

Constraints on Extra-Dimensional Theories from Virtual-Graviton Exchange

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Abstract

We study the effective interactions induced by loops of extra-dimensional gravitons and show the special rôle of a specific dimension-6 four-fermion operator, product of two flavour-universal axial currents. By introducing an ultraviolet cut-off, we compare the present constraints on low-scale quantum gravity from various processes involving real-graviton emission and virtual-graviton exchange. The LEP2 limits on dimension-6 four-fermion interactions give one of the strongest constraint on the theory, in particular excluding the case of *strongly-interacting* gravity at the weak scale.

1 Introduction

In this paper we consider scenarios in which the Standard Model fields are confined on a 3-dimensional brane, while gravity propagates in the full D -dimensional space, with δ flat and compactified extra spatial dimensions ($D = 4 + \delta$) [1]. Since the theory is non-renormalizable, it lacks much predictivity in the quantum domain, as a consequence of our ignorance of the short-distance regime. Nevertheless, at energies much smaller than M_D (the Planck mass of the D -dimensional theory) we can use an effective theory to correctly describe the interactions of the extra-dimensional gravitons with gauge and matter fields [2, 3, 4]. The effective theory, in particular, allows the calculation of the rate for soft-graviton emission in high-energy colliders. Also, at energies much larger than M_D , one can use a semi-classical approach to compute gravitational scattering with small momentum transfer [5] and estimate black-hole production [6].

Observables dominated by ultraviolet contributions are experimentally interesting but remain unpredictable, lacking a knowledge of the underlying theory at short distances. In particular this is the case for effects induced by virtual graviton exchange, both at tree and loop levels. A general, but not very restrictive, parametrization of all virtual effects can be done in terms of higher-dimensional effective operators. Many theoretical and experimental analyses have concentrated on the dimension-8 operator $T_{\mu\nu}T^{\mu\nu}$ induced by tree-level graviton exchange [2, 4, 7].

Here we want to describe a parametrization which, to a certain extent, allows a comparison among different observables induced by graviton quanta [8]. This requires the introduction of a new mass scale, besides the fundamental parameters M_D and δ , which we will call Λ and which represents the validity cut-off scale of the Einstein gravitational action. As an analogy, consider the case of the Standard Model, in which the complete theory is known and perturbative. If we compare radiative corrections computed in the full theory with the results obtained by cutting off power-divergent loops in the Fermi theory, we find that the appropriate cut off is given by the smaller scale $\Lambda \sim M_W \approx gG_F^{-1/2}$ rather than by the larger Fermi scale $\Lambda \approx G_F^{-1/2}$. Similarly, we will estimate graviton-loop effects by introducing an explicit ultraviolet cutoff Λ , that can be different from $G_D^{1/(2-D)}$, where G_D is the D -dimensional Newton constant. We expect that Λ represents the mass of the new states introduced by the short-distance theory to cure the ultraviolet behavior of gravity. However, for

our purposes, Λ is just a phenomenological parameter. From the point of view of the effective theory, Λ/M_D parametrizes how strongly (or weakly) coupled quantum gravity is, and therefore controls the unknown relative importance of tree-level versus loop graviton effects.

We should immediately emphasize that the cut-off procedure is arbitrary and therefore the comparison between different observables has, at best, a semi-quantitative meaning. We will discuss how the “geometrical” factors in front of the divergent loop integrals can be estimated with the language of naïve dimensional analysis [9] and we will compare them with the results obtained with specific regulators. Of course, order-one coefficients are unpredictable, and therefore our results are only to be viewed as “reasonable estimates”.

Nevertheless, some interesting results can be obtained. First of all, even in a weakly-coupled theory, graviton loops can be more important than tree-level graviton exchange. This is because tree-level effects generate a dimension-8 operator, while loop effects can generate operators of lower dimension. Secondly, using the symmetry properties of gravity, we will show that graviton loops can generate only a single dimension-6 operator involving fermions, here called Υ , which is the product of two flavour-universal axial-vector currents, see eq. (17). This operator, in conjunction with graviton emission and the $T_{\mu\nu}T^{\mu\nu}$ operator, should be the object of analysis in complete experimental studies of graviton effects at colliders. Indeed, we will show that the LEP2 limit on Υ at present provides one of the strongest constraints on extra-dimensional gravity, in particular excluding the case of a fully strongly-interacting theory within the range of near-future colliders. Finally, graviton loops can also generate some other dimension-6 operators relevant only for Higgs physics.

This paper is organized as follows. In sect. 2 we discuss different approaches to estimate divergent loops. Then, in sects. 3 to 6 we compute the effects from graviton emission, graviton exchange at tree and loop level, and loops with both graviton and gauge bosons. The different constraints are compared in sect. 7 and our conclusions are presented in sect. 8.

2 Dealing with divergences

We start by discussing the procedure we follow to estimate divergent loop coefficients. Naïve Dimensional Analysis (NDA) [9] represents a successful way to estimate the geometrical factors multiplying the coefficients of effective operators generated by strongly-interacting dynamics. The rules of NDA dictate that, working in units of the strongly-interacting scale Λ_S , the value of the dimensionless coupling g of the interaction (at the scale Λ_S) is such that any loop order gives an equal contribution. Each additional loop carries a factor $g^2/(\ell_4\ell_\delta)$, where the D -dimensional loop factor is defined as*

$$\ell_D = (4\pi)^{D/2}\Gamma(D/2). \quad (1)$$

Notice that, for simplicity, we have chosen to factorize the D -dimensional loop integration in the usual 4-dimensional coordinates (with $\ell_4 = 16\pi^2$) and in the δ extra coordinates (where the integration actually corresponds to the summation over the Kaluza-Klein modes of the compactified space). Then the value of g^2 that corresponds to a strongly-coupled interaction is given by

$$g^2 = g_\star^2 \equiv \ell_4\ell_\delta. \quad (2)$$

The coupling of the D -dimensional graviton in the effective theory can be written as

$$g^2 = c_\delta(E/M_D)^{2+\delta}, \quad c_\delta \equiv (2\pi)^\delta, \quad (3)$$

where E is the typical energy and c_δ is the coefficient relating the D -dimensional Planck mass to the reduced Planck mass, given here following the notations of ref. [2]. Therefore the energy scale where gravity becomes strong, as defined by NDA, is

$$\Lambda_S = \left(\frac{\ell_4\ell_\delta}{c_\delta}\right)^{\frac{1}{2+\delta}} M_D = \left[16\pi^{2-\frac{\delta}{2}} \Gamma\left(\frac{\delta}{2}\right)\right]^{\frac{1}{2+\delta}} M_D. \quad (4)$$

*Starting from the equation

$$\int \frac{d^D k}{(2\pi)^D} f(k^2) = \frac{S_{D-1}}{2(2\pi)^D} \int dk^2 (k^2)^{\frac{D}{2}-1} f(k^2),$$

where $S_{D-1} = 2\pi^{D/2}/\Gamma(D/2)$ is the surface of the unit-radius sphere in D dimensions, we define as $1/\ell_D$ the coefficient of the integral in the right-hand side. We find $\ell_D = 2^D \pi^{D/2} (\frac{D}{2} - 1)!$ for D even, and $\ell_D = 2^D \pi^{(D+1)/2} [\prod_{k=0}^{(D-3)/2} (k + 1/2)]$ for D odd.

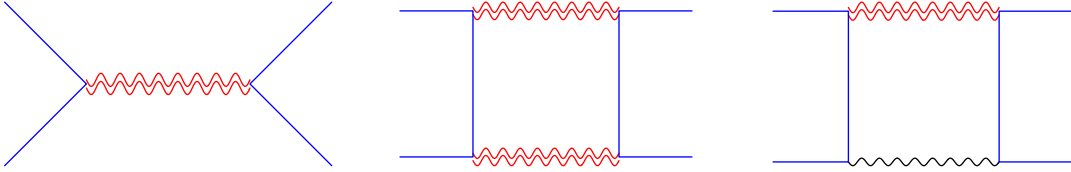


Figure 1: *Fig. 1a: tree-level graviton exchange generating the dimension-8 operator τ . Fig. 1b: one-loop graviton exchange generating the dimension-6 operator Υ . Fig. 1c: exchange of gravitons and vector bosons at one loop generating dimension-6 operators that affect electroweak precision data.*

As mentioned before, we are interested in the possibility that the ultraviolet behavior of gravity is modified before the theory becomes strongly interacting. In this case, the NDA estimates can still be used by taking $g^2 = \epsilon g_*^2$, where ϵ is a small parameter that measures the weakness of the couplings [10]. If we parametrize this weakness by introducing an ultraviolet cut off Λ , we can take $\epsilon = (\Lambda/\Lambda_S)^{2+\delta}$. Therefore, the coefficient of any desired operator is estimated by multiplying the appropriate power of the graviton coupling $(c_\delta/M_D^{2+\delta})^{1/2}$ times the appropriate 4-dimensional and extra dimensional loop factors ℓ_4 and ℓ_δ , times the appropriate power of Λ (and not Λ_S) needed to match the correct dimensions of the effective operator. Concrete examples are discussed in the next sections, where we will use this “modified NDA” to estimate the effects of virtual graviton exchange and to make a “reasonable” comparison between virtual and real graviton effects.

In the following we will also compare the results from “modified NDA” with the results obtained from explicit perturbative calculations with ultraviolet regulators. One possibility we consider is to cut-off each divergent integral at a common scale Λ . This procedure should somehow reproduce the appearance of the new states with mass Λ that tame the ultraviolet behavior of the theory. Another example we study is to multiply each divergent integrand by a factor $\exp(-\Lambda^2/k^2)$, and integrate over the full range of internal momenta k . This could mimic the exponential suppression at a string scale of order Λ . Of course these procedures, which are not unambiguously defined and in particular break gauge invariance, can only be viewed as tools to obtain estimates and order-unity factors in the results cannot be trusted. Moreover, the cutoff parameter is not necessarily universal, i.e. contributions to different processes might be cut off by different values of Λ .

Another procedure we will follow is to assume that the graviton exchange (at tree- or loop-level) mediates interactions between two currents located at different points along the extra-dimensional coordinates. In such a case, the coefficient of any contact operator involving different particles separated by a distance r is finite and computable. This is because quantum gravity, despite being non-renormalizable, is a local theory, and non-local operators cannot receive ultraviolet-divergent corrections. The divergence is recovered by letting $r \rightarrow 0$; therefore we can use the splitting as a physical regulator by identifying r^{-1} with the cut-off Λ . This procedure is well-defined and gauge-invariant, and it corresponds to the assumption that different matter fields are located on different branes in the extra-dimensional space. This hypothesis was proposed in ref. [11] in order to alleviate the problems of baryon-number and flavour-symmetry violations and to understand the fermion-mass pattern. However, this procedure does not regularize graviton contributions to multiple self-interactions of the same matter field, *e.g.* like effective operators mediating Bhabha scattering.

3 Graviton emission

Graviton emission in particle scattering can be computed using the infrared properties of gravity. Therefore its theoretical prediction is independent of Λ , as long as the relevant energy of the process is smaller than Λ . Checking the validity of this assumption, necessary for employing the effective-theory result, is non-trivial in the case of hadron colliders, where parton scattering can occur at very different center-of-mass energies. Applying the method suggested in ref. [2], one can define a minimum value of M_D , for a given δ , below which the effective theory cannot be trusted. For instance, the window in M_D values where graviton emission is observable at the LHC and is reliably estimated by the theory disappears for $\delta \geq 5$. Graviton emission rates in LHC collisions are suppressed if Λ is smaller than the LHC center of mass energy.

Graviton emission has been studied in the processes $e^+e^- \rightarrow \gamma \cancel{E}$ and $e^+e^- \rightarrow Z \cancel{E}$ at LEP, and $p\bar{p} \rightarrow \text{jet} + \cancel{E}_T$ and $p\bar{p} \rightarrow \gamma + \cancel{E}_T$ at the Tevatron. The present limits at 95% CL are summarized in table 1[†], together with the

[†]We do not include a CDF analysis [12] on single-photon events, since the obtained limits on M_D grow with δ , signalling that the relevant kinematics are probably outside the validity range of the effective theory.

Experiment	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta = 6$
ALEPH [13]	1.26	0.95	0.77	0.65	0.57
DELPHI [14]	1.36	1.05	0.84	0.69	0.59
L3 [15]	1.02	0.81	0.67	0.58	0.51
OPAL [16]	1.09	0.86	0.71	0.61	0.53
DØ [17]	0.89	0.73	0.68	0.64	0.63
combined	1.45	1.09	0.87	0.72	0.65

Table 1: 95% CL limits on the D -dimensional Planck mass M_D (in TeV), for some values of the number of extra dimensions δ , from graviton-emission processes in different experiments.

combined bound. We have combined the bounds from the different experiments assuming that, whenever the experiment does not report the χ^2 dependence, the best-fit value occurs at $M_D \rightarrow \infty$. Therefore, our procedure is only approximate. At any rate, the effect of the combination gives only a moderate improvement of the strongest single bound.

4 Virtual graviton exchange at tree level

Tree-level exchange of gravitons generates the effective dimension-8 operator τ [2, 4, 7] (see fig. 1a)

$$\mathcal{L}_{\text{int}} = c_\tau \tau, \quad \tau = \frac{1}{2} \left(T_{\mu\nu} T^{\mu\nu} - \frac{T_\mu^\mu T_\nu^\nu}{\delta + 2} \right), \quad (5)$$

where $T_{\mu\nu}$ is the energy-momentum tensor. Its coefficient c_τ is divergent because, although we are dealing with a tree-level contribution, we have to sum over all possible intermediate graviton configurations that conserve 3-dimensional momentum and energy, but not necessarily the extra-dimensional momentum. Therefore c_τ depends on the cutoff Λ , and it cannot be computed from the low-energy effective theory. We can estimate it by using the rules of “modified NDA” presented before:

$$c_\tau^{\text{NDA}} = \frac{c_\delta \Lambda_{\text{NDA}}^{\delta-2}}{\ell_\delta M_D^{\delta+2}} = \frac{\pi^{\delta/2}}{\Gamma(\delta/2)} \frac{\Lambda_{\text{NDA}}^{\delta-2}}{M_D^{\delta+2}}. \quad (6)$$

We have introduced here a subscript in the definition of the cut-off Λ_{NDA} to distinguish it from the cut-off parameter used in the different regularization procedures that we will now study.

An explicit calculation of c_τ gives

$$c_\tau = \frac{c_\delta}{M_D^{\delta+2}} G(k \rightarrow 0, r = 0) \quad (7)$$

where $G(k, r)$ is the scalar propagator with momentum k between two points separated by a distance $r \equiv |\vec{r}|$ in the extra dimensions:

$$G(k, r) = \frac{1}{V} \sum_{\vec{n}} \frac{e^{i\vec{n}\cdot\vec{r}/R}}{k^2 + |\vec{n}|^2/R^2}. \quad (8)$$

Here V is the volume of the extra dimensions. For simplicity we assume a toroidal compactification, so that $V = (2\pi R)^\delta$ where R is the radius. For sufficiently large extra dimensions, we can convert the sum into an integral over a continuous Kaluza-Klein mass distribution (with $m = |\vec{n}|/R$) and over the angle θ formed by the two vectors \vec{n} and \vec{r} :

$$G(k, r) = \frac{S_{\delta-2}}{c_\delta} \int_0^\infty dm m^{\delta-1} \int_0^\pi d\theta \sin^{\delta-2} \theta \frac{e^{imr \cos \theta}}{k^2 + m^2}. \quad (9)$$

For $r \neq 0$, we can perform the integrals in eq. (9) and find

$$G(k, r) = \frac{1}{(2\pi)^{\delta/2}} \left(\frac{k}{r} \right)^{\frac{\delta}{2}-1} K_{\frac{\delta}{2}-1}(kr) \stackrel{kr \rightarrow 0}{\simeq} \frac{\Gamma\left(\frac{\delta}{2} - 1\right) r^{2-\delta}}{4\pi^{\delta/2}}, \quad (10)$$

where K_ν is a modified Bessel function.

If we take $r = 0$, we obtain

$$G(k, 0) = \frac{S_{\delta-1}}{2c_\delta} \int_0^\infty dm^2 \frac{(m^2)^{\frac{\delta}{2}-1}}{k^2 + m^2}. \quad (11)$$

For $\delta \geq 2$, the integral is divergent and must be regularized by some *ad hoc* procedure. We can introduce an unknown function $f(m^2/\Lambda^2)$ that acts as an ultraviolet regulator at the scale Λ . From eq. (11) we find

$$c_\tau = c_\tau^{\text{NDA}} \left(\frac{\Lambda}{\Lambda_{\text{NDA}}} \right)^{\delta-2} \int_0^\infty dx f(x) x^{\frac{\delta}{2}-2}, \quad x \equiv m^2/\Lambda^2. \quad (12)$$

a) If we choose $f(x) = e^{-x}$, as proposed in sect. 2, we obtain

$$\frac{c_\tau}{c_\tau^{\text{NDA}}} \left(\frac{\Lambda_{\text{NDA}}}{\Lambda} \right)^{\delta-2} = \frac{2\Gamma(\delta/2)}{\delta-2}. \quad (13)$$

The right-hand side of eq. (13) varies between $\sqrt{\pi}$ and 1, for δ between 3 and 6.

b) If we assume a sharp cut-off at the scale Λ and take $f(x) = 1$ for $x < 1$ and $f(x) = 0$ otherwise, we obtain

$$\frac{c_\tau}{c_\tau^{\text{NDA}}} \left(\frac{\Lambda_{\text{NDA}}}{\Lambda} \right)^{\delta-2} = \frac{2}{\delta-2}. \quad (14)$$

Equation (14) varies between 2 and 1/2, for δ between 3 and 6. Therefore, for both choices of regulators a) and b), the results for c_τ (with $\delta \leq 6$) are in fair agreement with the estimates using “modified NDA”, for equal values of Λ and Λ_{NDA} .

c) Finally, we assume that different matter particles interacting with the gravitons are localized at different points in the extra dimensions. Using eq. (10) and identifying $r = 1/\Lambda \ll R$, we find

$$\frac{c_\tau}{c_\tau^{\text{NDA}}} \left(\frac{\Lambda_{\text{NDA}}}{\Lambda} \right)^{\delta-2} = \frac{2^{\delta-1}\Gamma^2(\delta/2)}{\delta-2}. \quad (15)$$

Here, with an abuse of notation, we have denoted by c_τ the terms in the operator τ involving splitted particles. The right-hand side of eq. (15) grows fast with δ and it is already equal to 32 for $\delta = 6$.

In the case $\delta = 2$, there is an infrared logarithmic divergence and, for all kinds of regulators considered here, c_τ becomes

$$c_\tau = \frac{\pi}{M_D^4} \ln \frac{\Lambda^2}{E^2}, \quad (16)$$

where E is the typical energy exchanged in the process.

The operator in eq. (5) does not affect precision observables at the Z -resonance, but it gives anomalous contributions to many ordinary particle processes. At LEP, the most sensitive channels are Bhabha scattering and diphoton productions, but experiments have also set limits on c_τ from dilepton, dijet, $b\bar{b}$, WW , and ZZ production. Experiments at the Tevatron have set constraints on c_τ from studying dielectron and diphoton final states. Experiments at HERA have also provided constraints from $e^\pm p \rightarrow e^\pm p$. Present bounds on c_τ are summarized in table 2[‡]. We have combined different experiments following the same approximation used in the previous section.

5 Graviton loops

Loops with exchange of virtual gravitons, like those illustrated in fig. 1b, can be important because generate operators with dimensionality lower than the operator in eq. (5), arising from tree-level graviton exchange. If we consider only fermions or gauge bosons, a single operator with dimension less than 8 can be generated, and it is given by

$$\mathcal{L}_{\text{int}} = c_\Upsilon \Upsilon, \quad \Upsilon = \frac{1}{2} \left(\sum_{f=q,\ell} \bar{f} \gamma_\mu \gamma_5 f \right) \left(\sum_{f=q,\ell} \bar{f} \gamma^\mu \gamma_5 f \right). \quad (17)$$

[‡]In the notations of refs. [2] and [7], we find $c_\tau = 4\pi/\Lambda_T^4 = 8/M_S^4$. We do not use the strong limit from the ZZ channel preliminarily reported by L3, since this result has not been confirmed by the full analysis.

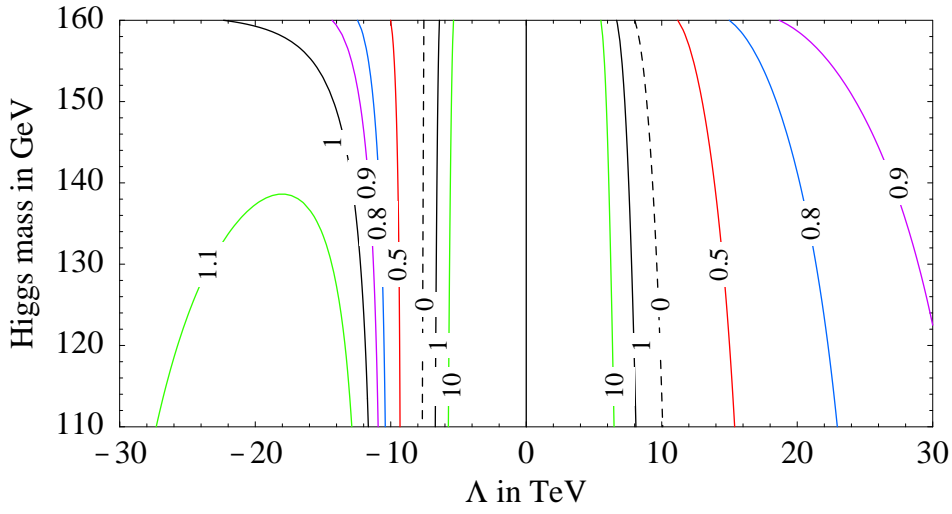


Figure 2: $\sigma(gg \rightarrow h) \times \text{BR}(h \rightarrow \gamma\gamma)$ in units of its SM value, in presence of the new operators $-(\pi/\Lambda^2)H^\dagger H F_{\mu\nu} F^{\mu\nu}$ and $-(\pi/\Lambda^2)H^\dagger H G_{\mu\nu}^a G^{a\mu\nu}$.

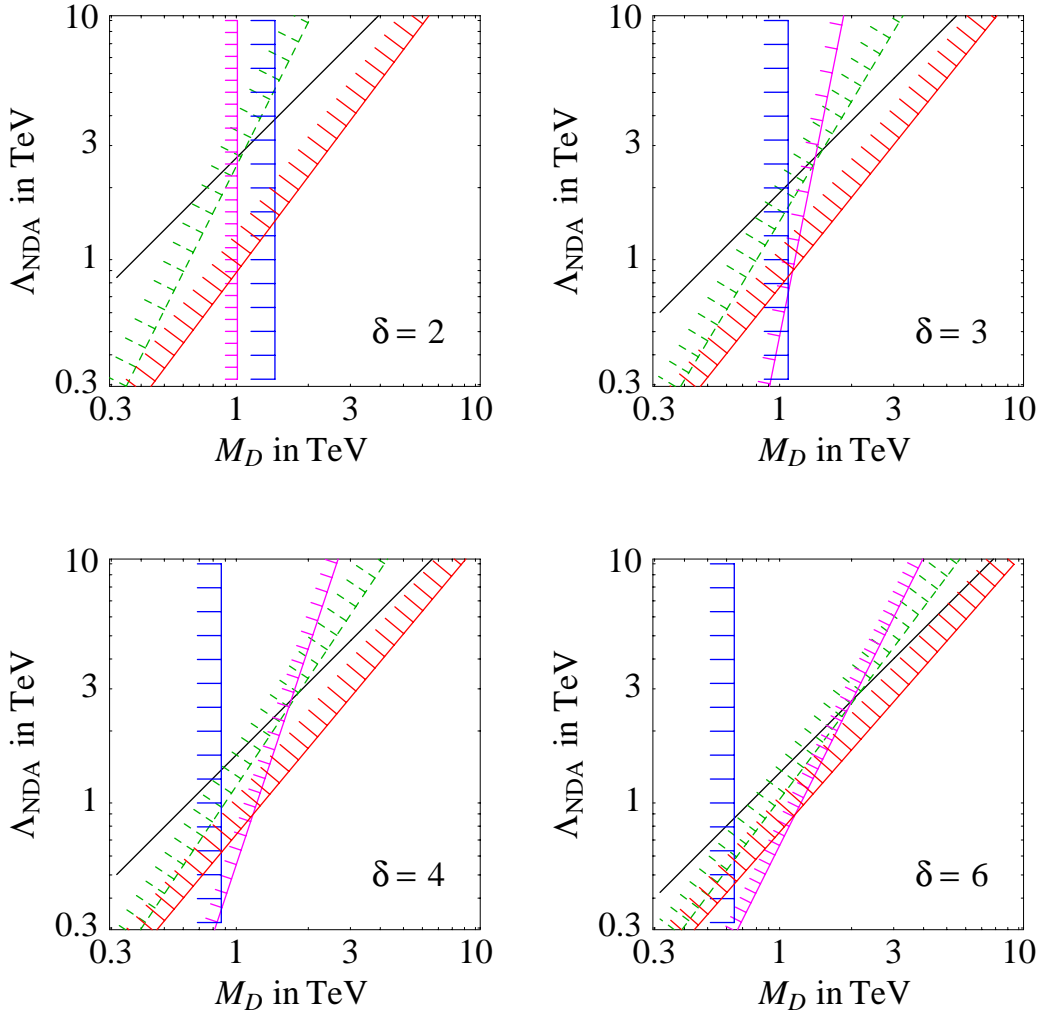


Figure 3: 95% CL collider bounds on graviton phenomenology in the plane $(M_D, \Lambda_{\text{NDA}})$ for $\delta = 2, 3, 4, 6$ flat extra dimensions. The solid black line shows the value of the cut-off Λ_{NDA} corresponding to a strongly-interacting gravitational theory, as defined from NDA, see eq. (4). The other lines mark the regions excluded by the bounds from graviton emission (vertical blue line), tree-level virtual graviton exchange (purple line with short borderlines), graviton loops (red solid line), graviton and gauge boson loops (green dashed line).

Experiment	Process	+	-
LEP combined [18]	$\gamma\gamma$	0.93	1.01
LEP combined [19]	e^+e^-	1.18	1.17
H1 [20]	e^+p and e^-p	0.74	0.71
ZEUS [21]	e^+p and e^-p	0.72	0.73
DØ [22]	e^+e^- and $\gamma\gamma$	1.08	1.01
CDF [23]	e^+e^- and $\gamma\gamma$	0.86	0.84
combined		1.28	1.25

Table 2: *Dimension-8 operator τ : 95% CL limits on $|8/c_\tau|^{1/4}$ (in TeV) for positive and negative values of c_τ , from different experiments.*

Here the sum extends over all quarks and leptons in the theory. It is easy to realize that graviton loops cannot produce any other operator of dimensions 6 or 7, involving only fermions and gauge bosons. Since the intermediate state involves virtual gravitons, any induced operator should be the product of two currents, which are singlets under all gauge and global symmetry groups, and are even under charge conjugation.

The operator Υ in eq. (17) is already present in the tree-level gravitational Lagrangian, if one employs a formalism in which the connection is not symmetric so that there is a torsion coupled to the spin density of matter. Within a given formalism its coefficient is not computable in the context of extra dimensions with matter localized on a infinitesimally thin brane [24]. In the case of supergravity with minimal Kähler potential, Υ is not present in the tree-level Lagrangian. In the context of extra dimensions, Υ was previously discussed in ref. [24]. Here we make the conservative assumption that the coefficient of the operator Υ vanishes at tree level and we estimate its contribution from quantum loops of gravitons.

The coefficient c_Υ is ultraviolet divergent and, using “modified NDA”, we estimate

$$c_\Upsilon^{\text{NDA}} = \frac{c_\delta^2}{\ell_4 \ell_\delta^2} \frac{\Lambda_{\text{NDA}}^{2+2\delta}}{M_D^{4+2\delta}} = \frac{\pi^{\delta-2}}{16\Gamma^2(\delta/2)} \frac{\Lambda_{\text{NDA}}^{2+2\delta}}{M_D^{4+2\delta}}. \quad (18)$$

An explicit calculation (outlined in the Appendix) gives

$$c_\Upsilon = \frac{15}{64} \frac{c_\delta^2}{\ell_4 M_D^{4+2\delta}} \int_0^\infty dk^2 k^4 G^2(k, r). \quad (19)$$

Taking $r = 0$ and introducing an explicit cut-off function f we get

$$\frac{c_\Upsilon}{c_\Upsilon^{\text{NDA}}} \left(\frac{\Lambda_{\text{NDA}}}{\Lambda} \right)^{2\delta+2} = \frac{15}{64} \int_0^\infty dx f(x) \int_0^\infty dy f(y) \int_0^\infty dz f(z) \frac{z^2(xy)^{\frac{\delta}{2}-1}}{(x+z)(y+z)}, \quad x \equiv \frac{m_1^2}{\Lambda^2}, \quad y \equiv \frac{m_2^2}{\Lambda^2}, \quad z \equiv \frac{k^2}{\Lambda^2}. \quad (20)$$

The three integrations correspond to the summations over the Kaluza-Klein modes of the two graviton propagators (with masses m_1 and m_2 , respectively) and to the 4-dimensional loop with internal momentum k .

- Choosing $f(x) = e^{-x}$, we find that $c_\Upsilon = c_\Upsilon^{\text{NDA}}$ for the values of $\Lambda/\Lambda_{\text{NDA}}$ listed in the first row of table 4.
- For the case of the sharp cut-off, $c_\Upsilon = c_\Upsilon^{\text{NDA}}$ for the values of $\Lambda/\Lambda_{\text{NDA}}$ listed in the second row of table 4.
- Finally, the coefficient of the single terms in Υ that involve fermions separated by a distance $r = 1/\Lambda \ll R$ is (with the usual abuse of notation)

$$\frac{c_\Upsilon}{c_\Upsilon^{\text{NDA}}} \left(\frac{\Lambda_{\text{NDA}}}{\Lambda} \right)^{2\delta+2} = \frac{15\delta^2(\delta+2)}{(\delta+1)(\delta+3)} 2^{2\delta-7} \Gamma^4(\delta/2). \quad (21)$$

This gives $c_\Upsilon = c_\Upsilon^{\text{NDA}}$ for the values of $\Lambda/\Lambda_{\text{NDA}}$ listed in the third row of table 4.

The axial-vector interaction in eq. (17) does not affect atomic parity violation nor μ decay and it is not constrained by high-precision LEP1 and SLD data. The most stringent limits on the coefficient of the operator Υ come from study of contact interactions at LEP, from dijet and Drell-Yan production at the Tevatron, from ep scattering at HERA, from neutrino-nucleon scattering. The present bounds are summarized in table 3[§].

[§]The terms in Υ and τ involving different fermions could be suppressed if the wave functions of fermions were significantly splitted in the extra dimensions. In table 3 we have separately shown the limit from Bhabha scattering to stress that a strong bound exists also for operators involving only electron fields, which would not be suppressed and are not regularized by the “splitting” cut-off. The data in the first two lines of table 3 are discarded from our combination.

Experiment	Process	+	-
LEP combined [25]	e^+e^-	11.3	11.5
LEP combined [25]	$\mu^+\mu^-$	16.4	12.7
LEP combined [25]	$\ell^+\ell^-$	17.2	15.1
LEP combined [25]	$b\bar{b}$	15.3	11.5
H1 [20]	e^+p and e^-p	2.5	3.9
ZEUS [21]	e^+p and e^-p	4.6	5.3
DØ [26]	dijet	3.2	3.1
DØ [26]	dielectron	4.7	5.5
CDF [26]	dilepton	4.5	5.6
CCFR [27]	νN scattering	3.7	5.9
combined		20.6	15.7

Table 3: Dimension 6 operator Υ : 95% CL limits on $|c_\Upsilon/4\pi|^{-1/2}$ (in TeV) for positive and negative values of c_Υ , from different experiments.

Graviton loops can also generate some new dimension-6 operators involving the Higgs doublet H , which have the general structure

$$\mathcal{L}_{\text{int}} = cH^\dagger H \mathcal{L}_{\text{SM}}, \quad (22)$$

where \mathcal{L}_{SM} represents any of the SM dimension-4 interaction terms. All coefficients c are zero unless we introduce a linear coupling between the 4-dimensional Ricci scalar and the Higgs bilinear, as considered in ref. [28]. On general grounds, one expects that such coupling is non-vanishing, and modified NDA gives the estimate $c \sim c_\Upsilon$ [¶].

The terms in eq. (22), with H replaced by its vacuum expectation value v , can be absorbed in a redefinition of the fields and the coupling constants. Once this is done, the interactions of the physical Higgs boson h ($H \rightarrow (v+h)/\sqrt{2}$) in a linear expansion are given by

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{SM}}(h) + cvh\mathcal{L}_{\text{SM}}(v). \quad (23)$$

This modifies the Higgs couplings to fermions and weak gauge bosons by a factor $1 + cv^2$. The bounds on c_Υ presented in table 3 imply that this modification differs from 1 by at most 3×10^{-3} .

More interesting is the case of operators coupling the Higgs fields to photons and gluons $\mathcal{O}_{h\gamma\gamma} = h F_{\mu\nu}F^{\mu\nu}$ and $\mathcal{O}_{hgg} = h G_{\mu\nu}^a G^{a\mu\nu}$, since the graviton-mediated interaction has to compete with a loop-induced SM term. We find that the coefficients of these effective operators are

$$g_{h\gamma\gamma} = \frac{47\alpha I_\gamma}{72\pi v} - \frac{cv}{4}, \quad g_{hgg} = -\frac{\alpha_s I_g}{12\pi v} - \frac{cv}{4}, \quad (24)$$

where I_γ and I_g are the ordinary SM loop functions [29]. We have normalized them such that, neglecting higher order corrections, $I_\gamma = I_g = 1$ for $m_h \ll m_t, m_W$, while $I_\gamma = 1.18$ and $I_g = 1.03$ for $m_h = 115$ GeV. In fig. 2 we show the quantity $\sigma(gg \rightarrow h) \times \text{BR}(h \rightarrow \gamma\gamma)$ in units of the SM value, as a function of the Higgs mass m_h and the new-physics scale $\Lambda \equiv \pm|c/(4\pi)|^{-1/2}$. Notice that the interference of the new-physics and SM contributions in $g_{h\gamma\gamma}$ and g_{hgg} is always constructive for one coupling and destructive for the other one, leading to a partial compensation in $\sigma(gg \rightarrow h) \times \text{BR}(h \rightarrow \gamma\gamma)$. This is because the dominant SM contribution to $g_{h\gamma\gamma}$ is from W exchange, while the one to g_{hgg} is from top exchange, and they have opposite sign. Experiments at the LHC can determine the quantity $\sigma(gg \rightarrow h) \times \text{BR}(h \rightarrow \gamma\gamma)$ with a precision of 10–15% [30], and therefore significantly probe the virtual-graviton contribution, as apparent from fig. 2. The implications of the new couplings $g_{h\gamma\gamma}$ and g_{hgg} have also been considered in ref. [31].

6 Gravitons and SM gauge bosons

Although the operators τ and Υ do not directly affect precision electroweak data, they contribute when they are dressed by gauge bosons. This corresponds to loops with exchange of virtual gravitons and vector bosons

[¶]An explicit computation of the coefficients c of the operator $H^\dagger H(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu})$ discussed below, using the three regulators introduced in sect. 2 gives $c = 0$, because of a cancellation between the relevant Feynman diagrams. Such cancellations are related to the fact that the trace of the energy-momentum tensor of massless spin-one particles vanishes in 4 dimensions. Other regularizations can give $c \neq 0$. We will encounter a similar ambiguous result in the next section when studying the corrections to the anomalous magnetic moments.

(such those illustrated in fig. 1c) leading to new dimension-6 operators [8]. We estimate them by making the conservative assumption that the SM gauge couplings remain weak up to the cutoff.

Since graviton loops are flavour universal (neglecting the bottom quark mass) gravitational corrections to the various electroweak precision measurements can be embedded in three parameters that are usually chosen to be $\epsilon_1, \epsilon_2, \epsilon_3$ [32]^{||}. Similarly, we can also estimate the contribution to the anomalous magnetic moment of the muon. The results from “modified NDA” are

$$\delta\epsilon_i^{\text{NDA}} \approx \frac{c_\delta M_Z^2 \Lambda_{\text{NDA}}^\delta}{\ell_4 \ell_\delta M_D^{\delta+2}} = \frac{\pi^{\frac{\delta-4}{2}}}{16\Gamma(\delta/2)} \frac{M_Z^2 \Lambda_{\text{NDA}}^\delta}{M_D^{\delta+2}}, \quad (25)$$

$$\delta a_\mu^{\text{NDA}} \approx \frac{c_\delta m_\mu^2 \Lambda_{\text{NDA}}^\delta}{\ell_4 \ell_\delta M_D^{\delta+2}} = \frac{\pi^{\frac{\delta-4}{2}}}{16\Gamma(\delta/2)} \frac{m_\mu^2 \Lambda_{\text{NDA}}^\delta}{M_D^{\delta+2}}. \quad (26)$$

Because of a cancellation between different Feynman diagrams, the one-loop graviton correction to δa_μ is zero when estimated with the regulators discussed in sect. 2 [8]. A finite result in agreement with the estimate in eq. (26) is obtained using other regularizations, like the non-supersymmetric dimensional regularization [34] that spoils the cancellation by acting in a different way on fermions and vector bosons.

In our analysis, we will take as a representative bound from electroweak data $|\delta\epsilon_i| < 10^{-3}$. Then, eq. (26) implies $|\delta a_\mu| < 10^{-9}$, which is about 1σ of the experimental value and of the SM theoretical estimate. Therefore, electroweak data and the anomalous magnetic moment of the muon give comparable bounds on virtual graviton effects.

7 Comparison of the different constraints

In this section we attempt a comparison between the constraints from the different collider observables. For virtual-graviton effects, we will use the estimates from “modified NDA”, and then comment on other regulators. Once again, we stress that this procedure does not predict order-one coefficients and that, in this sense, our bounds are only semi-quantitative.

Notice also that the dominant effects at colliders are produced by the heaviest Kaluza-Klein modes allowed by the kinematics of the relevant process. On the other hand, the lightest Kaluza-Klein modes affect astrophysical processes, giving bounds that for $\delta = 2, 3$ are much stronger than collider bounds [35]. In particular, supernova bounds directly exclude observable deviations from the Newton law at sub-millimeter scales caused by extra-dimensional gravitons. Reasonable modifications of the low-energy part of the spectrum may remove the lightest Kaluza-Klein modes, without affecting the high-energy signals that we consider.

Our results are summarized in fig. 3. The scale Λ that we have introduced in this analysis plays an important rôle in comparing the different observables, since it parametrizes the relative strength of tree-level versus loop effects or, in other words, it defines how strongly-interacting gravity is. Setting $\Lambda = M_D$, as assumed in many analyses, is an arbitrary restriction: Λ is a relevant free parameter.

As apparent from fig. 3, the most stringent bounds arise from graviton emission and from the dimension-6 operator Υ , which is severely constrained by the recent LEP2 data. The limit on the operator τ is less significant than the bound from Υ at large values of M_D , and weaker than the bound from graviton emission at small M_D . However, it rules out a small additional region of parameter space at intermediate values of M_D . The LEP2 bounds on Υ give a stronger constraint than electroweak precision measurements or $(g-2)_\mu$.

In order to study the significance of the $\mathcal{O}(1)$ factors not controlled by NDA, in the previous sections we have computed the coefficients of the relevant operators with three different arbitrary cut-off procedures. For $\Lambda = \Lambda_{\text{NDA}}$, regulators with an exponential functional behavior or a sharp cut-off give smaller values of c_Υ than the “modified NDA” estimate, while the splitting regulator gives larger values. However, these factors can be reabsorbed in the definition of Λ . For each regularization procedure, we choose to define the ratio $\Lambda/\Lambda_{\text{NDA}}$ by enforcing $c_\Upsilon = c_\Upsilon^{\text{NDA}}$, separately for all values of δ . The results are shown in table 4. With this definition, the bounds from the Υ operator, using explicit regulators, are equal to those plotted in fig. 3. The bound from graviton emission (vertical lines in fig. 3) does not depend on Λ , and therefore remains unmodified. The coefficient of the τ operator is then determined and its value in units of c_τ^{NDA} is given in table 5, for the different regulators. Since c_τ scales as M_D^{-4} , for all the considered cut-off procedures the bounds from τ are only

^{||}Graviton corrections to the S, T, U parameters [33] are plagued by large gauge-dependent infrared effects that, in the unitary gauge, unphysically increase with increasing M_D . The reason is that S, T, U parameterize new physics present only in the vector boson sector and therefore are not physical observables in the case of gravity, which couples to everything.

$\Lambda/\Lambda_{\text{NDA}}$	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta = 6$
a) exponential cut-off	1.57	1.51	1.40	1.29	1.19
b) sharp cut-off	1.58	1.60	1.56	1.52	1.48
c) splitting cut-off	0.89	0.76	0.64	0.55	0.47

Table 4: The values of $\Lambda/\Lambda_{\text{NDA}}$ in different regularization procedures obtained by imposing $c_{\Upsilon} = c_{\Upsilon}^{\text{NDA}}$.

$c_{\tau}/c_{\tau}^{\text{NDA}}$	$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta = 6$
a) exponential cut-off	2.68	1.97	1.91	2.04
b) sharp cut-off	3.20	2.45	2.34	2.37
c) splitting cut-off	2.40	1.66	1.55	1.61

Table 5: The values of $c_{\tau}/c_{\tau}^{\text{NDA}}$ in different regularization procedures obtained by imposing $c_{\Upsilon} = c_{\Upsilon}^{\text{NDA}}$. For $\delta = 2$ the comparison is affected by the infrared divergence, since $c_{\tau}/c_{\tau}^{\text{NDA}} = \ln(\Lambda^2/E^2)$. However, this is also of order unity for relevant values of the typical energy E .

slightly stronger than in the NDA case, shown in fig. 3. In this respect, once the different cut-off parameters are appropriately compared, our results are rather insensitive of the regularization procedure. We do not need to discuss how the other bounds change, since they are always sub-dominant.

The limits on Υ from LEP2 imply that the gravitational theory can never become strongly interacting, in the experimentally relevant range of M_D , since the solid line in fig. 3 corresponding to $\Lambda = \Lambda_S$ lies in the excluded region. A low cut-off Λ , necessary to reduce the contributions from graviton loops, has important implications for collider experiments. It reduces the sensitivity region from graviton emission, but it opens up the possibility of discovering the new states which presumably lie at the scale Λ and are responsible for the softening of the ultraviolet behaviour of gravitational interactions. Especially if $\Lambda \ll M_D$, the new physical phenomena related to the scale Λ could give the discovering signatures in present and future experiments. Ignoring them is clearly a strong limitation of graviton phenomenology.

The physics at the scale Λ could itself generate new effective operators, potentially giving stronger constraints than those considered here. One possibility is that Λ be the scale of some string model, and the interaction strength be related to the string coupling. Although one cannot make model-independent statements, it is plausible to expect that Λ has to be larger than few TeV. Alternatively, Λ could be the mass scale of supersymmetric particles. Assuming conserved matter parity, physics at the scale Λ generates effective dimension-6 operators only at one-loop order, and the lower bounds on Λ are very weak, of the order of 100 GeV.

Finally, we recall that a reparametrization invariant extra-dimensional theory containing SM fields localized on a 3-dimensional brane contains not only gravitons but also brane fluctuations, described by δ neutral scalars named ‘branons’. This can be understood geometrically: a brane straight in some system of coordinate is bent when described using different coordinates. (An exception is when the brane is located at special singular points of the extra dimensional space; such pathological spaces are often studied because of their mathematical simplicity). Branon couplings are controlled by the tension τ of the brane, just like graviton couplings are controlled by the Planck mass M_D . Therefore, just like it is possible to study effects generated by gravitons alone, one can also study effects generated by branons alone. Colliders provide the most stringent constraints. With δ flat extra dimensions branon missing energy signals are equal to missing energy signals produced by gravitons in 6 extra dimensions, rescaled by the factor $\pi\delta c_6 M_{10}^8/30\tau^2$ (predicted to be 1/5 by the simplest toy string models) [36]. The same rescaling factor applies to branon virtual effects: tree level (one loop) graviton exchange corresponds to one loop (three loops) branon exchange. Therefore fig. 3d also describes branon phenomenology after reinterpreting $M_D \rightarrow \tau$ as described above. If $\tau \ll M_D^4$ brane fluctuations suppress graviton effects [37].

8 Conclusions

Collider experiments probe theories with extra-dimensional gravity in a model-independent way through graviton emission, and in a model-dependent way through studies of contact interactions. We have given some tools that allow for a comparison between the two, under the assumption that the coefficients of the new contact interactions are dominated by graviton effects (at tree or loop level). It is important to emphasize that our

analysis has two important limitations: *i*) the coefficients of the effective operators induced by graviton exchange (at tree or loop level) can only be estimated, since they are sensitive to the ultraviolet; *ii*) because of this sensitivity, other contributions from unknown new physics can be equally or more important.

Nevertheless, we believe that some interesting results have been reached by this analysis. Because of the different dimensionality of the operators involved, graviton loop corrections can be more effective than tree-level exchange in constraining the theory from present data. We have found that the structure of effective operators generated by graviton loops is relatively simple. At the level of dimension-6 operators, loop diagrams with an intermediate state of virtual gravitons generate only the operator Υ and some operators relevant for Higgs physics. Mixed loops with graviton and gauge bosons intermediate states can generate other operators which affect electroweak precision measurements, but which are less effective in constraining the theory. Therefore global analyses of extra-dimensional gravitons at colliders should simultaneously take into account graviton emission and the effects of the operators τ and Υ .

LEP2 data constrain the coefficient of the operator Υ and thus put one of the strongest limit on the existence of extra-dimensional gravity at the weak scale. In particular, they disfavour strongly-interacting gravity at accessible energies. This has important implications for future searches at high-energy colliders, limiting the available parameter space where graviton emission can be observed, but leaving open the possibility of detecting the physics responsible for the premature softening of quantum gravity.

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A Graviton-fermion loops

In order to write reparametrization invariant Lagrangians involving fermions we introduce the D -bein E_M^A relating generic coordinates x_M to locally free-fall systems ξ_A , so that the metric is given by $g_{MN} = \eta_{AB} E_M^A E_N^B$. The D -bein basis definition introduces an additional gauge symmetry, besides diffeomorphisms, due to the freedom of rotating ξ_A with a local Lorentz transformation. With a clever gauge choice [38] Lorentz ghosts are absent and the D -bein can be reexpressed in terms of the metrics. Expanding the metrics in fluctuations around flat space, $g_{MN} = \eta_{MN} + \kappa h_{MN}$ one has ($\kappa^2 = 4c_\delta/M_D^{2+\delta}$)

$$E_M^A = \delta_{AM} + \frac{\kappa}{2} h_{AM} - \frac{\kappa^2}{8} (h \cdot h)_{AM} + \frac{\kappa^3}{16} (h \cdot h \cdot h)_{AM} + \dots \quad (27)$$

This gauge choice can be generalized to systems involving branes [8] in such a way that the brane vierbein e_m^a keeps the same form as E_M^A in eq. (27) with the metrics replaced by the induced metrics on the brane:

$$h_{MN}(x^R) \rightarrow h_{mn}(x^r) = h_{mn}^0 + \frac{1}{\kappa} (\partial_m \xi_i) (\partial_n \xi^i) + (\xi^i \partial_i h_{mn}^0 + h_{im}^0 \partial_n \xi^i + h_{in}^0 \partial_m \xi^i) + \dots \quad (28)$$

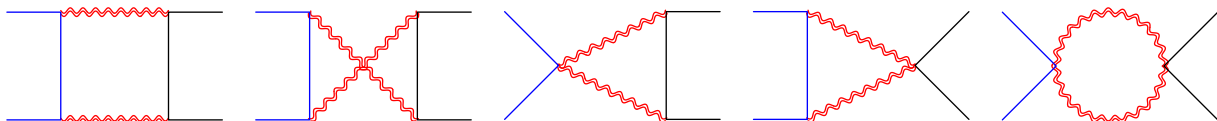
where $h^0(x^r)$ denotes $h(x^r, x^i = 0)$, the D -dimensional graviton field evaluated at the brane rest position $x^i = 0$. We have splitted D dimensional indices M, N into their four-dimensional components m, n plus their extra dimensional components $i, j = \{1, \dots, \delta\}$. The ‘branons’ $\xi^i(x_m)$ are δ scalar fields that fix the position of each point of the brane in the extra dimensions as $x^i(x_m) = \xi^i(x_m)$. This choice completely fixes brane reparametrizations without giving ghost fields [39]. The kinetic term of a Dirac field localized on a brane with tension τ is

$$S = \int d^4x e \left[-\tau + e_a^m \frac{i}{2} [\bar{\Psi} \gamma_a (D_m \Psi) - (\overline{D_m \Psi}) \gamma_a \Psi] \right], \quad D_m = \partial_m + \frac{1}{8} [\gamma_a, \gamma_b] \omega_m^{ab}, \quad (29)$$

where $e = \det e_m^a$ and the graviton contribution to the spin-connection is

$$\omega_m^{ab} = \frac{\kappa}{2} \partial_b h_{am} + \frac{\kappa^2}{8} (h_{bn} \partial_m h_{an} + 2h_{an} \partial_n h_{bm} + 2h_{bn} \partial_a h_{mn}) - (a \leftrightarrow b) + \dots$$

By expanding the action up to second order in κ one finds the off-shell $\bar{\Psi} \Psi h$ and $\bar{\Psi} \Psi h h$ couplings. Taking into account that Feynman graphs with branons or with corrections to fermionic gravitational vertices give no one-loop contribution to the dimension-6 operator Υ , we only need to compute the following graphs:



We computed them using two different gauges for the graviton KK, the deDonder and the unitary gauge (see ref. [8]), verifying that their sum is gauge invariant mode by mode (so that we can perform this check without introducing any regulator). The result is given by eq. (19). It contains no factor δ because the δ -dependent ‘scalar’ part in the graviton/radion propagator gives no contribution.

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