Neutrino dipole moments and charge radii in noncommutative space-time

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In this paper we obtain a bound $\Lambda_{\rm NC} \stackrel{<}{\sim} 150$ TeV on the scale of space-time noncommutativity considering photon-neutrino interactions. We compute "$\star$-dipole moments" and "$\star$-charge radii" originating from space-time noncommutativity and compare them with the dipole moments calculated in the neutrino-mass extended standard model (SM). The computation depends on the nature of the neutrinos, Dirac versus Majorana, their mass and the energy scale. We focus on Majorana neutrinos. The "$\star$–charge radius" is found to be $r^* = \sqrt{|\langle r_\nu^2 \rangle_{\rm NC}|} = \left| 3 \sum_{i=1}^{3} (\theta^{0i})^2 \right|^{1/4} \stackrel{<}{\sim} 1.6 \times 10^{-19}$ cm at $\Lambda_{\rm NC} = 150$ TeV.

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In this paper we compare the consequences of the neutrino-photon interaction that can be induced by space-time non-commutativity [1], with characteristic electromagnetic properties of neutrinos: charge radii and dipole moments. These miniscule dipole moments are sensitive probes of fluctuations at scales as small as 10^{-35} cm, as seen through electromagnetic interactions at long range.

The action of the model that we would like to study, differs from commutative theory by the presence of $\star$-products and Seiberg-Witten (SW) maps [2, 3, 4, 5]. In the presence of space-time noncommutativity, neutral particles can couple to gauge bosons via a $\star$-commutator [6]

$$D_\mu^{\rm NC}\widehat{\psi} = \partial_\mu\widehat{\psi} - ie\kappa\widehat{A}_\mu \star \widehat{\psi} + ie\kappa\widehat{\psi} \star \widehat{A}_\mu , \qquad (1)$$

where a hat denotes noncommutative fields that are expanded in terms of regular fields via SW maps. The $\star$-products originate from antisymmetric tensor fields that can conceivably be traced back to an extension of gravity. In the language of quantized gravity, noncommutative effects belong to target space, a quantum deformation of the classical base space. On target space, the $\star$-products induce an algebraic structure of position operators that define noncommutative space-time. In any case, observable effects are not necessarily fixed to the Planck scale.

The effective model of neutrino and photon interactions in noncommutative space-time [1] provides a description of the interaction of particles that enter from an asymptotically commutative region into a noncommutative interaction region [3]. The expansion in the form proposed in [2, 3, 4, 5] is understood as a perturbative description of non-commutativity of the target space variables. Currently, this approach neither tries to describe dynamics of noncommutative structures nor includes nonperturbative effects. The action, written in terms of commutative fields, is gauge invariant under $U(1)_{\rm em}$-gauge transformations. The requirements satisfied by our model are summarized in [7]. For related work on noncommutative gauge theory and phenomenology, see [6, 7, 8, 9, 10].

Expanding the $\star$-product in (1) to first order in the antisymmetric (Poisson) tensor $\theta^{\mu\nu}$, we find the following covariant derivative on neutral spinor fields:

$$D_\mu^{\rm NC}\widehat{\psi} = \partial_\mu\widehat{\psi} + e\kappa\theta^{\nu\rho}\,\partial_\nu\widehat{A}_\mu\,\partial_\rho\widehat{\psi} . \qquad (2)$$

We treat $\theta^{\mu\nu}$ as a constant background field of strength $|\theta^{\mu\nu}| = 1/\Lambda_{\rm NC}^2$ that models the non-commutative structure of space-time in the neighborhood of the interaction region [9]. The scale of noncommutativity $\Lambda_{\rm NC}$ enters by choosing dimensionless matrix elements $c^{\mu\nu} = \Lambda_{\rm NC}^2\theta^{\mu\nu}$ of order one. As θ is not invariant under Lorentz transformations, the neutrino field can pick up angular momentum in the interaction. In the following we will assume θ to be constant in the mean over as large an interaction region as acceptable. The bounds that we are going to derive rely on this assumption which we do not debate any further. How this regional restriction can be derived is not of our concern here. However, if string theory leads consistently to associated Seiberg-Witten maps, it is for this theory to answer those questions precisely.

The gauge-invariant action for a neutral fermion that couples to an Abelian gauge boson via (2) is

$$\begin{aligned} S &= \int d^4x\,\bar{\psi}\left[\left(\,i\gamma^\mu\partial_\mu - m\right)\right. \qquad (3) \\ &\quad - \left.\frac{e}{2}\kappa F_{\mu\nu}\left(i\theta^{\mu\nu\rho}\partial_\rho - \theta^{\mu\nu}m\right)\right]\psi , \\ \theta^{\mu\nu\rho} &= \theta^{\mu\nu}\gamma^\rho + \theta^{\nu\rho}\gamma^\mu + \theta^{\rho\mu}\gamma^\nu , \end{aligned}$$

up to first order in θ [1, 10]. The noncommutative part of (3) induces a *force*, proportional to the gradient of the field strengths, which represents an interaction of the

Stern-Gerlach type [11]. This interaction is non-zero even for $m_\nu = 0$ and in this case reduces to the coupling between the stress-energy tensor of the neutrino $T^{\mu\nu}$ and the symmetric tensor composed from θ and F [1]. Above interaction has also been derived in Ref. [12], following the general discussion of the photon–electron noncommutative interaction in [13]. For a discussion of energy dependent Stern-Gerlach type of interactions, we refer to [12].

Following the general arguments of [14, 15, 16, 17] only the Dirac neutrino can have a magnetic moment. However, the transition matrix elements relevant for $\nu_i \longrightarrow \nu_j$ may exist for both Dirac and Majorana neutrinos. In the neutrino-mass extended standard model [17], the photon-neutrino effective vertex is determined from the $\nu_i \longrightarrow \nu_j + \gamma$ transition, which is generated through 1-loop electroweak processes that arise from the so-called "neutrino–penguin" diagrams via the exchange of $\ell = e, \mu, \tau$ leptons and weak bosons, and is given by [10, 14]

$$
\begin{aligned}
J_\mu^{\text{eff}}(\gamma\nu\bar\nu)\epsilon^\mu(q) = & \left\{ F_1(q^2)\bar\nu_j(p')(\gamma_\mu q^2 - q_\mu \slashed{q})\nu_i(p)_L \right. \\
& - iF_2(q^2)\left[m_{\nu_j}\bar\nu_j(p')\sigma_{\mu\nu}q^\nu\nu_i(p)_L \right. \\
& + \left.\left. m_{\nu_i}\bar\nu_j(p')\sigma_{\mu\nu}q^\nu\nu_i(p)_R \right] \right\}\epsilon^\mu(q). \quad (4)
\end{aligned}
$$

The above effective interaction is invariant under electromagnetic gauge transformations. The first term in (4) vanishes identically for real photon due to the electromagnetic gauge condition.

From the general decomposition of the second term of the transition matrix element T obtained from (4),

$$
\text{T} = -i\epsilon^\mu(q)\bar\nu(p')\left[A(q^2) - B(q^2)\gamma_5 \right]\sigma_{\mu\nu}q^\nu\nu(p), \quad (5)
$$

we found the following expression for the electric and magnetic dipole moments

$$
d_{ji}^{\text{el}} \equiv B(0) = \frac{-e}{M^{*2}}\left(m_{\nu_i} - m_{\nu_j} \right)\sum_{\ell=e,\mu,\tau}\text{U}_{jk}^\dagger \text{U}_{ki}F_2\left(\frac{m_{\ell_k}^2}{m_W^2}\right), (6)
$$

$$
\mu_{ji} \equiv A(0) = \frac{-e}{M^{*2}}\left(m_{\nu_i} + m_{\nu_j} \right)\sum_{\ell=e,\mu,\tau}\text{U}_{jk}^\dagger \text{U}_{ki}F_2\left(\frac{m_{\ell_k}^2}{m_W^2}\right), (7)
$$

where $i, j, k = 1, 2, 3$ denotes neutrino species, and

$$
F_2\left(\frac{m_{\ell_k}^2}{m_W^2}\right) \simeq -\frac{3}{2} + \frac{3}{4}\frac{m_{\ell_k}^2}{m_W^2}, \quad \frac{m_{\ell_k}^2}{m_W^2} \ll 1, \quad (8)
$$

was obtained after the loop integration. In Eqs. (6) and (7) $M^* = 4\pi v = 3.1$ TeV, where $v = (\sqrt{2}\,G_F)^{-1/2} = 246$ GeV represents the vacuum expectation value of the scalar Higgs field [18].

The neutrino mixing matrix U [19] is governing the decomposition of a coherently produced left-handed neutrino $\widetilde\nu_{L,\ell}$ associated with charged-lepton-flavor $\ell = e, \mu, \tau$ into the mass eigenstates $\nu_{L,i}$:

$$
|\widetilde\nu_{L,\ell}; \vec{p}\rangle = \sum_i \text{U}_{\ell i}|\nu_{L,i}; \vec{p}, m_i\rangle, \quad (9)
$$

For a Dirac neutrino $i = j$ [15, 20]. Using $m_\nu = 0.05$ eV [21], and with the definition $\mu_{ii} \equiv \mu_{\nu_i}$, from (7), in units of [e cm] and Bohr magneton, we obtain

$$
\begin{aligned}
\mu_{\nu_i} = & \frac{3e}{M^{*2}}m_{\nu_i}\left[1 - \frac{1}{2}\sum_{\ell=e,\mu,\tau}\frac{m_\ell^2}{m_W^2}|\text{U}_{\ell i}|^2 \right] \\
\simeq & 1.56 \times 10^{-26}[\text{e/eV}] = 0.29 \times 10^{-30} \text{ [e cm]} \\
= & 1.60 \times 10^{-20}\mu_B. \quad (10)
\end{aligned}
$$

From formula (10) it is clear that the chirality flip, which is necessary to induce the magnetic moment, arises only from the neutrino masses. The Dirac neutrino magnetic moment (10) is still much smaller than the bounds obtained from astrophysics [22, 23]. More details about Dirac neutrinos can be found in [24, 25].

In the case of off-diagonal transition moments, the first term in (8) vanishes in the summation over ℓ due to the orthogonality condition of U (GIM cancellation)

$$
d_{\bar\nu_j\nu_i}^{\text{el}} = \frac{3e}{2M^{*2}}\left(m_{\nu_i} - m_{\nu_j} \right)\sum_{\ell=e,\mu,\tau}\frac{m_{\ell_k}^2}{m_W^2}\text{U}_{jk}^\dagger \text{U}_{ki}, \quad (11)
$$

$$
\mu_{\bar\nu_j\nu_i} = \frac{3e}{2M^{*2}}\left(m_{\nu_i} + m_{\nu_j} \right)\sum_{\ell=e,\mu,\tau}\frac{m_{\ell_k}^2}{m_W^2}\text{U}_{jk}^\dagger \text{U}_{ki}. \quad (12)
$$

In Majorana 4-component notation the Hermitian, neutrino-flavor antisymmetric, electric and magnetic dipole operators are

$$
\begin{pmatrix} D_5 \\ D \end{pmatrix}_{ij}^{\mu\nu} = e\,\psi_i^\top\left[\text{C}\,\sigma^{\mu\nu}\begin{pmatrix} \gamma_5 \\ i\mathbf{1} \end{pmatrix} \right]\psi_j. \quad (13)
$$

The characterizing feature of Majorana neutrinos, i.e. fields that do not distinguish particle from antiparticle $(\psi_i = \psi_i^c)$, forces one to use both charged lepton and antilepton propagators in the loop calculation of neutrino-penguin diagrams, producing a transition matrix element T which is a complex antisymmetric quantity in lepton-flavor space:

$$
\begin{aligned}
\text{T}_{ji} = & -i\epsilon^\mu\bar\nu_j\left[(A_{ji} - A_{ij}) - (B_{ji} - B_{ij})\gamma_5 \right]\sigma_{\mu\nu}q^\nu\nu_i \\
= & -i\epsilon^\mu\bar\nu_j\left[2i\text{Im}A_{ji} - 2\text{Re}B_{ji}\gamma_5 \right]\sigma_{\mu\nu}q^\nu\nu_i. \quad (14)
\end{aligned}
$$

From this equation it is explicitly clear that for $i = j$, $d_{\nu_i}^{\text{el}} = \mu_{\nu_i} = 0$. Also, considering transition moments, only one of two terms in (14) is non-vanishing if the interaction respects CP invariance: The first term vanishes if the relative CP of ν_i and ν_j is even, and the second term vanishes if it is odd [16]. Finally, the dipole moments describing the transition from Majorana neutrino mass eigenstate-flavor ν_j to ν_k in the mass extended standard model are:

$$
d_{\nu_i\nu_j}^{\text{el}} = \frac{3e}{2M^{*2}}\left(m_{\nu_i} - m_{\nu_j} \right)\sum_{\ell=e,\mu,\tau}\frac{m_{\ell_k}^2}{m_W^2}\text{Re}\text{U}_{jk}^\dagger \text{U}_{ki}, \quad (15)
$$

$$
\mu_{\nu_i\nu_j} = \frac{3e}{2M^{*2}}\left(m_{\nu_i} + m_{\nu_j} \right)\sum_{\ell=e,\mu,\tau}\frac{m_{\ell_k}^2}{m_W^2}i\,\text{Im}\text{U}_{jk}^\dagger \text{U}_{ki}, (16)
$$

For the Majorana case the neutrino-flavor mixing matrix U is approximatively unitary, i.e it is necessarily of the following form [18]

$$\sum_{i=1}^{3} \mathrm{U}_{jk}^{\dagger} \mathrm{U}_{ki} = \delta_{ji} - \varepsilon_{ji}, \qquad (17)$$

where ε is a hermitian nonnegative matrix (i.e. with all eigenvalues nonnegative) and

$$|\varepsilon| = \sqrt{\mathrm{Tr}\,\varepsilon^2} = \mathcal{O}\left(m_{\nu_{\mathrm{light}}}/m_{\nu_{\mathrm{heavy}}}\right),$$
$$\sim 10^{-22} \text{ to } 10^{-21}. \qquad (18)$$

The case $|\varepsilon| = 0$ is excluded by the very existence of oscillation effects.

The neutrino dipole moments (15,16) violate lepton number by ± 2 and for a general neutrino mass matrix, they independently violate CP. Finally, note that there is no exact GIM cancellation in lepton-flavor space, at least in the Majorana case considered here, unlike the case for off–diagonal transition matrix elements (11,12), and for quark-flavors [10].

The flavor cancellation mechanism operates partly for the Majorana light neutrino dipole moments. It is generated through the smallness of light neutrino masses controlled by their heavy counterparts, combined with the quadratic charged lepton mass asymmetry in (8,18).

Note that expressions (4) to (18) are exact in their bilinear dependence on the mixing matrix U_{kj} to all orders of light and inverse heavy neutrino masses. However, the mixing matrix U is subunitary, as it must be a submatrix of the truly unitary 6×6 neutrino flavor mixing matrix.

The electromagnetic dipole moments universally reduce to the three light mass eigenfields (eigenstates) of neutrinos and antineutrinos, even though the mediating interactions proceed through light and heavy weak-interaction eigenfields (eigenstates) involving minimally six flavors. This extends the analogous situation pertaining to neutrino and antineutrino oscillations, valid at energies, in production and detection, far below the masses of heavy–neutrino flavors [17]. The MNS parameterization is generated by diagonalizing the light–flavor mass matrix in (6,7). The corresponding analytic structure is quite definite, yet often globally referred to as the see–saw mechanism [26]. Specific mass patterns for light-neutrino flavors arising from approximate discrete symmetries are discussed in [27].

The transition dipole moments in general receive very small contributions because of the smallness of the neutrino mass, $|m_\nu| \simeq 10^{-2}$ eV [21]. The largest contribution amog them is proportional to Re and Im part of $\mathrm{U}_{3\tau}^{\dagger}\mathrm{U}_{\tau 2}$, which corresponds to the $2 \to 3$ transition.

For the sum and difference of neutrino masses we assume hierarchical structure and take $|m_3 + m_2| \simeq |m_3 - m_2| \simeq |\Delta m_{32}^2|^{1/2} = 0.05$ eV [21]. For the MNS matrix elements we set $|\mathrm{ReU}_{3\tau}^{\dagger}\mathrm{U}_{\tau 2}| \simeq |\mathrm{ImU}_{3\tau}^{\dagger}\mathrm{U}_{\tau 2}| \leq 0.5$.

The electric and magnetic transition dipole moments of neutrinos $d_{\nu_2\nu_3}^{\mathrm{el}}$ and $\mu_{\nu_2\nu_3}$ are then denoted as $\left(d_{\mathrm{mag}}^{\mathrm{el}}\right)_{23}$ and are given by

$$\left|\left(d_{\mathrm{mag}}^{\mathrm{el}}\right)_{23}\right| = \frac{3e}{2M^{*2}}\frac{m_\tau^2}{m_W^2}\sqrt{|\Delta m_{32}^2|}\binom{|\mathrm{ReU}_{3\tau}^{\dagger}\mathrm{U}_{\tau 2}|}{|\mathrm{ImU}_{3\tau}^{\dagger}\mathrm{U}_{\tau 2}|},$$
$$\lesssim 2.03 \times 10^{-30}[e/\mathrm{eV}] = 0.38 \times 10^{-34}\,[\mathrm{e\,cm}],$$
$$= 2.07 \times 10^{-24}\,\mu_B. \qquad (19)$$

The electric transition dipole moments of light neutrinos are smaller than the ones of the d-quark. This is *the* order of magnitude of light neutrino transition dipole moments underlying the see–saw mechanism. It is by orders of magnitude smaller than in unprotected SUSY models.

Now we extract an upper limit on the $\star$-gradient interaction. The strength of the interaction (3) becomes $|m_\nu\,e\,\kappa\,\theta\,F|$. We compare it with the dipole transition interactions $|F\,d_{\mathrm{mag}}^{\mathrm{el}}|$ for Majorana case (15,16). Assuming that contributions from the neutrino-mass extended standard model are at least as large as those from noncommutativity, for $\kappa = 1$ we derive the following bound on noncommutativity arising from the Majorana nature of neutrinos:

$$\Lambda_{\mathrm{NC}} \gtrsim \left|\frac{e\,\kappa\,m_\nu}{\left(d_{\mathrm{mag}}^{\mathrm{el}}\right)_{23}}\right|^{1/2} \simeq 150\,\mathrm{TeV}. \qquad (20)$$

This is the main result of our considerations [28]. In Ref. [12] the neutrino energy dependence was taken into account. The bound on noncommutativity thus obtained is not a strict lower limit, but rather indicate the scale Λ_{NC} at which the expected values of the neutrino electromagnetic dipole moments due to noncommutativity in our model matches the standard model contributions. We would like to point out that on the scale of noncommutativity this bound involves only the basic properties of neutrinos and photons.

We proceed with determination of the radius of the photon–neutrino interaction, by evaluating the quantity that we shall call the neutrino $\star$-charge radius $r^{*2} = \langle r_\nu^2\rangle_{\mathrm{NC}}$. Since noncommutativity can be a source of "transvers plasmon" decay into neutrino–antineutrino pairs [1], this is to be compared with the same process induced by the neutrino charge radii defined by the axial electromagnetic interaction form factor [15, 16, 29, 30, 31, 32, 33, 34] in the neutrino-mass extended standard model:

$$\langle r_\nu^2\rangle = 6\left[\frac{\partial F_1(q^2)}{\partial q^2}\right]_{q^2=0}; \left[F_1(q^2)\right]_{q^2\to 0}\longrightarrow\frac{q^2}{6}\langle r_\nu^2\rangle, (21)$$

which in the limit of massless neutrinos corresponds to

$$\langle r_{\nu_\ell}^2\rangle \simeq \frac{2}{M^{*2}}\left(3 - 2\log\frac{m_\ell^2}{m_W^2}\right) = \frac{G_F}{\sqrt{2}\,\pi^2}\left(\frac{3}{4} + \log\frac{m_W}{m_\ell}\right). (22)$$

We estimate the charge radii in the standard model from (22) by taking $\ell = e$: $\sqrt{|\langle r_{\nu_e}^2\rangle|} \simeq 0.64 \times 10^{-16}$ [cm]. Here

we remark that astrophysical estimates give interesting bounds [36, 37, 38, 39]. These calculations should implement all neutrino flavor properties. The so derived bounds may also help in establishing the Majorana nature of light neutrinos.

To estimate the $\star$-charge radii we first evaluate the partial width

$$\sum_{\ell=e,\mu,\tau} \Gamma_{\text{SM}}(\gamma \to \bar{\nu}_\ell^{\text{L}} \nu_\ell^{\text{L}}) = \frac{\alpha}{144} \frac{q^6}{E_\gamma} \sum_{\ell=e,\mu,\tau} \left|\langle r^2_{\nu_\ell}\rangle\right|^2, \quad (23)$$

which gives the SM rate induced by the charge radii [31].

For plasmon at rest $q^2 = E_\gamma^2 = \omega_{\text{pl}}^2$. Taking the average value of the plasmon frequencies of red-giant and white-dwarf stars $\omega_{\text{pl}} = 15$ keV [31, 32], we obtain

$$\Gamma_{\text{SM}}^{-1}(\gamma \to \bar{\nu}\nu) = \left(\frac{1\,\text{keV}}{\omega_{\text{pl}}}\right)^5 \times 0.25 \times 10^{13} \text{ years}$$
$$\simeq 3 \times 10^6 \text{ years}. \quad (24)$$

This value has to be compared with astrophysical observations.

The next step is to compare (23) with the noncommutative rate $\sum \Gamma_{\text{NC}}(\gamma \to \bar{\nu}_\ell^{\text{L}} \nu_\ell^{\text{L}})$.

From Eq. (3) we extract the following gauge-invariant amplitude for the $\gamma(q) \to \nu(k')\bar{\nu}(k)$ vertex in momentum space for the left–chiral neutrinos:

$$\mathcal{M}_{\gamma\nu\bar{\nu}} = ie\,\kappa\,\bar{\psi}_L(\theta^{\mu\nu\rho}k_\nu q_\rho)\psi_L\,\epsilon_\mu(q). \quad (25)$$

The amplitude (24) for the off-shell photon decay to massless Majorana neutrinos, leads to the following rate in the c.m. system [1]:

$$\sum_{\ell=e,\mu,\tau} \Gamma_{\text{NC}}(\gamma \to \bar{\nu}_\ell^{\text{L}} \nu_\ell^{\text{L}}) = \frac{\alpha}{16} \frac{\kappa^2 q^6}{E_\gamma \Lambda_{\text{NC}}^4} \sum_{i=1}^{3}(c^{0i})^2. \quad (26)$$

The coefficients, (c^{0i}), are not independent. In pulling out the overall scale Λ_{NC}, we can always impose the constraint $\sum_{i=1}^{3}(c^{0i})^2 \equiv \vec{E}_\theta^2 = 1$ [8].

We obtain the $\star$–charge radii, which in fact could represent the range of noncommutativity, to be a simple function of the scale of noncommutativity:

$$r^* = \sqrt{|\langle r^2_\nu\rangle_{\text{NC}}|} = \frac{\sqrt{\sqrt{3}\,\kappa}}{\Lambda_{\text{NC}}}. \quad (27)$$

The bound from Majorana neutrino induced scale of noncommutativity (20), for $\kappa = 1$ implies [35]

$$r^* \stackrel{<}{\sim} 1.6 \times 10^{-19}[\text{cm}]. \quad (28)$$

This means that the $\star$–induced charge radii r^* at the $\Lambda_{\text{NC}} \stackrel{>}{\sim} 150$ TeV scale, are dominated by the neutrino-mass extended standard model physics and are practically unobservable.

Note that there are polarization phenomena induced by the noncommutativity tensor $\theta^{\mu\nu}$, which would involve correlations between spin and momenta. These, however, have been integrated out in our estimate. The motivation to do so lies in the fact that our model for the photon–neutrino interaction represents only the tree-level effective noncommutative gauge field theory in which the question of renormalization is not addressed [1, 12, 13, 40, 41].

In conclusion, we have compared the neutrino-mass extended standard model charge radii and electromagnetic dipole moments of neutrinos with their analogs arising from a theory of noncommutative space-time. If the charge radii and electromagnetic dipole moments should be found experimentally different from those predicted by the neutrino-mass extended standard model, as indicated from astrophysics [36, 37, 38, 39], then this could be a signature of noncommutativity. We observe that the sensitivity to noncommutativity in our model involving neutrinos [1] appears to be a function of the scale of energy involved in the physical process, running from the weak scale [1] up to a few hundreds of TeV's [this work]. In this way we can "understand" neutrinos as particles which manifest them self as Majorana objects at the short distances (high energies).

Independently of the noncommutative part of the story, we believe that the difference between (10) and (19), produced by standard model physics, points toward the right direction for the determination of the real nature of neutrinos. We hope that this comparison sheds light on the magnitude of associated phenomena thus induced. Our results relate to a set of physics problems that involve mass and electromagnetic properties of neutrinos.

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We assume θ to be constant in the first approximation, but only in the interaction region. A NC background "ether" that is truely constant at macroscopic distances and time scales is essentially ruled out. This is compatible with our expectation that NC effects are relevant only at very short distances and high energies.

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