200 partially reconstructed $\bar{B}^0 \to D^{*+}a_1^-$ events, combining this mode with its charge-conjugate, was reported very recently by the BABAR collaboration [38]. The a_1 was reconstructed via the decay chain $a_1^- \to \rho^0\pi^-$, $\rho^0 \to \pi^+\pi^-$, while the D^* was identified by a slow pion. We will now show that the D^* chirality can be determined from a suitable angular decay distribution [39]. We will only assume an S-wave $\rho^0\pi^-$ structure, without using the a_1 resonance shape and width, which involve a large uncertainty [14]. A small D-wave correction can also be incorporated in the calculation.

The decay amplitude for this process is written in terms of weak helicity amplitudes H'_i, in analogy with (14),

$$A(\bar{B}^0 \to D^{*+}\pi^-(p_1)\pi^-(p_2)\pi^+(p_3)) = \sum_{i=0,+,-} H'_i A_i \ \ . \tag{19}$$

The strong amplitude A_i involves two terms, corresponding to two possible ways of forming a ρ meson from $\pi^+\pi^-$ pairs, each of which can be written in terms of two invariant amplitudes:

$$A(a_1(p,\varepsilon) \to \rho(p',\varepsilon')\pi) = A(\varepsilon\cdot\varepsilon'^*) + B(\varepsilon\cdot p')(\varepsilon'^*\cdot p) \ , \tag{20}$$

convoluted with the amplitude for $\rho^0(\varepsilon') \to \pi^+(p_i)\pi^-(p_j)$, which is proportional to $\varepsilon' \cdot (p_i - p_j)$. One finds

$$A(a_1^-(p,\varepsilon) \to \pi^-(p_1)\pi^-(p_2)\pi^+(p_3)) \propto C(s_{13}, s_{23})(\varepsilon \cdot p_1) + (p_1 \leftrightarrow p_2) \ , \tag{21}$$

$$C(s_{13}, s_{23}) = [A + Bm_{a_1}(E_3 - E_2)]B_\rho(s_{23}) + 2AB_\rho(s_{13}) \ , \tag{22}$$

where $s_{ij} = (p_i+p_j)^2$, $B_\rho(s_{ij}) = (s_{ij} - m_\rho^2 - im_\rho\Gamma_\rho)^{-1}$, and pion energies are given in the a_1 rest frame. The amplitudes A and B are related to S- and D-wave $\rho\pi$ amplitudes. When neglecting the small D-wave amplitude [14], they obey [40]

$$B = -A\left(1 - \frac{m_\rho}{E_\rho}\right)\frac{E_\rho}{m_\rho \vec{p}_\rho^{\ 2}} \ . \tag{23}$$

Defining an angle θ between the normal to the a_1 decay plane and the direction opposite to the D^* in the a_1 rest frame, one calculates the $B \to D^* 3\pi$ decay distribution,

$$\begin{aligned}\frac{d\Gamma}{ds_{13}ds_{23}d\cos\theta} &\propto |H_0'|^2\sin^2\theta|\vec{J}|^2 + (|H_+'|^2 + |H_-'|^2)\frac{1}{2}(1+\cos^2\theta)|\vec{J}|^2 \\ &+ (|H_+'|^2 - |H_-'|^2)\cos\theta\,\mathrm{Im}[(\vec{J}\times\vec{J}^*)\cdot\hat{n}] \ ,\end{aligned} \tag{24}$$

where

$$\vec{J} = C(s_{13}, s_{23})\vec{p}_1 + C(s_{23}, s_{13})\vec{p}_2 \ . \tag{25}$$

A fit to the angular decay distribution enables separate measurements of the three terms $|H_0'|^2$, $|H_+'|^2 + |H_-'|^2$ and $|H_+'|^2 - |H_-'|^2$. In particular, one can measure the P-odd up–down asymmetry of the D^* momentum direction with respect to the a_1 decay plane,

$$\mathcal{A} = \frac{3}{4}\frac{\langle \mathrm{Im}\ [\hat{n}\cdot(\vec{J}\times\vec{J}^*)]\mathrm{sgn}\ (s_{13} - s_{23})\rangle}{\langle|\vec{J}|^2\rangle}\frac{|H_+'|^2 - |H_-'|^2}{|H_0'|^2 + |H_+'|^2 + |H_-'|^2} \ , \tag{26}$$

which determines $|H_+'|^2 - |H_-'|^2$. Integration over the entire Dalitz plot gives

$$\mathcal{A} = -0.237\frac{|H_+'|^2 - |H_-'|^2}{|H_0'|^2 + |H_+'|^2 + |H_-'|^2} \ . \tag{27}$$

In the heavy quark symmetry and factorization approximation [37], using (17) where $y \equiv (m_B^2 + m_{D^*}^2 - m_{a_1}^2)/2m_Bm_{D^*} = 1.432$, $\epsilon \equiv m_{a_1}/(m_B - m_{D^*}) = 0.376$, the results are

$$|H_0'| = 0.866 \ , \quad |H_+'| = 0.188 \ , \quad |H_-'| = 0.463 \ . \tag{28}$$

These values, which depend somewhat on m_{a_1} and on neglecting corrections to factorization in $H_\pm'$, imply $\mathcal{A} = 0.042$. The sign of the asymmetry, which is not expected to change under these corrections, provides an unambiguous signature for a $V - A$ coupling in contrast to $V + A$. In the a_1^- rest frame *the* $\bar{B}^0$ *and* D^{*+} *prefer to move in the hemisphere opposite to* $\vec{p}(\pi^-)_{\rm slow} \times \vec{p}(\pi^-)_{\rm fast}$. Present statistics seem to be sufficient for determining this sign.

4 Determining $2\beta+\gamma$ in CP asymmetries

Both $\bar{B}^0 \to D^{*+}\rho^-$ and $\bar{B}^0 \to D^{*+}a_1^-$ belong to a class of processes, which also contains $\bar{B}^0 \to D^+\pi^-,\ D^{*+}\pi^-,\ D^+\rho^-$ [41], from which the weak phase $2\beta+\gamma$ can be determined with no hadronic uncertainty. Using the well-measured value of β [5] this would fix γ. The difficulty in these methods lies in having to measure a very small time-dependent interference between $b \to c\bar{u}d$ and doubly-CKM-suppressed $\bar{b} \to \bar{u}c\bar{d}$ transitions, where $|V_{ub}^*V_{cd}/V_{cb}V_{ud}^*| \simeq 0.02$. In decays to $D^+\pi^-,\ D^{*+}\pi^-,\ D^+\rho^-$ the resulting analyses are sensitive to the doubly-CKM-suppressed rate, a precise measurement of which is extremely challenging. In the case of decays to two vector mesons, $\bar{B}^0 \to D^{*+}\rho^-$, one avoids the need to determine this small rate by using an interference between helicity amplitudes of CKM-allowed and doubly-CKM-suppressed decays [42]. This requires a detailed angular analysis in addition to time-dependent measurements. The feasibility of using an angular analysis for measuring the helicity amplitudes in the dominant CKM-allowed channel was demonstrated in [35]. It will be more difficult, but not impossible, to measure the time-dependent interference of helicity amplitudes with such disparate magnitudes. Here we will describe this method for determining $2\beta+\gamma$, first in $\bar{B}^0 \to D^{*+}\rho^-$ [42], and then in $\bar{B}^0 \to D^{*+}a_1^-$ [39] where a discrete ambiguity in the weak phase can be resolved. Our considerations will not depend on an assumption of factorization.

4.1 $\bar{B}^0(t) \to D^{*+}\rho^-$

It is convenient to write the amplitude $A \equiv A(\bar{B}^0 \to D^{*+}(\to D^0\pi^+)\rho^-(\to \pi^-\pi^0))$ in a linear polarization basis [43], in which the D^* and ρ transverse polarizations are either parallel or perpendicular to one another, $H_{\parallel,\perp} = (H_+ \pm H_-)/\sqrt{2}$, and to similarly expand $a \equiv A(B^0 \to D^{*+}\rho^-)$ in terms of $h_{0,\parallel,\perp}$:

$$A = \frac{3}{2\sqrt{2\pi}}(H_0 g_0 + H_\parallel g_\parallel + iH_\perp g_\perp)\ ,\quad a = \frac{3}{2\sqrt{2\pi}}(h_0 g_0 + h_\parallel g_\parallel + ih_\perp g_\perp)\ , \tag{29}$$

$$g_0 = \cos\theta_1\cos\theta_2\ ,\ g_\parallel = \frac{1}{\sqrt{2}}\sin\theta_1\sin\theta_2\cos\phi\ ,\ g_\perp = \frac{1}{\sqrt{2}}\sin\theta_1\sin\theta_2\sin\phi \tag{30}$$

The transversity amplitudes can be written as,

$$H_t = |H_t|\exp(i\Delta_t)\ ,\qquad h_t = |h_t|\exp(i\delta_t)\exp(i\gamma)\ . \tag{31}$$

The time-dependent rate for $\bar{B}^0(t) \to D^{*+}\rho^-$ has the general form

$$\begin{aligned}\Gamma(t) &\propto e^{-\Gamma t}\left[(|A|^2+|a|^2) + (|A|^2-|a|^2)\cos\Delta mt + 2\mathrm{Im}\left(e^{-2i\beta}Aa^*\right)\sin\Delta mt\right] \\ &= e^{-\Gamma t}\sum_{t\le t'}\left(\Lambda_{tt'} + \Sigma_{tt'}\cos\Delta mt + \rho_{tt'}\sin\Delta mt\right) g_t g_t'\ .\end{aligned} \tag{32}$$

Each of the coefficients in the sum can be measured by performing a time-dependent angular analysis. Denoting by $\Phi \equiv 2\beta + \gamma$, this determines the following quantities:

$$\begin{aligned}
&|H_t|^2 \ , \quad |H_0||H_\perp| \sin(\Delta_0 - \Delta_\perp) \ , \quad |H_\parallel||H_\perp| \sin(\Delta_\parallel - \Delta_\perp) \ , \\
&|H_t||h_t| \sin(\Phi + \Delta_t - \delta_t) \ , \qquad\qquad\qquad (t = 0, \parallel, \perp) \ , \\
&|H_\perp|h_0| \cos(\Phi + \Delta_\perp - \delta_0) - |H_0|h_\perp| \cos(\Phi + \Delta_0 - \delta_\perp) \ , \\
&|H_\perp|h_\parallel| \cos(\Phi + \Delta_\perp - \delta_\parallel) - |H_\parallel|h_\perp| \cos(\Phi + \Delta_\parallel - \delta_\perp) \ .
\end{aligned} \tag{33}$$

One does not rely on knowledge of the small $|h_t|^2$ terms [42], in which uncertainties would be large. Decays into the charge-conjugate state $D^{*-}\rho^+$ determine similar quantities, where Φ is replaced by $-\Phi$. It is then easy to show that this overall information is sufficient for determining $\sin\Phi$ *up to a sign ambiguity*.

4.2 What is new in $\bar{B}^0(t) \to D^{*+}a_1^-$?

The amplitudes $A' \equiv A(\bar{B}^0 \to D^{*+}(3\pi)^-_{a_1})$ and $a' \equiv A(B^0 \to D^{*+}(3\pi)^-_{a_1})$ are written in analogy with (29):

$$A' = \sum_{t=0,\parallel,\perp} H'_t A_t \ , \qquad a' = \sum_{t=0,\parallel,\perp} h'_t A_t \ . \tag{34}$$

Instead of *real* functions g_t of the angular variables, one has calculable *complex* amplitudes A_t defined in Eq. (21), which are functions of corresponding angles [39]. One measures $\Gamma(\bar{B}^0(t) \to D^{*+}(3\pi)^-_{a_1})$ and $\Gamma(\bar{B}^0(t) \to D^{*-}(3\pi)^+_{a_1})$ as a function of θ and an angle ψ that defines the direction of the D^* decay plane. The complex A_t, in contrast to the real g_t, imply that one can measure also interference terms between helicity amplitudes H'_t and $h'_{t'}$ in which the cosines and sines in (33) are interchanged. This additional information has the effect of resolving the ambiguity in the sign of $\sin\Phi$ [39].

The advantage of $B \to D^* a_1$ in determining *unambiguously* the CP-violating phase $2\beta + \gamma$ can be traced back to the parity-odd measurables that occur in this process but not in $B \to D^*\rho$. As noted, $|H'_+|^2 - H'_-|^2 = 2\mathrm{Re}(H'_\parallel H'^*_\perp)$ is P-odd, and so is $\mathrm{Im}[e^{2i\beta}(H'_\parallel h'^*_\perp + H_\perp h^*_\parallel)]$. These terms, which do not occur in the time-dependent rate of $\bar{B}^0 \to D^{*+}\rho^-$, do occur in $\bar{B}^0(t) \to D^{*+}a_1^-$ multiplying a P-odd function of θ, $\cos\theta \, \mathrm{Im}[(\vec{J} \times \vec{J}^*) \cdot \hat{n}]$. A practical advantage of $\bar{B}^- \to D^{*+}a_1^-$ over $\bar{B}^0 \to D^{*+}\rho^-$ is the occurrence of only charged pions in the first process. A slight disadvantage of the first process may be an intrinsic uncertainty in the amplitudes A_t obtained in (21).

5 Conclusion

Parity-odd measurables in hadronic and photonic B decays were shown to test the chiral structure of weak b quark couplings at tree level and at the one-loop level, respectively. Time-dependent CP asymmetries in $B \to D^* a_1$ complement measurements in $B \to D^* \rho$, and resolve a discrete ambiguity in a clean determination of the CP-violating phase $2\beta + \gamma$.

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