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Resolution of Gordon Ambiguity of Nucleon Current in Relativistic Nuclear Matter

K. Miyazaki

Abstract

We investigate the electromagnetic vertex function for a nucleon in relativistic nuclear medium. The effect of mean fields on the internal nucleon lines (propagators) connected to external lines in the corresponding Feynman diagram offsets the difference between the familiar CC2 current and the so-called CC1 or CC3 current. It is therefore found that the CC2 current is physically reasonable. Consequently, the famous Gordon ambiguity of the nuclear current has been resolved.

In the electro (or photo) nuclear scatterings or reactions, nucleon current has the familiar Dirac plus Pauli form (called as CC2) determined phenomenologically in free space,

$$\Gamma^{\mu}_{CC2}\left(p_f, p_i\right) = F_1\left(q^2\right)\gamma^{\mu} + F_2\left(q^2\right)\left(i\sigma^{\mu\nu}q_{\nu}\right)/\left(2M\right), \tag{1}$$

where $p_{i(f)}$ is the initial and final momentum of a nucleon, $q = p_f - p_i$, M is free nucleon mass and $F_{1(2)}\left(q^2\right)$ is the Dirac (Pauli) form factor. Using the Gordon decomposition

$$\left(i\sigma^{\mu\nu}q_{\nu}\right)/\left(2M\right) = \gamma^{\mu} - \left(p_f + p_i\right)^{\mu}/\left(2M\right), \tag{2}$$

Eq. (1) is rewritten in the so-called CC1 form

$$\Gamma^{\mu}_{CC1}\left(p_f, p_i\right) = \left[F_1\left(q^2\right) + F_2\left(q^2\right)\right]\gamma^{\mu} - F_2\left(q^2\right)\left(p_f + p_i\right)^{\mu}/\left(2M\right), \tag{3}$$

or in the so-called CC3 form

$$\Gamma^{\mu}_{CC3}\left(p_f, p_i\right) = F_1\left(q^2\right)\left(p_f + p_i\right)^{\mu}/\left(2M\right) + \left[F_1\left(q^2\right) + F_2\left(q^2\right)\right]\left(i\sigma^{\mu\nu}q_{\nu}\right)/\left(2M\right). \tag{4}$$

However, Eqs. (1), (3) and (4) are not equivalent for a nuclear nucleon. This famous Gordon ambiguity of nuclear current is the renewed interests in recent investigations of $\left(e, e'p\right)$ reaction [1-4] based on the Walecka $\sigma - \omega$ model [5] and the relativistic optical potential model.

Equation (2) is not an identity, while using the true identity

$$i\frac{\sigma^{\mu\nu}q_\nu}{2M}=\gamma^\mu-\frac{p_f^\mu+p_i^\mu}{2M}+\frac{\not{p}_f-M}{2M}\gamma^\mu+\gamma^\mu\frac{\not{p}_i-M}{2M}, \tag{5}$$

Eq. (3) is rewritten as

$$\Gamma^\mu_{CC1}\left(p_f,p_i\right)=\Gamma^\mu_{CC2}\left(p_f,p_i\right)-F_2\left(q^2\right)\left(\frac{\not{p}_f-M}{2M}\gamma^\mu+\gamma^\mu\frac{\not{p}_i-M}{2M}\right), \tag{6}$$

and Eq. (4) is

$$\Gamma^\mu_{CC3}\left(p_f,p_i\right)=\Gamma^\mu_{CC2}\left(p_f,p_i\right)+F_1\left(q^2\right)\left(\frac{\not{p}_f-M}{2M}\gamma^\mu+\gamma^\mu\frac{\not{p}_i-M}{2M}\right). \tag{7}$$

We can see that the difference between CC1 (CC3) and CC2 is just the second term of Eq. (6)((7)). It has no contributions to free positive-energy Dirac spinors (the $++$ coupling). However, the Dirac spinor for a nucleon in nuclear medium described by the Walecka model contains negative-energy state [6] due to large scalar and vector mean fields. Thus, the second terms of Eqs. (6) and (7) contribute to the coupling between positive and negative-energy state (the $+-$ coupling). It is an essential ingredient in the relativistic investigations of Refs. [1-4]. (In nucleon knock-out reactions, initial bound nucleon, or the missing energy-momentum, is generally off (the mass) shell regardless of relativistic or non-relativistic model. Usually, an appropriate on-shell prescription is employed. As a result, there is an additional term [1] to Eq. (6). It however contributes to both the $++$ and $+-$ couplings and so is not our main interest. Hereafter it is neglected.)

However, recent fully relativistic DWIA analyses of $\left(e,e'p\right)$ [7] and $\left(\gamma,p\right)$ [8] reactions suggest that CC2 form is more appropriate than CC1 and CC3. Do they mean that the second terms of Eqs. (6) and (7) should be suppressed by other medium effects? To answer the problem, we first consider the isoscalar current of a nucleon in symmetric nuclear matter. In this case, $q=0$ and the Dirac equation for a nuclear nucleon is

$$\left(\not{p}-M-U\right)\psi=0, \tag{8}$$

where the mean-field (or potential) U does not depend on the momentum and is composed of the scalar and vector parts:

$$U=S+\gamma^0V. \tag{9}$$

Therefore, the CC1 form (6) indicates that the isoscalar current (vertex) γ^{μ} is transformed in nuclear medium as

$$\gamma^{\mu} \to \gamma^{\mu} - (1/2)\,\xi\left(\bar{U}\gamma^{\mu} + \gamma^{\mu}\bar{U}\right), \tag{10}$$

where $\bar{U} = U/M$ and ξ is the isoscalar anomalous magnetic moment.

Here, we take the renormalized Walecka model developed in Ref. [9]. In this model, the wave function is renormalized as (See Eq. (40) in Ref. [9].)

$$\psi_{(0)} = Z^{1/2}\psi_R, \tag{11}$$

where $\psi_{(0)}$ is the wave function in the un-renormalized Walecka model and the renormalization factor is given by (See Eqs. (39) and (63) in Ref. [9].)

$$Z^{-1} = 1 + \xi\bar{U}. \tag{12}$$

It is noted that this renormalization is due to nuclear medium not to the vacuum in free space. If we first consider the isoscalar current $\bar{\psi}_{(0)}\gamma^{\mu}\psi_{(0)}$ in the un-renormalized model and then apply the wave function renormalization of Eq. (11),

$$\bar{\psi}_{(0)}\gamma^{\mu}\psi_{(0)} = \bar{\psi}_R Z^{1/2}\gamma^{\mu}Z^{1/2}\psi_R. \tag{13}$$

Because $\left|\xi\bar{U}\right| \ll 1$ (See Eqs. (26) and (107) in Ref. [9].), the current becomes

$$Z^{1/2}\gamma^{\mu}Z^{1/2} \approx \left(1 - \frac{1}{2}\xi\bar{U}\right)\gamma^{\mu}\left(1 - \frac{1}{2}\xi\bar{U}\right) \approx \gamma^{\mu} - \frac{1}{2}\xi\left(\bar{U}\gamma^{\mu} + \gamma^{\mu}\bar{U}\right). \tag{14}$$

This is just Eq. (10) or the CC1 current.

However, the vertex should be also renormalized. For the purpose, we use the Ward identity

$$\Gamma^{\mu} = \left(\partial/\partial p_{\mu}\right)G(p)^{-1}, \tag{15}$$

where Γ^{μ} is the renormalized vertex and $G(p)$ is the propagator of a nucleon in the nuclear medium:

$$G(p)^{-1} = \not{p} - M - \Sigma(p). \tag{16}$$

Here, the self-energy of the nucleon $\Sigma(p)$ is given by U_{tot} of Eq. (28) in Ref. [9]:

$$\Sigma(p) = U\left(1 + \xi\bar{U}\right) - \frac{1}{2}\xi\left[\bar{U}\left(\not{p} - M\right) + \left(\not{p} - M\right)\bar{U}\right]. \tag{17}$$

Therefore,

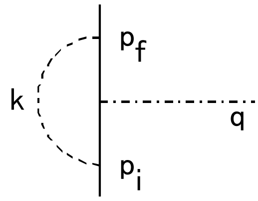
$$\Gamma^{\mu} = \gamma^{\mu} + \frac{1}{2}\xi\left(\bar{U}\gamma^{\mu} + \gamma^{\mu}\bar{U}\right) \approx \left(1+\frac{1}{2}\xi\bar{U}\right)\gamma^{\mu}\left(1+\frac{1}{2}\xi\bar{U}\right) \approx Z^{-1/2}\gamma^{\mu}Z^{-1/2} \qquad (18)$$

Replacing γ^{μ} in Eq. (13) by Γ^{μ},

$$\bar{\psi}_R Z^{1/2}\gamma^{\mu}Z^{1/2}\psi_R \to \bar{\psi}_R Z^{1/2}\Gamma^{\mu}Z^{1/2}\psi_R = \bar{\psi}_R\gamma^{\mu}\psi_R . \qquad (19)$$

Consequently, for the renormalized wave function ψ_R, the original γ^{μ} vertex or the CC2 current is recovered. We have found that the difference between the CC1 and CC2 current is just the effect of wave function renormalization but it is canceled out by the effect of vertex renormalization due to the Ward identity.

So as to generalize the above consideration to the full form of CC1 current (3), we consider the first-order quantum correction, which is depicted by the next Feynman diagram, to the electromagnetic vertex function for a nucleon in symmetric nuclear matter:



Here, the solid, dashed and dotted-dashed lines indicate nucleon ($G(p)$), meson ($D(k)$) and photon propagator, respectively. This is expressed by

$$\Gamma^{\mu}\left(p_f, p_i\right) = \sum_a \int d^4k\, D_a(k)\Lambda_a G_f(p_f - k)\frac{1}{2}(1+\tau_3)\gamma^{\mu} G_i(p_i - k)\Lambda_a , \qquad (20)$$

where the index a indicates any of all necessary mesons and Λ_a is its vertex. The nucleon propagator satisfies Dyson equation [5,10]

$$G_{i(f)}(p) = G^{(0)}(p) + G^{(0)}(p)U_{i(f)}G_{i(f)}(p), \qquad (21)$$

where $G^{(0)}(p)$ is the non-interacting Green's function. We assume that the mean field does not depend on the momentum and that different one-body Hartree and optical potentials describe the initial and final states:

$$U_{i(f)} = S_{i(f)} + \gamma^0 V_{i(f)} . \qquad (22)$$

Substituting the iteration expansion of Eq. (21), Eq. (20) is expanded as

$$\Gamma^{\mu}\left(p_f, p_i\right) = \sum_{n=0}^{\infty} \Gamma^{\mu}_{(n)}\left(p_f, p_i\right), \tag{23}$$

$$\Gamma^{\mu}_{(n)}\left(p_f, p_i\right) = \sum_a \int d^4k\, D_a\left(k\right) \Lambda_a \Upsilon^{\mu}_{(n)}\left(p_f, p_i, k\right) \Lambda_a . \tag{24}$$

Using identities [10],

$$\left(G^{(0)}\left(p\right)\right)^2 = -\frac{\partial}{\partial \not{p}} G^{(0)}\left(p\right), \tag{25}$$

$$G^{(0)}\left(p\right) \gamma^0 G^{(0)}\left(p\right) = -\frac{\partial}{\partial p_0} G^{(0)}\left(p\right), \tag{26}$$

$\Upsilon^{\mu}_{(n)}$ for $n \geq 1$ is given by

$$\Upsilon^{\mu}_{(n)}\left(p_f, p_i, k\right) = \frac{1}{n!}\left[\Delta_i(p_i) + \Delta_f(p_f)\right]^n \Upsilon^{\mu}_{(0)}\left(p_f, p_i, k\right), \tag{27}$$

where

$$\Delta_{i(f)}\left(p_{i(f)}\right) = -\left(S_{i(f)} \frac{\partial}{\partial \not{p}_{i(f)}} + V_{i(f)} \frac{\partial}{\partial p^0_{i(f)}}\right), \tag{28}$$

and

$$\Upsilon^{\mu}_{(0)}\left(p_f, p_i, k\right) = G^{(0)}(p_f - k) \frac{1}{2}\left(1 + \tau_3\right) \gamma^{\mu} G^{(0)}\left(p_i - k\right). \tag{29}$$

Consequently,

$$\Gamma^{\mu}\left(p_f, p_i\right) = \sum_{n=0}^{\infty} \frac{1}{n!}\left[\Delta_i(p_i) + \Delta_f(p_f)\right]^n \Gamma^{\mu}_{(0)}\left(p_f, p_i\right). \tag{30}$$

The second terms of Eqs. (6) and (7) operate on the external nucleon lines in the Feynman diagram. Therefore, Eq. (30) explicitly extracted the effects of mean fields on the internal nucleon lines connected to external lines. Although other medium corrections are still incorporated in $\Gamma^{\mu}_{(0)}\left(p_f, p_i\right)$ implicitly, they are not relevant to Gordon ambiguity. In deriving Eq. (30), only the general relations, the Dyson equation (21) and the identities (25) and (26), were used. Therefore, the above procedure can be applied to any higher-order corrections including charged-meson current, and so Eq. (30) is valid for the complete expression of the nucleon current in a nuclear medium within the meson field theory. Because

$$\left[\Delta_i(p_i)+\Delta_f(p_f)\right]^n q^\mu = 0, \tag{31}$$

after all, we obtain

$$q_\mu \Gamma^\mu\left(p_f, p_i\right) = \sum_{n=0}^{\infty} \frac{1}{n!}\left[\Delta_i(p_i)+\Delta_f(p_f)\right]^n q_\mu \Gamma^\mu_{(0)}\left(p_f, p_i\right). \tag{32}$$

Then, we calculate the proper current in nuclear medium. Replacing $q_\mu \Gamma^\mu_{(0)}\left(p_f, p_i\right)$ in Eq. (32) by

$$q_\mu \Gamma^\mu_{CC1}\left(p_f, p_i\right) = F_1\left(q^2\right)\left[\left(\not{p}_f - M\right) - \left(\not{p}_i - M\right)\right] - \frac{1}{2M} F_2\left(q^2\right)\left[\left(\not{p}_f - M\right)^2 - \left(\not{p}_i - M\right)^2\right]. \tag{33}$$

we have

$$q_\mu \Gamma^\mu\left(p_f, p_i\right) = q_\mu \Gamma^\mu_{CC1}\left(p_f, p_i\right) + \Pi_1\left(p_f, p_i\right) + \Pi_2\left(p_f, p_i\right) + \Pi_3\left(p_f, p_i\right), \tag{34}$$

where

$$\Pi_1\left(p_f, p_i\right) = F_1\left(q^2\right)\left[\Delta_i(p_i)+\Delta_f(p_f)\right]\left[\left(\not{p}_f - M\right) - \left(\not{p}_i - M\right)\right], \tag{35}$$

$$= -F_1\left(q^2\right)\left(U_f - U_i\right), \tag{36}$$

$$\Pi_2\left(p_f, p_i\right) = -\frac{1}{2M} F_2\left(q^2\right)\left[\Delta_i(p_i)+\Delta_f(p_f)\right]\left[\left(\not{p}_f - M\right)^2 - \left(\not{p}_i - M\right)^2\right], \tag{37}$$

$$= \frac{1}{2M} F_2\left(q^2\right)\left(U_f \not{q} + \not{q} U_i\right) + \frac{1}{2M} F_2\left(q^2\right)\left[\left(\not{p}_f - M\right)\left(U_f - U_i\right) + \left(U_f - U_i\right)\left(\not{p}_i - M\right)\right], \tag{38}$$

$$\Pi_3\left(p_f, p_i\right) = -\frac{1}{4M} F_2\left(q^2\right)\left[\Delta_i(p_i)+\Delta_f(p_f)\right]^2\left[\left(\not{p}_f - M\right)^2 - \left(\not{p}_i - M\right)^2\right], \tag{39}$$

$$= -\frac{1}{2M} F_2\left(q^2\right)\left(U_f^2 - U_i^2\right). \tag{40}$$

Here, $\Delta_{i(f)}\left(p_{i(f)}\right)$ does not operate on the phenomenological form factors $F_{1(2)}\left(q^2\right)$.

Using the Dirac equation for the initial and final (positive-energy) state wave function,

$$\left(\not{p}_i - M - U_i\right)\psi_i = 0, \tag{41}$$

$$\bar{\psi}_f\left(\not{p}_f - M - U_f\right) = 0, \tag{42}$$

the second term of Eq. (38) cancels out Eq. (40). Therefore, we have the current expression as

$$\Gamma^{\mu}\left(p_f, p_i\right) = \Gamma^{\mu}_{CC1}\left(p_f, p_i\right) + \frac{1}{2M} F_2\left(q^2\right)\left(U_f \gamma^{\mu} + \gamma^{\mu} U_i\right) - F_1\left(q^2\right)\left(U_f - U_i\right)\frac{q^{\mu}}{q^2}. \tag{43}$$

The second and third terms are the corrections to the CC1 current. Substituting Eq. (6) and using Eqs. (41) and (42) again, we can see that the second term of Eq. (43), which corresponds to the vertex renormalization of Eq. (17), offsets the second term of Eq. (6), which correspond to the wave function renormalization of Eq. (14):

$$\Gamma^{\mu}\left(p_f, p_i\right) = \Gamma^{\mu}_{CC2}\left(p_f, p_i\right) - F_1\left(q^2\right)\left(U_f - U_i\right)\left(q^{\mu}/q^2\right). \tag{44}$$

$$= \Gamma^{\mu}_{CC2}\left(p_f, p_i\right) - \left(q^{\mu}/q^2\right)\left[q \cdot \Gamma_{CC2}\left(p_f, p_i\right)\right]. \tag{45}$$

Equation (45) is also derived if $\Gamma^{\mu}_{(0)}\left(p_f, p_i\right)$ in Eq. (32) is replaced by $\Gamma^{\mu}_{CC2}\left(p_f, p_i\right)$. In that case, Π_2 and Π_3 in Eq. (34) disappear while Π_1, which produces the second term of Eq. (45), still remains. This fact indicates that the second term of Eq. (45) is not a vertex correction but a term which guarantees the current conservation

$$\left\langle \bar{\psi}_f \middle| q_{\mu}\Gamma^{\mu}\left(p_f, p_i\right) \middle| \psi_i \right\rangle = 0. \tag{46}$$

It corresponds to the Landau gauge prescription [1,11] for restoring current conservation. However, the proper current Γ^{μ} has such a term naturally. There are no needs to include it by hand. In the practical calculations of physical quantities, the second term of Eq. (45) has no effects [1,11]. Therefore, we recover the CC2 current. In the recent full DWIA analysis of (γ, n) reaction [8], the CC1 current can reproduce experimental data fairly well, while the CC2 current failed. As the authors pointed out, this success of the CC1 current is spurious due to the serious effect of two-body mechanism as meson exchange current on the (γ, n) reaction [12].

Because the above calculation is quite formal and general, we can apply it to the CC3 current by using

$$q_{\mu}\Gamma^{\mu}_{CC3}\left(p_f, p_i\right) = F_1\left(q^2\right)\left[\left(\not{p}_f - M\right) - \left(\not{p}_i - M\right)\right] + \frac{1}{2M} F_1\left(q^2\right)\left[\left(\not{p}_f - M\right)^2 - \left(\not{p}_i - M\right)^2\right] \tag{47}$$

in place of $q_{\mu}\Gamma^{\mu}_{(0)}\left(p_f, p_i\right)$ in Eq. (32). In this case, $F_2\left(q^2\right)$ in Eqs. (38) and (40) are replaced by

$-F_1\left(q^2\right)$. Thus, Eq. (43) becomes

$$\Gamma^{\mu}\left(p_f, p_i\right)=\Gamma^{\mu}_{CC3}\left(p_f, p_i\right)-\frac{1}{2M}F_1\left(q^2\right)\left(U_f\gamma^{\mu}+\gamma^{\mu}U_i\right)-F_1\left(q^2\right)\left(U_f-U_i\right)\frac{q^{\mu}}{q^2}. \qquad (48)$$

Using Eqs. (41) and (42), the second term of Eq. (48) offsets the second term of Eq. (7). Consequently, CC2 current is also restored. It is not strange to obtain the same result for both CC1 and CC3 current. It is essentially due to the fact that the Ward identity itself does not depend on the value of ξ in Eq. (12).

We studied the Gordon ambiguity of the nucleon current in nuclear medium described by the relativistic mean-field model. First, for the isoscalar current at $q=0$, it is shown that the difference between the CC1 and CC2 current is just the effect of wave function renormalization, but is cancelled out by the effect of vertex renormalization due to the Ward identity. Next, this result is generalized by investigating the electromagnetic vertex function. We explicitly extracted the effects of mean fields on the internal nucleon lines (propagators) connected to external lines in the corresponding Feynman diagram. Consequently, Eq. (32) is obtained, in which the terms of $n\geq 1$ are the effects of nuclear medium on the current. This equation is valid for the complete vertex in nuclear medium. Then, we calculated the current and obtained Eq. (45) regardless of CC1, CC2 or CC3 as $\Gamma^{\mu}_{(0)}\left(p_f, p_i\right)$ in Eq. (32). Important point is a consistent treatment of the internal and external lines of a nucleon in Feynman diagram. In conclusion, the proper current is never worried about the Gordon ambiguity and the familiar CC2 current is reasonable. This naturally explains the results of recent fully RDWIA analyses of $\left(e,e'p\right)$ [7] and $\left(\gamma,p\right)$ [8] reactions.

References

[1] J.A.Caballero, T.W.Donnelly, E.Moya de Guerra and J.M.Udias, Nucl.Phys. **A632** (1998) 323, [arXiv:nucl-th/9710038].

[2] J.J.Kelly, Phys.Rev. **C60** (1999) 044609, [arXiv:nucl-th/9905024].

[3] D.Debruyne, J.Ryckebusch, W.Van Nespen and S.Janssen, Phys.Rev. **C62** (2000) 024611, [arXiv:nucl-th/0005058].

[4] Andrea Meucci, Phys.Rev. **C65** (2002) 044601, [arXiv:nucl-th/0111077].

[5] B.D.Serot and J.D.Walecka, *Advances in Nuclear Physics*, Vol.**16** (Plenum, New York, 1986).

[6] L.S.Celenza and C.M.Shakin, *Relativistic Nuclear Physics* (World Scientific, Singapore, 1986).

[7] J.M.Udias, J.A.Caballero, E.Moya de Guerra, Javier R.Vignote and A.Escuderos, Phys.Rev. **C64** (2001) 024614, [arXiv:nucl-th/0101038].

[8] A.Meucci, C.Giusti and F.D.Pacati, Phys.Rev. **C64** (2001) 064615, [arXiv:nucl-th/0107052].

[9] K.Miyazaki, CERN Document Server (CDS), Ext-2002-056.

[10] S.A.Chin, Ann. of Phys. **108** (1977) 301.

[11] S.Pollock, H.W.L.Naus and J.H.Koch, Phys.Rev. **C53** (1996) 2304, [arXiv:nucl-th/9512043].

[12] A.Meucci, C.Giusti and F.D.Pacati, Phys.Rev. **C66** (2002) 034610, [arXiv:nucl-th/0205055].