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Nonanomalous Discrete R-Symmetry and Light Gravitino

Koichi Hamaguchi¹ and Nobuhito Maru²

¹ *Deutsches Elektronen-Synchrotron DESY, D-22603, Hamburg, Germany*

² *Department of Physics, University of Tokyo, Tokyo 113-0033, Japan*

Abstract

We discuss nonanomalous R-symmetry in the supersymmetric grand unified theories. In particular, we explore anomaly-free solutions predicting the gravitino mass in the range of $10^{-3} \text{ eV} \lesssim m_{3/2} \lesssim 1 \text{ TeV}$ when the μ -parameter is fixed to be $\mu \simeq 1 \text{ TeV}$. In the minimal SU(5) GUT, we have shown that $\mu \simeq 1 \text{ TeV}$ is obtained only if the gravitino is ultralight with mass $m_{3/2} \sim 10^{-3} \text{ eV}$. If extra fields $\mathbf{5} \oplus \mathbf{5}^*$ or $\mathbf{10} \oplus \mathbf{10}^*$ are introduced, many solutions predicting $m_{3/2} \gtrsim 10^{-3} \text{ eV}$ are found. The R-parity is violated due to the vacuum expectation value of the superpotential, but it is controlled by the discrete R-symmetry. We find that the R-parity violating couplings are naturally suppressed much below the experimental bounds for some charge assignments. These charge assignments predict light gravitino with masses of order $\mathcal{O}(10^{-3} \text{ eV})$ – $\mathcal{O}(1 \text{ MeV})$. These discrete R-symmetries can be considered as solutions to the μ -problem in low energy supersymmetry breaking models such as the gauge mediation.

1 Introduction

A discrete R-symmetry Z_{NR} often appears as a remnant of the rotational symmetry of the compactified extra space in higher dimensional supergravity or string theory [1, 2]. This discrete R-symmetry should be nonanomalous since this is a gauge symmetry. An R-symmetry plays a crucial role in the phenomenology of supersymmetric (SUSY) theory. First, it can suppress the cosmological constant compared to the Planck scale. Second, the SUSY-invariant mass term (the μ -term) of the Higgs chiral multiplet can be forbidden so that the Higgs mass not be the Planck scale. If an R-symmetry breaking is related to SUSY breaking, the Higgs chiral multiplet can obtain a mass of the order of the gravitino mass $m_{3/2} \simeq 1$ TeV by the Giudice-Masiero (GM) mechanism [3]. Third, an R-parity forbids the dimension-four baryon and lepton number violating operators causing too rapid proton decay [4, 5]. These observations motivate us to ask whether we can find a nonanomalous discrete R-symmetry with the above properties. In a paper by Kurosawa, Maru and Yanagida [6], nonanomalous discrete R-symmetries in the minimal SUSY standard model (MSSM) and the SUSY grand unified theory (GUT) were found under the situation that the GM mechanism works. These solutions can also forbid the dimension-five baryon- and lepton-number violating operators. Furthermore, extra fields $\mathbf{5} \oplus \mathbf{5}^*$ with the mass of order 1 TeV, which can be testable in collider experiments, are predicted from the anomaly cancellations in the GUT case.

In this paper, we consider nonanomalous R-symmetries *without* the GM mechanism.¹ In particular, we consider that the μ -term is induced by the vacuum expectation value (VEV) of the superpotential $\langle W \rangle$, assuming that a fractional power of $\langle W \rangle$ is allowed in the superpotential. The fractional power makes it possible to obtain the correct size of the μ -term even for gravitino mass smaller than the electroweak scale. We find that the $\mu \simeq 1$ TeV is obtained only if the gravitino is extremely light as $m_{3/2} \simeq 10^{-3}$ eV in the minimal SU(5) GUT. If extra fields such as $\mathbf{5} \oplus \mathbf{5}^*$ or $\mathbf{10} \oplus \mathbf{10}^*$ are added to the minimal SU(5) GUT, we find many charge assignments predicting $m_{3/2} \gtrsim 10^{-3}$ eV. These charge assignments can be considered as solutions to the μ -problem in low energy SUSY breaking models such as the gauge mediated SUSY breaking (GMSB) models [8].²

In our framework, the R-parity is in general violated due to the fractional powers of $\langle W \rangle$. However, the R-parity violating couplings are well controlled by the symmetry. In fact, it turns out that R-parity violation is small enough for some charge assignments. It is further shown that the dimension-five baryon- and lepton-number violating operators are naturally suppressed. In these charge assignments the gravitino masses are predicted

¹We do not consider a possibility of anomaly cancellations by Green-Schwarz mechanism [7].

²Solutions to the μ -problem in the GMSB models have been reported so far in Refs. [9, 10, 11, 12]. Also interesting mechanisms for the μ -problem in various mediation mechanisms of SUSY breaking are recently proposed [13, 14].

in the range $m_{3/2} \sim \mathcal{O}(10^{-3} \text{ eV})\text{--}\mathcal{O}(1 \text{ MeV})$, and hence the gravitino is the lightest supersymmetric particle (LSP). Its lifetime is much longer than the age of the universe, and the gravitino can be the dominant component of the dark matter.

This paper is organized as follows. In the next section, after discussing the general constraint on the power of the superpotential, we search for anomaly-free solutions of the minimal SU(5), and the minimal SU(5) with $\mathbf{5} \oplus \mathbf{5}^*$ or $\mathbf{10} \oplus \mathbf{10}^*$. In our analysis, constraints on the R-parity violating operators from the proton decay and neutrino masses are taken into account. A brief discussion on the cosmology of the gravitino LSP with R-parity violation is also given. The last section contains a summary of our paper. In the appendix, a subtle issue between the fractional power and the discrete symmetry is discussed.

2 Discrete R-symmetry in GUT

In this section, we consider the discrete R-symmetry in the GUT. Before discussing anomaly cancellations in detail, we give a general constraint on powers of the VEV of the superpotential in the next subsection.

2.1 General constraint on the powers of W

We assume that the μ -term is generated from

$$W \simeq \left(\frac{\langle W \rangle}{M_P^3} \right)^y M_P H \bar{H}, \quad (1)$$

where y is a non-negative number and $M_P = 2.4 \times 10^{18} \text{ GeV}$ is the reduced Planck scale. Here and hereafter, we omit coefficients of order unity. Then, the following conditions should be satisfied,

$$2\alpha y + h + \bar{h} = 2\alpha \pmod{N}, \quad (2)$$

$$h + \bar{h} \neq 2\alpha \pmod{N}, \quad (3)$$

where h, \bar{h} and α denote the R-charge of H, \bar{H} and the Grassmann coordinate θ , respectively and they are all integers. (See Table 1.) The second condition (3) is necessary to forbid the Higgs mass term with Planck scale. From Eq. (1), we can predict the gravitino mass because

$$\mu \simeq \left(\frac{\langle W \rangle}{M_P^3} \right)^y M_P \simeq \left(\frac{m_{3/2}}{M_P} \right)^y M_P, \quad (4)$$

and hence

$$m_{3/2} \simeq \left(\frac{\mu}{M_P} \right)^{1/y} M_P \simeq 10^{18.4-15.4/y} \text{ GeV}, \quad (5)$$

	T	\bar{F}	\bar{N}	H	\bar{H}	θ
$SU(5)$	$\mathbf{10}$	$\mathbf{5}^*$	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{5}^*$	
Z_{NR}	t	\bar{f}	\bar{n}	h	\bar{h}	α

Table 1: The matter content of GUT. Z_{NR} charges of the fields denote those of the scalar components and are integers. We take the R-charge of the Grassmann coordinate θ to be an integer α .

where $\mu \simeq \text{TeV}$ is assumed throughout this paper. Eq. (5) can be rewritten as

$$y \simeq \frac{\log(M_P/\mu)}{\log(M_P/m_{3/2})}. \quad (6)$$

From this equation, we can derive lower and upper bounds on y . We assume

$$m_{3/2} \lesssim \mu \ll M_P. \quad (7)$$

Thus, Eqs.(6) and (7) lead to

$$0 < y \lesssim 1. \quad (8)$$

Here, one might wonder if the fractional power $y < 1$ is incompatible with the discrete symmetry. This issue is briefly discussed in the Appendix. As can be seen from Eq. (4), the fractional power $y < 1$ is crucial to obtain the correct size of μ -term for a gravitino mass smaller than the weak scale.

Besides the constraint in Eq.(8), there is a lower bound on y coming from the lower bound on the gravitino mass $m_{3/2}$. In the GMSB model, when we fix the soft mass scale m_{soft} , the SUSY breaking F -term is bounded from below as $\sqrt{F} \gtrsim \mathcal{O}(10 \text{ TeV}) \times (m_{\text{soft}}/100 \text{ GeV})$ in order to avoid the negative mass squared for the scalar field in the messenger sector [8]. This leads to a lower bound on the gravitino mass, $m_{3/2} \gtrsim \mathcal{O}(0.01 \text{ eV})$. Even lighter gravitino mass $m_{3/2} \sim \mathcal{O}(10^{-3} \text{ eV})$ can be allowed in some SUSY breaking models in higher dimensional spacetime [15]. For $m_{3/2} \simeq 10^{-3} \text{ eV}$, one can see from Eq. (6) that y can be as small as $y \simeq 1/2$. From the above arguments, we impose the following constraints on the parameter y :

$$\frac{1}{2} \leq y \leq 1. \quad (9)$$

2.2 Anomaly cancellation

We are now at the position to discuss the anomaly cancellation. Let us first take the minimal $SU(5)$ GUT. Its matter content is described in Table 1. Z_{NR} charge of the

fields, which is taken to be generation independent for simplicity, denotes those of the scalar component and are integers. Note that we take the R-charge of the Grassmann coordinate θ to be an arbitrary integer α . The Yukawa couplings and the Majorana mass term are given by³

$$W = TTH + T\bar{F}\bar{H} + \bar{F}\bar{N}H + \frac{1}{2}M_m\bar{N}^2, \quad (10)$$

where M_m is a Majorana mass for the right-handed neutrinos. For Eq. (10) to be allowed, the corresponding R-charges have to satisfy

$$2t + h = 2\alpha \pmod{N}, \quad (11)$$

$$t + \bar{f} + \bar{h} = 2\alpha \pmod{N}, \quad (12)$$

$$\bar{f} + \bar{n} + h = 2\alpha \pmod{N}, \quad (13)$$

$$2\bar{n} = 2\alpha \pmod{N}. \quad (14)$$

Anomaly cancellation conditions for $Z_{NR}[SU(3)_C]^2$ and $Z_{NR}[SU(2)_L]^2$ are [16]

$$Z_{NR}[SU(3)_C]^2 = \frac{3}{2}\{3(t - \alpha) + (\bar{f} - \alpha)\} + 3\alpha = \frac{N}{2}k \quad (k \in \mathbf{Z}), \quad (15)$$

$$\begin{aligned} Z_{NR}[SU(2)_L]^2 &= \frac{1}{2}\{9(t - \alpha) + 3(\bar{f} - \alpha)\} + \frac{1}{2}\{(h - \alpha) + (\bar{h} - \alpha)\} + 2\alpha \\ &= \frac{N}{2}k' \quad (k' \in \mathbf{Z}). \end{aligned} \quad (16)$$

These conditions are simplified to

$$h + \bar{h} = 2\alpha \pmod{\frac{N}{3}}, \quad (17)$$

$$h + \bar{h} = \alpha \pmod{\frac{N}{2}}. \quad (18)$$

Eqs. (17) and (18) lead to

$$h + \bar{h} = 4\alpha \pmod{N}. \quad (19)$$

Substituting this back into (17) or (18) results in $6\alpha = 0 \pmod{N}$, or equivalently

$$6\alpha = Nk \quad (k \in \mathbf{Z}). \quad (20)$$

Taking into account $0 < \alpha < N$, $0 < k < 6$ is obtained. If we take $k = 3$, then $2\alpha = N$, and

$$h + \bar{h} = 4\alpha = 2\alpha \pmod{2\alpha}, \quad (21)$$

³We have suppressed $\mathcal{O}(1)$ coefficients of the terms.

is derived and contradicts the condition (3). Thus, we find $k = 1, 2, 4$ and 5 . In other words, N is classified as,

$$N = 6\alpha, 3\alpha, 3\left(\frac{\alpha}{2}\right), 6\left(\frac{\alpha}{5}\right). \quad (22)$$

First, $N = 6\alpha$ case is considered. From Eqs. (2) and (19),

$$2\alpha y + 4\alpha = 2\alpha \pmod{6\alpha} \rightarrow y = -1 + 3n \quad (n \in \mathbf{Z}). \quad (23)$$

This has no solution for $1/2 \leq y \leq 1$. Second case is $N = 3\alpha$. From Eqs. (2) and (19), we obtain

$$2\alpha y + 4\alpha = 2\alpha \pmod{3\alpha} \rightarrow y = -1 + \frac{3}{2}n \quad (n \in \mathbf{Z}). \quad (24)$$

This has a solution $y = 1/2$ for $n = 1$. Third case is $N = 3(\alpha/2) = 3\alpha'$ ($\alpha = 2\alpha'$, $\alpha' \in \mathbf{Z}$). From Eqs. (2) and (19), we obtain

$$2\alpha y + 4\alpha = 2\alpha \pmod{3\alpha'}, \quad (25)$$

$$\Leftrightarrow 4\alpha'y + 8\alpha' = 4\alpha' \pmod{3\alpha'} \rightarrow y = -1 + \frac{3}{4}n \quad (n \in \mathbf{Z}). \quad (26)$$

This also has a solution $y = 1/2$ for $n = 2$. The last case is $N = 6(\alpha/5) = 6\alpha'$ ($\alpha = 5\alpha'$, $\alpha' \in \mathbf{Z}$). From Eqs. (2) and (19), we obtain

$$2\alpha y + 4\alpha = 2\alpha \pmod{6\alpha'}, \quad (27)$$

$$\Leftrightarrow 10\alpha'y + 20\alpha' = 10\alpha' \pmod{6\alpha'} \rightarrow y = -1 + \frac{3}{5}n \quad (n \in \mathbf{Z}). \quad (28)$$

Taking into account $1/2 \leq y \leq 1$, $5/2 < n \leq 10/3$ is obtained. This has a solution $n = 3$, but it does not correspond to the minimum solution for positive y . The dominant μ -term comes from the term with $y = 1/5$ (i.e. $n = 2$), and it would cause $\mu \gg 1$ TeV unless $m_{3/2} \ll 10^{-3}$ eV. Thus, this case has no solution.

Therefore, the correct size of the μ -term can be obtained for $y = 1/2$ when $N = 3\alpha$ and $N = 3(\alpha/2)$, which predict an extremely light gravitino with mass $m_{3/2} \simeq \mathcal{O}(10^{-3}$ eV).

Baryon- and lepton-number violating operators

We have shown that the correct size of μ -term can be obtained for ultralight gravitino $m_{3/2} \simeq 10^{-3}$ eV, by means of the fractional power of the superpotential's VEV, $\langle W \rangle^{1/2}$. However, if we allow general interaction terms including the fractional power of $\langle W \rangle$, there appear baryon- and lepton-number violating operators as well. Therefore we have next to consider constraints on these operators.

Let us first consider the following superpotential

$$W \simeq \left(\frac{\langle W \rangle}{M_P^3}\right)^z M_P \bar{F} H, \quad (29)$$

which includes the so-called bilinear R-parity violation, $W = \hat{\mu}_i L_i H_u$. The coupling is given by

$$\hat{\mu} \simeq \left(\frac{m_{3/2}}{M_P} \right)^z M_P. \quad (30)$$

The bilinear R-parity violation can generate the neutrino mass [17] which explains the atmospheric neutrino oscillation if $\hat{\mu}/\mu \sim \mathcal{O}(10^{-4}-10^{-7})$ [18, 19]. In other words, $\hat{\mu}$ should be smaller than $\mathcal{O}(10^{-1} \text{ GeV}-10^{-4} \text{ GeV})$ in order to avoid a too large neutrino mass.

The power z is determined by the symmetry:

$$\begin{aligned} 2\alpha z + \bar{f} + h &= 2\alpha \pmod{N}, \\ \rightarrow 2\alpha z &= \bar{n} \pmod{N}, \\ \rightarrow z &= \frac{\bar{n} + rN}{2\alpha} \quad (r \in \mathbf{Z}), \end{aligned} \quad (31)$$

where we have used Eq. (13). From Eq. (14), the charge of the right-handed neutrino should be either $\bar{n} = \alpha$ or $\bar{n} = \alpha + N/2 \pmod{N}$. If $\bar{n} = \alpha$, the minimum non-negative z is given by $z = 1/2$ since $N > \alpha$. In this case, the bilinear coupling is given by

$$\hat{\mu} \simeq \left(\frac{m_{3/2}}{M_P} \right)^{1/2} M_P \simeq 10^3 \text{ GeV} \left(\frac{m_{3/2}}{10^{-3} \text{ eV}} \right)^{1/2}. \quad (32)$$

This generates too large neutrino mass and hence is excluded. Thus, the charge of the right-handed neutrino should be $\bar{n} = \alpha + N/2 \pmod{N}$. This also means that the case of $N = \text{odd}$ is excluded. If $N = \text{even}$ and $\bar{n} = \alpha + N/2 \pmod{N}$, Eq. (31) gives rise to

$$z = \begin{cases} \frac{5+6r}{4} & \text{for } N = 3\alpha, \\ \frac{7+6r}{8} & \text{for } N = \frac{3\alpha}{2}. \end{cases} \quad (33)$$

The minimum non-negative z are given by $z = 5/4$ ($r = 0$) and $z = 1/8$ ($r = -1$), respectively. Therefore, the bilinear R-parity violating coupling is given by

$$\hat{\mu} \simeq \begin{cases} \left(\frac{m_{3/2}}{M_P} \right)^{5/4} M_P \simeq 3 \times 10^{-20} \text{ GeV} \left(\frac{m_{3/2}}{10^{-3} \text{ eV}} \right)^{5/4} & \text{for } N = 3\alpha, \\ \left(\frac{m_{3/2}}{M_P} \right)^{1/8} M_P \simeq 4 \times 10^{14} \text{ GeV} \left(\frac{m_{3/2}}{10^{-3} \text{ eV}} \right)^{1/8} & \text{for } N = \frac{3\alpha}{2}. \end{cases} \quad (34)$$

Thus, the $N = 3(\alpha/2)$ case is clearly excluded. On the other hand, for $N = 3\alpha$ the contribution to the neutrino mass from the R-parity violation is extremely small. Therefore, the dominant contribution to the neutrino masses are understood to be generated by the standard seesaw mechanism [20].

Next we consider the trilinear R-parity violation caused by the following superpotential

$$W \simeq \left(\frac{\langle W \rangle}{M_P^3} \right)^{z'} T\bar{F}\bar{F} \simeq \left(\frac{m_{3/2}}{M_P} \right)^{z'} T\bar{F}\bar{F} \equiv \lambda_{\text{eff}} T\bar{F}\bar{F}, \quad (35)$$

which includes both of the baryon- and lepton-number violating operators, $\lambda_{ijk} L_i L_j E_k^c$, $\lambda'_{ijk} L_i Q_j D_k^c$ and $\lambda''_{ijk} U_i^c D_j^c D_k^c$. The power z' is determined by

$$\begin{aligned} 2\alpha z' + t + 2\bar{f} &= 2\alpha \pmod{N}, \\ \rightarrow 2\alpha z' &= 2\alpha + \bar{n} \pmod{N}, \end{aligned} \quad (36)$$

where we have used Eqs. (12), (13) and $h + \bar{h} = 4\alpha$. For $N = 3\alpha$ ($=$ even) and $\bar{n} = \alpha + N/2 \pmod{N}$, z' is given by

$$z' = \frac{9 + 6r'}{4} \quad (r' \in \mathbf{Z}). \quad (37)$$

The minimum non-negative z' is given by $z' = 3/4$ ($r' = -1$) and the effective coupling becomes

$$\lambda_{\text{eff}} \simeq 10^{-23} \left(\frac{m_{3/2}}{10^{-3} \text{ eV}} \right)^{3/4}, \quad (38)$$

which easily satisfy the constraint from the proton decay, $\lambda'_{11j} \lambda''_{11j} \lesssim 2 \times 10^{-27} (m_{\text{soft}}/100 \text{ GeV})^2$ ($j = 2, 3$) [21].

Let us also discuss the dimension five operator $W \simeq (1/M_{\text{eff}}) TTT\bar{F}$, which causes the proton decay [5]. In order to suppress the proton decay rate below the experimental bound, the effective mass scale should be $M_{\text{eff}} > \mathcal{O}(10^{25} \text{ GeV})$ [22], i.e., much larger than M_P . Notice that the R-parity cannot forbid this operator. In our framework, the operator is given by

$$W \simeq \frac{1}{M_P} \left(\frac{\langle W \rangle}{M_P^3} \right)^{z''} TTT\bar{F} \simeq \frac{1}{M_P} \left(\frac{m_{3/2}}{M_P} \right)^{z''} TTT\bar{F} \equiv \frac{1}{M_{\text{eff}}} TTT\bar{F}, \quad (39)$$

and z'' should satisfy

$$\begin{aligned} 2\alpha z'' + 3t + \bar{f} &= 2\alpha \pmod{N}, \\ \rightarrow 2\alpha z'' &= 2\alpha \pmod{N}, \\ \rightarrow z'' &= \frac{2\alpha + r''N}{2\alpha} = \frac{2 + 3r''}{2} \quad (r'' \in \mathbf{Z}), \end{aligned} \quad (40)$$

where we have used Eqs. (11), (12), $h + \bar{h} = 4\alpha$, and $N = 3\alpha$. The minimum non-negative z'' is given by $z'' = 1$ ($r'' = 0$), and hence the effective mass scale $M_{\text{eff}} \sim M_P (M_P/m_{3/2}) \sim 10^{48} \text{ GeV} (10^{-3} \text{ eV}/m_{3/2})$ is much above the experimental bound. Therefore, the discrete R-symmetry naturally suppresses the dimension five proton decay operator.

	t	\bar{f}	\bar{n}	h	\bar{h}	θ
P	1	1	1	0	0	α
V	1	-3	5	-2	2	
A	0	-1	1	0	1	

Table 2: The charges of the generators P, V, A .

explicit Z_{NR} charge assignments

Before closing this subsection, we comment on the explicit form of Z_{NR} . Nonanomalous R-symmetries are represented by $Z_{NR} = P^\alpha V^\beta A^\gamma$ ($\alpha, \beta, \gamma \in \mathbf{Z}$) [16] where the generators P, V and A are summarized in Table 2. One can easily check that the charge assignments in Table 2 satisfy Yukawa conditions (11)-(13).⁴ For $N = 3\alpha$ (= even) and $\bar{n} = \alpha + N/2 \pmod{N}$, we obtain

$$\bar{n} = \alpha + 5\beta + \gamma = \alpha + \frac{N}{2} \pmod{N} \Leftrightarrow 5\beta + \gamma = \frac{N}{2} = \frac{3}{2}\alpha \pmod{N}, \quad (41)$$

while Eq. (19) leads to

$$-2\beta + (2\beta + \gamma) = \gamma = 4\alpha \pmod{N}. \quad (42)$$

From Eqs. (41) and (42), we obtain

$$Z_{3\alpha R} = (PA^4)^\alpha V^\beta, \quad (43)$$

$$\begin{aligned} 5\beta &= -\frac{5}{2}\alpha \pmod{N = 3\alpha} \\ &= -\frac{5}{2}\alpha + 3\alpha m \quad (m \in \mathbf{Z}). \end{aligned} \quad (44)$$

Therefore,

$$\alpha : \beta : N = 10 : (-5 + 6m) : 30, \quad (45)$$

and the explicit forms of the Z_{NR} are given by

$$Z_{30R} = (PA^4)^{10} V^{6m-5} \quad (m = 1, 2, 3, 4), \quad (46)$$

$$Z_{6R} = (PA^4)^2 V^5. \quad (47)$$

⁴By imposing 3 conditions (11)-(13) on the 6 parameters $t, \bar{f}, \bar{n}, h, \bar{h}$ and θ , the charge assignments can be represented in terms of 3 parameters α, β and γ .

2.3 Introducing $\mathbf{5} \oplus \mathbf{5}^*$

In this subsection and the next, we consider the possibility that $10^{-3} \text{ eV} \lesssim m_{3/2} \lesssim 1 \text{ TeV}$ is predicted by introducing a pair of $\mathbf{5} \oplus \mathbf{5}^*$ or $\mathbf{10} \oplus \mathbf{10}^*$. If we introduce a pair of $\mathbf{5} \oplus \mathbf{5}^*$ with Z_{NR} charges ξ and $\bar{\xi}$, respectively, the anomaly cancellation conditions are modified as

$$\begin{aligned} Z_{NR}[SU(3)_C]^2 &= \frac{3}{2}\{3(t - \alpha) + (\bar{f} - \alpha)\} + \frac{1}{2}\{(\xi - \alpha) + (\bar{\xi} - \alpha)\} + 3\alpha = \frac{N}{2}k \quad (k \in \mathbf{Z}) \\ &\Leftrightarrow 3(h + \bar{h}) - (\xi + \bar{\xi}) = 4\alpha \pmod{N}, \end{aligned} \quad (48)$$

$$\begin{aligned} Z_{NR}[SU(2)_L]^2 &= \frac{1}{2}\{9(t - \alpha) + 3(\bar{f} - \alpha)\} + \frac{1}{2}(h + \bar{h} - 2\alpha) \\ &\quad + \frac{1}{2}\{(\xi - \alpha) + (\bar{\xi} - \alpha)\} + 2\alpha = \frac{N}{2}k' \quad (k' \in \mathbf{Z}) \\ &\Leftrightarrow 2(h + \bar{h}) = \xi + \bar{\xi} \pmod{N}. \end{aligned} \quad (49)$$

From these conditions,

$$h + \bar{h} = 4\alpha \pmod{N}, \quad (50)$$

$$\xi + \bar{\xi} = 8\alpha \pmod{N}, \quad (51)$$

are obtained. At this stage, N is undetermined. In order to fix N , let us take into account the mixed gravitational anomaly cancellation [16],

$$\begin{aligned} Z_{NR}[\text{gravity}]^2 &= 30(t - \alpha) + 15(\bar{f} - \alpha) + 3(\bar{n} - \alpha) + 2(h - \alpha) + 2(\bar{h} - \alpha) \\ &\quad + 5(\xi - \alpha) + 5(\bar{\xi} - \alpha) + (8 + 3 + 1)\alpha - 21\alpha \\ &= -23\alpha \pmod{N \text{ or } N/2}. \end{aligned} \quad (52)$$

Mod N or $N/2$ depends on whether N is odd or even. In the following, we consider whether the mixed anomaly can be canceled *without introducing singlets*.

If N is odd,

$$N = \frac{23}{k}\alpha \quad (k = 1, 2, \dots, 22). \quad (53)$$

Eq. (5) tells us

$$m_{3/2} \simeq 10^{18.4 - \frac{15.4}{-1 + \frac{23}{2k}n}} \text{ GeV} \quad (n \in \mathbf{Z}), \quad (54)$$

because

$$2\alpha y + 4\alpha = 2\alpha \pmod{23\alpha/k} \rightarrow y = -1 + \frac{23}{2k}n. \quad (55)$$

Taking into account that $1/2 \leq y \leq 1$ and y is a minimum of non-negative value, we obtain

$$k = 6, 7 \quad (\text{for } n = 1), \quad (56)$$

$$k = 12, 13, 14, 15 \quad (\text{for } n = 2). \quad (57)$$

$N : \text{odd}$					
(n, k)	$m_{3/2}$	(n, k)	$m_{3/2}$	(n, k)	$m_{3/2}$
(1,6)	40 GeV	(2, 12)	40 GeV	(2, 14)	2.9 keV
(1,7)	2.9 keV	(2, 13)	24 MeV	(2, 15)	0.035 eV
$N : \text{even}$					
(1, 12)	40 GeV	(2, 24)	40 GeV	(2, 28)	2.9 keV
(1, 13)	24 MeV	(2, 25)	1.2 GeV	(2, 29)	14 eV
(1, 14)	2.9 keV	(2, 26)	24 MeV	(2, 30)	0.035 eV
(1, 15)	0.035 eV	(2, 27)	330 keV		

Table 3: The gravitino mass for GUT with $5 \oplus 5^*$.

The gravitino mass is summarized in Table 3.

If N is even,

$$N = \frac{46}{k}\alpha \quad (k = 1, 2, \dots, 45), \quad (58)$$

$$2\alpha y + 4\alpha = 2\alpha \pmod{46\alpha/k} \rightarrow y = -1 + \frac{23}{k}n, \quad (59)$$

and Eq. (5) leads to

$$m_{3/2} \simeq 10^{18.4 - \frac{15.4}{-1 + \frac{23}{k}n}} \text{ GeV} \quad (n \in \mathbf{Z}). \quad (60)$$

Taking into account that $1/2 \leq y \leq 1$ and y is a minimum of non-negative value, we obtain

$$k = 12 \sim 15 \quad (\text{for } n = 1), \quad (61)$$

$$k = 24 \sim 30 \quad (\text{for } n = 2). \quad (62)$$

The gravitino mass in this case is also summarized in Table 3.

Now let us turn to discuss the baryon- and lepton-number violating operators. First, the bilinear R-parity violating coupling $\hat{\mu}$ is given by Eq. (30), with a fractional power z in Eq. (31). One can show in the same way as before that the case of $\bar{n} = \alpha$ is excluded since it results in $z = 1/2$, which induces too large neutrino masses. If $N = \text{even}$ and $\bar{n} = \alpha + N/2 \pmod{N}$, Eq. (31) and Eq. (58) give rise to

$$z = \frac{k + 23 + 46r}{2k} \quad (r \in \mathbf{Z}). \quad (63)$$

The minimum non-negative z is given by $r = 0$ for $k = 12\text{--}15$, and $r = -1$ for $k = 24\text{--}30$. The resultant bilinear couplings are given by $\hat{\mu} \simeq (8 \times 10^{-7} \text{--} 7 \times 10^{-19}) \text{ GeV}$ for $k = 12\text{--}15$, and $\hat{\mu} \simeq (10^{15}\text{--}10^{18}) \text{ GeV}$ for $k = 24\text{--}30$. Hence, $k = 24\text{--}30$ cases are excluded. Namely,

k	$m_{3/2}$	$\hat{\mu}$	λ_{eff}
14	2.9 keV	6×10^{-14} GeV	6×10^{-17}
15	0.035 eV	7×10^{-19} GeV	7×10^{-22}

Table 4: The gravitino mass for GUT with $5 \oplus 5^*$ and R-parity violating couplings $\hat{\mu}$ and λ_{eff} . $N = \text{even}$ and $\bar{n} = \alpha + N/2 \pmod{N}$. Only the cases which predict sufficiently small R-parity violation are listed.

the remaining charge assignments are $N = (46/k)\alpha = \text{even}$, $\bar{n} = \alpha + N/2 \pmod{N}$, and $k = 12-15$.

Next we consider the trilinear R-parity violating coupling $W \sim \lambda_{\text{eff}} T \bar{F} \bar{F}$. The effective coupling λ_{eff} is given by $\lambda_{\text{eff}} \simeq (m_{3/2}/M_P)^{z'}$. The power z' is again determined by Eq. (36), $2\alpha z' = 2\alpha + \bar{n} \pmod{N}$. (Notice that $h + \bar{h} = 4\alpha$ is satisfied also in the present case. See Eq. (50).) Therefore, for $N = (46/k)\alpha = \text{even}$ and $\bar{n} = \alpha + N/2 \pmod{N}$, z' is given by

$$z' = \frac{3k + 23 + 46r'}{2k} \quad (r' \in \mathbf{Z}). \quad (64)$$

The minimum non-negative z' is given by $r' = 0$ for $k = 12-15$, and the trilinear couplings are $\lambda_{\text{eff}} \simeq 8 \times 10^{-10}$ ($k = 12$), 5×10^{-13} ($k = 13$), 6×10^{-17} ($k = 14$), and 7×10^{-22} ($k = 15$). Thus, the cases of $k = 12, 13$ are excluded by the constraint from the proton decay, $\lambda'_{11j} \lambda''_{11j} \lesssim 2 \times 10^{-27} (m_{\text{soft}}/100 \text{ GeV})^2$ ($j = 2, 3$) [21].

The R-parity violating couplings are listed in Table. 4 for the remaining charge assignments $k = 14, 15$ together with the gravitino masses. We find that the neutrino mass induced by these R-parity violations is too small to explain the mass scale observed in the atmospheric and solar neutrino oscillation experiments, and hence the dominant contribution to the neutrino masses should be generated by the seesaw mechanism.

As for the dimension five operator $W \simeq (1/M_{\text{eff}}) T T T \bar{F}$, the effective mass scale is given by Eq. (39), $M_{\text{eff}} \simeq M_P (M_P/m_{3/2})^{z''}$. It is easy to show that the power is again given by $z'' = 1$ for $N = (46/k)\alpha = \text{even}$, $\bar{n} = \alpha + N/2 \pmod{N}$ and $k = 14, 15$. Thus, the effective mass scale $M_{\text{eff}} \sim M_P (M_P/m_{3/2}) \sim 10^{39} \text{ GeV} (1 \text{ keV}/m_{3/2})$ is much above the experimental bound and the proton decay via the dimension five operator is naturally suppressed by the discrete R-symmetry also in these charge assignments.

Finally, we comment on the explicit form of $Z_{NR} = P^\alpha V^\beta A^\gamma$ ($\alpha, \beta, \gamma \in \mathbf{Z}$). (See Table 2). We only consider the cases of $N = \text{even}$, $\bar{n} = \alpha + N/2 \pmod{N}$ and $k = 14, 15$, since other cases are excluded by the constraints on R-parity violation, as we have shown. Thus,

$$\bar{n} = \alpha + 5\beta + \gamma = \alpha + \frac{N}{2} \pmod{N} \Leftrightarrow 5\beta + \gamma = \frac{N}{2} \pmod{N}. \quad (65)$$

Eq. (50) leads to

$$-2\beta + (2\beta + \gamma) = \gamma = 4\alpha \pmod{N}. \quad (66)$$

From Eqs. (65) and (66),

$$5\beta + 4\alpha = \frac{N}{2} \pmod{N}, \quad (67)$$

is obtained. Thus, we have

$$Z_{\frac{46}{k}\alpha R} = (PA^4)^\alpha V^\beta, \quad (68)$$

$$5\beta + 4\alpha = \frac{23}{k}\alpha \pmod{\frac{46}{k}\alpha} = \frac{23}{k}\alpha + \frac{46}{k}\alpha m \quad (m \in \mathbf{Z}). \quad (69)$$

Therefore,

$$\alpha : \beta : N = 5k : (46m + 23 - 4k) : 230, \quad (70)$$

and we obtain

$$\begin{aligned} Z_{230R} &= (PA^4)^{70} V^{46m-33} \quad (k = 14, m = 1, 2, 4, 5), \\ &\quad (PA^4)^{75} V^{46m-37} \quad (k = 15, m = 1, 3, 4, 5). \end{aligned} \quad (71)$$

$$Z_{46R} = (PA^4)^{14} V^{21} \quad (k = 14), \quad (PA^4)^{15} V^{11} \quad (k = 15). \quad (72)$$

2.4 Introducing $10 \oplus 10^*$

Next, we consider the case with $\xi(\mathbf{10}) \oplus \bar{\xi}(\mathbf{10}^*)$. The anomaly cancellation conditions are modified as

$$Z_{NR}[SU(3)_C]^2 : -\frac{3}{2}(h + \bar{h}) + 3\alpha + \frac{3}{2}(\xi + \bar{\xi}) + \frac{3}{2}Nk'' - 3\alpha = \frac{N}{2}k, \quad (73)$$

$$Z_{NR}[SU(2)_L]^2 : -(h + \bar{h}) + \alpha + \frac{3}{2}(\xi + \bar{\xi}) - 3\alpha = \frac{N}{2}k'. \quad (74)$$

These are simplified to

$$3(h + \bar{h}) = 3(\xi + \bar{\xi}) \pmod{N}, \quad (75)$$

$$2(h + \bar{h}) = 3(\xi + \bar{\xi}) - 4\alpha \pmod{N}, \quad (76)$$

and then

$$h + \bar{h} = 4\alpha \pmod{N}, \quad (77)$$

$$3(\xi + \bar{\xi}) = 12\alpha \pmod{N}, \quad (78)$$

are obtained. In order to fix N , let us take into account the mixed gravitational anomaly cancellation,

$$\begin{aligned} Z_{NR}[\text{gravity}]^2 &= 30(t - \alpha) + 15(\bar{f} - \alpha) + 3(\bar{n} - \alpha) + 2(h - \alpha) + 2(\bar{h} - \alpha) + 10(\xi - \alpha) \\ &\quad + 10(\bar{\xi} - \alpha) + (8 + 3 + 1)\alpha - 21\alpha \\ &= (\xi + \bar{\xi}) - 37\alpha \pmod{N \text{ or } N/2}. \end{aligned} \quad (79)$$

In the same manner as the $\xi(\mathbf{5}) \oplus \bar{\xi}(\mathbf{5}^*)$ case, one can show that the $\bar{n} = \alpha$ case is excluded because it would generate too large neutrino mass from the bilinear R-parity violation. Therefore, we consider the case of $N = \text{even}$ and $\bar{n} = \alpha + N/2$. Then, Eqs. (78) and (79) lead to

$$3Z_{NR}[\text{gravity}]^2 = -99\alpha \pmod{N/2}, \quad (80)$$

$$\rightarrow N = \frac{198}{k}\alpha \quad (k = 1, 2, \dots, 197). \quad (81)$$

From Eq. (2), we obtain

$$y = -1 + \frac{99}{k}n \quad (n \in \mathbf{Z}). \quad (82)$$

Taking into account that $1/2 \leq y \leq 1$ and y is a minimum of positive values,

$$k = 50 \sim 66 \quad (\text{for } n = 1), \quad (83)$$

$$k = 100 \sim 132 \quad (\text{for } n = 2), \quad (84)$$

are obtained.

Some of these charge assignments cause too large baryon- or lepton-number violation via the R-parity violation. The orders of magnitudes of bilinear ($\hat{\mu}$) and trilinear (λ_{eff}) couplings can be estimated in the same way as $\mathbf{5} \oplus \mathbf{5}^*$ case. In Table. 5, we show the gravitino mass and these couplings for the cases in which the R-parity violations are below the experimental bounds ($k = 57\text{--}66$). In these cases, the dimension five operator $W \simeq (1/M_{\text{eff}})TTT\bar{F}$ are naturally suppressed as $M_{\text{eff}} \gg 10^{36}$ GeV like the $\mathbf{5} \oplus \mathbf{5}^*$ case.

Finally, we comment on the concrete form of Z_{NR} . From Eqs. (77), (81) and $\bar{n} = \alpha + N/2$, we obtain

$$Z_{\frac{198}{k}\alpha R} = (PA^4)^\alpha V^\beta, \quad (85)$$

$$5\beta + 4\alpha = \frac{99}{k}\alpha \pmod{N = \frac{198}{k}\alpha} = \frac{99}{k}\alpha + \frac{198}{k}\alpha m \quad (m \in \mathbf{Z}), \quad (86)$$

and hence

$$\alpha : \beta : N = 5k : (198m + 99 - 4k) : 990. \quad (87)$$

k	$m_{3/2}$	$\hat{\mu}$	λ_{eff}
57	3.2 MeV	7×10^{-11} GeV	7×10^{-14}
58	420 keV	9×10^{-12} GeV	9×10^{-15}
59	49 keV	1×10^{-12} GeV	1×10^{-15}
60	5.2 keV	1×10^{-13} GeV	1×10^{-16}
61	490 eV	1×10^{-14} GeV	1×10^{-17}
62	41 eV	8×10^{-16} GeV	8×10^{-19}
63	2.9 eV	6×10^{-17} GeV	6×10^{-20}
64	0.18 eV	4×10^{-18} GeV	4×10^{-21}
65	0.95×10^{-2} eV	2×10^{-19} GeV	2×10^{-22}
66	0.4×10^{-3} eV	9×10^{-21} GeV	9×10^{-24}

Table 5: The gravitino mass for GUT with $10 \oplus 10^*$ and R-parity violating couplings $\hat{\mu}$ and λ_{eff} . $N = \text{even}$ and $\bar{n} = \alpha + N/2 \pmod{N}$. Only the cases which predict sufficiently small R-parity violation are listed.

For $k = 66$, it reduces to Z_{30R} or Z_{6R} given in Eqs. (46) and (47). For $k = 57\text{--}65$, we obtain

$$Z_{110R} = (PA^4)^{35} V^{22m-17} \quad (k = 63), \quad (88)$$

$$Z_{330R} = (PA^4)^{95} V^{66m-43} \quad (k = 57), \quad (PA^4)^{100} V^{66m-47} \quad (k = 60), \quad (89)$$

$$Z_{990R} = (PA^4)^{5k} V^{198m+99-4k} \quad (\text{other } k), \quad (90)$$

where $m = 1, 2, 3, 4, 5$. Among these cases, for $198m + 99 - 4k = 5m'$ ($m' \in \mathbf{Z}$), they reduce to Z_{22R} , Z_{66R} , and Z_{198R} .

2.5 Gravitino LSP with R-parity violation

As can be seen from Table. 4 and 5, the predicted masses of the gravitino are $\mathcal{O}(10^{-3} \text{ eV} - 1 \text{ MeV})$. Therefore, the gravitino is the LSP. Here, let us comment on the cosmology of this gravitino LSP. Gravitino LSP dark matter without R-parity was investigated in Ref. [23] under the assumption that the R-parity violation is the dominant contribution to the neutrino masses. According to them, we have found that the lifetime of the gravitino is much longer than the age of the universe,⁵ since the decay rate is suppressed by the small R-parity violating coupling in addition to the Planck scale. (Notice that the R-parity violation in our scenario is even smaller than that considered in Ref. [23]. Thus, the lifetime of the gravitino in our case is much longer than that in their case.) For such a long lifetime, the flux of the diffuse gamma ray generated by the gravitino decay is

⁵The lifetime of the LSP gravitino could be shorter than the age of the universe [24] if there is a large trilinear coupling $\lambda \sim \mathcal{O}(0.1\text{--}1)$ close to the experimental bound and if the gravitino mass is relatively large, $m_{3/2} \gtrsim \mathcal{O}(1 \text{ GeV})$. However, this is not the case in our scenario.

smaller than the observed value [23]. Therefore, for $m_{3/2} > \mathcal{O}(1 \text{ keV})$, the gravitino can be the dominant component of the dark matter in spite of the presence of the R-parity violation.⁶

As for the next-to-lightest SUSY particle (NLSP), it can decay either (i) into gravitino via the usual R-parity conserving interaction, or (ii) into the standard model particles via R-parity violating couplings. We have found that the partial decay rate of the latter one is much smaller than that of the former one, since the R-parity violations are extremely small. (See Table. 4 and 5.) Thus, the NLSP decay rate is determined by the former channel. If the decay occurs during or after the big-bang nucleosynthesis (BBN), $t \simeq 1\text{--}100 \text{ sec}$, it might spoil the success of the BBN [25]. However, for $m_{3/2} < \mathcal{O}(1 \text{ MeV})$ the lifetime of the NLSP is shorter than 1 sec and this problem is avoided.

3 Summary

In this paper, we have studied nonanomalous discrete R-symmetry in GUT without imposing the Giudice-Masiero condition. In the minimal SU(5) GUT, $\mu \simeq 1 \text{ TeV}$ is obtained only if the gravitino is ultralight as $m_{3/2} \simeq 10^{-3} \text{ eV}$, and we find simple solutions Z_{6R} and Z_{30R} . If a pair of $\mathbf{5} \oplus \mathbf{5}^*$ or $\mathbf{10} \oplus \mathbf{10}^*$ are added to the minimal SU(5) GUT, we can find many solutions predicting $m_{3/2} \gtrsim 10^{-3} \text{ eV}$. Here, we comment on the mass of these additional multiplets, $\xi + \bar{\xi}$, which can be estimated since their charges are determined by the anomaly cancellation condition. [See Eqs.(51) and (78).] The effective operator which induces the mass of $\xi + \bar{\xi}$ is given by $W \simeq (\langle W \rangle / M_P^3)^y M_P \xi \bar{\xi}$. We have checked that the mass $m_\xi \simeq (m_{3/2} / M_P)^y M_P$ is larger than the electroweak scale in all cases we have discussed.

Since the fractional power of $\langle W \rangle$ is considered in this paper, R-parity is necessarily violated, but R-parity violating couplings are controlled by the symmetry. In fact, the couplings were found to be small enough to avoid the constraints from proton decay and neutrino masses for some charge assignments. Furthermore, it has also been shown that dimension-five baryon- and lepton-number violating operators are naturally suppressed. Therefore, the proton stability is ensured by the symmetry.

Low energy baryon- and/or lepton-number violating interactions might cause a difficulty for baryogenesis since they might wash out the baryon asymmetry together with the sphaleron [26] process. However, we found that this is not the case for our scenario because the R-parity violating couplings are so small that their interactions have never been in thermal equilibrium.

The predicted gravitino masses were found to be in the range $m_{3/2} \sim \mathcal{O}(10^{-3} \text{ eV})\text{--}$

⁶For $m_{3/2} < \mathcal{O}(1 \text{ keV})$ the gravitino cannot be a cold dark matter, and hence the dominant component of the dark matter should be another particle or object.

$\mathcal{O}(1 \text{ MeV})$ and the gravitino is the LSP. Since the lifetime of the gravitino is much longer than the age of the universe, the gravitino can be the dominant component of the dark matter.

As for the neutrino mass, there are two contributions in our framework, i.e., the seesaw mechanism and the R-parity violation. The R-parity violation can explain the neutrino mass scale if $\hat{\mu}/\mu \sim \mathcal{O}(10^{-4}-10^{-7})$ and/or $\lambda_{\text{eff}} \sim \mathcal{O}(10^{-4}-10^{-5})$ [19, 27, 28]. However, the predicted values of these couplings are either much larger or much smaller than these ranges. The charge assignments predicting too large couplings are excluded. Therefore, the neutrino mass scale should be generated by the seesaw mechanism, and contributions from the R-parity violation gives only tiny perturbation to it.

The discrete R-symmetry considered in this paper can explain the order of magnitude of the μ -term for light gravitinos. Hence, they can be considered as solutions to the μ -problem in low energy SUSY breaking models such as the gauge mediation.

Note Added

1 In our scenario, the predicted masses of the gravitino are very small. We would like to stress that, if the gravitino is lighter than about 100 keV, there is an interesting possibility to detect the slow decay of the lightest neutralino into gravitino in future collider [29].

2 If the gravitino is lighter than the proton, proton can decay into the gravitino via the R-parity violating coupling $\lambda'' U^c D^c D^c$. This leads to a stringent limit on the coupling as $\lambda'' < 10^{-15} (m_{3/2}/\text{eV})$ [30]. We found, however, that all the charge assignments which satisfy the constraints discussed in the text also satisfy this constraint, and hence the conclusion does not change. (We are grateful to Ryuichiro Kitano for pointing out this constraint.)

3 B -parameter in our case is at most of order loop suppression factor times gaugino mass, which is induced at 2-loops in gauge mediation. This implies that large $\tan\beta$ is preferred. (We wish to thank Stephan Huber for useful comments and discussion.)

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Appendix

In this appendix we briefly comment on the fractional power and discrete symmetry. Although we have derived Eq.(2) from Eq.(1), it is nontrivial in the case of fractional power $y < 1$ since the R-charges are defined under mod N in the framework of discrete Z_{NR} symmetry.

Let us first discuss the vacuum expectation value of the superpotential, $\langle W \rangle$. Suppose that a gauge singlet operator X has a Z_{NR} charge 2α and it develops a vacuum expectation value $\langle X \rangle \ll M_P$. Under the Z_{NR} symmetry, it is also possible to consider that X has a charge $2\alpha + nN$ with $n \in \mathbf{Z}$. Then, in general, the superpotential can have the following vacuum expectation value:

$$\langle W \rangle = \sum_{n > -2\alpha/N} c_n \left(\frac{\langle X \rangle}{M_P} \right)^{\frac{2\alpha}{2\alpha+nN}} M_P^3, \quad (91)$$

where we have renormalized the mass dimension of X operator to be 1. If all of the coefficients c_n are of order one, the vacuum expectation value of the superpotential becomes Planck scale, which is inconsistent with almost vanishing cosmological constant or low energy SUSY. Thus we expect that the right-hand side of the above equation is dominated by a certain term with $n = n_0$:

$$\langle W \rangle \simeq c_{n_0} \left(\frac{\langle X \rangle}{M_P} \right)^{\frac{2\alpha}{2\alpha+n_0N}} M_P^3. \quad (92)$$

The number n_0 is likely to depend on the operator X as well as on the origin of the Z_{NR} symmetry, which we do not discuss in this paper. On the other hand, the operator relevant to the μ -term is also written in terms of $\langle X \rangle$:

$$W = \sum_{n > -2\alpha/N} c'_n \left(\frac{\langle X \rangle}{M_P} \right)^{\frac{2\alpha-h-\bar{h}+rN}{2\alpha+nN}} M_P H \bar{H}, \quad (93)$$

where r is the minimum integer which gives $2\alpha - h - \bar{h} + rN > 0$. Then, we naturally expect that the above operator is also dominated by the term with $n = n_0$:

$$W \simeq c'_{n_0} \left(\frac{\langle X \rangle}{M_P} \right)^{\frac{2\alpha-h-\bar{h}+rN}{2\alpha+n_0N}} M_P H \bar{H}. \quad (94)$$

Though it is possible that the fractional power which gives rise to the μ -term is different from the one responsible for the $\langle W \rangle$, we argue that it is unnatural and assume that both of them are dominated by the term with same $n = n_0$. Then, from Eqs.(92) and (94), we obtain the following expression of the μ -term,

$$W \simeq \left(\frac{\langle W \rangle}{M_P^3} \right)^{\frac{2\alpha-h-\bar{h}+rN}{2\alpha}} M_P H \bar{H}, \quad (95)$$

which leads to Eq.(1) with the y given in (2). The same argument is also applied to the cases of baryon- and lepton-number violating operators discussed in the text.

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