

A probe of the Radion-Higgs mixing in the Randall-Sundrum model at e^+e^- colliders

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Abstract

In the Randall-Sundrum model, the radion-Higgs mixing is weakly suppressed by the effective electroweak scale. A novel feature of the existence of gravity-scalar mixing would be a sizable three-point vertex, $h_{\mu\nu}^{(n)}-h-\phi$. We study this vertex in the process $e^+e^- \rightarrow h_{\mu\nu}^{(n)} \rightarrow h\phi$, which is allowed only with a non-zero radion-Higgs mixing. It is shown that the angular distribution is a unique characteristic of the exchange of massive spin-2 gravitons, and the total cross section at the future e^+e^- colliders is big enough to cover a large portion of the parameter space where the LEP/LEP II data cannot constrain.

I. INTRODUCTION

Although the standard model (SM) has been very successful in describing the electroweak interactions of the gauge bosons and fermions, an important ingredient, the Higgs boson, awaits to be experimentally discovered [1]. In the SM, the Higgs boson plays a central role of the electroweak symmetry breaking. Since its mass should be lighter than the scale $(8\pi\sqrt{2}/3G_F)^{1/2} \sim 1$ TeV for the preservation of unitarity in the $W_L W_L \rightarrow W_L W_L$ process [2], the primary efforts of future collider experiments are directed toward the search for the Higgs boson.

Radiative corrections to the mass of the Higgs boson give rise to the gauge hierarchy problem, which has motivated a number of models for physics beyond the SM. Recently, it was known that the enormous hierarchy between the electroweak and Planck scale can be explained, without resort to any new symmetry, by introducing extra dimensional space [3, 4]. In particular, a scenario proposed by Randall and Sundrum (RS) [4] has drawn a lot of interests, in which an additional spatial dimension of a S^1/Z_2 orbifold is introduced with two 3-branes at the fixed points. A geometrical suppression factor, called the warp factor, can naturally explain the gauge hierarchy with moderate values of the model parameters. The original RS model has a four-dimensional massless scalar field, the modulus or radion, about the background geometry. In order to avoid unconventional cosmological equations, a stabilization mechanism is required [5, 6], through which the brane configuration is stabilized and the radion attains a mass. Moreover, the radion is likely to be lighter than the Kaluza-Klein states of any bulk fields. Therefore, the radion will probably be the first sign of the warp-geometry.

Various phenomenological aspects of the radion have been studied in the literature, including its decay modes [7, 8], its effects on the oblique parameters of the electroweak precision observations [9], and other effects on phenomenological signatures at present and future colliders [10]. Above all, a possible mixing between the radion and Higgs boson may cause some deviations in phenomenological observables of the Higgs boson from the SM ones even in the minimal scenario in which all the SM fields are confined on the TeV brane. The mixing can come from the simplest example of gravity-scalar mixing, $\xi R(g_{\text{vis}})\widehat{H}^\dagger\widehat{H}$, where $R(g_{\text{vis}})$ is the Ricci scalar of the induced metric $g_{\text{vis}}^{\mu\nu}$ and \widehat{H} is the Higgs field in the five-dimensional context. Some studies of the effects of the radion-Higgs mixing have also been performed,

e.g., the production and decay of scalar particles at the LHC [11], the unitarity bounds [12], and the total and partial decay widths of the Higgs boson [13].

Another interesting way to probe the radion-Higgs mixing is to search for new couplings emerging as the mixing turns on: Good examples are the triple vertices linear in the Higgs field. In Ref. [14], it was shown that in the effective potential for the SM Higgs boson interacting with the KK gravitons and the radion, but without the radion-Higgs mixing, the SM minimum of $\partial V_h/\partial h_0|_{(h_0)=0} = 0$ is a unique one, where h_0 is the SM Higgs boson without the mixing. Therefore, the vertices linear in the Higgs field h_0 are absent when the radion-Higgs mixing vanishes. As the radion-Higgs mixing turns on, non-zero vertices of h - ϕ - $h_{\mu\nu}^{(n)}$ and h - ϕ - ϕ are generated, where h (ϕ) is physical Higgs (radion) state and $h_{\mu\nu}^{(n)}$ is the KK graviton. The vertex of h - ϕ - ϕ has been examined in the decay of $h \rightarrow \phi\phi$, which can be sizable in some parameter space [14]. However, investigating a vertex in a specific decay mode depends crucially on the mass spectrum of the Higgs boson and the radion. The decay $h \rightarrow \phi\phi$ is only possible when $m_h > 2m_\phi$. Moreover, the branching ratio of $h \rightarrow \phi\phi$ for a light radion (*e.g.*, for $m_\phi = 30$ GeV and $m_h = 120$ GeV) is below 10^{-3} , which may be too small for detection.

Instead, here we focus on the h - ϕ - $h_{\mu\nu}^{(n)}$ vertex by studying the associated production of the radion with the Higgs boson at the future e^+e^- colliders. As shall be shown below, this high-energy process has several advantages: (i) the observation of this rare process would exclusively probe the radion-Higgs mixing, (ii) the angular distribution could reveal the exchange of massive KK gravitons, (iii) the coupling strength of h - ϕ - $h_{\mu\nu}^{(n)}$ is much larger than the other KK-graviton-involved coupling ϕ - ϕ - $h_{\mu\nu}^{(n)}$, which also vanishes as $\xi \rightarrow 0$, and (iv) the SM background of $e^+e^- \rightarrow b\bar{b}b\bar{b}$ is small enough to easily detect the signal.

This paper is organized as follows. Section II summarizes the RS model and the basic properties of the radion-Higgs mixing. In Sec. III, the process of $e^+e^- \rightarrow h_{\mu\nu}^{(n)} \rightarrow h\phi$ is studied in detail. Section IV deals with the summary and conclusions.

II. REVIEW OF THE RANDALL-SUNDRUM MODEL AND RADION-HIGGS MIXING

In the RS scenario, a single extra dimension is introduced with non-factorizable geometry, which is compactified on a S^1/Z_2 orbifold with size b_0 [4]. Two orbifold fixed points

accommodate two three-branes, the Planck brane at $y = 0$ and our visible brane at $y = 1/2$. If the bulk cosmological constant Λ and the brane cosmological constants $V_{hid, vis}$ satisfy the relation of $\Lambda/m_0 = -V_{hid} = V_{vis} = -12m_0/\epsilon^2$, the following classical solution to Einstein equations respects the four-dimensional Poincare invariance:

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - b_0^2 dy^2, \quad (1)$$

where $\eta_{\mu\nu}$ is the Minkowski metric, $\sigma(y) = m_0 b_0 |y|$, and $y \in [0, 1/2]$. Here the five-dimensional Planck mass M_5 is denoted by $\epsilon \equiv 1/M_5^3$. The four-dimensional Planck mass on the visible brane, defined by $M_{Pl} \equiv 1/\sqrt{8\pi G_N}$, is

$$\frac{M_{Pl}^2}{2} = \frac{1 - \Omega_0^2}{\epsilon^2 m_0}, \quad (2)$$

where $\Omega_0 \equiv e^{-m_0 b_0/2}$ is known as the warp factor. Since our brane is arranged to be at $y = 1/2$, a canonically normalized scalar field has the mass multiplied by the warp factor, *i.e.*, $m_{phys} = \Omega_0 m_0$. Since the moderate value of $m_b b_0/2 \simeq 35$ can generate TeV scale physical mass, the gauge hierarchy problem is explained.

In the original RS scenario, the compactification radius b_0 is assumed to be constant: No quantum fluctuation about the extra-dimension size was considered [4]. However, the cosmological evolution in the RS scenario requires a fine-tuning between the densities on the two branes, otherwise the two branes blow apart, *i.e.*, $b_0 \rightarrow \infty$. The problem is that this fine-tuning leads to unconventional cosmology [6]. It is shown that a stabilization mechanism can naturally avoid this problematic fine-tuning, inducing a massive scalar corresponding to the gravitational fluctuation in the distance between two branes.

In the minimal RS model, in which all the SM fields are confined on the visible brane, new phenomenological ingredients are two kinds of gravitational fluctuations about the RS metric:

$$\eta_{\mu\nu} \rightarrow \eta_{\mu\nu} + \epsilon h_{\mu\nu}(x, y), \quad b_0 \rightarrow b_0 + b(x). \quad (3)$$

The five-dimensional gravitational field is expanded as a sum of KK modes given by

$$h_{\mu\nu}(x, y) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x) \frac{\chi^{(n)}(y)}{\sqrt{b_0}}, \quad (4)$$

and the canonically normalized radion field, $\phi_0(x)$, is

$$\phi_0(x) \equiv \left(\frac{12}{\epsilon^2 m_0}\right)^{1/2} \Omega_b(x) \simeq \sqrt{6} M_{Pl} \Omega_b(x), \quad (5)$$

where

$$\Omega_b(x) \equiv e^{-m_0[b_0+b(x)]/2}. \quad (6)$$

The compactification of the fifth dimension yields the four-dimensional interaction Lagrangian of the KK gravitons and the radion as

$$\mathcal{L} = -\frac{\phi_0}{\Lambda_\phi} T^\mu{}_\mu - \frac{1}{\widehat{\Lambda}_W} T^{\mu\nu}(x) \sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}(x), \quad (7)$$

where Λ_ϕ is the vacuum expectation value (VEV) of the radion field, $T^\mu{}_\mu$ is the trace of the symmetric energy-momentum tensor $T^{\mu\nu}$, and $\widehat{\Lambda}_W = \sqrt{2}M_{\text{Pl}}\Omega_0$.

Note that all the known gauge and discrete symmetries of the SM as well as the Poincare invariance on the visible brane do not prohibit the following gravity-scalar mixing [8, 14]:

$$S_\xi = \xi \int d^4x \sqrt{g_{\text{vis}}} R(g_{\text{vis}}) \widehat{H}^\dagger \widehat{H}, \quad (8)$$

where $R(g_{\text{vis}})$ is the Ricci scalar for the induced metric on the visible brane, $g_{\text{vis}}^{\mu\nu} = \Omega_b^2(x)(\eta^{\mu\nu} + \epsilon h^{\mu\nu})$, \widehat{H} is the Higgs field before re-scaling, i.e., $H_0 = \Omega_0 \widehat{H}$, and ξ quantifies the size of the mixing term. The free-field Lagrangian of the Higgs boson and radion is given by [14]

$$\mathcal{L}_0 = -\frac{1}{2} \{1 + 6\gamma^2 \xi\} \phi_0 \square \phi_0 - \frac{1}{2} \phi_0 m_{\phi_0}^2 \phi_0 - \frac{1}{2} h_0 (\square + m_{h_0}^2) h_0 - 6\gamma \xi \phi_0 \square h_0, \quad (9)$$

where

$$\gamma \equiv v_0/\Lambda_\phi, \quad (10)$$

and v_0 is the VEV of the Higgs boson around 246 GeV.

We introduce the states h and ϕ which diagonalize \mathcal{L}_0 , defined by

$$\begin{pmatrix} h_0 \\ \phi_0 \end{pmatrix} = \begin{pmatrix} 1 & 6\xi\gamma/Z \\ 0 & -1/Z \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix} \quad (11)$$

$$\equiv \begin{pmatrix} d & c \\ b & a \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix}, \quad (12)$$

where

$$Z^2 \equiv 1 + 6\xi\gamma^2(1 - 6\xi) \equiv \beta - 36\xi^2\gamma^2. \quad (13)$$

The first matrix in Eq. (11) diagonalizes the kinetic terms of the Lagrangian, which leads to the mass matrix of

$$\mathcal{L}_m = -\frac{1}{2} \begin{pmatrix} h' & \phi' \end{pmatrix} \begin{pmatrix} m_{h_0}^2 & 6\xi\gamma m_{h_0}^2/Z \\ 6\xi\gamma m_{h_0}^2/Z & (m_{\phi_0}^2 + 36\xi^2\gamma^2 m_{h_0}^2)/Z^2 \end{pmatrix} \begin{pmatrix} h' \\ \phi' \end{pmatrix}. \quad (14)$$

This symmetric mass matrix is further diagonalized by an orthogonal mass matrix with the mixing angle θ given by

$$\tan 2\theta = 12\gamma\xi Z \frac{m_{h_0}^2}{m_{\phi_0}^2 - m_{h_0}^2(Z^2 - 36\xi^2\gamma^2)}. \quad (15)$$

Note that in the RS scenario the radion-Higgs mixing is only suppressed by the $1/\Lambda_\phi$, which is of order of the electroweak scale, while in the large extra-dimensional model the mixing is severely suppressed by the Planck scale.

The eigenvalues for the square of masses are

$$m_\pm^2 = \frac{1}{2Z^2} \left\{ m_{\phi_0}^2 + \beta m_{h_0}^2 \pm \sqrt{(m_{\phi_0}^2 + \beta m_{h_0}^2)^2 - 4Z^2 m_{\phi_0}^2 m_{h_0}^2} \right\}, \quad (16)$$

where m_+ is the larger of the Higgs mass m_h and the radion mass m_ϕ . Since the mixing matrix in Eq. (12) is not unitary, there is an ambiguity which particle should be called the Higgs or radion. In the following, m_{h_0} is set to be the Higgs mass in the limit of $\xi \rightarrow 0$. We refer the reader to Ref. [14] for the detailed recipe to obtain m_{h_0} , m_{ϕ_0} , θ (thus a , b , c , and d in Eq. (12)) from the given γ , ξ , m_h and m_ϕ .

There are two algebraic constraints on the value of ξ . One is from the requirement that the square root of the inverse function of Eq. (16) is positive definite, leading to

$$\frac{m_+^2}{m_-^2} > 1 + \frac{2\beta}{Z^2} \left(1 - \frac{Z^2}{\beta} \right) + \frac{2\beta}{Z^2} \left[1 - \frac{Z^2}{\beta} \right]^{1/2}. \quad (17)$$

The second one comes from the condition $Z^2 > 0$:

$$\frac{1}{12} \left(1 - \sqrt{1 + \frac{4}{\gamma^2}} \right) \leq \xi \leq \frac{1}{12} \left(1 + \sqrt{1 + \frac{4}{\gamma^2}} \right). \quad (18)$$

All phenomenological signatures of the RS model including the radion-Higgs mixing are specified by five parameters

$$\xi, \quad \Lambda_\phi, \quad \frac{m_0}{M_{\text{Pl}}}, \quad m_\phi, \quad m_h, \quad (19)$$

which in turns determine $\hat{\Lambda}_W$ and KK graviton masses $m_G^{(n)}$ as

$$\hat{\Lambda}_W = \frac{\Lambda_\phi}{\sqrt{3}}, \quad m_G^{(n)} = x_n \frac{m_0}{M_{\text{Pl}}} \frac{\hat{\Lambda}_W}{\sqrt{2}}. \quad (20)$$

Here x_n is the n -th root of the first order Bessel function.

Some comments on the parameters in Eq. (19) are in order here. First the dimensionless coefficient of the radion-Higgs mixing, ξ , approaches $1/6$ in the conformal limit with $m_h \rightarrow 0$.

In general, the ξ is expected to be of order one. In most of the parameter space, Eq. (17) is crucial to bound ξ . The Λ_ϕ is also constrained since it is related by Eq. (20) to the masses and effective couplings of KK gravitons. The Tevatron Run I data of Drell-Yan process and the electroweak precision data were analyzed to constrain $m_G^{(1)} \gtrsim 600$ GeV, which corresponds $\Lambda_\phi \gtrsim 4$ TeV [15]. A reasonable range of the ratio m_0/M_{Pl} is believed to be $0.01 \lesssim m_0/M_{\text{Pl}} \lesssim 0.1$ since a large value of m_0/M_{Pl} would yield a large bulk curvature damaging the reliability of the RS solution [16]. In what follows, we consider the case of $\Lambda_\phi = 5$ TeV and $m_0/M_{\text{Pl}} \sim 0.1$ where the effect of radion on the oblique parameters is small [9]. About the Higgs boson mass, we could wonder if the radion-Higgs mixing can weaken the Z - Z - h coupling such that the LEP/LEP II may miss the Higgs boson with a mass below the current bound of 113 GeV. The answer is mostly negative due to an exact sum rule that the sum of the squares of Z - Z - h and Z - Z - ϕ couplings should be larger than the SM one [12]. Both couplings cannot be suppressed. For $m_h = 110$ GeV, LEP/LEP II data on the Higgs search exclude a large part of the parameter space while for $m_h = 120$ GeV they allow most of the parameter space [14]. In the following, we safely take the Higgs mass to be 120 GeV. The mass scale of the radion depends on a specific stabilization mechanism. As the simplest mechanism by Goldberger and Wise predicts the relation $m_{\phi_0} \sim \hat{\Lambda}_W/40$, the radion is generically light [5]. We notice that the decay mode of $h \rightarrow \phi\phi$, exclusively allowed for $\xi \neq 0$, can have sizable branching ratios for $m_\phi = 40 \sim 60$ GeV if $m_h = 120$. In order to provide complementary information through the process $e^+e^- \rightarrow h\phi$, we consider the cases of $m_\phi = 30, 70$ and 170 GeV.

The gravity-scalar mixing $\xi R \widehat{H}^\dagger \widehat{H}$ modifies the couplings among the h , ϕ and $h_{\mu\nu}^{(n)}$. In particular, the following four tri-linear vertices emerge, which would vanish if the mixing goes to zero,

$$h-\phi-\phi, \quad h_{\mu\nu}^{(n)}-h-\phi, \quad \phi-\phi-\phi, \quad h_{\mu\nu}^{(n)}-\phi-\phi. \quad (21)$$

Without the radion-Higgs mixing, the first two couplings linear in the Higgs field are prohibited since the unique minimum of the effective potential of the Higgs boson is the SM one, i.e., $\partial V_h/\partial h_0|_{(h_0)=0} = 0$. The triple coupling $\phi-\phi-\phi$, though allowed in a stabilization mechanism, is suppressed by a factor of m_ϕ/M_{Pl} [14]. The vertex of $h-\phi-\phi$ has been investigated in the decay $h \rightarrow \phi\phi$ [14]. Apart from the fact that the measurement of this decay mode is only possible when $m_h > 2m_\phi$, the branching ratio of $h \rightarrow \phi\phi$ for a light radion ($m_\phi \lesssim 30$ GeV and $m_h = 120$ GeV) is too small for detection. Therefore, it is important

$$\begin{aligned}
& \text{Diagram 1: } h_{\mu\nu}^{(n)} \text{ vertex with } k_3 \text{ and } k_2 \text{ incoming, } h \text{ and } \phi \text{ outgoing.} \\
& i \frac{4k_{1\mu}k_{2\nu}}{\Lambda_W} g_{Gh\phi} \equiv i \frac{4k_{1\mu}k_{2\nu}}{\Lambda_W} \left\{ 3\gamma\xi [a(\gamma b + d) + bc] + \frac{1}{2}cd \right\} \\
& \text{Diagram 2: } h_{\mu\nu}^{(n)} \text{ vertex with } k_3 \text{ and } k_2 \text{ incoming, } \phi \text{ and } \phi \text{ outgoing.} \\
& i \frac{4k_{1\mu}k_{2\nu}}{\Lambda_W} g_{G\phi\phi} \equiv i \frac{4k_{1\mu}k_{2\nu}}{\Lambda_W} \left\{ 3a\gamma\xi [a\gamma + 2c] + \frac{1}{2}c^2 \right\}
\end{aligned}$$

FIG. 1: Feynman rules for the tri-linear vertices involving $h_{\mu\nu}^n$, where we have made use of the symmetry of $h_{\mu\nu}^n$ under the interchange of $\mu \leftrightarrow \nu$ indices.

to study the scattering processes to uniquely probe the other vertices in Eq. (21), especially those involving the KK gravitons. In Fig. 1, we present the Feynman rules for the vertices $h_{\mu\nu}^{(n)}-\phi-\phi$ and $h_{\mu\nu}^{(n)}-h-\phi$.

In most of the parameter space, the coupling strength of $h_{\mu\nu}^{(n)}-h-\phi$ is much larger than that of $h_{\mu\nu}^{(n)}-\phi-\phi$. This can be easily shown by noticing that the parameter $\gamma \equiv v_0/\Lambda_\phi$ is very small with $\Lambda_\phi = 5$ TeV. In the limit of $\gamma \ll 1$, we have $a, d \sim \mathcal{O}(1)$ but $b, c \sim \mathcal{O}(\gamma)$, implying

$$g_{Gh\phi} \sim \mathcal{O}(\gamma), \quad g_{G\phi\phi} \sim \mathcal{O}(\gamma^2). \quad (22)$$

Figure 2 shows the ratio of $g_{Gh\phi}^2$ to $g_{G\phi\phi}^2$ as a function of ξ , for $m_\phi = 30$ and 70 GeV. The $g_{Gh\phi}^2$ is at least a few times to more than an order of magnitude larger than $g_{G\phi\phi}^2$. Even though numerically $g_{G\phi\phi}$ with $\Lambda_\phi = 5$ TeV ($\gamma \simeq 0.05$) is still not negligible, the $h_{\mu\nu}^{(n)}-h-\phi$ vertex has largest chance to probe the radion-Higgs mixing through high-energy scattering processes.

III. NUMERICAL ANALYSIS ON $e^+e^- \rightarrow h_{\mu\nu}^{(n)} \rightarrow h\phi$

For the process

$$e^-(p_1, \lambda_{e^-}) + e^+(p_2, \lambda_{e^+}) \rightarrow h(k_1) + \phi(k_2), \quad (23)$$

the helicity amplitudes $\mathcal{M}(\lambda_{e^-}, \lambda_{e^+})$ are

$$\begin{aligned}
\mathcal{M}(++) &= \mathcal{M}(--) = 0, \\
\mathcal{M}(+-) &= \mathcal{M}(-+) = \frac{g_{Gh\phi}}{2} \sum_n \frac{1}{1 - m_{(n)}^2/s} \frac{s}{\widehat{\Lambda}_W^2} \beta \sin 2\Theta.
\end{aligned} \quad (24)$$

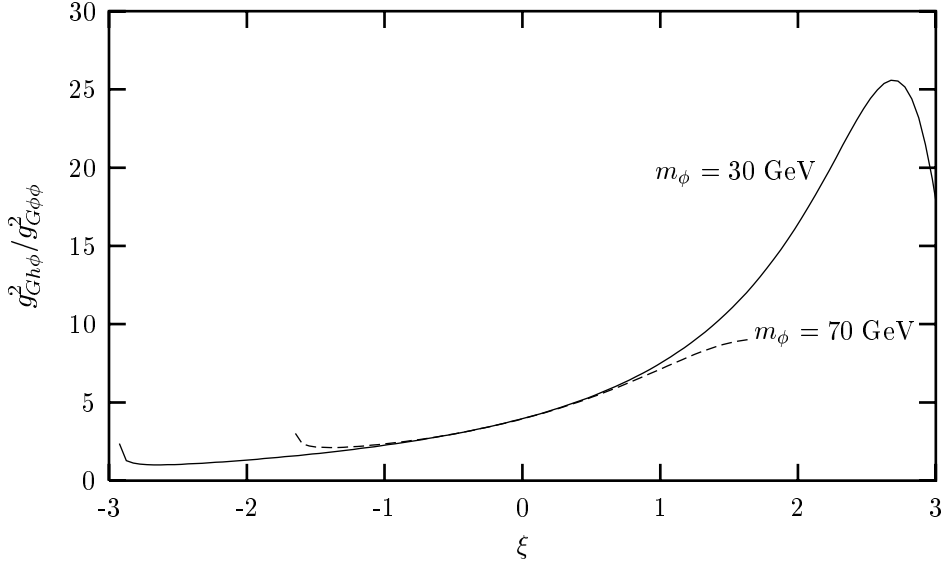


FIG. 2: The ratio of $g_{Gh\phi}^2$ to $g_{G\phi\phi}^2$ as a function of ξ with $\Lambda_\phi = 5$ TeV and $m_0/M_{\text{Pl}} = 0.1$.

Here λ_{e^-} (λ_{e^+}) is the helicity of the electron (positron), $\beta = 1 + \mu_h^4 + \mu_\phi^4 - 2\mu_h^2 - 2\mu_\phi^2 - 2\mu_h^2\mu_\phi^2$, $\mu_{h,\phi} \equiv m_{h,\phi}/\sqrt{s}$, and Θ is the scattering angle of the Higgs boson with respect to the electron beam.

The differential cross section is

$$\frac{d\sigma}{d\cos\Theta} = \frac{g_{Gh\phi}^2 \beta^{5/2}}{256\pi s} \left(\frac{s}{\widehat{\Lambda}_W^2} \right)^2 \left(\sum_n \frac{1}{1 - m_G^{(n)2}/s} \right)^2 \sin^2 2\Theta. \quad (25)$$

In the above equation, the sum is over the KK states of the graviton. With the input parameters, the mass $m_G^{(1)}$ of the first KK state is about 782 GeV and $m_G^{(2)}$ is about 1.43 TeV. Although the sum is over all states, the majority of the contributions comes from the first state. Figure 3 presents $d\sigma/d\cos\Theta$ in fb as a function of $\cos\Theta$ for $\xi = 0.5, 1$, and 1.6. We have set $m_\phi = 70$ GeV. The angular distribution proportional to $\sin^2 2\Theta$ is characteristic of the spin-2 KK graviton exchange, which is useful to discriminate from any new vector-boson-exchange or scalar-exchange contributions.

The total cross section is

$$\sigma_{tot} = \frac{g_{Gh\phi}^2 \beta^{5/2}}{240\pi s} \left(\frac{s}{\widehat{\Lambda}_W^2} \right)^2 \left(\sum_n \frac{1}{1 - m_G^{(n)2}/s} \right)^2. \quad (26)$$

In Fig. 4, we present the total cross section in fb as a function of ξ for $m_\phi = 30, 70, 170$ GeV in the parameter space of ξ allowed in Eq. (17). The σ_{tot} decreases with increasing m_ϕ due to kinematic suppression. With the anticipated luminosity of 1000 fb^{-1} , we have about a hundred events for $e^+e^- \rightarrow h\phi$ if the mixing parameter ξ is of order one.

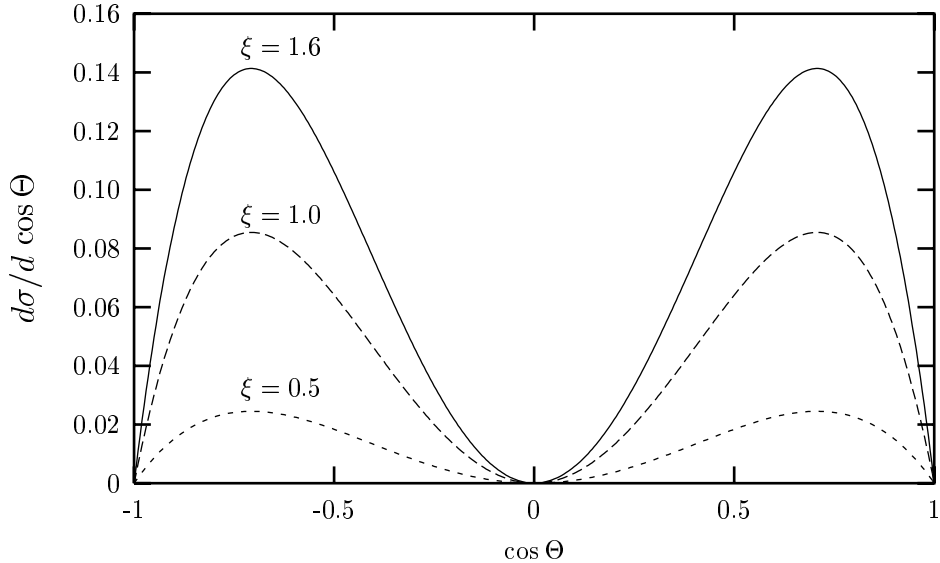


FIG. 3: The $d\sigma/d \cos \Theta$ in fb as a function of $\cos \Theta$ for $\xi = 0.5, 1,$ and 1.6 . We set $m_\phi = 70$ GeV with $\Lambda_\phi = 5$ TeV and $m_0/M_{\text{Pl}} = 0.1$.

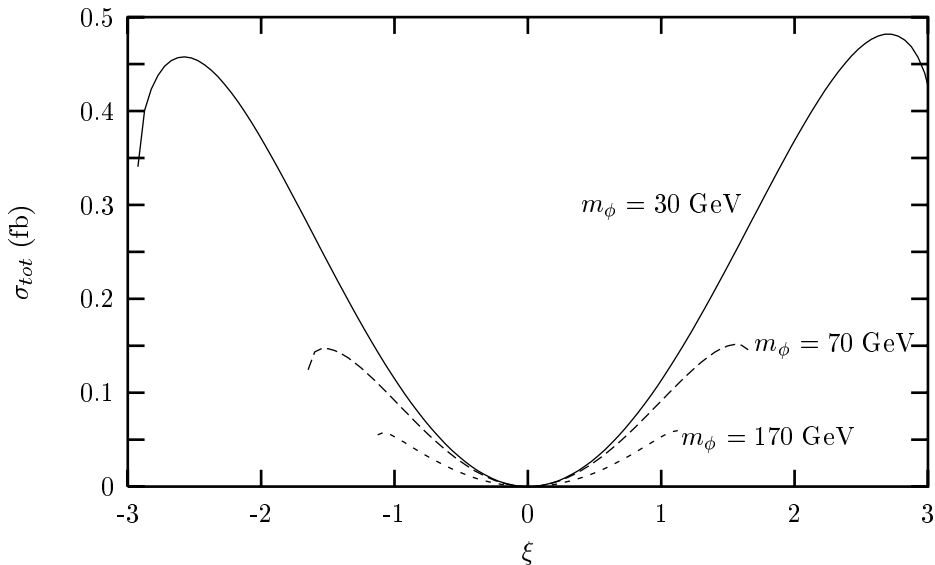


FIG. 4: The total cross section σ_{tot} in fb as a function of ξ for $m_\phi = 30, 70,$ and 170 GeV.

Some discussions on the SM backgrounds are in order here. Since the radion and Higgs boson decay promptly, the final state will consist of at least four particles. For $m_h = 120$ GeV, the radion-Higgs mixing does not change the dominant decay mode of the Higgs boson of $h \rightarrow b\bar{b}$ [11, 14]. For the radion with mass $m_\phi \lesssim 2m_W$, the decay mode into $b\bar{b}$ is dominant or second dominant, depending on the value of ξ . Therefore, the final state will likely be

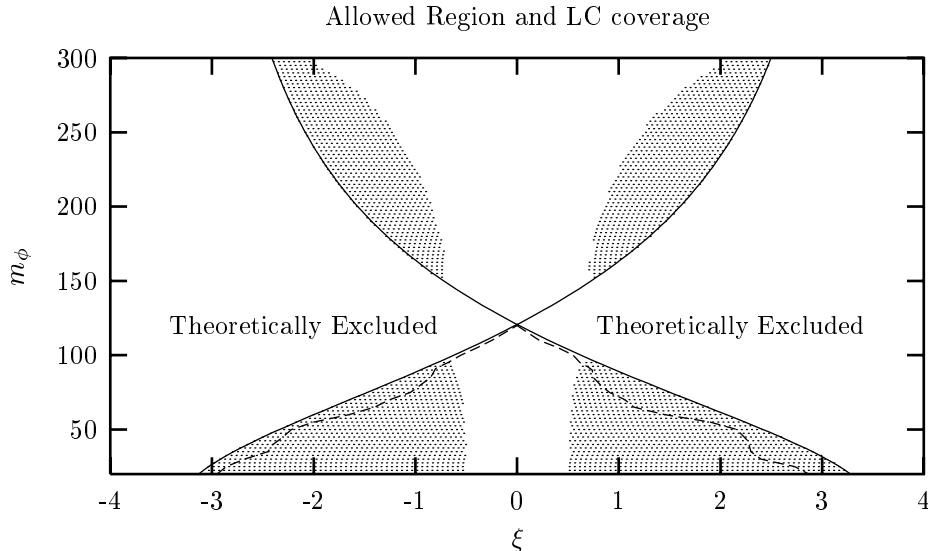


FIG. 5: The allowed region of (ξ, m_ϕ) by the algebraic constraints (inside the solid line) and the LEP/LEP2 data (inside the dashed line). The Higgs mass m_h is set to be 120 GeV. For $m_\phi > m_h$, the dashed line almost overlaps the solid line. The dotted regions are for $\sigma_{tot}(e^+e^- \rightarrow h\phi) > 0.03$ fb.

$b\bar{b}b\bar{b}$. We consider the SM background of $e^+e^- \rightarrow b\bar{b}b\bar{b}$. In order to regulate the collinear divergences and afford realistic detection criteria, we impose the selection cuts

$$p_{T_i} > 10 \text{ GeV}, \quad |\cos \theta_i| < 0.95, \quad \cos(\theta_{ij}) < 0.9, \quad (27)$$

where p_{T_i} is the transverse momentum of each jet, θ_i is the scattering angle of the final particle i , and θ_{ij} is the angle between the particles i and j . Additional cut on the invariant mass of two jets such as $m_{h,\phi} - 5 \text{ GeV} < m_{b\bar{b}} < m_{h,\phi} + 5 \text{ GeV}$ will further reduce the background cross section to the order of 0.001 fb at $\sqrt{s} = 500 \text{ GeV}$. The SM background is almost negligible. If the process $e^-e^+ \rightarrow h\phi$ has a total cross section of order of 0.01 fb, the future linear colliders (LC) will be able to probe the process exclusively sensitive to the radion-Higgs mixing.

Figure 5 summarizes the allowed parameter space of (ξ, m_ϕ) . First, the algebraic constraints in Eqs. (17) and (18) exclude substantial regions denoted by ‘Theoretically excluded’ in Fig. 5. The LEP and LEP II data exclude the region between the solid and dashed line. The dotted regions correspond to $\sigma_{tot}(e^+e^- \rightarrow h\phi) > 0.03$ fb. We have chosen a rather conservative criteria for sensitivity reach for such a small SM background. If $m_\phi < m_h$, the process of $e^+e^- \rightarrow h_{\mu\nu}^{(n)} \rightarrow h\phi$ at the future LC can probe the parameter space with

$|\xi| \gtrsim 0.5$. Even when $m_\phi > m_h$, although the cross section is limited by the kinematics, LC will be able to cover a quite substantial portion of the parameter space which the LEP/LEP II data cannot constrain.

If the Higgs boson mass is in fact heavier, say, 150 GeV, the sensitivity regions in the (ξ, m_ϕ) plane will be reduced, especially for the region $m_\phi > m_h$. An obvious reason is that the allowed kinematic phase space is reduced. Whereas in the region $m_\phi < m_h$, the sensitivity regions stay about the same.

IV. CONCLUSIONS

In the original Randall-Sundrum scenario in which all the SM fields are confined on the visible brane, the phenomenological signatures of the radion-Higgs mixing at e^+e^- colliders have been studied. In the warped geometry, the radion-Higgs mixing is weakly suppressed by the VEV of the radion at electroweak scale. It is known that the production and decay of the Higgs boson are modified due to the radion-Higgs mixing and thus the Higgs search strategy in the future colliders needs refinement. Complementarily, high energy processes exclusively allowed for non-zero mixing can also provide valuable information. We pointed out that there are four triple-vertices which would vanish without the radion-Higgs mixing. In particular, the vertex of $h_{\mu\nu}^{(n)}$ - h - ϕ has a large interaction strength, and involves the KK graviton. We studied the scattering process of $e^+e^- \rightarrow h_{\mu\nu}^{(n)} \rightarrow h\phi$ at the future linear e^+e^- colliders. The angular distribution proportional to $\sin^2 2\Theta$ has shown a unique feature of the exchange of massive spin-2 gravitons. Furthermore, in a large portion of the parameter space where LEP/LEP II data cannot constrain, the total cross section is large enough for sensitivity reach. We conclude that $e^+e^- \rightarrow h_{\mu\nu}^{(n)} \rightarrow h\phi$ is a very sensitive process to probe the radion-Higgs mixing in the Randall-Sundrum model.

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REFERENCES

- [1] See, *e.g.*, J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Reading, MA, 1990).
- [2] M. J. Duncan, G. L. Kane and W. W. Repko, Nucl. Phys. **B272**, 517 (1986).
- [3] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B436**, 257 (1998); N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. **B429**, 263 (1998).
- [4] L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
- [5] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999); W. D. Goldberger and M. B. Wise, Phys. Lett. **B475**, 275 (2000).
- [6] C. Csaki, M. Graesser, L. Randall and J. Terning, Phys. Rev. **D62**, 045015 (2000).
- [7] S. B. Bae, P. Ko, H. S. Lee and J. Lee, Phys. Lett. **B487**, 299 (2000).
- [8] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. **B595**, 250 (2001).
- [9] C. Csaki, M. L. Graesser and G. D. Kribs, Phys. Rev. **D63**, 065002 (2001); C. S. Kim, J. D. Kim and Jeonghyeon Song, Phys. Lett. **B511**, 251 (2001); C. S. Kim, J. D. Kim and Jeong-hyeon Song, arXiv:hep-ph/0204002 (Phys. Rev. **D**, in press).
- [10] U. Mahanta and S. Rakshit, Phys. Lett. **B480**, 176 (2000); K. Cheung, Phys. Rev. **D63**, 056007 (2001); U. Mahanta and A. Datta, Phys. Lett. **B483**, 196 (2000); S. C. Park, H. S. Song and J. Song, Phys. Rev. **D65**, 075008 (2002); S. C. Park, H. S. Song and J. Song, Phys. Rev. **D63**, 077701 (2001); C. S. Kim, Kang Young Lee and Jeonghyeon Song, Phys. Rev. **D64**, 015009 (2001).
- [11] M. Chaichian, A. Datta, K. Huitu and Z. h. Yu, Phys. Lett. **B524**, 161 (2002).
- [12] T. Han, G. D. Kribs and B. McElrath, Phys. Rev. **D64**, 076003 (2001).
- [13] J. L. Hewett and T. G. Rizzo, arXiv:hep-ph/0202155.

- [14] D. Dominici, B. Grzadkowski, J. F. Gunion and M. Toharia, arXiv:hep-ph/0206192.
- [15] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. **D63**, 075004 (2001).
- [16] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Lett. **B473**, 43 (2000).