Real-time D-brane condensation

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Abstract

Unstable D-branes or brane-antibrane systems can decay to lower-dimensional branes. In the effective field theory description, the final state branes are defects in the tachyon field which describes the initial instability. We study the dynamical formation of codimension one defects (kinks) using Sen's ansatz for the tachyon Lagrangian. It is shown that the slope of the kink diverges within a finite amount of time after the tachyon starts to roll. We discuss the relevance for reheating after brane-antibrane inflation.

1. Introduction. In the last few years, significant progress has been made in understanding the nature of transitions in unstable systems of D-branes [1]. Most of the work has been motivated by formal considerations, but there is also an important application, namely inflation from brane-antibrane collisions [2, 3]. This is an appealing application of genuine string theoretic constructions to cosmology, but it may have problems with reheating [4, 5, 6]. It seems likely that the energy density liberated from the brane collisions is efficiently converted into closed string states (ultimately gravitons) [7, 8], and not necessarily into visible radiation. To quantitatively address this potential problem, one should consider how the evolving tachyon condensate couples to photons, which being an open string excitation should reside on some stable brane remaining in the final state.

One way in which branes could be created in the collision is through the Kibble mechanism [9, 10]; the tachyon field forms topological defects, which are known to be a consistent description of branes whose dimension is lower than that of the original branes [11]. For example, a brane-antibrane system has a complex tachyon field, leading to vortices which represent codimension-two branes. On the other hand an unstable brane has a real tachyon field, which can form kinks in some direction, representing codimension one branes. For simplicity we are going to study the latter situation in this letter. It should be noted however that this process could also originate from a brane-antibrane collision, in which one component of the complex tachyon first undergoes condensation to form unstable codimension-one kinks, followed by the second, orthogonal component; the intersection of these two kinks is the vortex. Our simplified situation describes the second step in this process.

The static properties of tachyon defects have been well-studied in the literature; the fact that their tensions match the known ones of D-branes is part of the evidence that they *are* D-branes [12, 13]. In addition, excitations around these defects have been shown to reproduce

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the excited states of strings in the case of p-adic strings, where such calculations are tractable [14]. However, not much is known about the dynamics of defect formation; precisely how do they form in space and time, starting from an unstable configuration? These details will be important for making a quantitative calculation of reheating on the branes which form (one of which is presumed to contain the standard model). We may then hope to settle the question of whether such a brane universe will be completely dominated by gravitational radiation at the end of inflation.

In the present work, we do not attempt to improve on the reheating computation, contenting ourselves with determining the tachyon profile $T(t, x)$ as a function of time and the extra dimension which is transverse to the kink. We will show that Sen's version of the effective action for the tachyon leads to a somewhat surprising result: the slope of the kink diverges in a finite time after the brane collision. We show that this is related to the observation that caustics can also form in this situation. We conclude by speculating that both this problem and that of the caustics is an artifact of ignoring higher derivative corrections to the tachyon action.

2. Action and equations of motion. Sen has proposed a simple form for the tachyon action which captures the essential qualitative features of exact computations from boundary string field theory (BSFT), and which quantitatively agrees with the exact result in certain limits [15, 16]. It has the form

$$\mathcal{L} = -\mathcal{T}V(T)\sqrt{-\det(g_{\mu\nu} - \partial_\mu T \partial_\nu T)} \tag{1}$$

where $\mathcal{T}$ is the tension of the original nonBPS brane, and the potential is $V(T) = e^{-|T|^2/a^2}$ for the superstring. T has dimensions of length and a is of order the string length scale. Evaluating the determinant, and assuming T varies in only one spatial dimension x, which we shall refer to as the bulk, the action becomes

$$\mathcal{L} = -\mathcal{T}V(T)\sqrt{1 - \dot{T}^2 + T'^2} \tag{2}$$

Let us compare this with the exact result from BSFT [12, 13]:

$$\mathcal{L} = -\mathcal{T}e^{-T^2/a^2}F[-(\partial_\mu T)^2] \tag{3}$$

where

$$F(x) = \frac{4^x x \Gamma(x)^2}{2\Gamma(2x)}, \tag{4}$$

Although this looks very different from (2), it has some essential similarities. For example both $F(x)$ and $\sqrt{1+x}$ approach $\sqrt{x}$ in the limit of large x, appropriate for describing the static kink solution, which has the property that $T' \to \infty$. Moreover, both functions have first derivatives which diverge as $x \to -1$, which puts a limit on how fast the field can roll for homogeneous configurations: $\dot{T} \to \pm 1$. (It turns out that the zeroes of $F(x)$ and $\sqrt{1+x}$ do not have special significance, so it is not important that they do not coincide.) We will work with Sen's form of the action since it captures the distinctive behavior of the exact one,

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but is much simpler to work with. We checked certain of the following numerical results also using the BSFT action to be sure that no significant differences arose.

The equation of motion which follows from (3) is

$$\ddot{T} = \frac{2a^{-2}T(1 - \dot{T}^2 + T'^2) + (1 - \dot{T}^2)T'' + 2\dot{T}T'\dot{T}'}{(1 + T'^2)} \tag{5}$$

In the homogeneous case, this simplifies to $\ddot{T} = 2a^{-2}T(1 - \dot{T}^2)$, which shows the limiting velocity $\dot{T} \to 1$. The asymptotic form of the solution is

$$T \cong T_i + t - \frac{\sqrt{2\pi}a}{8}\mathrm{erf}(\frac{\sqrt{2}t}{a}) \tag{6}$$

at large times.

3. Dynamical solutions. For nonhomogeneous tachyon configurations in which T has reached large values $T \gg a$, analytic approximations to the solution have been developed in reference [17]. However in the present study we are interested in formation of kinks, near which T remains zero. Before looking for analytic approximations, let us consider numerical evolution of the equation of motion (5). We assume the extra dimension is compactified on a circle of circumference $2L$, and approximate it by N discretize points. We put periodicd boundary conditions on T, and replace spatial derivatives in eq. (5) by finite differences. This results in a set of N coupled ordinary differential equations which can be solved numerically.

The initial condition is taken to be $T(0, x) = \epsilon \cos(\pi x/L)$ where $\epsilon \ll a$. This is an idealization of random initial conditions where $T(0, x)$ happened to cross zero at two locations in the compact bulk. Fig. 1 shows the how the spatial profiles evolve in time.
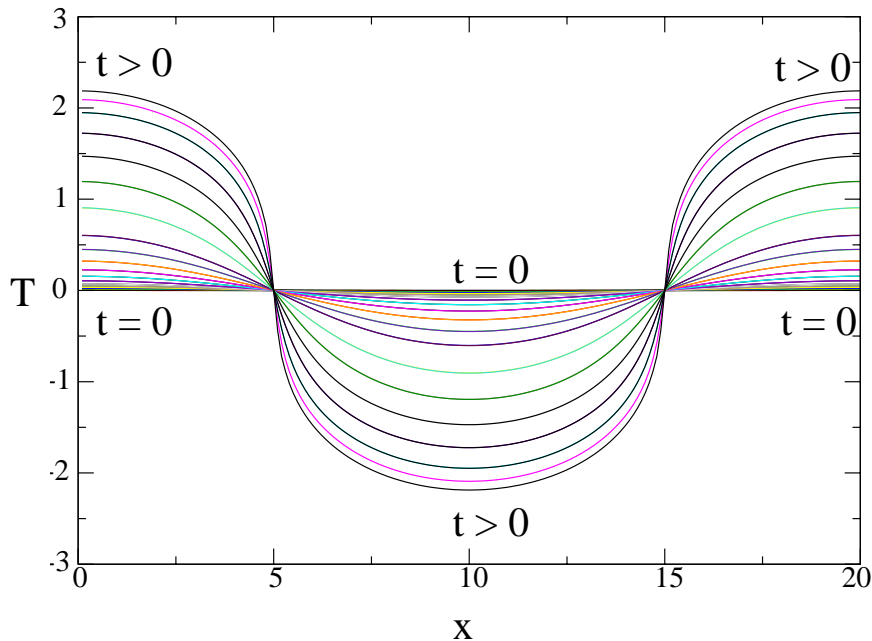

Figure 1. Sequence of spatial tachyon profiles $T(x)$ for a series of increasing times, showing formation of a kink-antikink pair.

A few features of the above solutions are noteworthy. At any position x apart from the kink locations, $T(t,x)$ approaches the behavior (6), where $T_0 = T_0(x)$. This is demonstrated in Fig. 2, which plots $\dot{T}(x)$ for a sequence of increasing times. It is clear that $|\dot{T}| \to 1$ at any x, though the approach to the asymptotic value takes longer for points closer to the kink locations. Secondly, the slope of the kink becomes quite steep on a short time scale. In fact, the code crashes at a certain finite time because the slope becomes singular.
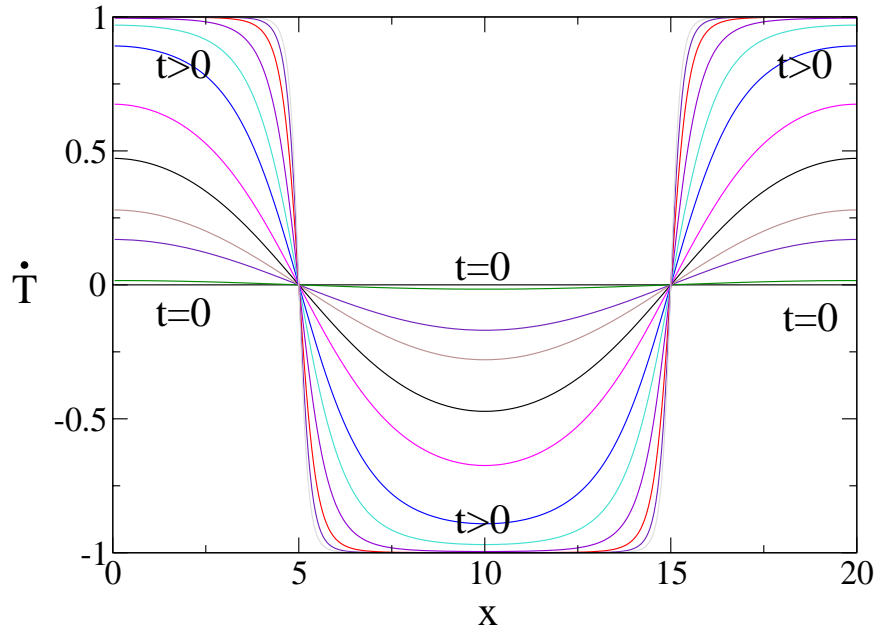

Figure 2. Sequence of spatial tachyon velocity profiles $\dot{T}(x)$ for a series of increasing times, showing that $|\dot{T}| \to 1$ in the bulk.

One might wonder if numerical error is the reason that the evolution cannot be followed past a certain time. However this seems not to be the case. Using finer lattices and time steps did not help the problem. Neither did higher order differencing schemes for discretizing the spatial derivatives. It is interesting to see in detail why the numerical evolution fails beyond the critical time. To show this, consider $T(t, x_i)$ at the lattice sites x_i which are in the vicinity of one of the kinks. The behavior of the quantity $1 - \dot{T}^2 + T'^2$ which appears in the tachyon action is shown in Fig. 3. The curious feature is that at the lattice sites neighboring the position of the kink (on either side), $1 - \dot{T}^2 + T'^2$ starts plunging toward negative values after a certain time. Such values are unphysical since the square root of $1 - \dot{T}^2 + T'^2$ appears in the action, and this is what causes the numerical evolution to crash. Again, one might suspect some sort of numerical problem, but this behavior proved to be completely insensitive to all manner of modifications to the program which were tried. We will give some analytical arguments for why this pathological end to the evolution is in fact inherent in the equations of motion, rather than a fluke of the numerics.
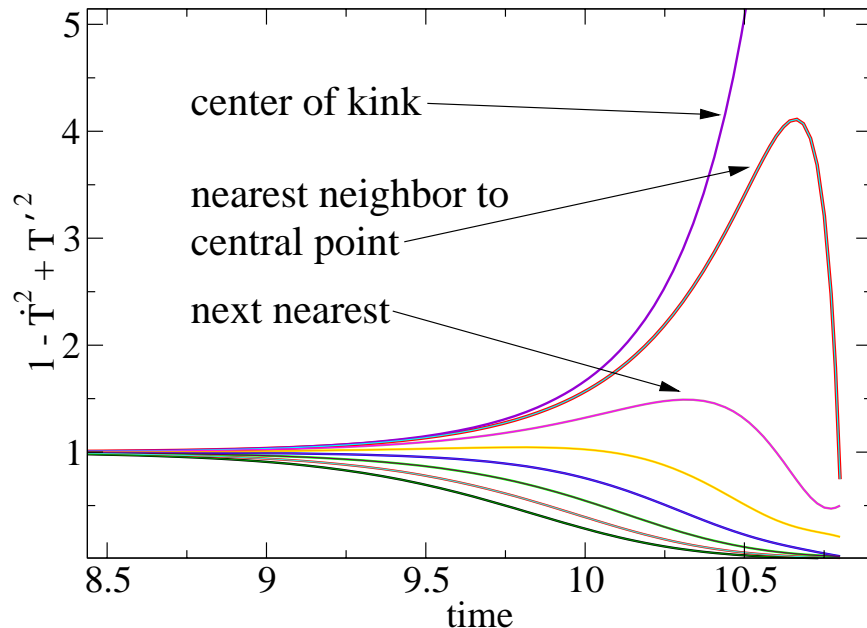
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Figure 3. The quantity $1 - \dot{T}^2 + T'^2$ as a function of time in units of a, for several lattice sites in the vicinity of a kink. The rapid descent of the curve for the nearest neighbor to the kink center signals a pathology in the time evolution.

4. Analytic approach. To analytically study the dynamics of $T(t, x)$ close to the kink, we can use the fact that it is odd in x and write the ansatz

$$T(t, x) \cong q(t)x + p(t)x^3 + r(t)x^5 + \cdots \tag{7}$$

Substituting this into the equation of motion and ignoring the terms which are subleading in x gives

$$\ddot{q} = \frac{2}{a^2}q + \frac{2q\dot{q}^2 + 6p}{1 + q^2} \tag{8}$$

This cannot be solved for q since it depends on p, whose solution depends on r, *etc.*, but let us suppose that p is initially zero and can thus be ignored at least for early times. The solution has two regimes. At early times, before $\dot{q}$ has become large, the second term is negligible and therefore

$$q(t) \cong q_+ e^{\sqrt{2}t/a} + q_- e^{-\sqrt{2}t/a} \tag{9}$$

However $\dot{q}$ quickly grows, so for $t \gg a$ it will no longer be consistent to ignore the second term in (8). One can show that the solution in the regime where the second term dominates has the behavior

$$q \sim \frac{c}{t - t_0} \tag{10}$$

This behavior is borne out by the numerical solutions, as shown in Fig. 4, which plots $\ln(T(t, x_i)/x_i) \cong q(t)$ versus t for the lattice points closest to the kink. At early times the kink formation starts very slowly (both terms in (9) are important), while the intermediate straight-line behavior confirms (9) when the positive exponential dominates. After a time

interval of approximately $2a$ from the start of the linear region, the curves start to diverge from each other, showing that T is no longer just linear in x over the range of x_i given. But the topmost curve is given by the site x_1 which is the nearest neighbor to the kink, so this gives the best approximation to the linear term in (7), and we see that it begins to rise faster than exponentially with time, in qualitative agreement with (10).
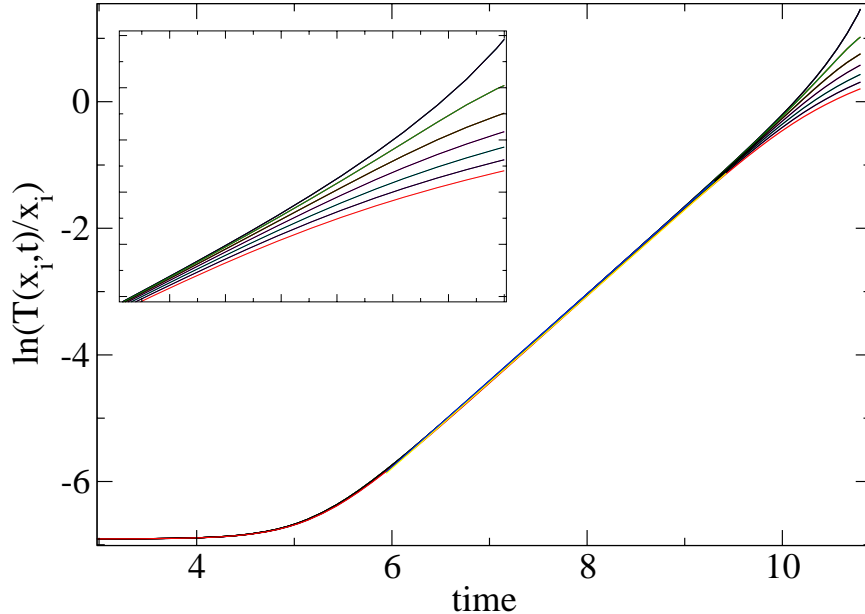

Figure 4. $\ln(T(t, x_i)/x_i)$, which should approximate $q(t)$, as a function of time (in units of a), for several lattice sites in the vicinity of a kink. Inset: close-up of the large t region.

5. Hamilton-Jacobi method. The preceding argument is not conclusive because we have ignored the undetermined function $p(t)$, which could conceivably soften the behavior of $q(t)$. Therefore we give an additional argument which bolsters the conclusion that the slope of the kink diverges in a finite time. We use the formalism of [17], in which Hamilton-Jacobi equations were used to demonstrate the formation of caustics in the tachyon profile when the initial conditions were inhomogeneous. At first sight their formalism appears to be inapplicable to the study of kinks because it is the leading term in an asymptotic expansion in powers of e^{-T^2/a^2}, which is not small very near the kink.

However, we have seen that no matter how close a given position x_i may be to the kink, if we simply wait long enough, the value of $T(t, x_i)$ will become sufficiently large that we *can* expand in e^{-T^2/a^2}. Suppose we carry out this procedure for two points x_i and $-x_i$ which bracket a kink at $x = 0$; we can then deduce the behavior at the kink by interpolation. An interesting aspect of the Hamilton-Jacobi method is that rather than determining $T(t, x_i)$ for later times, it instead yields $T(t, x_c[t, x_i])$ where $x_c[t, x_i]$ is a characteristic curve that depends on which initial value x_i was chosen. We will show that $x_c[t, x_i]$ actually crosses $x = 0$ from the right for positive x_i, with a nonzero value for T at this event. Therefore the tachyon becomes multivalued, since $T = 0$ at the origin by construction. Its slope must diverge at this moment.

To see this, let us consider some x_i close to the kink, but starting at a late enough time

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$t_i \equiv 0$ that $T(t_i, x_i) \equiv T_i \gg a$. The characteristic curve associated with this event is

$$x_c[t, x_i] = x_i - \frac{T_i' t}{\sqrt{1 + T_i'^2}} \tag{11}$$

where $T_i' = T'(t_i, x_i)$, and the solution for T along this curve is

$$T(t, x_c[t, x_i]) = T_i + \frac{t}{\sqrt{1 + T_i'^2}} \tag{12}$$

Clearly, as long as $T_i' \neq 0$ which will be true sufficiently close to the kink, x_c will cross $x = 0$ in a finite time, $t = x_i \sqrt{1 + T_i'^2}/T_i'$, and T will be nonzero. The procedure is illustrated in Fig. 5
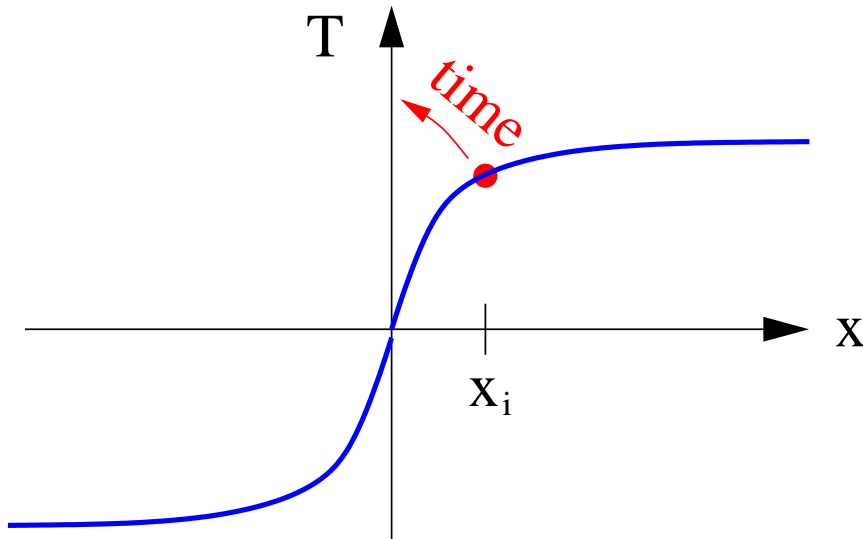

Figure 5. Schematic representation of the temporal evolution of tachyon spatial profiles in the Hamilton-Jacobi method; the point on the curve at x_i evolves toward the T axis as shown.

6. Discussion. In this letter we have quantified the time-dependence of tachyon defect formation, hence formation of a D-brane, in the decay of an unstable nonBPS brane to a stable one. Numerical and analytic methods indicate that the profile of the kink becomes infinitely steep in a finite amount of time of order the string scale, bringing an end to our ability to follow the evolution. Notice that T should continue to roll toward ∞ in the bulk at large times. This situation is reminiscent of the problem of caustic formation, where the appearance of cusps also creates an obstacle to evolving the fields further in time.

In the case of caustics, the second derivative T'' ceases to exist at some point. It is tempting to speculate that this problem is an artifact of the approximations which led to the action (3). The BSFT result is only exact for tachyon configurations with vanishing spatial derivatives higher than first order. There are corrections of order T''^2 which must exist, but are difficult to compute in BSFT. The worldsheet partition function which must be evaluated to find the effective action is no longer Gaussian when $T'' \neq 0$, so such corrections would

have to be computed perturbatively. So far such calculations have not been done, so we are not able to test the conjecture that the new terms cure the caustic problem. If they do, it also seems likely that they would ameliorate the problems of kink formation. A fully string theoretic calculation might be needed to settle the issue.

For the problem of reheating, the divergence of the slope of the kink is not necessarily a serious obstacle to doing computations; if most of the production of gauge bosons on the final brane takes place before the singularity occurs, then it is not so important. Intuitively we might guess that the rapid increase of the kink slope will decrease the efficiency of reheating however. The reason is that the bulk motion of the tachyon fluid should not be influenced much by the presence of the defect if the latter is highly localized. Therefore we might expect that most of the energy in the tachyon condensate will be released into gravitons, just as if the final state branes were not present. This could be an argument for alternative scenarios of brane-antibrane inflation, such as branes at angles, where the final state brane has the same dimensionality as the colliding ones [18].

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Note Added: After this work was completed, we became aware of [19], which found the formation of a singularity in the energy density of the tachyon configuration at a finite time. Since this was an exact string theoretic calculation, it means that the effect we found is likely to be real and not an artifact of truncating the expansion in derivatives of the fields, as suggested above. It was suggested in [19] that the origin of the singularity is the formation of a delta function source of brane tension whose presence cannot easily be inferred from the evolution of the field equations alone. We thank A. Sen for these observations.

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