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From weak-scale observables to leptogenesis

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Abstract

Thermal leptogenesis is an attractive mechanism for generating the baryon asymmetry of the Universe. However, in supersymmetric models, the parameter space is severely restricted by the gravitino bound on the reheat temperature T_{RH}. For hierarchical light neutrino masses, it is shown that thermal leptogenesis *can* work when $T_{RH} \sim 10^9$ GeV. The low-energy observable consequences of this scenario are $BR(\tau \to \ell\gamma) \sim 10^{-8} - 10^{-9}$. For higher T_{RH}, thermal leptogenesis works in a larger area of parameter space, whose observable consequences are more ambiguous. A parametrisation of the seesaw in terms of weak-scale inputs is used, so the results are independent of the texture chosen for the GUT-scale Yukawa matrices.

1 Introduction

Leptogenesis [1] is an appealing mechanism for producing the baryon asymmetry of the Universe[2]. In the seesaw model[3], heavy singlet ("right-handed") neutrinos ν_R decay out-of-equilibrium, producing a net lepton asymmetry, which is reprocessed by Standard Model (SM) $B + L$ violating processes [4] into a baryon asymmetry. A natural and cosmology-independent way to produce the ν_R is by scattering in the thermal plasma. This scenario is refered to as "thermal leptogenesis". However, the lightest ν_R can be produced only if their mass M_1 is less than the reheat temperature T_{RH} of the plasma after inflation. In addition, the asymmetry is proportional to M_1[5], so there is a lower bound on M_1 to get a large enough asymmetry. This implies 10^8 GeV $< M_1 < T_{RH}$.

The seesaw is an attractive minimal extension of the SM that generates the observed small ν masses. Three right-handed neutrinos, with large majorana masses M_i, are added to the Standard Model, along with a Yukawa matrix for the neutrinos. It is desirable to supersymmetrise the seesaw, to address the hierarchy between the weak scale and the M_i. In the SUSY seesaw, T_{RH} must be low enough to avoid over-producing gravitinos [6, 7]—the canonical bound for gravity mediated SUSY breaking is $T_{RH} \lesssim 10^9$ GeV. The aim of this paper is to identify the parameter space where thermal leptogenesis can work, taking $M_1 \sim T_{RH} \sim 10^9$ GeV.

The SUSY seesaw has more low-energy consequences than the non-SUSY version. It induces lepton flavour violating (LFV) entries in the slepton mass matrix, which can lead to radiative lepton decays[8], such as $\mu \to e\gamma$, at experimentally accessible rates. Eighteen parameters are required to define the neutrino and sneutrino mass matrices (in the charged lepton mass eigenstate basis), which is the same number as there are high scale inputs for the seesaw model [9]. It can be shown that the SUSY seesaw can be parametrised with the sneutrino and light neutrino mass matrices[10], in a texture model independent way. That is, the high-scale physical inputs of the SUSY seesaw—the ν_R masses M_i and Yukawa coupling

$\mathbf{Y}_\nu$—can be "reconstructed" from the neutrino and sneutrino mass matrices[1]. The baryon asymmetry can therefore be expressed as a function of weak scale observables. In this paper we identify the ranges of experimentally measurable quantities which are consistent with thermal leptogenesis. This phenomenological analysis differs from previous work [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22] by making minimal assumptions about the high scale theory: we assume the SUSY seesaw and universal soft masses at the GUT scale. The usual approach is to assume a GUT-scale texture that generates the desired neutrino mass matrix, and discuss leptogenesis—the aim here is to input the slepton mass matrix instead of a texture. This analysis should be consistent with all GUTS and texture choices covered by these assumptions.

Section 2 includes notation, and a review of leptogenesis and our parametrisation of the seesaw model. Section 3 presents approximate analytic expressions for the quantities on which leptogenesis depends. The low energy signatures of the parameter space where thermal leptogenesis works are discussed in section 4. CP violation is briefly discussed in section 5. The results are discussed and summarised in section 6.

2 Review

The observed deficits in muon neutrinos from the atmosphere [23] and in electron neutrinos from the sun[24, 25, 26] can be fit with small neutrino mass differences. The recent KamLAND observation of a $\bar{\nu}_e$ deficit from reactors confirms the neutrino mass explanation of the solar neutrino puzzle[27]. The small Δm^2 are consistent with three patterns of neutrino mass: hierarchical ($\Delta m^2_{atm} = m_3^2$, $\Delta m^2_{sol} = m_2^2$), degenerate ($m_3 \simeq m_2 \simeq m_1 \gg \Delta m^2_{atm}$) and quasi-Dirac ($m_3^2 \simeq m_2^2 \simeq \Delta m^2_{atm}$, $\Delta m^2_{sol} = m_3^2 - m_2^2$). The leptogenesis scenario considered in this paper, where the ν_R are produced by scattering in the plasma, does not work for degenerate m_i [5] (see also [28] for a detailed discussion). The quasi-Dirac spectrum could be interesting, although it is possibly disfavoured by supernova data [29]. We assume the m_i are hierarchical, so the neutrino masses are much smaller than the charged lepton and quark masses. These small masses can be naturally understood in the seesaw model.

In subsection 2.1, the seesaw is reviewed from the top-down; introducing new physics at a high scale M_X, and seeing its low energy implications. This approach has been followed by many model builders who construct a natural or symmetry-motivated structure of the high-scale mass and Yukawa matrices, and then study its low energy consequences. See *e.g.* [30] for early works that produce neutrino mass matrices with small mixing angles, and [31] for more complete up-to-date references. Lepton flavour violation due to the SUSY seesaw, which could be observed in $\ell_j \to \ell_i \gamma$ [8, 32] or in slepton production and decay at colliders [33] has also been extensively studied from a top down approach (see *e.g* citations of [8, 32]). Recent studies (for instance [34]) have considered the branching ratios for $\ell_j \to \ell_i \gamma$ in models that induce the two observed large mixing angles among the light leptons.[2].

Subsection 2.3, is a "top-down" review of leptogenesis, which is the obvious approach [11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22] since the asymmetry is generated at high scales. See [11] for examples and models. The translation between this approach and our bottom-up phenomenological analysis is not obvious, so it is difficult to relate our work to these papers. A phenomenological analysis of leptogenesis in non-SUSY (so no LFV) SO(10) models was discussed in [16, 22], with particular attention to the low-energy CP violation. A Yukawa-matrix independent analysis has also been done in the case where there are only two right-handed neutrinos[17].

[1] This "reconstruction" would require universal soft masses at the GUT scale, and improbable experimental accuracy at the weak scale, so is in practice impossible.

[2] see also [35] for a more phenomenological discussion of $\mu \to e\gamma$

2.1 Notation and Numbers

We consider the supersymmetric see-saw for two reasons: first, supersymmetry stabilizes the Higgs mass against the quadratic divergences that appear due to heavy particles (*e.g.* the right-handed neutrinos). Secondly, the slepton masses enter our bottom-up parametrisation of the see-saw.

The leptonic part of the superpotential reads

$$W_{lep} = {e_R^c}^T \mathbf{Y_e} L \cdot H_d + {\nu_R^c}^T \mathbf{Y}_\nu L \cdot H_u - \frac{1}{2} {\nu_R^c}^T \mathcal{M} \nu_R^c, \tag{1}$$

where L_i and e_{Ri} ($i = e, \mu, \tau$) are the left-handed lepton doublet and the right-handed charged-lepton singlet, respectively, and H_d (H_u) is the hypercharge $-1/2$ ($+1/2$) Higgs doublet. $\mathbf{Y_e}$ and $\mathbf{Y}_\nu$ are the Yukawa couplings that give masses to the charged leptons and generate the neutrino Dirac mass, and $\mathcal{M}$ is a 3×3 Majorana mass matrix. This is the minimal seesaw; additional terms are possible, for instance in SO(10) models a small triplet *vev* $\langle T \rangle$ is probable[36], leading to a $\nu_L \langle T \rangle \nu_L$ mass term.

We work in the left-handed basis where the charged lepton mass matrix is diagonal, and in a basis of right-handed neutrinos where $\mathcal{M}$ is diagonal

$$D_{\mathcal{M}} \equiv \mathrm{d}iag(M_1, M_2, M_3), \tag{2}$$

with $M_i \geq 0$, and $M_1 < M_2 < M_3$. In this basis, the neutrino Yukawa matrix must be non-diagonal, but can always be diagonalized by two unitary transformations:

$$\mathbf{Y}_\nu = V_R^\dagger D_Y V_L, \tag{3}$$

where $D_{\mathbf{Y}_\nu} \equiv \mathrm{d}iag(y_1, y_2, y_3)$ and $y_1 \ll y_2 \ll y_3$. Later in the paper, we will assume that $D_{\mathbf{Y}_\nu}$ is hierarchical, with a steeper hierarchy than is in the light neutrino mass matrix: $(y_1/y_2)^2 \ll m_1/m_2$.

It is natural to assume that the overall scale of $\mathcal{M}$ is much larger than the electroweak scale or any soft mass. Therefore, at low energies the right-handed neutrinos are decoupled and the corresponding effective Lagrangian contains a Majorana mass term for the left-handed neutrinos: $\delta \mathcal{L}_{lep} = -\frac{1}{2} \nu^T m_\nu \nu +$ h.c., with

$$m_\nu = {\mathbf{m_D}}^T \mathcal{M}^{-1} \mathbf{m_D} = {\mathbf{Y}_\nu}^T \mathcal{M}^{-1} \mathbf{Y}_\nu \langle H_u^0 \rangle^2. \tag{4}$$

We define the Higgs *vev* $\langle H_u^0 \rangle^2 = v_u^2 = v^2 \sin^2 \beta$, where $v = 174$ GeV. In the basis where the charged-lepton Yukawa matrix, $\mathbf{Y_e}$ and the gauge interactions are diagonal, the $[m_\nu]$ matrix can be diagonalized by the MNS [38] matrix U according to

$$U^T [m_\nu] U = \mathrm{d}iag(m_1, m_2, m_3) \equiv D_{m_\nu}, \tag{5}$$

where U is a unitary matrix that relates flavour to mass eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{6}$$

and the m_i can be chosen real and positive, and ordered such that $m_1 < m_2 < m_3$. Assuming hierarchical left-handed ν masses, we take $m_3^2 = \Delta m^2_{\mathrm{atm}} = 2.7 \times 10^{-3} eV^2$ [39] and $m_2^2 = \Delta m^2_{\mathrm{solar}} = 7.0 \times 10^{-5} eV^2$ [40]. This corresponds to $m_3 = 5.2 \times 10^{-2}$ eV ($3.9 - 6.3 \times 10^{-2}$ eV at 90% C.L.), and $m_2 = 8.2 \times 10^{-3}$ eV ($7 - 15 \times 10^{-3}$ eV at 3 σ). m_1 is unknown, usually unimportant, and we take it to be $m_2/10$. As we shall see, the baryon asymmetry is weakly dependent on $\tan\beta$ in the parametrisation we use, so we set $\sin\beta = 1$.

U can be written as

$$U = V \cdot \mathrm{diag}(e^{-i\phi/2}, e^{-i\phi'/2}, 1) \quad , \tag{7}$$

where ϕ and ϕ' are CP violating phases, and V has the form of the CKM matrix

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} . \tag{8}$$

The numerical values of the angles are $.28 \leq \tan^2\theta_{sol} \leq .91$ (3σ), with best fit point $\tan^2\theta_{sol} = .44$ [40], so $\theta_{sol} = .41$. We take $\theta_{atm} = \pi/4$. The CHOOZ angle θ_{13} is experimentally constrained $\sin\theta_{13} \leq .2$ [41]. Considerable effort and thought has gone into designing experiments sensitive to smaller values of θ_{13}. J-PARC hopes to reach $O(0.05)$ [42], and a neutrino factory could detect θ_{13} as small as $0.02 \to 0.001$[42, 43].

We assume a simple gravity-mediated SUSY breaking scenario, with universal soft masses at the scale M_X. The sneutrino mass matrix (in the charged lepton mass eigenstate basis) can be written in the leading log approximation as

$$\left(m^2_{\tilde{\nu}}\right)_{ij} \quad \simeq \quad (\text{diagonal part}) - \frac{3m_0^2 + A_0^2}{8\pi^2}[\mathbf{Y}_\nu^\dagger]_{ik}[\mathbf{Y}_\nu]_{kj} \ln\frac{M_X}{M_k} \ , \tag{9}$$

where "diagonal-part" includes the tree level soft mass matrix, the radiative corrections from gauge and charged lepton Yukawa interactions, and the mass contributions from F- and D-terms.

The branching ratio for $\ell_j \to \ell_i\gamma$ can be estimated

$$\frac{BR(\ell_j \to \ell_i\gamma)}{BR(\ell_j \to \ell_i\bar{\nu}_i\nu_j)} \sim C\frac{\alpha^3}{G_F^2 m^4_{SUSY}}|y_k^2\tilde{V}^*_{Lkj}\tilde{V}_{Lki}|^2\tan^2\beta \simeq 10^{-7}|y_3^2\tilde{V}^*_{L3j}\tilde{V}_{L3i}|^2\left(\frac{100GeV}{m_{SUSY}}\right)^4\left(\frac{\tan\beta}{2}\right)^2 \tag{10}$$

where $C \sim O(0.001 \div 0.01)$, and $\tilde{V}_L$ diagonalises the second term of eqn (9). More accurate formulae for the branching ratios can be found in [32]. To further simplify these estimates, it would be convenient to assume that $V_L = \tilde{V}_L$. That is, the lepton asymmetry will be a function of the angles of V_L, and it would be simplest to estimate $\ell_j \to \ell_i\gamma$ using the angles of V_L for those of $\tilde{V}_L$. This a reasonable approximation when $\theta_{Lij} \gg \frac{y_i}{y_j}\theta_{Rij}$ $(i < j)$, where θ_{Rij} (θ_{Lij}) is an angle of $V_R(V_L)$. For hierarchical Yukawa eigenvalues, this is likely to be true, even if an angle θ_R in V_R is large, because the usual texture estimate for θ_{Lij} would be $\sqrt{y_i/y_j}$. We assume this condition is verified, so the principle contribution to $[m^2_{\tilde{\nu}}]_{ij}$ is $\propto y_k^2 V^*_{Lki}V_{Lkj}$.

Table 2.1 lists the current upper limits on the $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ branching ratios, and the corresponding bounds on V_{L3j} that can be estimated from eqn (10) [44]. It also contains the hoped for sensitivity of some anticipated rare decay searches. Colliders could also be sensitive to flavour violating slepton masses [33].

Leptogenesis will depend on angles of V_L. In the remainder of the paper, we will claim that "leptogenesis predicts an observable $BR(\ell_j \to \ell_i\gamma)$", if the V_L elements required exceeed the last column of table 2.1 (last three rows), with $y_3 = 1$. It is clear that the branching ratios can be decreased, for fixed V_L, by decreasing y_3 and adjusting weak scale SUSY parameters. However, if SUSY is discovered, these masses and mixing angles could in principle be measured at colliders, and some information about the magnitude of y_3 could be available through the renormalisation group equations [45]. This assumption of universal soft masses at the scale M_X will not be crucial for our conclusions. Additional contributions to the off-diagonal soft masses are unlikely to cancel the ones we discuss, so the lower bounds we set on LFV branching ratios, from requiring leptogenesis to work, should remain.

$BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$ (PSI)	$y_3^2 V_{L32}^* V_{L31} < .006$
$BR(\tau \to e\gamma) < 2.7 \times 10^{-6}$ (CLEO)	$y_3^2 V_{L33}^* V_{L31} \lesssim 12$
$BR(\tau \to \mu\gamma) < 1.1 \times 10^{-6}$ (CLEO)	$y_3^2 V_{L33}^* V_{L32} < 9$
$BR(\mu \to e\gamma) \sim 10^{-14\div15}$ (PSI/nufact)	$y_3^2 V_{L32}^* V_{L31} \sim 3 \times 10^{-4}$
$BR(\tau \to e\gamma) \sim 10^{-9}$ (BABAR/BELLE)	$y_3^2 V_{L33}^* V_{L31} \sim 0.2$
$BR(\tau \to \mu\gamma) \sim 10^{-9}$ (BABAR/BELLE/LHC)	$y_3^2 V_{L33}^* V_{L32} \sim 0.2$

Table 1: Current limits [46] and hoped for sensitivities [47] of some experiments. The numerical bounds in the right column are multiplied by $\left(\frac{m_{SUSY}}{100 \text{ GeV}}\right)^2 \left(\frac{2}{\tan\beta}\right)$.

Various CP violating phases in the neutrino and slepton mass matrices could be measured in upcoming experiments. However, the experimental sensitivity to the phases depends on the magnitude of unmeasured real parameters. Anticipated $0\nu\beta\beta$ experiments may be sensitive to a maximal phase ϕ', for the neutrino mass spectrum we consider. The minimum value of the angle δ that could be measured at a ν factory depends on $\Delta m_{32}^2, \Delta m_{21}^2$ and θ_{13} (see *e.g.* [43]), so there is no forseeable clear upper bound. The imaginary part of the product of off-diagonal slepton masses $\Im\{[m_{\tilde{\nu}}^2]_{12}[m_{\tilde{\nu}}^2]_{23}[m_{\tilde{\nu}}^2]_{31}\} = \tilde{J}(\tilde{m}_2^2 - \tilde{m}_1^2)(\tilde{m}_3^2 - \tilde{m}_2^2)(\tilde{m}_1^2 - \tilde{m}_3^2)$ could be measured in slepton flavour oscillations down to $\tilde{J} = 10^{-3}$ [48]. $\tilde{J}$ depends on the magnitude of the $[m_{\tilde{\nu}}^2]_{ij}$ as well as their phases, so all the phases in the (s)lepton sector can be of order 1.

2.2 Review of the parametrisation

It was shown in [10] that the seesaw can be parametrised from the bottom-up, using the neutrino and sneutrino mass matrices. See [49] for applications. A similar phenomenological parametrisation of the non-SUSY seesaw [50] could be used, if the scale M of right-handed masses was low enough to measure dimension six operators $\propto 1/M^2$. We briefly review [10] here.

It is in principle possible to extract the matrix

$$P \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu = V_L^\dagger D_Y^2 V_L \tag{11}$$

from its contribution to the renormalisation group running of the slepton mass matrix. This relies critically on having universal soft masses at the GUT scale, and on very precise measurements of sneutrino masses and decays. It is therefore unrealistic[10]. However, since SUSY has not yet been discovered, D_Y and V_L can be used as inputs in a "bottom-up" parametrisation of the seesaw.

The aim is to determine $\mathbf{Y}_\nu$ and $\mathcal{M}$ from $[m_\nu]$ and P. V_L and D_Y can be determined from P, and used to strip the Yukawas off $[m_\nu]$:

$$D_Y^{-1} V_L^* \frac{[m_\nu]}{v_u^2} V_L^\dagger D_Y^{-1} = V_R^* D_{\mathcal{M}}^{-1} V_R^\dagger = \mathcal{M}^{-1}, \tag{12}$$

where the left hand side of this equation is known ($[m_\nu]$ is one of the inputs, and V_L and D_Y were obtained from eq. (11)). Therefore, V_R and $D_{\mathcal{M}}$ can also be determined. This shows that, working in the basis where the charged lepton Yukawa coupling, $\mathbf{Y_e}$, the right-handed Majorana mass matrix, $\mathcal{M}$, and the gauge interactions are all diagonal, it is possible to determine *uniquely* the heavy Majorana mass matrix, $\mathcal{M}$, and the neutrino Yukawa coupling, $\mathbf{Y}_\nu = V_R^\dagger D_Y V_L$, starting from $[m_\nu]$ and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$.

2.3 Leptogenesis, and the upper bound

The see-saw mechanism provides a natural framework to generate the baryon asymmetry of the Universe, defined as $\eta_B = (n_B - n_{\bar{B}})/s$, where s is the entropy density. As was shown by Sakharov[53], generating a baryon asymmetry requires baryon number violation, C and CP violation, and a deviation from thermal equilibrium. These three conditions are fulfilled in the out-of-equilibrium decay of the right-handed neutrinos and sneutrinos in the early Universe. In the remainder of this paper, "right-handed neutrinos", and the shorthand notation ν_R, refer to both right-handed neutrinos and right-handed sneutrinos.

In gravity mediated SUSY breaking scenarios, these is an upper bound from gravitino production on the reheat temperature T_{RH} of the Universe after inflation. The gravitino has a mass $m_{3/2} \sim m_{SUSY}$ and only gravitational interactions with SM particles, so it is very weakly coupled, and long-lived. If a significant number of them decay at or after Big Bang Nucleosynthesis, they could disrupt the predicted abundances of light elements[6, 7]. Gravitinos can be created by various mechanisms in the early Universe, such as scattering in the thermal plasma[6, 7], or direct coupling to the inflaton (preheating) [54, 55]. The latter is effective, but avoidable [55]. The number density of gravitinos produced in scattering increases with the plasma temperature, so the bound on $n_{3/2}$ sets an upper bound on the reheat temperature of the Universe after inflation of

$$T_{RH} \lesssim 10^9 \to 10^{12} GeV \tag{13}$$

(corresponding to $m_{3/2} \sim 100$ GeV $\to 10$ TeV [7]). This bound assumes that the gravitino decays; there are models where the gravitino is the LSP, which allow $T_{RH} \lesssim 10^{11}$ GeV [56].

Let us briefly review the mechanism of generation of the BAU through leptogenesis [1, 11]. At the end of inflation, a certain number density of right-handed neutrinos, n_{ν_R}, is somehow produced. If these right-handed neutrinos ν_{R_i} decay out of equilibrium, a lepton asymmetry can be created. The subsequent ratio of the lepton excess to the entropy density s is given by

$$\eta_L = \frac{n_\ell - n_{\bar{\ell}}}{s} = \sum_i \frac{n_{\nu_{R_i}}}{s} \ \epsilon_i \ \tilde{d}_i. \tag{14}$$

The CP-violating parameter ϵ_i is determined by the particle physics model that gives the masses and couplings of the ν_R. The value of n_{ν_R}/s depends on the mechanism to generate the right-handed neutrinos. We assume the ν_{R_1} are generated by Yukawa scattering in the thermal plasma, in which case $n_{\nu_R}/s \lesssim n^{eq}/s \simeq .2/g_*$, where n^{eq} is the equilibrium number density of massless particles, and $g_* \simeq 230$ is the number of propagating states in the supersymmetric plasma [3]. This also implies an

[3] in our conventions, $n_{\nu_R} = (n_{\nu_R} + n_{\bar{\nu}_R})/2$. The .2 is an approximation to $g_* n^{eq}/s = \zeta(3)135/(8\pi^4)$.

upper bound on the ν_R mass: $M_1 \lesssim T_{RH}$. Finally, $\tilde{d}_1$ is the fraction of the produced asymmetry that survives after ν_R decay. To ensure $\tilde{d}_1 \sim 1$, lepton number violating interactions (decays, inverse decays and scatterings) must be out of equilibrium when the right-handed neutrinos decay. In the case of the lightest right-handed neutrino ν_{R_1}, this corresponds approximately to

$$K = \frac{\Gamma_{D_1}}{2H|_{T\simeq M_1}} < 1 \tag{15}$$

where H is the Hubble parameter at the temperature T, and Γ_{D_1} the ν_{R_1} decay rate. There are two competing requirements on the ν_R parameters—the couplings must be large enough to produce a thermal distribution, but small enough that the ν_R decay out of equilibrium. Thermal leptogenesis has been carefully studied in [18][4]. The numerical results of [18, 11] suggest that $n_{\nu_R}\tilde{d}_1/s < n^{eq}_{\nu_R}/s$: either n_{ν_R} does not attain its equilibrium number density, or lepton number violating interactions wash out a significant fraction of the asymmetry as it is produced. Defining an effective light neutrino "mass"

$$\frac{\tilde{m}_1}{v_u^2} = 8\pi\frac{\Gamma_{D_1}}{M_1^2} = \frac{(\mathbf{Y}_\nu\mathbf{Y}_\nu{}^\dagger)_{11}}{M_1} \tag{16}$$

$n_{\nu_R}\tilde{d}_1/s \gtrsim 10^{-4}$ is realised for [28] 5×10^{-5} eV $\lesssim \tilde{m}_1 \lesssim 10^{-2}$ eV. The precise numerical bound on $\widetilde{m}_1$ depends on M_1, and can be found in [18].

For $\tilde{m}_1 > 10^{-4}$ eV and $M_1 \sim 10^9$ GeV, the dilution factor d_1 can be approximated [57, 58]

$$\frac{n_{\nu_R}\tilde{d}_1}{s} \equiv d_1 \simeq \frac{1}{6g_*}\frac{1}{\sqrt{K^2+1}} \tag{17}$$

with $K \simeq 910\tilde{m}_1/eV$ from eqn (15). This is a slight modification of the approximation, to ensure that it falls between the $M_1 = 10^8$ GeV and 10^{10} GeV lines of [11], in the relevant range .001 eV $\lesssim \tilde{m}_1 \lesssim$ .1 eV. The exact numerical factor is important, because it is difficult to get a large enough asymmetry. Multiplying d_1 by a factor of a few significantly increases the parameter space where thermal leptogenesis can work. The approximation (17) neglects the decrease in d_1 at $\tilde{m}_1 \lesssim 10^{-4}$ eV, which is due to underproduction of ν_{R_i} in scattering. This is reasonable, because $\tilde{m}_1 \geq m_1$ [5], and we take $m_1 = m_2/10$.

The last step is the transformation of the lepton asymmetry into a baryon asymmetry by non-perturbative B+L violating (sphaleron) processes [4], giving

$$\eta_B = C\eta_{B-L} = (3-9)\times10^{-11}, \tag{18}$$

where $C = 8/23$ in the Minimal Supersymmetric Standard Model. Big Bang Nucleosynthesis constrains η_B to lie in the range of eqn (18). In a flat Universe, the CMB determines $\eta_B \simeq (0.75-1.0)\times10^{-10}$ [52]. The wider BBN range is used in this paper, because it is difficult to generate a large enough η_B.

The CP asymmetry can be approximated as

$$\epsilon_1 \simeq -\frac{3}{8\pi}\frac{1}{[\mathbf{Y}_\nu\mathbf{Y}_\nu{}^\dagger]_{11}}\sum_j \mathrm{Im}\left\{[\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger]^2_{1j}\right\}\left(\frac{M_1}{M_j}\right) \tag{19}$$

$$= -\frac{3}{8\pi}\frac{M_1}{[\mathbf{Y}_\nu\mathbf{Y}_\nu{}^\dagger]_{11}}\mathrm{Im}\left\{[\mathbf{Y}_\nu\frac{[m_\nu]^\dagger}{v_u^2}\mathbf{Y}_\nu{}^T]_{11}\right\}. \tag{20}$$

[4] See [20] for a detailed analysis of thermal leptogenesis at higher temperatures, including the effects of ν_{R2} and ν_{R3}

if the lepton asymmetry is generated in the decay of the lightest right-handed neutrino, and if the masses of the right-handed neutrinos are hierarchical [5]. Were the asymmetry produced in the decay of ν_{R_2} or ν_{R_3}, it would depend on a different combination of couplings.

It is straightforward to show [5] that if ϵ_1 is written

$$|\epsilon_1| = \frac{3}{8\pi v_u^2} M_1 m_3 \delta_{HMY} \tag{21}$$

then eqn (20) implies the upper bound $\delta_{HMY} \leq 1$. The numerical results of [21, 59] agree with this constraint. Using eqns (14) and (18), this can be transformed into a lower bound on M_1:

$$M_1 \gtrsim \frac{\eta_B}{C} \left[\frac{n_{\nu_R} + n_{\tilde{\nu}_R}}{s} \; \frac{3}{8\pi} \frac{m_3}{v_u^2} \; \tilde{d}_1 \right]^{-1} = 10^9 \left(\frac{\eta_B}{3 \times 10^{-11}} \right) \left(\frac{.05 eV}{m_3} \right) \left(\frac{4 \times 10^{-4}}{d_1} \right) \text{ GeV.} \tag{22}$$

Setting m_3 to its 90% C.L. upper bound 0.063 eV, and d_1 to its maximum value $n^{eq}/s \simeq 45/(2\pi^4 g_*)$, implies $M_1 > 3 \times 10^8$ GeV.

This lower bound on M_1 comes very close to the gravitino bound eqn (13) on the reheat temperature. Thermal production of the ν_R requires $M_1 \lesssim T_{RH}$ so either ϵ is close to its upper bound, or $M_1, T_{RH} > 10^9$ GeV, or thermal leptogenesis does not generate the observed baryon asymmetry. We explore the first option, and somewhat the second. The third possibility, non-thermal ν_R production, has been discussed by many authors (see *e.g.* references of [60]).

3 Analytic approximations for δ_{HMY}, M_1, $\tilde{m}_1$

At least three inputs are required to parametrise thermal leptogenesis [18, 11, 28]. A possible choice would be the mass M_1 and decay rate $\propto \tilde{m}_1$ of the ν_R, and the CP asymmetry ϵ_1. However, $\epsilon \propto M_1$, so we use M_1, $\tilde{m}_1$ and δ_{HMY} (introduced by Hamaguchi, Murayama and Yanagida), where δ_{HMY} measures how close ϵ comes to saturating its upper bound. Note however, that δ_{HMY} is not a CP phase.

This section contains simple analytic approximations indicating the dependence of leptogenesis parameters on measurable quantities, such as neutrino masses and rare LFV decays. We used this approximation, with attention to the phases, in [37]. A similar, somewhat simplified version was introduced in [16].

The inputs for the analytic approximation are:

$$V_L, \; D_Y, \; U, \; [m_\nu] \tag{23}$$

Two of the angles of U are known, and the CHOOZ angle is bounded above. The eigenvalues y_i of the neutrino Yukawa matrix are unknown, and realistically cannot be determined from the sneutrino mass matrix. It seems reasonable to assume a hierarchy for the $\{y_i\}$, since we measure hierarchical Yukawas for the quarks and charged leptons. The y_i remain as variables in the equations; we will discover that only the smallest eigenvalue y_1 is relevant, and can be "traded" for the mass M_1 of the lightest ν_R, which is tightly constrained. V_L contains three unknown angles, related to the lepton flavour violating decays $\ell_j \to \ell_i \gamma$. There are three phases in both U and V_L, all are unknown, and assumed to be chosen to maximise the baryon asymmetry.

The lightest eigenvalue and corresponding eigenvector of $\mathcal{M}$ are estimated in the first Appendix, which also contains some simple (but illuminating) 3-d plots of leptogenesis parameters. The mass of

[5] If the hierarchy in Y_ν is similar to that of the quarks and charged leptons, then a hierarchy in the M_i is natural.

the lightest ν_R is

$$|M_1| \simeq \frac{y_1^2 v_u^2}{|W_{1j}^2 m_j|}. \tag{24}$$

where the matrix $W = V_L U$ is the rotation from the basis where the ν_L masses are diagonal to the basis where the neutrino Yukawa matrix $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ is diagonal. There are three limiting values for M_1, corresponding to $M_1 \simeq y_1^2 v_u^2/m_i$: $M_1 \to y_1^2 v_u^2/m_3$ when $W_{13} \to 1$, $M_1 \to y_1^2 v_u^2/m_1$ when $W_{13}, W_{12} \to 0$, and $M_1 \to y_1^2 v_u^2/m_2$ when $W_{13} < m_2/m_3$, $W_{12} \to 1$. This is easy to see in figure 4.

M_1 is the only quantity relevant for leptogenesis which depends on y_1. The latter is effectively unmeasurable; it is constrained by theoretical expectations, and by the requirement that the analytic approximation be self-consistent. Theoretically, the eigenvalues of Y_ν are expected to be hierarchical, and of order the quark or lepton Yukawas, so figure 4 is plotted with $y_1 \sim 10^{-4}$. The approximations of this section are consistent, provided that the dropped $O(y_1^2, y_1^2/y_2^2)$ terms are smaller than the $O(m_1/m_3)$ terms which are kept. This is the case for $y_1 \sim 10^{-4}$. Since y_1 is unmeasurable and only weakly constrained, it can be adjusted, as function of m_i and W_{1j}, to obtain a value for M_1 where leptogenesis could work. In fact, since $M_1 \propto y_1^2$ is tightly constrained, the requirement $M_1 \sim 10^9$ GeV "determines" y_1.

The eigenvector (50) can be used to evaluate the ν_{R1} decay rate: eqn. (16) becomes

$$\tilde{m}_1 \simeq \frac{\sum_k |W_{1k}^2| m_k^2}{|\sum_n W_{1n}^2 m_n|} \tag{25}$$

$\tilde{m}_1$ has various limits: $\tilde{m}_1 \to m_3$ for W_{13} large, $\tilde{m}_1 \to m_2$ for W_{12} large and $W_{13} < m_2/m_3$, and $\tilde{m}_1 \to m_1$ when $W \to 1$. This is easy to see from the RHS of figure 4. In the $W \to 1$ limit, washout is minimised, because the dilution factor $d_1 \propto 1/\tilde{m}_1$.

To saturate the upper bound (21), δ_{HMY} needs to approach 1. Evaluating eq. (20) with the eigenvector(50), gives

$$\begin{aligned}
\delta_{HMY} &= \frac{\mathrm{Im}\left\{\sum_{\ell,m} W_{1\ell}^2 m_\ell^3 W_{1m}^{*2} m_m\right\}}{m_3 |\sum_n W_{1n}^2 m_n| (\sum_j |W_{1j}|^2 m_j^2)} \\
&\simeq \frac{|W_{11}W_{12}|^2 m_1 m_2^3 + |W_{11}W_{13}|^2 m_1 m_3^3 + |W_{12}W_{13}|^2 m_2 m_3^3}{m_3 (\sum_n |W_{1n}|^2 m_n)(\sum_j |W_{1j}|^2 m_j^2)}
\end{aligned} \tag{26}$$

This paper is about the relation between real low energy observables (such as $BR(\mu \to e\gamma)$) and the baryon asymmetry, so scant attention will be paid to the phases in U and V_L. For most of parameter space [6], the phases can be chosen such that δ_{HMY} is larger than the second expression in eqn (26).

This second expression is plotted on the LHS in figure 5. δ_{HMY} can approach 1 if the numerator (upstairs) is dominated by $m_3^3 m_2$ or by $m_3^3 m_1$. This is because of the m_3 in the denominator. If the $m_3^3 m_1$ dominates upstairs, then δ_{HMY} will approach 1 when $W_{1j}^2 m_j \simeq W_{11}^2 m_1$ and $|W_{1n}|^2 m_n^2 \simeq |W_{13}|^2 m_3^2$, or equivalently, when

$$\frac{m_1^2}{m_3^2} < W_{13}^2 < \frac{m_1}{m_3} \quad and \quad W_{12}^2 < \frac{m_1}{m_2}, \frac{m_2^2}{m_3^2} \tag{27}$$

This corresponds to the highest ridge in δ_{HMY} in figure 5. Notice that the position of the peak depends sensitively on the lightest neutrino mass m_1.

[6] everywhere but when the three terms upstairs have equal magnitude

If $m_3^3 m_2$ dominates upstairs, then $\delta_{HMY} \to 1$ when

$$W_{12}^2 \frac{m_2^2}{m_3^2} < W_{13}^2 < W_{12}^2 \frac{m_2}{m_3} \quad and \quad W_{11}^2 < W_{12}^2 \frac{m_2}{m_1}, W_{13}^2 \frac{m_3^2}{m_1^2} \tag{28}$$

This corresponds to the shoulder at slightly large W_{13}, which is cut by $W_{12} \sim 1$ in the LH plot of figure 5.

Finally, for W_{13} very small, $\delta \to m_2/m_3 \sim 0.1$ along the ridge at $W_{12} \sim 0.1$. This corresponds to the $m_2^3 m_1$ term dominating upstairs, and arises when

$$W_{11}^2 \frac{m_1^2}{m_2^2} < W_{12}^2 < \frac{m_1}{m_2} W_{11}^2 \quad and \quad W_{13}^2 < W_{11}^2 \frac{m_1}{m_3}, W_{12}^2 \frac{m_2^2}{m_3^2} \tag{29}$$

Although δ_{HMY} does not reach its maximum value for these parameters, the washout is small, so the baryon asymmetry generated is only slightly too small. As we will see in figure 3, it is large enough if $M_1, T_{RH} \sim 10^{10}$ GeV are allowed.

4 When does thermal leptogenesis work?

The baryon asymmetry can be approximated as

$$\eta_B \simeq \frac{8d_1}{23} \frac{3}{8\pi v_u^2} M_1 m_3 \delta_{HMY} \tag{30}$$

by combining eqns (14), (18), and (21). This is plotted in figure 1, which suggests that η_B *can* be large enough.

The issue is whether a large enough asymmetry can be generated, so the observational upper limit on η_B is unimportant. Also, the asymmetry calculated here is the upper bound corresponding to maximal CP violation, so it can be reduced by taking smaller phases. We use the one-σ observational lower bound on η_B from nucleosynthesis: $\eta_B \gtrsim 3 \times 10^{-11}$. To obtain a large enough baryon asymmetry by thermal leptogenesis, the parameters M_1, δ_{HMY}, and d_1 must occupy narrow ranges. The washout effects are minimised when the ν_R decay rate is small, which corresponds to $W \to 1$. In this case, $n_{\nu_R} \tilde{d}_1 / s = d_1 < 10^{-3}$ which implies the lower bound $\epsilon \gtrsim 10^{-7}$. (If $\epsilon \gtrsim 10^{-6}$ can be obtained, then $d_1 \sim 10^{-4}$ is large enough.) Eqn (21) implies a lower bound on M_1 to get ϵ_1 large enough. In addition, $M_1 \lesssim T_{RH}$ is required for thermal production; the canonical SUSY gravitino bound is $T_{RH} \lesssim 10^9$ GeV, so $5\epsilon \times 10^{15}\text{GeV} \lesssim M_1 \lesssim T_{RH}$. Since $\epsilon \simeq 10^{-7}$ is required, for $M_1 \lesssim 10^9$ GeV one must have $\delta_{HMY} \to 1$. The parameter space of choice can be summarised as

$$\begin{array}{rcl} few \times 10^8 GeV & \lesssim M_1 \lesssim & few \times 10^9 GeV \\ d_1 & \to & \frac{45}{2\pi^4 g_*} \quad , g_* = 230 \\ \delta_{HMY} & \to & 1 \end{array} \tag{31}$$

As can be seen from figures 4 — 5, it is difficult to simultaneously satisfy these conditions. M_1 and d_1 increase as $W_{13}, W_{12} \to 0$, but δ decreases.

The analytic approximations of the previous section show that the baryon asymmetry depends on U and the first row of V_L (via W_{1j}), on the light neutrino masses m_i, on the lightest neutrino Yukawa y_1, and on phases. These real parameters are known, or could be experimentally constrained in the next

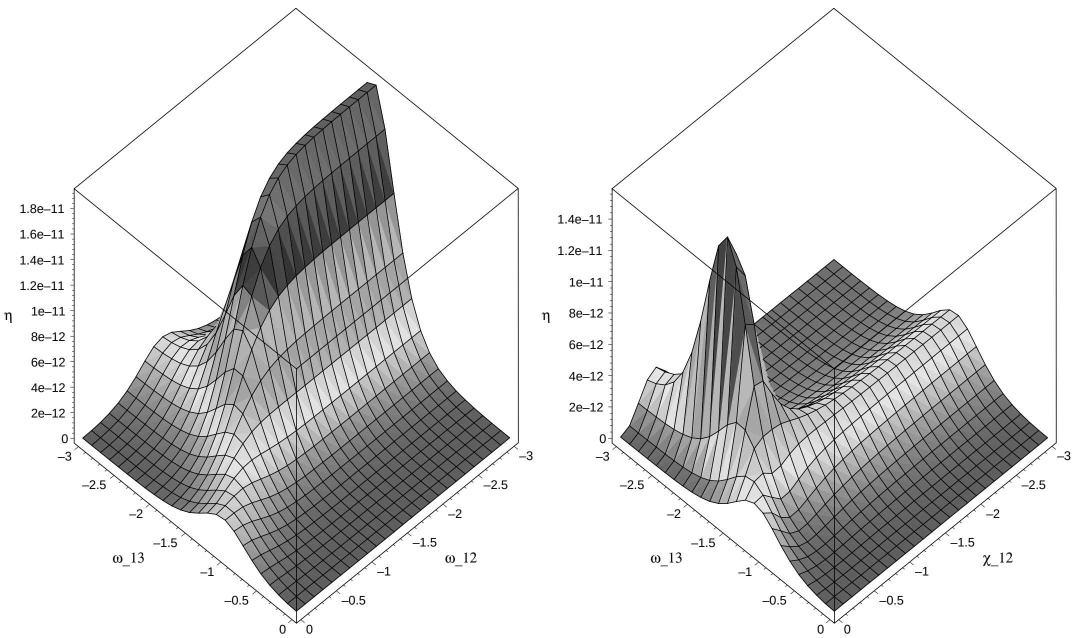

Figure 1: 3-d plot of $\eta_B = 8d_1\epsilon/23$, for central neutrino mass values, $m_1 = m_2/10$ and $M_1 = 10^9$ GeV. On the left, η_B is plotted as a function of $\omega_{12} \simeq log[W_{12}]$ and $\omega_{13} \simeq log[W_{13}]$. On the right, η_B is plotted as a function of ω_{13} and χ_{12}, defined such that $W_{12} = \cos\theta_{W13}\sin(\theta_{sol} - 10^{\chi_{12}}\pi/2)$. The RHS measure on parameter space is more sensible, see the discussion after eqn(34).

20 years—with the exception of m_1, y_1, and V_{L12}. For the purposes of this paper, V_{L12} is included with the measurable angles, and y_1 is *determined* as a function of m_1, by requiring that M_1 be in the range (31) where leptogenesis could work. The baryon asymmetry then becomes independent of y_1. Some subtle dependence on m_1 remains: the area and location of the high ridge in figure 1 depend on m_1, but the baryon asymmetry and low energy footprints do not. This is discussed in the Appendix about m_1.

Notice also that it could be expected to have a similar hierarchy in the neutrino Yukawas as in the other fermions, in which case $y_1 \sim h_u, h_e$ or h_d. This gives

$$M_1 \sim \left(\frac{y_1}{h_u}\right)^2 \left(\frac{m_2}{W_{1j}^2 m_j}\right) 3 \times 10^6 \text{ GeV} \tag{32}$$

where $W_{1j}^2 m_j$ is usually of order m_2. If $y_1 \simeq h_u$, then η_B is too small over most of parameter space. This was found in some models by [15]. The baryon asymmetry can be large enough, for $y_1 \simeq h_u$, in the small area of parameter space where $W_{1j}^2 m_j \simeq m_1 \lesssim m_2/100$. This is in the m_1 Appendix too.

The baryon asymmetry depends weakly on $\tan\beta$, when M_1 is taken as an input, and d_1 is approximated as $\propto 1/\tilde{m}_1 \propto \sin^2\beta$. The m_i are experimentally measured, and therefore independent of $\sin^2\beta$, so it is clear from eqn (30) that the $\sin\beta$ dependence arises entirely from $\tilde{m}_1$. If instead $M_1 = (y_1^2 v_u^2)/|W_{1j}^2 m_j|$, then $\eta_B \propto \sin^4\beta$. In both cases, larger $\sin\beta$ is marginally favoured.

The parameters W_{12} and W_{13} are convenient, because they summarise the unknown mixing angles and phases. The physically relevant quantities for leptogenesis (M_1, ϵ,...) can be plotted as a function of the two real unknowns $|W_{12}|$ and $|W_{13}|$. However, the W_{1j} are not observable—the matrix W is related to the more physical matrices V_L and U by $W = V_L U$. Recall that V_L rotates from the basis where the neutrino Yukawa matrix Y_ν is diagonal to the basis where Y_e is diagonal, and U rotates from the basis where $[m_\nu]$ is diagonal to the basis where Y_e is diagonal. W can be written

$$\begin{aligned} W_{13} &= V_{L11} \sin\theta_{13} e^{-i\delta} + V_{L12}/\sqrt{2} + V_{L12}/\sqrt{2} && (33) \\ W_{12} &= V_{L11} \sin\theta_{sol} + V_{L12}(\cos\theta_{sol} - \sin\theta_{sol}\sin\theta_{13}e^{i\delta})/\sqrt{2} \\ &\quad -V_{L13}(\cos\theta_{sol} + \sin\theta_{sol}\sin\theta_{13}e^{i\delta})/\sqrt{2} && (34) \end{aligned}$$

where $\theta_{12} = \theta_{sol}$ and $\theta_{23} = \pi/4$ in the MNS matrix.

From a model building perspective[31], there are two natural limits for W. The most popular is for the large leptonic mixing angles to come from the seesaw structure of the light neutrino mass matrix. In this case, $V_L \sim 1$ can easily arise, so $W \sim U$. This is similar to the quark sector, where the CKM matrix (the analogue of V_L) has small angles. An example of this is texture models where the large atmospheric mixing angle is due to a ν_R mass eigenstate having approximately equal Yukawa couplings to ν_μ and ν_τ [61]. Alternatively, the electron Yukawa matrix Y_e could be "the odd man out" ; it could have large off-diagonal elements in a basis where the neutrino mass matrix $[m_\nu]$ and Y_ν are simultaneously almost diagonal[62]. In this case $V_L \sim U^\dagger$ and $W \sim 1$. These two cases are discussed in the following two subsections. Figure 1 suggests that thermal leptogenesis can work for $W \sim 1$. As we shall see, a large enough asymmetry is also possible in the $V_L \to 1$ limit, if a slightly larger T_{RH} is allowed.

The plots are functions of $\log W_{13}$ and $\log W_{12}$, rather than, *e.g.*W_{12} and W_{13}. It is sensible to use logarithmic measure on unknown physical parameters[7] because it is equally probable for mixing angles to have any order of magnitude between *e.g.*10^{-3} and 1. However, W_{1j} are not physical parameters, so this reasoning does not apply to them. Specifically, values of $W_{12} \lesssim \sin\theta_{sol}$ arise in the presumably small area of parameter space where $V_L \simeq U^\dagger$. This reasoning does apply to the CHOOZ angle and the unknown angles of V_L, but we prefer to plot η_B as a function of two unknowns, rather than four. So a more appropriate measure on W_{12} might be logarithmic in the difference away from $U_{12} \simeq \sin\theta_{sol}$. Therefore, on the RHS of figure 1 is plotted the same function as on the LHS, but as a function of $\omega_{13} \simeq \log(W_{13})$, and χ_{12}, the latter defined such that

$$W_{12} = \sin(\theta_{sol} - 10^{\chi_{12}}\pi/2). \tag{35}$$

$\chi_{12} \simeq \log[W_{12} - U_{12}]$ is an approximation to the log of the unknown angles of V_L (see eqn (34)). So the RHS plot tells us the same information as its twin on the left: the asymmetry is largest if a large angle in V_L cancels the large solar angle in the MNS matrix U.

A final technical comment: $W \sim 1$ and $V_L \sim 1$ mean the 12 and 13 matrix elements are small $\lesssim .1$. $W \sim 1$ means that W maximises η_B, so $W_{12} \lesssim .1$ and $.01 \lesssim W_{13} \lesssim .1$. $V_L \sim 1$ includes both the possibilities that the angles of V_L are smaller, or larger, than the CHOOZ angle.

Section 4.1 studies the phenomenological consequences of sitting in the region where thermal leptogenesis works easily, which corresponds approximately to $W_{12} \lesssim .1$, $.01 \lesssim W_{13} \lesssim .1$. Then in section 4.2, some of the parameters which are fixed in figure 1 are varied, so a large enough baryon asymmetry can be generated for $V_L = 1$. The parameter space between these two limits is discussed in section 4.3.

4.1 $W \sim 1$

The parameter space where η_B is largest in figure 1 corresponds roughly to

$$.01 \lesssim W_{13} \lesssim .1 \quad W_{12} \lesssim .1 \quad . \tag{36}$$

[7] this choice of measure is neither unique nor universally agreed on

This can be understood from the analytic approximation (30). We fix $M_1 \simeq 10^9$ GeV, so $\eta_B \propto d_1 \delta_{HMY}$. The factor d_1 is largest when the ν_R decay rate $\Gamma \propto \tilde{m}_1$ is smallest, so more of the asymmetry survives when $W \to 1$ (see the expression 25). δ_{HMY} parametrises how close ϵ can come to its upper bound (21). For $m_1 \sim m_2/10$, the values of W_{12}, W_{13} where δ_{HMY} is maximised (eqn 27) correspond to eqn (36). η_B is maximal at smaller W_{13} than δ_{HMY}, as can be seen by comparing figures 1 and 4. This is due to the lepton number washout encoded in d_1, which is faster at larger W_{13}.

To obtain W_{12} and W_{13} in this region, V_L must have the form

$$V_L = R_{23}[U_{\approx}]^\dagger \tag{37}$$

where R_{23} is an unspecified complex rotation in the 23 plane (written in the form of eqn (8) with $\theta_{12} = \theta_{13} = 0$, $S \equiv \sin\theta_{23}$, $C \equiv \cos\theta_{23}$, and taking a 23 phase α), and $[U_{\approx}]$ is a matrix whose angles are roughly those of the MNS matrix $\pm.1$. The unknown R_{23} appears because leptogenesis only depends on the first row of W.

It is interesting to study the implications for $\ell_j \to \ell_i \gamma$ of eqn (37). Taking $[U_{\approx}] = U$

$$\begin{aligned}
V_{L31} &= Se^{i(\alpha+\phi'/2)}c_{13}s_{12} + Cs_{13}e^{i\delta} \\
&\simeq Se^{i(\alpha+\phi'/2)}s_{sol} + Cs_{13}e^{i\delta} \\
V_{L32} &= Se^{i(\alpha+\phi'/2)}(c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta}) + Cs_{23}c_{13} \\
&\simeq Se^{i(\alpha+\phi'/2)}c_{sol}/\sqrt{2} + C/\sqrt{2}
\end{aligned}$$

For generic values of S, this implies $V_{L32} \sim 1$, so an experimentally accessible $\tau \to \mu\gamma$ branching ratio. If, on the other hand, S is tuned to make $V_{L32} \to 0$, then $BR(\tau \to \mu\gamma)$ would be unobservable. However, in this case $V_{L31} \sim -\sin\theta_{sol}/\sqrt{1+\cos^2\theta_{sol}} \simeq 1/\sqrt{3}$, so $\tau \to e\gamma$ should be observable. One can conclude that if leptogenesis takes place in the $W \sim 1$ peak of figure 1, then one or both of $\tau \to \mu\gamma$ and $\tau \to e\gamma$ should have a branching ratio $\gtrsim 10^{-9}$ (according to the leading log approximation of the introduction). Similarly, $BR(\mu \to e\gamma)$ should be $\gtrsim 10^{-14}$ if S or $s_{13} \gtrsim 10^{-3}$.

4.2 $V_L = 1$

It is barely possible to get a large enough baryon asymmetry when the angles in V_L are small, although this is not evident from figure 1. In the limit $V_L \to 1$, the matrix $W \to U$, so $W_{12} \simeq \sin\theta_{sol}$ and $W_{13} \simeq \sin\theta_{13}$. In figure 1, η_B is at least a factor of 6 to small at $\log W_{12} \sim -0.5$ (equivalently, χ_{12} small), but there is a bump at $W_{13} \sim .1$. In this section, η_B at $V_L = 1$ is increased by varying m_3, m_2 and M_1; V_L close to the identity is discussed in the following subsection.

If $V_L = 1$, eqns (24) and (25) give

$$\begin{aligned}
M_1 &= \frac{y_1^2 v_u^2}{m_2 s_{12}^2} \\
\tilde{m}_1 &= m_2
\end{aligned}$$

To maximise the asymmetry, M_1 is taken to be $f \times 10^9$ GeV, where f is a few (this determines $y_1^2 = M_1 m_2 s_{12}^2/v_u^2 \simeq 7.2f \times 10^{-8}$). In figure 10 of [11], the lepton asymmetry is plotted as a function of $\tilde{m}_1$ for various values of M_1 and $\epsilon_1 = 10^{-6}$. This plot shows that for $\epsilon \simeq 10^{-6}$, a large enough asymmetry can be generated when $\tilde{m}_1 \simeq m_2$. Note that the washout effects are correctly included in this plot of [11], so this result does not depend on the analytic approximation of eqn (17). Also, the asymmetry *increases* as $\tilde{m}_1$ decreases, so small values of the solar mass are prefered.

The upper bound of eqn (21) implies that for $\epsilon = 10^{-6}$, $fm_3\delta_{HMY} = 0.25$eV. From the experimentally allowed range of m_3 given after eqn (6), we see that $M_1 \sim 3 \times 10^9$ GeV is required, assuming $\delta \sim 1$ is also possible. For $V_L = 1$,

$$\delta_{HMY} \simeq \frac{s_{13}^2 m_3^3 s_{12}^2 m_2 \sin(\phi' - 2\delta)}{m_3(s_{12}^2 m_2^2 + s_{13}^2 m_3^2) s_{12}^2 m_2} \tag{38}$$

which approaches 1 when $s_{13} \simeq s_{12}m_2/m_3$. In the RH plot of figure (1), $V_L = 1$ corresponds to $W_{13} = \sin\theta_{13}$ and $\chi_{12} \to -\infty$. In figure 2, δ_{HMY} is plotted as a function of θ_{13} on the LHS; the analytic approximation to η_B (eqn (30)) is plotted on the RHS. So thermal leptogenesis "works" at $V_L = 1$, for $M_1 \sim 6 \times 10^9$ GeV.

Phrased another way: for arbitrarily small $\ell_j \to \ell_i\gamma$ branching ratios, requiring thermal leptogenesis to work *predicts* the CHOOZ angle θ_{13}. If m_3 is taken at its $90\% C.L.$ upper bound, and $M_1 \sim 6 \times 10^9$ GeV, then $\eta_B \sim 3 \times 10^{-11}$ can be obtained. This requires a CHOOZ angle of $\theta_{13} \sim 4 \times 10^{-2}$, and phases which satisfy $2\delta - \phi' = \pi/2$.

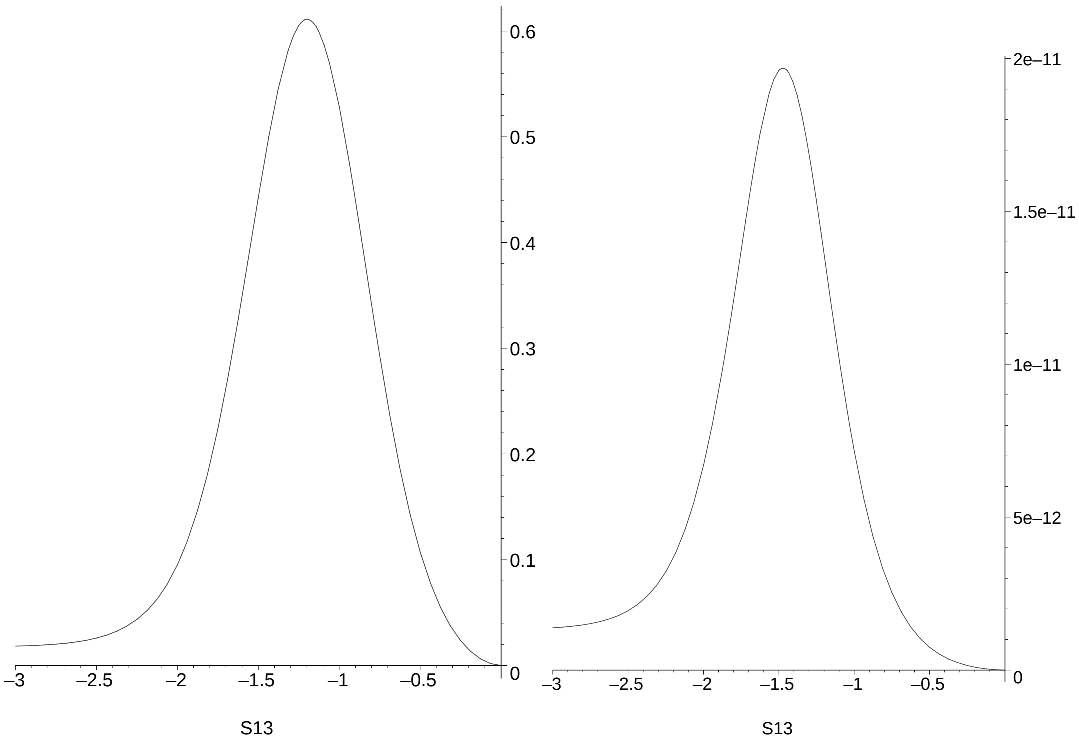

Figure 2: On the LHS δ_{HMY}, and on the RHS the analytic approximation to η_B, as a function of $S13 = log[\sin\theta_{13}]$. $\delta_{HMY} \sim 1$ is required to get a large enough asymmetry, see the discussion in section 4.2. The remaining parameters are $\tan^2\theta_{sol} = 0.44$, $m_2 = 7 \times 10^{-3}$ eV, $m_3 = 6.3 \times 10^{-2}$ eV, and for the η_B plot, $V_L = 1$ and $M_1 = 4 \times 10^9$ GeV.

4.3 V_L from 1 to $U^\dagger$

It is clear from the RHS of figure 1, that if a large enough asymmetry can be generated at $V_L = 1$ ($\chi_{12} \sim -3$), then enough baryons can be generated along the ridge leading to the peak, and also along the "other" ridge at $W_{12} \sim m_1/m_2$. To sit on this second ridge requires $W \sim 1$, so has the same experimental signatures as discussed in section 4.1. This section is about the $W_{13} \sim m_2/m_3$ ridge stretching from $V_L \sim 1$ to the $W \sim 1$ peak.

Starting from the small χ_{12}, flat section of the ridge and moving towards the peak corresponds to allowing small matrix elements $V_{L12}, V_{L13} \lesssim .1$. In this limit,

$$W_{13} \simeq \sin\theta_{13} + V_{L12}/\sqrt{2} + V_{L13}/\sqrt{2}$$

and $W_{13} \sim 0.04$ is required to get a large enough asymmetry. Unfortunately, $W_{13} \sim 0.04$ determines a sum of three unknowns: $\theta_{13} \gtrsim .04$ could be observed, $V_{L13} \sim 0.04$ induces a potentially observable $\tau \to e\gamma$ signal, but V_{L12} has no observable consequences. For $V_L \sim 1$, the V_{L12} contribution to m_ν^2 (eqn (9)) is suppressed by $y_2^2 \sim 10^{-4}$.

Figure 3 is a contour plot in ω_{13} and χ_{12} space of the approximation (30) to η_B. The contours enclose the area when $\eta_B > 2\times 10^{-11}$, for $M_1 = f \times 10^9$ GeV, central values of m_3 and m_2, $m_1 = m_2/10$, and are labelled by f. Allowing $f > 1$ significantly increases the available parameter space. This corresponds to increasing M_1 (which should be $\lesssim T_{RH}$), or increasing m_3, which is constrained by atmospheric neurtino oscillations, or for $\chi_{12} \lesssim 0.5$, to decreasing m_2, which is constrained by solar neutrino experiments and KamLAND. Perhaps the most palatable way to increase f is to allow $T_{RH} \sim 10^{10}$ GeV. The value of η_B chosen, $\eta_B = 2 \times 10^{-11}$, is minimal. To obtain the CMB favoured $\eta_B \simeq 9 \times 10^{-11}$, would require values of f that were four times larger.

5 CP violation

In a previous paper[37], we discussed the relation between the leptonic phases that could be measured at low energy, and the CP violation required for leptogenesis. We assumed that ϵ was large enough, and studied the relative importance of the neutrino factory phase δ and the double beta decay phase ϕ' for leptogenesis. If the right-handed neutrinos ν_{R1} are produced *non*-thermally, getting ϵ large enough may not be a significant constraint (see *e.g.* [60] for a discussion and references). However, we have seen that it is a challenge when the ν_{R1} are produced thermally. So in this section, we briefly discuss the relative importance of low-energy phases for thermal leptogenesis—imposing the constraint that ϵ is large enough.

It is well known that there is no linear relation between the "leptogenesis phase" and δ or ϕ' [63]. That is, the lepton asymmetry can be non-zero when $\delta = \phi' = 0$, and it can be zero when $\delta, \phi' \neq 0$. To overcome this, we introduced a statistical notion of "overlap" between the leptogenesis phase and the low energy phases of our parametrisation. The overlap O_δ aimed to quantify the relative importance of the phase δ for leptogenesis, assuming that all the low-energy phases were $O(1)$. In [37], we considered the cases where $W_{13}^2 W_{12}^2 m_3^3 m_2$, or $W_{12}^2 W_{11}^2 m_2^3 m_1$, is the most important term upstairs in δ_{HMY}. That is, we consider $V_L = 1$, $V_L \sim 1$ and the case of large V_L angles that do not exactly cancel those in U. This occurs over most of the parameter space where ϵ could be large enough. However, ϵ is largest in the small area of parameter space where $W \sim 1$ and $W_{13}^2 W_{11}^{2*} m_3^3 m_1$ dominates upstairs in δ_{HMY}. So let us now consider which low-energy phases are important for leptogenesis in this case.

Writing the phases explicitely gives

$$\epsilon \propto \Im\{W_{11}^2 W_{13}^{*2}\} = \Im\{e^{i\phi}[V_{L11}c_{13}c_{12} + |V_{L12}|e^{i2\varphi_{12}}(-c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta})$$

Figure 3: Contour plot of η_B, as a function of $\omega_{13} \simeq \log[W_{13}]$ and $\chi_{12} \simeq \log[V_{L12} + V_{L13}]$. The contours enclose the area when $\eta_B > 2 \times 10^{-11}$, for $M_1 = f \times 10^9$ GeV, central values of m_3 and m_2, and $m_1 = m_2/10$. In the direction of increasing area, the lines correpond to $f = 1, 3, 6$ and 9.

$$+|V_{L13}|e^{i2\varphi_{13}}(s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta})]^2 \times [V_{L11}s_{13}e^{-i\delta} + |V_{L12}|e^{i2\varphi_{12}}s_{23}c_{13}$$
$$+|V_{L13}|e^{i2\varphi_{13}}c_{23}c_{13}]^2\} \tag{39}$$

where φ_{1j} is the phase of V_{1j}. $V_{1j} \sim U^*_{j1}e^{i\omega_1}$, because $W \sim diag\{e^{i\omega_1}, e^{i\omega_2}, 1\}$ [8]. The "neutrinoless double beta decay phase" ϕ' is irrelevant for leptogenesis, because it only enters into W_{12}. The phase ϕ of m_1 will always be important, because $W_{11} \propto e^{-i\phi/2}$, so ϵ will be a sum of terms $\propto \sin(m\phi + ...)$. Both the phases φ_{12} and φ_{13} of V_{L12} and V_{L13} are likely to have significant overlap with the leptogenesis phase, because $V_L \simeq U^\dagger$ so the $|V_{L1j}|$ are large. The "neutrino factory phase" δ always multiplies the CHOOZ angle, which suppresses its contribution to ϵ.

The three weak-scale phases which have significant "overlap" with the leptogenesis phase, in the area of parameter space near $W \sim 1$, are therefore φ_{12}, φ_{13} and ϕ. This is unfortunate, because although

[8]This constraint on W is what we usually refer to as $W \sim 1$. Since V_{11} is real, $\omega_1 \simeq -\phi/2$.

there is some hope of measuring ϕ' and δ, there is no foreseeable experiment to determine any of these three.

6 Discussion and summary

This paper has discussed leptogenesis in a minimal model of the SUSY seesaw, with gravity mediated SUSY breaking and universal soft masses at a high scale. It uses a parametrisation of the model in terms of

$$D_{m_\nu}, U, D_Y, V_L \tag{40}$$

where D_{m_ν} is the diagonal light majorana neutrino mass matrix (assumed hierarchical), U is the MNS matrix, D_Y is the diagonal neutrino Yukawa matrix, and V_L diagonalises $Y_\nu^\dagger Y_\nu$. The notation is briefly defined in table, 6. giving the equation numbers of more detailed definitions. The angles of the unitary matrix V_L can be related, in SUSY models, to the rates for $\ell_j \to \ell_i \gamma$ because $Y_\nu^\dagger Y_\nu$ contributes to the renormalisation group equations for the slepton masses. This allows the right handed neutrino masses and Yukawa couplings to be expressed as a function of quantities which could be measured, in principle or in practise, at the weak scale. This parametrisation is briefly reviewed in section 2.2.

$\mathbf{Y}_\nu = V_R^\dagger D_Y V_L$, y_i	neutrino Yukawa, eigenvalues	1
$\mathcal{M}, M_1$	ν_R mass matrix, lightest eigenvalue	1,24
V_L	$V_L \mathbf{Y}_\nu{}^\dagger \mathbf{Y}_\nu V_L^\dagger = D_Y^2$	3
φ_{ij}	phases of V_L	section 5
$[m_\nu]$, m_i	light neutrino mass matrix	4
U	MNS matrix	6
$\theta_{ij}; \phi, \phi', \delta$	angles; phases of U	7, 8
W	$V_L U$	45
ω_{1j}	$\simeq$log $[W_{1j}]$	51
χ_{12}	$\simeq$log $[U_{12} - W_{12}]$	35
η_L	lepton asymmetry	14
ϵ	CP asymmetry	20, 52
δ_{HMY}	ϵ/ϵ_{max}	21,26
$\widetilde{m}_1$	$\propto \nu_{R1}$ decay rate	16,25
$d_1, \tilde{d}_1$	dilution factor of lepton asymmetry	17,14
η_B	baryon asymmetry	18, 30

Table 2: Table of notation, with a brief description and the equation number of a more complete defintion.

The baryon asymmetry produced in leptogenesis depends on the number density of right-handed neutrinos which decay, on $\epsilon \equiv$ the average lepton asymmetry produced per decay, and on the survival probability of the asymmetry in the thermal plasma after it is produced. We consider the "thermal leptogensis" scenario, in which the right-handed neutrinos are produced by scattering interactions in the plasma. Non-thermal production mechanisms are also possible, perhaps even probable, but depend on additional parameters from the sector which produces the right-handed neutrinos. Both the thermally produced ν_R number density, and the survival probability of the lepton asymmetry in the plasma after it is produced, can be computed from the reheat temperature of the plasma after inflation T_{RH}, and from the seesaw parameters. These processes have been carefully studied in [18, 11].

A convenient analytic approximation to the numerical results of [18] is used in this paper. A single function d_1 (see equation (17)) is defined as the number density of $\nu_R \times$ the survival probabilty of the lepton asymmetry once it is produced. So the baryon to entropy ratio today is $\eta_B \simeq 8 d_1 \epsilon/23$. [9]

The CP asymmetry produced in the decay of the lightest ν_R is bounded above (for hierarchical M_i and m_j):

$$\epsilon < \frac{3 M_1 m_3}{8 \pi v_u^2} \tag{41}$$

where M_1 is the mass of the ν_{R1} and $m_3 = \sqrt{\Delta m^2_{atm}}$. Since d_1 cannot exceed $45/(2\pi^4 g_*)$ for thermally produced ν_{R1}, obtaining $\eta_B > 3 \times 10^{-11}$ requires $M_1 > 3 \times 10^8$ GeV.

In section 3, approximate analytic formulae for the lightest ν_R mass M_1, for the CP asymmetry ϵ and for the baryon asymmetry η_B, are given in terms of our weak-scale parameters. These approximations are valid for hierarchical M_i and neutrino Yukawa eigenvalues y_j. The baryon asymmetry can be written as a function

$$\eta_B(m_2, m_3, \theta_{23}, \theta_{12}; \theta_{13}, V_{L12}, V_{L13}, y_1, m_1, phases) \tag{42}$$

where "known" low-energy parameters precede the semi-colon. We concentrate on the dependence of η_B on real parameters, assuming that the phases can be chosen to maximise the asymmetry.

It is interesting that the baryon asymmetry only depends on 5 of the 8 unknown real parameters in eqn (40). Two of these, θ_{13} and V_{L13}, are possibly measurable; the constraints that thermal leptogenesis imposes on them will be discussed later for different areas of parameter space. On the other hand, there are no foreseen experiments that could determine y_1, m_1, and V_{L12}. V_{L12} is included in the discussion with θ_{13} and V_{L13}, because these three unknowns can be exchanged for the 12 and 23 elements of $W = V_L U$. This simplifies expressions and is convenient for plotting. The dependence of η_B on m_1 is subtle, comparatively unimportant, and discussed in an Appendix. The lightest right-handed neutrino mass M_1, and therefore the baryon asymmetry, is proportional to y_1^2. So y_1^2 is exchanged for M_1. This is a peculiar exchange— why do we want to use a GUT-scale mass as input in our weak-scale parametrisation? The off-diagonal elements of V_L, and the y_i, are related to lepton flavour violating off-diagonal slepton mass matrix entries (to which processes like $\ell_j \to \ell_i \gamma$ are sensitive), and to slepton mass differences. The smallest neutrino Yukawa y_1 makes negligeable contributions to both these effects. However, $M_1 > 3 \times 10^8$ GeV is required for thermal leptogenesis to have any hope of working, and if SUSY is discovered, the sparticle spectrum could give some indication of the gravitino mass, and therefore the allowed reheat temperature $T_{RH} > M_1$. So we "determine" y_1 by requiring that thermal leptogenesis *could* produce a large enough asymmetry: 3×10^8 GeV $< M_1 < T_{RH}$. Then we study the requirements on the remaining parameters such that the asymmetry *is* large enough. These additional requirements may have observable consequences.

The analytic formulae of section 3 are simple and compact, but nonetheless difficult to visualise. The asymmetry depends on the first row of the matrix W, so for qualitative understanding, we show 3-dimensional figures of leptogenesis parameters as a function of $\omega_{13} \simeq \log W_{13}$, and $\omega_{12} \simeq \log W_{12}$. Logarithmic measure is reasonable for unmeasured but observable matrix elements—which the W_{1j} are *not*. For small angles in V_L (see section 4 for a general discussion), the W_{1j} can be related to the more physical matrix elements V_{L1k} and U_{ij}: $W_{13} \sim \theta_{13} + V_{L12} + V_{L13}$, $W_{12} \sim \sin\theta_{sol} + V_{L12} + V_{L13}$. To present the area of parameter space where leptogenesis works with a more physical measure, we therefore plot the baryon asymmetry as a function of ω_{13} and $\chi_{12} \sim \log[V_{L12} + V_{L13}]$ in figures 1 and 3.

We define thermal leptogenesis to "work" if it can produce $\eta_B \gtrsim 3 \times 10^{-11}$, as required by Big Bang Nucleosynthesis. For $M_1 \simeq 10^9$ GeV (consistent with the canonical gravitino bound $T_{RH} \sim 10^9$

[9]where the 8/23 arises in the transformation of the lepton asymmetry into a baryon asymmetry by the electroweak $B + L$ violating processes.

GeV), leptogenesis *can* work: there is a limited parameter space where the upper bound on ϵ is almost saturated, *and* d_1 is close to maximal. This can be seen in figure 3, where the baryon asymmetry is large enough inside the contours, which are labelled by f, where $M_1 = f \times 10^9$ GeV. Increasing T_{RH} (and thereby the allowed M_1) enlarges the parameter space where thermal leptogenesis works.

We now come to the aim of the paper— what are the weak scale foot prints of thermal leptogenesis? What parameter values must be observed, if thermal leptogenesis works in an MSUGRA model?

Suppose first that $f \simeq 1$, which corresponds to $M_1 \sim 10^9$ GeV for central values of the light neutrino masses. Thermal leptogenesis works in the area of figure 3 at $\omega_{13} \sim -2$ and $\chi_{12} \sim -0.5$. This small area of parameter space is discussed in section 4.1, and occurs if $W = V_L U \sim 1$. The phenomenological consequences of this area of parameter space are unambiguous: the branching ratio of $\tau \to \mu\gamma$, or $\tau \to e\gamma$, should be large. More concretely, at least one of V_{L32} or V_{L31} is $O(1/\sqrt{2})$, so according to the estimates of table 2.1, $BR(\tau \to \ell\gamma) \gtrsim 10^{-8}$.

From a theoretical model building perspective, this area of parameter space corresponds to the neutrino Yukawa and light mass matrices Y_ν and $[m_\nu]$ being almost simultaneously diagonalisable. The large leptonic mixing angles arise in the rotation from this basis to the one where the charged lepton Yukawa matrix Y_e is diagonal.

The baryon asymmetry is largest at this point for two reasons. The ν_R decay rate (eqn (16)) is small, so lepton number violation is slow after the asymmetry is produced, and more of the asymmetry survives. Secondly, the asymmetry produced is almost maximal; it comes within a factor of $O(1)$ of the upper bound eqn (41). This is discussed after eqn (26).

The *area* of this parameter space, where η_B is largest, depends on the smallest neutrino mass m_1. The plots are made with $m_1 = m_2/10$; the area shrinks as m_1 decreases. This peak in η_B only exists for $m_1 \neq 0$. It is interesting that the CP asymmetry and the low-energy footprints of this area of parameter space are *independent* of m_1. However, the number density of ν_R (and therefore the baryon asymmetry) decreases for $m_1 \lesssim 10^{-5}$ eV, and our approximation fails. See the Appendix for a discussion.

In brief, if a sparticle spectrum consistent with gravity mediated SUSY breaking is measured, with a gravitino mass of $m_{3/2} \sim 100$ GeV ($T_{RH} \sim 10^9$ GeV), then $\tau \to \mu\gamma$ or $\tau \to e\gamma$ must be observable for thermal leptogenesis to work.

Now consider the enlarged parameter space allowed by $M_1/(10^9 \text{ GeV}) \equiv f > 1$ in figure 3: thermal leptogenesis works for $W_{13} \sim m_2/m_3$ and pretty much all values of W_{12}. This sets one constraint on the three "physical" matrix elements $\sin\theta_{13}$, V_{L13} and V_{L12}. As discussed in sections 4.2 and 4.3, it can be satisfied if any one of the angles is $O(m_2/m_3)$. These possibilities have different weak-scale implications.

If V_L has small angles like the CKM matrix, $V_{L13}, V_{L12} \ll 0.1$, then leptogenesis requires that the CHOOZ angle $\theta_{13} \simeq 0.04$, which is close to its current experimental bound. This implies that the baryon asymmetry is determined by parameters which can be measured in the neutrino sector [10]: m_3, m_2, θ_{13} and the phases δ and ϕ'.

If the CHOOZ angle $\theta_{13} \ll 0.1$, then it is still possible to sit on the $W_{13} \sim 0.04$ ridge, by having V_{L13}, or $V_{L12} \sim 0.04$. The former angle is related to $\tau \to e\gamma$, and could perhaps be measured in this process. Unfortunately, V_{L12} appears in the RGEs multiplying y_2^2, the middle Yukawa eigenvalue, so has no observable consequences in the slepton mass matrix. So thermal leptogenesis can "work" when θ_{13} and $BR(\ell_j \to \ell_i\gamma)$ are unobservably small.

The matrix elements V_{L13}, and V_{L12} are small along most of the W_{13} ridge currently under discussion. Many texture models occupy this area of parameters space, where the CKM-like matrix V_L (between the bases where Y_ν and Y_e are diagonal) has small angles. The large angles of the MNS matrix then arise from the majorana structure of $Y^T \mathcal{M}^{-1} Y$.

[10]Caveat: $\eta_B \propto M_1 m_3/m_2$ in this case, and we "determine" $M_1 \simeq 6 \times 10^9$ GeV by requiring η_B large enough. If m_3 (m_2) is larger (smaller) than the current best-fit values, η_B increases.

The baryon asymmetry is larger along the ridge than in the rest of parameter space, because the washout is moderate—$\tilde{m} \sim m_2$—and because the CP asymmetry ϵ approaches its upper bound (41). Notice that the baryon asymmetry on this ridge is independent of m_1—the solution remains as $m_1 \to 0$. As discussed after eqn (26), there are two limits in which ϵ is maximal: the ridge where $W_{13} \sim m_2/m_3$, and the previously discussed peak. There is an orthogonal ridge in figure 3, at $\chi_{12} \sim -0.5$ ($W_{12} \sim 0.1$), where $\delta_{HMY} \lesssim m_2/m_3$, but washout is minimised. It has the same observable footprints as the peak.

Until now we have considered *if* thermal leptogenesis works, then *what should we see at low energy?* Allowing $T_{RH} \sim 10^{10}$ GeV, it seems just about all phenomenology is consistent with thermal leptogenesis: large $\tau \to \mu\gamma$, $\theta_{13} \sim 0.04$, observable $\tau \to e\gamma$, nothing observable at all...So now consider the inverse question: are there weak-scale observations that can rule out thermal leptogenesis? The previous discussion is vague because SUSY has not been discovered. Clearly thermal leptogenesis does not work if W_{13} is too big or too small. Since all the terms which contribute to W_{13} cannot be measured, no experimental lower bound can be set. However, one could tell that W_{13} is too *large*, for instance if large V_{L13} ($\tau \to e\gamma$) is measured[11].

The analysis of this paper relies crucially on the assumption that the ν_R are produced thermally. A larger number density of the lightest ν_R, n_{ν_R}/s, could be produced non-thermally, so a large enough baryon asymmetry could be produced with a smaller ϵ. This would enlarge the available parameter space. Furthermore, if the ν_R are produced non-thermally, they could be ν_{R2} or ν_{R3}, making the formulae for ϵ inapplicable.

In summary, we study the baryon asymmetry resulting from the decay of the lightest right-handed neutrino ν_{R_1}, assuming the ν_{R_1}s are produced thermally. We present compact analytic approximations for the quantities relevant to thermal leptogenesis, in terms of the light neutrino masses, the MNS matrix, the smallest eigenvalue of the neutrino Yukawa matrix Y_ν, and the matrix V_L which diagonalises Y_ν on its SU(2) doublet indices. In the MSUGRA scenario, we can trade these parameters for the neutrino and sneutrino mass matrices (m_ν and $m^2_{\tilde{\nu}}$), or more usefully, for m_ν, for the branching ratios of lepton flavour violating decays $\ell_j \to \ell_i\gamma$, and for the lightest right-handed neutrino mass $M_1 \lesssim T_{RH}$. We find a small area of parameter space where a large enough baryon asymmetry is generated for $T_{RH} \sim 10^9$ GeV. It corresponds to large off-diagonal elements in $m^2_{\tilde{\nu}}$, and therefore observable $\tau \to e\gamma$. For $T_{RH} \sim 10^{10}$ GeV, leptogenesis can also work for smaller off-diagonal elements in $m^2_{\tilde{\nu}}$.

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Note added

After this work was completed, related analyses [64] appeared.

[11]If there are additional sources of flavour violation in the slepton masses, (*e.g* non-universal soft masses) this does not work.

7 Appendix:the approximation and plots

In this Appendix, the lightest eigenvalue and corresponding eigenvector of $\mathcal{M}$ are estimated, using an approximation borrowed from diagonalising neutrino mass matrices in R-parity violating theories.

It is first convenient to scale some powers of the smallest Yukawa out of the hermitian matrix $\mathcal{M}^{-1\dagger}\mathcal{M}^{-1}$:

$$v_u^4 \mathcal{M}^{-1\dagger}\mathcal{M}^{-1} = D_Y^{-1} V_L [m_\nu]^\dagger V_L^T D_Y^{-2} V_L^* [m_\nu] V_L^\dagger D_Y^{-1} \equiv \frac{\Lambda}{y_1^4} \quad . \tag{43}$$

This can be written more compactly as

$$\frac{\Lambda}{y_1^4} = \mathbf{D_Y^{-1}} \Delta^\dagger \mathbf{D_Y}^{-2} \Delta \mathbf{D_Y^{-1}} \quad , \tag{44}$$

by defining

$$\Delta = V_L^* [m_\nu] V_L^\dagger = V_L^* U^* D_{m_\nu} U^\dagger V_L^\dagger \equiv W^* D_{m_\nu} W^\dagger \quad , \tag{45}$$

where the matrix $W = V_L U$ is the rotation from the basis where the ν_L masses are diagonal to the basis where the neutrino Yukawa matrix $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ is diagonal.

The matrix Λ can be written

$$[\Lambda]_{ij} = (\vec{\lambda}_i) \cdot (\vec{\lambda}_j^\dagger) = \sum_k (\lambda_i)_k (\lambda_j^*)_k \quad , \tag{46}$$

where

$$\vec{\lambda}_i \equiv \frac{y_1}{y_i} \begin{pmatrix} \Delta_{1i}^* \\ y_1 \Delta_{2i}^*/y_2 \\ y_1 \Delta_{3i}^*/y_3 \end{pmatrix} \quad . \tag{47}$$

If the hierarchy in the y_i is steeper than in the m_j, and/or that the angles in W are large, then

$$|\vec{\lambda}_1|^2 \gg |\vec{\lambda}_2|^2, |\vec{\lambda}_3|^2 \tag{48}$$

so the largest eigenvalue of Λ ($= v_u^4 y_1^4/|M_1|^2$) is

$$|M_1| \simeq \frac{y_1^2 v_u^2}{\sqrt{|\lambda_1|^2}} \simeq \frac{y_1^2 v_u^2}{|\Delta_{11}|} = \frac{y_1^2 v_u^2}{|W_{1j}^2 m_j|} \tag{49}$$

with associated eingevector (normalised $\vec{\lambda}_1$):

$$\hat{\lambda}_1 \simeq \begin{pmatrix} \Delta_{11}^* \\ y_1 \Delta_{21}^*/y_2 \\ y_1 \Delta_{31}^*/y_3 \end{pmatrix} \times \frac{1}{\Delta_{11}^*} \tag{50}$$

In figure 4, M_1 is plotted as a function of $\omega_{12} \simeq \log W_{12}$ and $\omega_{13} \simeq \log W_{13}$, for $y_1 = 10^{-4}$, and using the central values of m_i listed after eqn (6). The precise definition is

$$W_{12} = \cos\theta_{W13} \sin\theta_{W12}, \qquad W_{13} = \sin\theta_{W13}$$
$$with \quad \theta_{W1j} = 10^{\omega_{1j}} \pi/2 \quad . \tag{51}$$

This Appendix contains many three dimensional plots of functions that will enter into the equation for the baryon asymmetry. The aim of these figures is to give a qualitative impression; quantitatively clearer

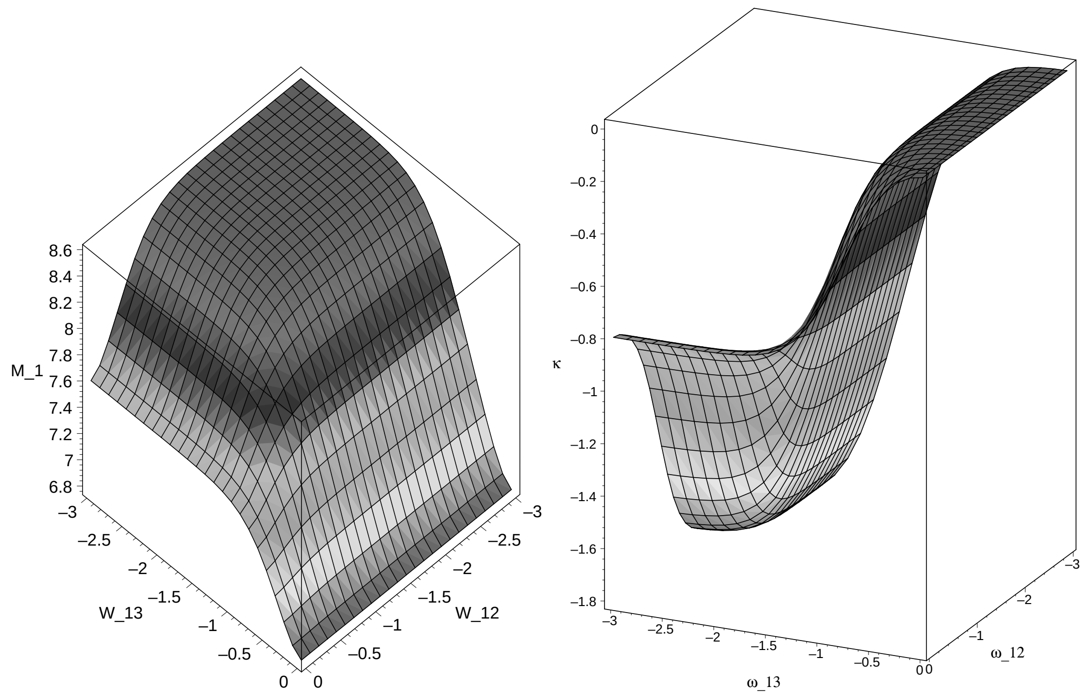

Figure 4: On the LHS, $\log[M_1/\mathrm{GeV}]$ as a function of $\omega_{12} \simeq log[W_{12}]$ and $\omega_{13} \simeq log[W_{13}]$. On the RHS, $\kappa = log_{10}(\tilde{m}_1/m_3)$. Recall we need $\kappa \lesssim -1.4$ to maximise the asymmetry. These plots are for central values of the neutrino masses: $y_1 = 10^{-4}, m_2 = 8.2 \times 10^{-3}$ eV, and $m_3 = 5.2 \times 10^{-2}$ eV.

contour plots of the baryon asymmetry are in the body of the paper. In figure 4, there are three limiting values for M_1, corresponding to $M_1 \simeq y_1^2 v_u^2/m_i$: $M_1 \to y_1^2 v_u^2/m_3$ when $W_{13} \to 1$, $M_1 \to y_1^2 v_u^2/m_1$ when $W_{13}, W_{12} \to 0$, and $M_1 \to y_1^2 v_u^2/m_2$ when $W_{13} < m_2/m_3$, $W_{12} \to 1$.

The ν_R decay rate can be evaluated with the eigenvector (50), which gives eqn (25). $\tilde{m}_1$ has three limits—m_1, m_2, m_3—depending ono the values of W_{1j}. The logarithm of $\tilde{m}_1/m_3$ is plotted on the RHS of figure 4. $\tilde{m}_1$ must be in the range given after eq. (16), which implies $\log{(\tilde{m}_1/m_3)} \lesssim -1.4$.

Finally, the CP asymmetry ϵ, eqn. (20), can be evaluated with the eigenvector (50) to obtain

$$\epsilon \simeq -\frac{3\Lambda_{11}^2}{8\pi[\Lambda D_Y^2 \Lambda]_{11}} \mathrm{Im}\left\{ \frac{[\Lambda D_Y \Delta^\dagger D_Y \Lambda^T]_{11}}{[\Lambda D_Y^{-1} \Delta^\dagger D_Y^{-1} \Lambda^T]_{11}} \right\} = \frac{3y_1^2}{8\pi \sum_j |W_{1j}|^2 m_{\nu_j}^2} \mathrm{Im}\left\{ \frac{\sum_k W_{1k}^2 m_{\nu_k}^3}{\sum_n W_{1n}^2 m_{\nu_n}} \right\} \quad , \tag{52}$$

where terms of order y_1/y_2 and y_1/y_3 have been dropped. $\delta_{HMY} \propto \epsilon_1/M_1$ is given in eqn (26), and plotted on the LHS of figure 5. ϵ_1 is plotted on the RHS; it peaks on the ridge of eqn (27) because this is where the larger values of M_1 and δ_{HMY} overlap.

The results in the remainder of the paper are based on the analytic approximations of this section. How reliable are these equations? The eqn (44) for $\Lambda = y_1^4[\mathcal{M}\mathcal{M}^\dagger]^{-1}$ is exact, but the formula we use for ϵ_1 assumes hierarchical M_i, so it is consistent to assume this in solving for the eigenvalues and eigenvectors of Λ. The approximation is that the first column (or row) of $y_1^2 D_Y^{-1} \cdot \Delta \cdot D_Y^{-1}$ is the lightest eigenvalue, multiplying its eigenvector. It breaks down if the elements of the second or third row/column become of order Δ_{11}, as one can see by writing the eigenvector in a basis rotated by a small angle from the eigenbasis. $y_1 \Delta_{12}/y_2, y_1\Delta_{13}/y_3 \simeq \Delta_{11}$ could occur if

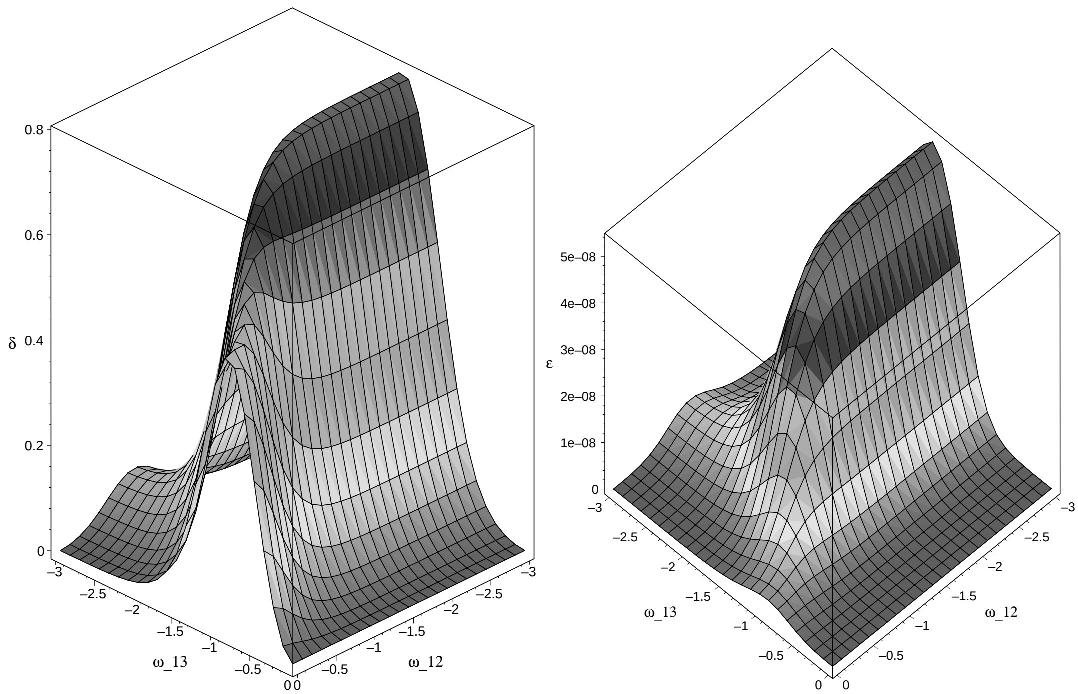

Figure 5: On the LHS (RHS), δ_{HMY} (ϵ) as a function of $\omega_{12} \simeq log[W_{12}]$ and $\omega_{13} \equiv log[W_{13}]$. On the RHS, M_1 is taken as a function of $y_1 = 10^{-4}$ and other inputs. Both plots are for central values of the neutrino masses.

1. the M_i were of similar magnitude, rather than hierarchical. This is "unlikely", because the hierarchy in D_Y is much steeper than in $[m_\nu]$.

2. m_1 too small—if $m_1/m_2, m_1/m_3 < y_2^2$, then the terms being kept are smaller than the neglected ones. This is discussed in an appendix.

8 Appendix: $m_1 \ll m_2/10$

In this paper, we assumed a hierarchical spectrum for the light neutrino masses: $\Delta m^2_{atm} = m_3^2$, $\Delta m^2_{sol} = m_2^2$, so the smallest neutrino mass m_1 is unlikely to be measured with anticipated data. However, it enters our formulae for the baryon asymmetry, as does the smallest Yukawa y_1. In the body of the paper, we fixed $m_1 = m_2/10$, and determined y_1 as a function of M_1 and our weak scale parameters, by requiring M_1 to be in the range allowed by leptogenesis. In this Appendix, we discuss the dependence of our results on m_1. For most values of W_{12} and W_{13}, m_1 is irrelevant because $|W_{11}|^2 m_1 \ll |W_{12}|^2 m_2, |W_{13}|^2 m_3$. However, m_1 cannot be dropped from our analytic expressions, for W close to the identity. This is the area of parameter space where η_B is maximal; the remainder of the Appendix is restricted to this area of parameter space. We are interested in how η_B scales with m_1, and in whether our analytic approximation is still valid.

The maximum value of ϵ, eqn (41), is independent of m_1, if M_1 is independent of m_1. We have fixed $M_1 \simeq T_{RH}$, which determines y_1^2 as a function of $W_{1n}^2 m_n \sim m_1$. So varying m_1 allows y_1 to vary:

$m_1 \sim m_2/100$ would allow leptogenesis to work for $y_1 \sim h_u$, which could be theoretically attractive.

As discussed after eqn (26), ϵ approaches its upper bound (equivalently $\delta_{HMY} \sim 1$) on the peak in figure 5, where $m_1^2/m_3^2 \sim W_{13}^2$ and $W_{12}^2 < m_1^2/m_2^2$. As m_1 decreases, the *area* in W_{12}, W_{13} space where $\delta_{HMY} \sim 1$ decreases, but the maximum value is unchanged. The numerical values of W_{12} and W_{13} where the maximum is reached will also decrease, making this parameter space increasingly "fine-tuned" (W very close to the identity is unlikely to be stable under renormalisation group running).

The ν_R decay rate must have values in the range given after eqn (16), to ensure η_B as large as possible. We first concern ourselves with the upper bound: $\tilde{m}_1 < 3 \times 10^{-3}$ eV. If $W_{1n}^2 m_n^2 \sim W_{13}^2 m_3^2$ and $W_{1j}^2 m_j \sim W_{11}^2 m_1$, as required to maximise δ_{HMY}, then from eqn (25), $\tilde{m}_1 \sim W_{13}^2 m_3^2/m_1$. To maximise $\eta_B \sim \delta_{HMY}/\tilde{m}_1$, requires $W_{13}^2 \simeq m_1^2/m_3^2$, so that the decay rate is slow enough, but δ_{HMY} is still $O(1)$. So as m_1 decreases from $m_2/10$ to $10^{-3} m_2$, the area of the peak on the RHS of figure 1 will shrink, but the height is unchanged.

For smaller values of m_1, the asymmetry will decrease. This is because $\tilde{m}_1$ is small, so ν_R production in the plasma is inefficient (see [11]).

It is straightforward to check that our analytic approximation holds, in the shrinking area of parameter space where $W_{13} \simeq m_1/m_3$ and $W_{12} < m_1/m_2$, provided that $y_1^2/m_1 \ll y_2^2/m_2$. This is the condition that $y_1^2 v_u^2/m_1$ is the lightest ν_R mass. So the analytic approximation fails as m_1 approaches $\frac{y_1^2}{y_2^2} m_2$.

Finally, the low energy prediction of the peak are independent of m_1, because they follow from requiring that $W \sim 1$. As m_1 decreases, W must approach the identity more and more closely, so V_L becomes more precisely $U^\dagger$. But whether $V_L \sim U^\dagger$, or $V_L = U^\dagger$, the expectation remains that $\tau \to \mu\gamma$ or $\tau \to e\gamma$ should be observable.

So in summary, the magnitude of the baryon asymmetry on the peak of figure 1 is independent of m_1 for $10^{-3} m_2 < m_1 < m_2/10$. As m_1 decreases, the location of the peak shifts to smaller W_{13}, and its area will shrink. We cannot say anything for $m_1 < 10^{-3} m_2$: our analytic formulae indicate that η_B will decrease, but the approximation they are based on is unreliable.

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