# Curvature force and dark energy

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**Abstract**

A curvature self-interaction of the cosmic gas is shown to mimic a cosmological constant or other forms of dark energy, such as a rolling tachyon condensate or a Chaplygin gas. Any given Hubble rate and deceleration parameter can be traced back to the action of an effective curvature force on the gas particles. This force self-consistently reacts back on the cosmological dynamics. The links between an imperfect fluid description, a kinetic description with effective antifriction forces, and curvature forces, which represent a non-minimal coupling of gravity to matter, are established.

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## 1 Introduction

An adequate description of the present Universe seems to require a cosmic substratum, which is characterized by a negative pressure [1]. In particular observations of supernovae at high redshift strongly suggest that the Universe is accelerating its expansion [2]. A possible explanation is the existence of a dominant component of dark energy, besides cold dark matter (pressureless). There exist a number of dark energy candidates, the best known being a cosmological constant and different quintessence scenarios ([3, 4, 5, 6]). Most of the latter

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rely on the dynamics of a minimally coupled scalar field. But also non-minimal "extended quintessence" models have been studied, which are characterized by an explicit coupling of the scalar field to the Ricci scalar [7]. This additional coupling results in a richer dynamical structure of the theory, which has been used to search for scaling and tracker field solutions [8]. A different type of non-minimal coupling is obtained from higher-order theories of gravity, which was shown to give rise to the concept of "curvature quintessence" [9]. Geometric terms in fourth-order gravity are interpreted as effective quantities within general relativity such as "curvature pressure" and "curvature density". Under certain conditions the curvature pressure may be sufficiently negative to generate a phase of accelerated expansion. This kind of modification of the gravitational action was previously used in connection with different problems, as for instance to avoid the initial singularity of homogeneous, isotropic universes [10].

A negative pressure may also be the consequence of self-interactions in gas models of the Universe [11, 12, 13, 14]. In particular, an "antifrictional" force, self-consistently exerted on the particles of the cosmic substratum, was shown to provide an alternative explanation for an accelerated expansion of the universe [12, 14]. This approach relies on the fact that the cosmological principle is compatible with the existence of a certain class of (hypothetical) microscopic one-particle forces, which manifest themselves as "source" terms in the macroscopic perfect-fluid balance equations. These sources can be mapped on an effective negative pressure of the cosmic medium. The energy-momentum tensor of the latter thus acquires an imperfect fluid structure. An advantage of this approach is the possibility to unify dark energy and dark matter, since just a single dark component has to be introduced to describe cosmological observations. However, a compelling microphysical explanation for antifrictional forces is still missing, as is the case for all other models.

The energy-momentum tensor of a non-minimally coupled scalar field has an imperfect fluid structure as well [15], which reduces to that of a perfect fluid in the limit of minimal coupling. This indicates that there might be a relation between imperfect fluid degrees of freedom and non-minimal coupling. Here we exploit the general idea of describing a non-minimal coupling within an imperfect fluid picture. We point out that effective antifrictional forces can be regarded as a specific non-minimal coupling of the cosmic gas to the Ricci scalar. Generally, a force that explicitly depends on curvature quantities describes a coupling of matter to the space-time curvature, which goes beyond Einstein's theory. However, mapping the non-minimal interaction on an imperfect fluid degree of freedom admits a self-consistent treatment on the basis of general relativity. This may be seen as a gas dynamical counterpart to the non-minimal couplings of scalar fields or those of higher-order gravity theories. We emphasize that the present coupling represents a new type of curvature self-interaction of the cosmic medium, which cannot be reduced to just a mapping of the described scalar field approaches to a fluid description. The starting points are quite different. The non-minimal scalar field approaches start with a given interaction term and then look for suitable solutions for the cosmological dynamics. It is not clear from the outset which coupling could provide a "successful" solution.

Here, we use an inverse strategy. We *design* a (non-minimal) fluid interaction such that it results in the desired cosmic evolution. Designing the coupling to obtain a specific dynamics has already been used for interacting two-component models [6]. We apply this idea to the case of a one-component fluid, which is self-consistently coupled to the Ricci scalar. As a characteristic feature of this approach, Hubble rate and deceleration parameter explicitly enter the microscopic dynamics, which gives rise to a self-consistent coupling of the latter to the gravitational field equations. We demonstrate this for a power-law behaviour of the scale factor, implying a specifically rolling tachyon field, for the ΛCDM model and for a (generalized) Chaplygin gas. All these cases may be understood as the result of specific curvature self-interactions in an otherwise pressureless gas.

A problem common to all unified models (dark energy and dark matter being the same component) is that a tiny, non-vanishing speed of sound can spoil the scenario by inducing acoustic oscillations of primordial, adiabatic fluctuations at late times [16, 17]. The fact that neither oscillations nor exponential instabilities are observed, at scales of galaxy clusters and below, puts severe constraints on the isentropic speed of sound in such models, i.e. $c_s^2 < 10^{-5}$ [16]. In fact, this seems to exclude any perfect fluid model, which does not mimic a ΛCDM model. As a consequence, the Chaplygin gas, say, cannot be considered a realistic model of the cosmic substratum. One should be aware, however, that the mentioned limits are derived under the assumption of an equation of state $P = P(\rho)$, where $P$ is the total pressure and $\rho$ is the energy density. Since a general (dissipative) fluid has to be described by an equation of state of the type $P = P(\rho, s)$, where $s$ is the (specific) entropy, it remains open whether or not these constraints apply in this more general case as well. The point is that dissipative processes in imperfect fluids give rise to entropy perturbations and we have $c_s^2 \neq \dot{P}/\dot{\rho}$. This implies that a simple relation between the perturbations of pressure and energy density, which was used to obtain the constraints in [16, 17], does not necessarily exist.

The paper is organized as follows. In section 2 we relate an effective, non-equilibrium type pressure to the Ricci scalar of a homogeneous and isotropic, spatially flat Universe. A gas dynamical motivation for this pressure as the result of a non-minimal curvature self-interaction is given in section 3. In section 4 the mentioned examples, a power-law behaviour, including a special case of a rolling tachyon, the ΛCDM model, and the Chaplygin gas are considered. A brief summary is given in section 5. Units are fixed by $c = k_B = h = 1$.

## 2 Field equations and viscous pressure

The field equations for a homogeneous, isotropic, and spatially flat Universe filled by an imperfect fluid are

$$3H^2 = 8\pi G\rho \ , \qquad \dot{H} = -4\pi G\left(\rho + P\right) \ , \qquad P = p + \Pi \ . \tag{1}$$

Here, $\rho$ is the energy density seen by a comoving observer. The fluid four-velocity $u^i$ is normalized by $u^i u_i = -1$. The Hubble rate is given by $H = \dot{a}/a$, where $a$ is the scale factor of the Robertson-Walker metric and a dot denotes a derivative with respect to cosmic time $t$. The pressure $P$ of the cosmic medium is assumed to be the sum of a kinetic part $p > 0$ [see Eq. (10) below] and an additional contribution $\Pi$. The derivation of the latter quantity from the type of self-interactions mentioned in the introduction is the main objective of the paper. We shall show that such a pressure appears as the result of an effective one-particle force $F^i$ of structure $mF^i = B\left(-Ep^i + m^2 u^i\right)$ [see Eq. (13) below], where $m$ is the mass of the gas particles, $p^i$ is their four momentum and $E \equiv -u_i p^i$ is the particle energy for a comoving (with the macroscopic four velocity) observer. The pressure $\Pi$ will directly be related to the force function $B$ [see Eqs. (15) and (16) below].

From the definition, $q \equiv -\ddot{a}/(aH^2)$, of the deceleration parameter, one has

$$q = -1 - \frac{\dot{H}}{H^2} \ . \tag{2}$$

For the special case of a constant $q > -1$, we find

$$H = \frac{1}{(1+q)\,t} \ , \quad a(t) \propto t^{\frac{1}{1+q}} \ , \tag{3}$$

and

$$H = H_0 \ , \qquad a(t) = a_0 e^{H_0 t} \ , \tag{4}$$

for $q = -1$. Often, an accelerated expansion of the Universe is traced back to a suitably designed scalar field potential. There exist other approaches, which imply a non-minimal coupling of a scalar field to the Ricci scalar [7, 9].

Here, we obtain an accelerating expansion of the Universe within a fluid picture, due to a sufficiently large negative effective non-equilibrium pressure $\Pi$. A conventional bulk viscous pressure of linear irreversible thermodynamics is inappropriate for this purpose, since it corresponds to a fluid configuration which is close to a fiducial equilibrium reference state such that the total pressure is positive. Non-standard self-interactions of the cosmic medium, however, have been considered as a potential mechanism to generate an accelerated expansion [11, 12, 13]. In this paper we demonstrate how these interactions can be obtained as the result of a non-minimal coupling of the underlying gas dynamics to the space-time curvature. For this purpose it is convenient to solve Eqs. (1) with (2) for $P/\rho$, which yields

$$\frac{P}{\rho} = \frac{1}{3}\left(-1 + 2q\right) \ . \tag{5}$$

In general, $q$ is time-dependent. The ratio $P/\rho$ may be related to the Ricci scalar, which in a homogeneous, isotropic, and spatially flat Universe is given by

$$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = 6\left(1 - q\right)H^2 \ . \tag{6}$$

Therefore we may also write

$$\frac{P}{\rho} = -\frac{1}{3}\left[\frac{R}{3H^2} - 1\right] \ . \tag{7}$$

The simplest way of obtaining the latter relation is to combine the trace of Einstein's equation, $-R = 8\pi GT = -8\pi G(\rho - 3P)$, with Friedmann's equation, $8\pi G\rho = 3H^2$. For the Einstein–de Sitter Universe one has $a \propto t^{2/3}$ and $R = 3H^2$, equivalent to $q = \frac{1}{2}$, i.e. $P = 0$. In the following we shall look for a mechanism that produces deviations from $R = 3H^2$, leading to a negative pressure. From now on we focus on non-relativistic matter and set $p = 0$, thus $P = \Pi$. This is motivated by observations of the large scale structure, which suggest that a non-relativistic equation of state ($p \ll \rho$) is required at the onset of structure formation and thereafter.

## 3 Kinetic theory and curvature self-interaction

Equation (7) shows that to have a non-vanishing dissipative pressure $\Pi$, a departure from $R = 3H^2$ is necessary. This comes about because $R = 3H^2$ characterizes a perfect fluid Universe with the equation of state for dust. Before focusing on such departures, we recall that from a gas dynamical point of view a perfect fluid consists of particles with mass $m$, which move on geodesics according to

$$m\frac{\mathrm{d}x^i}{\mathrm{d}\tau} = p^i \ , \qquad \frac{\mathrm{D}p^i}{\mathrm{d}\tau} = 0 \ . \tag{8}$$

The parameter $\tau$ denotes the proper time. This corresponds to a Boltzmann equation for the one-particle distribution function $f = f\left(x, p\right)$ (see, e.g. [18, 19, 20, 21]),

$$p^i f_{,i} - \Gamma^i_{kl} p^k p^l \frac{\partial f}{\partial p^i} = C\left[f\right] \ , \tag{9}$$

where $C[f]$ is Boltzmann's collision integral. The latter describes elastic binary collisions between the particles. The second moment of the distribution function provides us with the energy-momentum tensor, which, in a spatially homogeneous and isotropic Universe, has necessarily a perfect fluid structure, i.e.

$$T^{ik} = \int \mathrm{d}P p^i p^k f\left(x, p\right) = \rho u^i u^k + p h^{ik} \ , \tag{10}$$

where $h^{ik} = g^{ik} + u^i u^k$. The continuity equation $\dot{\rho} + 3H\left(\rho + p\right) = 0$ follows, with a pressure in the range $0 < p \leq \rho/3$. The special case of a dust universe is approached for $p \ll \rho$, which is obtained for $T \ll m$, where $T$ is the fluid equilibrium temperature ([18, 19, 20, 21]). In particular, the kinetic pressure is always non-negative.

Our strategy now is the following. Under the assumption that a gaseous fluid description makes sense, we attribute the accelerated expansion of the Universe to the existence of a non-vanishing dynamical pressure $P$ in Eq. (7). We ask for a suitable modification of the above perfect fluid description, which might give rise to a negative pressure of a substantial amount. A natural option for such a modification consists of additional interparticle interactions, not taken into account by Boltzmann's collision integral (e.g. inelastic interactions or many-particle effects). The currently unknown properties of dark energy (and dark matter as well) are then mapped onto non-standard interactions between the microscopic constituents of the fluid. That is, we shall look for those interactions that are able to reproduce the observed cosmological dynamics. This strategy resembles the more familiar scalar field approach according to which one tries to "explain" the dynamics of the Universe by designing a potential term in order to reproduce the given dynamics.

Following previous work [12], we introduce additional interactions, which cannot be reduced to elastic, binary collisions. There are specific interactions, which may be mapped onto a quantity $F^i$ such that the Boltzmann equation (9) is generalized to

$$p^i f_{,i} - \Gamma^i_{kl} p^k p^l \frac{\partial f}{\partial p^i} + m F^i \frac{\partial f}{\partial p^i} = C\left[f\right] \ . \tag{11}$$

The left-hand side of this equation can be regarded as

$$\frac{\mathrm{d}f\left(x,p\right)}{\mathrm{d}\tau} \equiv \frac{\partial f}{\partial x^{i}}\frac{\mathrm{d}x^{i}}{\mathrm{d}\tau} + \frac{\partial f}{\partial p^{i}}\frac{\mathrm{d}p^{i}}{\mathrm{d}\tau} \ ,$$

with

$$m\frac{\mathrm{d}x^{i}}{\mathrm{d}\tau} = p^{i} \ , \qquad \frac{\mathrm{D}p^{i}}{\mathrm{d}\tau} = F^{i} \ , \tag{12}$$

the equations of motion for gas particles moving under the influence of a force field $F^{i} = F^{i}\left(x,p\right)$. As a consequence, the particle motion is no longer geodesic. However, describing interactions in terms of a four-force raises the question of the extent to which such a procedure is consistent with the assumption of a spatially homogeneous and isotropic Universe.

To answer this question it is convenient to split the microscopic particle momentum according to $p^{i} = Eu^{i} + \lambda e^{i}$, where $u^{i}e_{i} = 0$ and $e^{i}e_{i} = 1$. Here, $E \equiv -p^{i}u_{i}$ is the particle energy as measured by an observer, comoving with the macroscopic four-velocity $u^{i}$. From $p^{i}p_{i} = -m^{2}$ we have $E^{2} = m^{2} + \lambda^{2}$. In general, the individual particles do not move with the mean velocity $u^{i}$. Apparently, homogeneous and isotropic models require a geodesic mean motion, but not necessarily a geodesic motion of the individual particles. To clarify the situation it is useful to introduce the *particle* velocity $u^{i}_{(p)}$, defined by $p^{i} \equiv mu^{i}_{(p)}$, which is not necessarily geodesic, and to contrast it with the velocity $u^{i}$ of the geodesic mean motion. The particle velocity is also normalized by $u^{i}_{(p)}u_{(p)i} = -1$.

In order to get an idea about the admissible forces, it seems suggestive to assume $F^{i}$ to be proportional to the difference $u^{i} - u^{i}_{(p)}$, i.e. to start with an ansatz $F^{i} \propto u^{i} - u^{i}_{(p)}$. On the other hand, the relation $F^{i}p_{i} = 0$ has to be satisfied. But the latter condition, together with the ansatz $F^{i} \propto u^{i} - u^{i}_{(p)}$, leads to $E = m$, the case that characterizes the mean motion with $u^{i}_{(p)} = u^{i}$, which is force-free. It follows that a non-vanishing force cannot simply be proportional to the difference between the macroscopic and the particle velocities. A more general ansatz is

$$\frac{F^{i}}{m} = Bu^{i} - Cu^{i}_{(p)} \ ,$$

where the quantities $B$ and $C$ are not constants but should depend on the particle and fluid quantities in such a way that $B = C$ only for $u^{i}_{(p)} = u^{i}$ in order to guarantee that the mean motion remains force-free. With this ansatz we obtain

$$F^{i}p_{i} = 0 \quad \Rightarrow \quad C = \frac{E}{m}B \ ,$$

which indeed provides us with $C = B$ for $E = m$, equivalent to $u^{i}_{(p)} = u^{i}$. For the force we find under such conditions

$$mF^{i} = B\left(-Ep^{i} + m^{2}u^{i}\right) = -Bu^{k}\left(g^{i}_{k}p^{m}p_{m} - p^{i}p_{k}\right) \ . \tag{13}$$

The expression in the parenthesis on the right-hand side of the second equation coincides with the projector orthogonal to the particle momentum. In the special case $p^{i} = mu^{i}$, we have $E = m$ and the force consistently vanishes. A force of the type (13) makes the individual particles move on non-geodesic trajectories, while the macroscopic mean motion remains geodesic. This force, which was used in [12], is compatible with the cosmological principle. Now we have to investigate whether, and under which circumstances, a deviation from the geodesic motion of the microscopic constituents due to a force (13) may result in an effective negative pressure of the cosmic medium. In the following we shall restrict ourselves to the case where $B$ does *not* depend on $E$.

An interaction term in the Boltzmann equation gives rise to "source" terms in the balances of the moments. In particular, from the balance for the second moment of $f$ we obtain

$$\dot{\rho} + 3H\left(\rho + p\right) = -3B\left(\rho + p\right) \ . \tag{14}$$

As before, $\rho$ and $p$ are defined by

$$\rho = u_i u_k \int \mathrm{d}P p^i p^k f\left(x,p\right) \quad \text{and} \quad p = \frac{1}{3} h_{ik} \int \mathrm{d}P p^i p^k f\left(x,p\right) \ ,$$

respectively. With the definition

$$\Pi H \equiv B\left(\rho + p\right) \ , \tag{15}$$

the energy balance (14) becomes

$$\dot{\rho} + 3H\left(\rho + p + \Pi\right) = 0 \ . \tag{16}$$

This proves that, macroscopically, the action of the force manifests itself as a dissipative pressure. The reinterpretation of the right-hand side of Eq. (14) in terms of an effective pressure is crucial for our approach. It maps the source in the energy balance, which is a consequence of the additional interaction, onto an imperfect fluid degree of freedom of a conserved energy-momentum tensor $T^{ik}_{\mathrm{eff}} = \rho u^i u^k + \left(p + \Pi\right) h^{ik}$. We emphasize that the quantity $\Pi$ does *not* coincide with the dissipative pressure of conventional, linear, irreversible fluid dynamics. The latter has its origin in Boltzmann's collision integral and may provide only small corrections in $p$. Here, it is the force (13), which, via the identification (15), generates an effective pressure of an entirely different kind. There is *no* restriction of the type $|\Pi| < p$, which is characteristic of conventional fluid dynamics.

In the following, we are interested in $\Pi \leq 0$, equivalent to $B \leq 0$. We assume the cosmic substratum to be non-relativistic matter ($p \ll \rho$). Then it follows from (15) that

$$\frac{P}{\rho} = \frac{B}{H} + \mathcal{O}\left(\frac{p}{\rho}\right) \ . \tag{17}$$

This may be regarded as the effective equation of state of the cosmic medium. The quantity $B$, which determines the strength of the force, is directly related to the effective fluid pressure. This opens the possibility to establish an explicit relation between the force function $B$, which quantifies the microscopic interaction and the cosmological parameters. Namely, comparing the result (17) from kinetic theory for particles in a force field with Eq. (5) [or (7)], which is a consequence of the field equations (1), we may simply read off the fraction $B/H$ which is equivalent to a given value of the deceleration parameter, namely

$$\frac{B}{H} = -\frac{1}{3}\left(1 - 2q\right) = -\frac{1}{3}\left[\frac{R}{3H^2} - 1\right] \ . \tag{18}$$

This relation is the key element of our approach. It relates the force function $B$ to the Hubble rate and to the deceleration parameter. Consequently, the effective one-particle force, which gives rise to a cosmological dynamics, characterized by a Hubble rate $H$ and a deceleration parameter $q$, is

$$\begin{aligned} mF^i &= -\frac{H}{3}(1-2q)\left[-Ep^i + m^2u^i\right] \\ &= -\frac{H}{3}\left[\frac{R}{3H^2} - 1\right]\left[-Ep^i + m^2u^i\right] \ . \end{aligned} \quad (19)$$

This quantity depends on the microscopic particle momenta but also on the Hubble rate $H$ and the deceleration parameter $q$. Through the expression (19), the microscopic particle motion, governed by Eq. (12), is self-consistently coupled to the cosmological dynamics. The parameters $H$ and $q$ enter the microscopic dynamics and determine the effective fluid pressure $\Pi$; in turn, via the field equations (1), $\Pi$ is coupled again to $H$ and $q$. According to Eqs. (5)–(7), the force is related to the Ricci scalar. It is proportional to the deviation from the flat dust Universe ($R = 3H^2$). This force describes an interaction of the individual particle with a space-time curvature, which is determined by the ensemble of particles itself, i.e. it represents a curvature self-interaction. All the properties of a force of the type (13) remain valid in this case. In particular, this self-interaction is compatible with the cosmological principle. For any given $H$ and $q$ we may construct a force field that produces the desired dynamics. The described procedure, which relies on identifying the quantities (5) [or (7)] and (17), couples the gas dynamics self-consistently to the Ricci scalar, more precisely to the quantity $R - 3H^2$. In a sense, this may be regarded as a gas-dynamical counterpart to corresponding couplings of a scalar field to $R$.

Curvature forces are generally not admitted in Einstein's theory since they represent a non-minimal coupling and violate the equivalence principle. Here, the mapping of the curvature interaction on an effective viscous pressure allows a treatment as an imperfect fluid within the framework of general relativity. We emphasize that our approach does not introduce new fundamental particles or fields and preserves Einstein gravity (the left-hand side of Einstein's equations). It remains open, however, whether the force (19) represents a physical reality or just a phenomenological fit to some other underlying microphysics. One might also think of an interpretation according to which averaging the inhomogeneous matter configuration (see, e.g., [22] for recent accounts) gives rise to a back-reaction on the homogeneous background dynamics, such that an epoch of accelerated expansion is induced by the process of structure formation [14]. A force of this type, being the result of an averaging procedure on cosmological scales, would hardly be detectable in accelerator experiments.

The force (19) may be split into components parallel and perpendicular to the comoving velocity:

$$mu_iF^i = B(E^2 - m^2) \ , \qquad me_iF^i = -BE\sqrt{E^2 - m^2} \ , \quad (20)$$

where

$$e^i \equiv \frac{1}{\sqrt{E^2 - m^2}}\left(p^i - Eu^i\right) \quad (21)$$

is the spatial direction of the particle momentum. In the non-relativistic limit, the spatial projection of the force becomes

$$e_i F^i \approx -Bmv \; . \tag{22}$$

For $q < 1/2$, the quantity $B = -\frac{1}{3}H(1-2q)$ plays the role of a negative friction coefficient. This allows us to interpret the previously discussed cosmic antifriction [12] as the result of a non-minimal coupling of the gas dynamics to the Ricci scalar, equivalent to a specific curvature self-interaction of the cosmic medium.

# 4 Curvature force and accelerated expansion

So far, we have established a link between the dynamical pressure $P \approx \Pi$ and the coefficient of antifriction $-B$, and we have shown that this antifriction can be interpreted as the result of a non-minimal coupling of matter to curvature. In order to study the dynamics for a model with given departure from the Einstein–de Sitter case, we have to integrate the equation

$$\frac{P}{\rho} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} \approx \frac{B}{H} \; , \tag{23}$$

which follows from (5) and (17). To solve this equation, an assumption on $B(H, \dot{H})$ is necessary. Alternatively, one might start from an assumption on the deceleration parameter $q$ and $B/H$ from Eq. (18). It is convenient to express the Hubble rate as a function of redshift $z = (a_0/a) - 1$. With

$$\dot{H} = -H' H (1+z) \; ,$$

where $H' \equiv \mathrm{d}H/\mathrm{d}z$, the resulting equation is

$$\frac{H'}{B+H} = \frac{3}{2(1+z)} \; . \tag{24}$$

## 4.1 Power-law expansion

The ansatz

$$B = \sigma \frac{\dot{H}}{H} - \nu H$$

with constant, non-negative parameters $\sigma$ and $\nu$ leads to

$$H(z) = H_0 (1+z)^{1+q}, \qquad q = \frac{1 - 3(\sigma + \nu)}{2 + 3\sigma} \; . \tag{25}$$

This is nothing but a power-law expansion, $a \propto t^n$, with $n = 1/(1+q)$. Since $q$ is the (constant) deceleration parameter, we consistently find that the exponent $n$ is larger than unity for any $-1 < q < 0$, equivalent to the conditions $3(\sigma + \nu) > 1$ and $\nu < 1$. This is the simplest case of a self-consistent solution of the cosmological dynamics with curvature self-interaction. Any power $1/(1+q)$ corresponds to a specific force function $B$ [cf. Eq. (18)], with some degeneracy $\sigma(\nu)$ for a given $q$.

### 4.2 The rolling tachyon

String-theory-inspired tachyon matter was introduced by Sen [23], and its cosmological consequences as an alternative to a minimally coupled scalar field were explored in [24, 25, 26, 27]. A rolling tachyon field $\varphi$ may lead to a power-law behaviour of the scale factor [25, 27] similar to a scalar field with exponential potential [28]. Tachyon matter, which is characterized by

$$\rho = \frac{V}{\sqrt{1-\dot{\varphi}^2}} \qquad \text{and} \qquad P = -V\sqrt{1-\dot{\varphi}^2} \ , \tag{26}$$

has recently received some attention as a possible candidate for dark matter and/or dark energy. Here we demonstrate how the corresponding dynamics may be related to our present approach. Assuming $\sigma = 0$, the relevant connection is established by

$$\nu = 1 - \dot{\varphi}^2 \ .$$

Since $\nu$ is assumed to be constant, $\dot{\varphi}$ has to be constant as well, which represents a special case of the tachyon dynamics, namely $\ddot{\varphi} = 0$ and $3HV\dot{\varphi} + \mathrm{d}V/\mathrm{d}\varphi = 0$, the quantity $V$ being the tachyon potential. The corresponding Hubble rate is determined by

$$H^2 = \frac{8\pi G}{3}\rho = H_0^2(1+z)^{3\dot{\varphi}^2} \ . \tag{27}$$

It follows that

$$\dot{\rho} = -3H\rho\dot{\varphi}^2 \ , \tag{28}$$

which implies

$$\frac{P}{\rho} = -(1-\dot{\varphi}^2) \ . \tag{29}$$

This is the general equation of state for tachyonic matter, here obtained for the special case $\varphi \propto t$ and $V \propto \varphi^{-2}$ [25, 27], which, according to (22), corresponds to a force field with spatial projection

$$e_i F^i \approx (1-\dot{\varphi}^2)Hmv \ . \tag{30}$$

As was pointed out in [29], the energy density $\rho$ and pressure $P$ of the tachyon field may be considered as the sum of two components according to

$$\rho = \rho_V + \rho_{\mathrm{DM}} \ , \qquad P = p_V + p_{\mathrm{DM}} \ , \tag{31}$$

where

$$\rho_{\mathrm{DM}} = \frac{V\dot{\varphi}^2}{\sqrt{1-\dot{\varphi}^2}} \ , \qquad p_{\mathrm{DM}} = 0 \ , \tag{32}$$

$$\rho_V = V\sqrt{1-\dot{\varphi}^2} \ , \qquad p_V = -\rho_V \ . \tag{33}$$

The first component behaves as a pressureless fluid, the second one has a negative pressure. The power $n$ (recall that $a \propto t^n$) is then related to the ratio $\rho_V/\rho_{\mathrm{DM}}$ by

$$n = \frac{1}{1+q} = \frac{2}{3}\frac{1}{1-\nu} = \frac{2}{3}\left(1 + \frac{\rho_V}{\rho_{\mathrm{DM}}}\right) \ . \tag{34}$$

It follows that this dynamics is realized for a force with

$$B = -\frac{\rho_V}{\rho}H \ .$$

## 4.3 The ΛCDM model

A constant deceleration parameter $q$, equivalent to a constant ratio $\Pi/\rho$, is not expected to provide a realistic description of the cosmological dynamics over a large range in redshift. Successful structure formation requires a period of decelerated, matter-dominated expansion of substantial length. Consistent with this requirement, the SNIa data suggest an onset of accelerated expansion at $z \sim 1$ [30]. (Notice, however, that there are models that allow structure formation also during acceleration [31]. In such a scenario the accelerated epoch could have started as early as $z \approx 5$ [32]). Therefore, a realistic model has to account for a transition from positive to negative values of the deceleration parameter. A simple choice which admits this kind of transition is $|B| \propto H^{-1}$, equivalent to the ansatz

$$\frac{B}{H} = -\frac{1}{\mu + 1}\frac{H_0^2}{H^2} \ , \tag{35}$$

where $\mu$ is a constant. Integration of Eq. (24) with the ansatz (35) yields

$$H = H_0\left[\Omega_{\rm CDM}(1+z)^3 + \Omega_\Lambda\right]^{1/2} \tag{36}$$

with $\Omega_{\rm CDM} = \mu/(\mu+1)$ and $\Omega_\Lambda = 1/(\mu+1)$. For $z \gg 1$ we have $H \propto (1+z)^{3/2}$, which is characteristic of a matter-dominated Universe. For the opposite case, $z \rightarrow -1$, the Hubble rate approaches the constant value $H \rightarrow H_0\Omega_\Lambda^{1/2}$. The Hubble rate (36) implies a transition from a matter-dominated Universe at $z \gg 1$ to a de Sitter universe as $z \rightarrow -1$. It reproduces the ΛCDM model. The observationally favoured value is $\Omega_\Lambda \approx 0.7$. One realizes by direct calculation that the Hubble rate (36) leads to

$$1 - 2q = 3\frac{\Omega_\Lambda}{\Omega_{\rm CDM}\left(1+z\right)^3 + \Omega_\Lambda} \ , \tag{37}$$

which is indeed consistent with the general relation (18). For large $z$, we have $q \rightarrow 1/2$, while $q \rightarrow -1$ for $z \rightarrow -1$. Consequently, the ΛCDM model is equivalent to a non-relativistic gas in which a curvature force of the type (22) is self-consistently exerted on the individual particles. Any ratio $\Omega_\Lambda/\Omega_{\rm CDM}$ can be traced back to a specific curvature self-interaction of the medium. The explicit expression for the antifriction coefficient $|B(z)| = (1-2q)H/3$ in Eq. (22) is

$$|B(z)| = H_0\frac{\Omega_\Lambda}{\left[\Omega_{\rm CDM}\left(1+z\right)^3 + \Omega_\Lambda\right]^{1/2}} \ . \tag{38}$$

For $z \gg 1$ we have $e_iF^i \rightarrow 0$, which corresponds to simple, non-interacting dust. For the opposite case $z \rightarrow -1$ the force projection approaches $e_iF^i \rightarrow H_0\Omega_\Lambda^{1/2}mv$, with the asymptotic Hubble rate [cf. Eq. (36) for $z \rightarrow -1$] as curvature antifriction constant. In other words, the interaction is gradually switched on during the cosmic expansion.

In our approach, the 'coincidence problem', i.e. the question, why $\Omega_\Lambda$ and $\Omega_{\rm CDM}$ happen to be of the same order today, is equivalent to the question: why is the cosmic force parameter $|B(z)|$ of the order of the Hubble rate just at the present epoch? There are tentative suggestions that the answer to this question might be related to the onset of the non-linear stage of the cosmic structure formation process [14].

### 4.4 The Chaplygin gas

A Chaplygin gas is defined by the equation of state (see [33, 34] and references therein) $P = -A/\rho$, where $A$ is a positive-definite constant. It can be generalized by putting an arbitrary power ($\alpha > 0$)

$$P = -\frac{A}{\rho^{\alpha}} \ . \tag{39}$$

This equation has the appealing feature of providing a negative pressure and at the same time a speed of sound that remains real and positive. It is reminiscent of certain cases of string-driven inflation—see Eq. (2.7) of [35] with $m < 0$. Support for this exotic fluid (with $\alpha = 1$) can also be found in higher dimensional theories [36]; likewise Bento *et al.* showed that Eq. (39) can be derived from a Lagrangian of the Born–Infeld type [37]. Integration of the continuity equation allows us to obtain the energy density

$$\rho = \left( A + \frac{D}{a^{3(1+\alpha)}} \right)^{1/(1+\alpha)} \ , \tag{40}$$

where $D$ is a constant. For large values of $a$ the energy density becomes a cosmological constant. For small values of $a$ it behaves like matter. This property has recently made the Chaplygin gas an interesting candidate for a one-component model of the cosmic substratum [33, 37, 38]. However, new observational constraints seem to restrict the parameter $\alpha$ to very small values ([16, 17]) for which the Chaplygin gas becomes indistinguishable from the $\Lambda$CDM model.

Let us now show that in our curvature force approach the Chaplygin gas can be obtained with an ansatz $B = -\beta H^{-2\alpha-1}$, where $\beta$ is a non-negative constant. From Eq. (17) and the Friedmann equation, we may immediately read off that for a choice $\beta = (8\pi G/3)^{(\alpha+1)}$ we have

$$\frac{P}{\rho} \approx \frac{B}{H} = -\frac{A}{\rho^{\alpha+1}} \tag{41}$$

and, consequently

$$B(z) = -\left(\frac{8\pi G}{3}\right)^{\alpha+1} \frac{A}{H^{2\alpha+1}} \ . \tag{42}$$

Thus the generalized Chaplygin gas is equivalent to a force field with spatial projection (according to Eq. (22))

$$e_i F^i \approx \left(\frac{8\pi G}{3}\right)^{\alpha+1} \frac{Amv}{H^{2\alpha+1}} \ . \tag{43}$$

We point out that the above relations for the Chaplygin gas (as well as those for the other examples given in this paper) rely on an assumption for the dependence $B = B(H)$ of the force function $B$ on the Hubble rate. Via Eq. (17) and the Friedmann equation, this is equivalent to an effective equation of state $P = P(\rho)$ (see the discussion in the introduction).

## 5 Conclusion

We have established a scheme that self-consistently relates the expansion behaviour of the Universe to curvature self-interactions in the cosmic gas. Assuming that a fluid picture is allowed for the description of the present Universe, we have constructed specific internal interactions, which may give rise to the observed cosmological evolution. The ΛCDM model may be regarded as the consequence of a non-relativistic particle motion, which is non-minimally coupled to the Ricci scalar. This corresponds to a curvature force, equivalent to a negative friction, characterized by Eqs. (22) and (38). Alternative dark energy candidates such as a rolling tachyon (here with $\phi \propto t$ and $V \propto \phi^{-2}$) or a Chaplygin gas have been traced back to curvature interactions in a similar manner. The corresponding negative friction coefficients are given by Eqs. (30) and (43), respectively. We conclude that actually any cosmological model with given $H(z)$ and $q(z)$ may be interpreted on the basis of a gas model with a self-consistent coupling to the space-time curvature. The presented approach rather than introducing new particles or fields introduces a new effective coupling of gravity to matter. The required non-minimal interaction can be incorporated into general relativity. Whether the corresponding force is a physical reality or the consequence of a back-reaction due to an averaging procedure, or whether it just provides a phenomenological fit to some other underlying microphysics, remains open at this stage. However, contrasting the CMB anisotropies with a perturbation analysis to be performed elsewhere, is already likely to constrain the admissible interactions.

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