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## $B_K$ from quenched overlap QCD\*

N. Garron<sup>a</sup><sub>cpt</sub> Centre de Physique Théorique<sup>†</sup>, Case 907, CNRS Luminy, F-13288 Marseille Cedex 9,  
 France, L. Cappi<sup>b</sup> and C. Thoburn<sup>c</sup> Dept. of Physics, Boston University, Boston, MA 02215, USA,  
 and C. Thoburn<sup>c</sup> Dept. of Physics, Boston University, Boston, MA 02215, USA, C. Howley<sup>a</sup><sub>cpt</sub>,  
 Avenue, Boston MA 02215, USA

<sup>a</sup>[

<sup>b</sup>[

<sup>c</sup>[

We present an exploratory calculation of the standard model  $\Delta S = 2$  matrix element relevant for indirect  $CP$  violation in  $K \rightarrow \pi\pi$  decays. The computation is performed with overlap fermions in the quenched approximation at  $\beta = 6.0$  on a  $16^3 \times 32$  lattice. The resulting bare matrix element is renormalized non-perturbatively. Our preliminary result is  $B_K^{\text{NDR}}(2 \text{ GeV}) = 0.61(7)$ , where the error does not yet include an estimate of systematic uncertainties.

### 1. Introduction

$K^0 - \bar{K}^0$  mixing induces indirect  $CP$  violation in  $K \rightarrow \pi\pi$  decays and is governed, in the standard model, by the matrix element

$$\langle \bar{K}^0 | \mathcal{O}_{\Delta S=2}(\mu) | K^0 \rangle = \frac{16}{3} M_K^2 F_K^2 \times B_K(\mu), \quad (1)$$

where  $\mathcal{O}_{\Delta S=2}$  is the four-quark operator given below (Eq. (2)) and where  $B_K$  parametrizes deviation from the vacuum saturation approximation.

The benefit of using overlap fermions [1] to compute this matrix element is their exact  $SU(N_f)_L \times SU(N_f)_R$  flavor-chiral symmetry at finite lattice spacing [2]. This implies that the  $\Delta S = 2$  operator is simple to construct on the lattice (unlike with staggered fermions) and does not mix with chirally dominant operators [3] (as it does with Wilson fermions). Overlap fermions further guarantee full  $\mathcal{O}(a)$ -improvement. On a different note, this exploratory computation tests the feasibility of calculating matrix elements of four-quark operators with overlap fermions.

### 2. Simulation details

We use Neuberger's overlap action [1]

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<sup>‡</sup>Unité Propre de Recherche 7061.

$$\mathcal{L} = a^4 \sum_x \bar{q} \left[ \left( 1 - \frac{1}{2\rho} am_q \right) D + m_q \right] q,$$

with

$$D = \frac{\rho}{a} \left( 1 + X / \sqrt{X^\dagger X} \right) \text{ and } X = D_W - \rho/a,$$

where  $D_W$  is the massless Wilson-Dirac operator.

The renormalized  $\Delta S = 2$  operator of Eq. (1) is related to the bare lattice operator through

$$\mathcal{O}_{\Delta S=2}(\mu) = Z_{\Delta S=2}(a\mu, g_0) \mathcal{O}_{\Delta S=2}^{\text{bare}}(g_0), \quad (2)$$

where the lattice operator with correct chiral properties is

$$\mathcal{O}_{\Delta S=2}^{\text{bare}} = [\bar{s}\gamma_\mu(1 - \gamma_5)\hat{d}][\bar{s}\gamma_\mu(1 - \gamma_5)\hat{d}],$$

with  $\hat{d} = (1 - aD/2\rho)d$ .

The computation is performed with the Wilson gauge action in the quenched approximation at  $\beta = 6.0$  ( $a^{-1}(r_0) \simeq 2.1 \text{ GeV}$ ) on a  $16^3 \times 32$  lattice with a statistics of 80 configurations and with  $\rho = 1.4$ . The pseudoscalar mesons simulated are composed of degenerate quarks with masses  $m_q = 0.04, 0.055, 0.07, 0.085, 0.1$  corresponding to a range from  $\sim m_s/2 \rightarrow 1.3m_s$ . Details regarding the implementation and the inversion of the overlap operator can be found in [4].

The three-point function used to extract the  $\Delta S = 2$  matrix element and its asymptotic time behavior are

$$\sum_{\mathbf{xy}} \langle J_0^L(x) \mathcal{O}_{\Delta S=2}^{bare}(0) J_0^L(y) \rangle \xrightarrow{T \gg y_0 > x_0 \gg 1} \quad (3)$$

$$\frac{|\langle K | J_0^L | 0 \rangle|^2}{(2M_K)^2} \langle \bar{K} | \mathcal{O}_{\Delta S=2}^{bare} | K \rangle \left\{ e^{M_K(y_0 - T - x_0)} + R_K \right. \\ \left. \times \left( e^{-M_K(x_0 + y_0) - \delta x_0} + e^{M_K(x_0 + y_0 - 2T) + \delta(y_0 - T)} \right) \right\},$$

with  $J_\mu^L = \bar{d} \gamma_\mu (1 - \gamma_5) \hat{s}$ . We allow for a time-reversed contribution because our lattice is rather short in the time direction and this contribution can be as large as 3-4% for our lightest quark mass at  $x_0 = T - y_0 = 8a$ , a point which is included in our fits. In Eq. (3)  $\delta$  measures the finite-volume shift in the energy of the two kaon state. This shift varies from about 2% to 5% in going from our heaviest to our lightest mass and is consistent with zero within roughly three to one standard deviations.  $R_K$  is clearly intimately related to  $\langle 0 | \mathcal{O}_{\Delta S=2}^{bare} | K K \rangle$ , but finite volume effects can be significant and we choose not to pursue the study of this time-reversed matrix element. The fit that we retain for determining the forward matrix element is to times  $x_0/a$  and  $y_0/a$  in the intervals  $6 \rightarrow 8$  and  $24 \rightarrow 26$  with  $y_0 > x_0$ . To obtain  $|\langle K | J_0^L | 0 \rangle|$  and  $M_K$  we compute two-point functions of left-handed currents which we symmetrize in time and fit in the time range  $5 \rightarrow 15$ .

The motivation for using left-handed currents is that they eliminate spurious zero-mode contributions which appear in finite volume and in the quenched approximation because the overlap operator satisfies an index theorem [5], as a good Dirac operator should. In our  $J_\mu^L$  correlators, topological zero modes,  $\phi_0(x)$ , contribute as

$$\gamma_\mu (1 - \gamma_5) \frac{\phi_0(x) \phi_0^\dagger(y)}{m_q} \gamma_\mu (1 - \gamma_5),$$

which vanishes because of the chirality of these modes. The use of left-handed currents is proposed in [6] where it is suggested that the weak interactions of Goldstone bosons be studied in the  $\epsilon$ -regime of Gasser and Leutwyler.

### 3. Chiral behavior of $B_K$

We obtain  $B_K^{bare}$  from

$$B_K^{bare} = \frac{3}{8} \frac{\langle \bar{K} | \mathcal{O}_{\Delta S=2}^{bare} | K \rangle}{\langle \bar{K} | \bar{J}_0^L | 0 \rangle \langle 0 | \bar{J}_0^L | K \rangle},$$

where  $\bar{J}_0^L$  is the conjugate of  $J_0^L$ . Quenched chiral perturbation theory (Q $\chi$ PT) predicts [7]

$$B_K = B \left[ 1 - 6 \left( \frac{M}{4\pi F} \right)^2 \ln \frac{M}{\Lambda_B} + b \left( \frac{M}{4\pi F} \right)^4 \right], \quad (4)$$

where  $B$ ,  $F$  and  $M$  are the leading order values of the  $B$ -parameter, leptonic decay constant and pseudoscalar mass, respectively.  $\Lambda_B$  is a scale which corresponds to  $\mathcal{O}(p^4)$  contact terms in the effective theory and the term proportional to  $b$  is added to parametrize likely higher order contributions. We fit this functional form to our results for  $B_K^{bare}$ , with  $M = M_K$ ,  $F = F_\pi^{phys}$  or  $F_K^{phys}$  and  $b \equiv 0$  or not. The fit parameters are  $B^{bare}$ ,  $\Lambda_B$  and  $b$  when it is not fixed to zero.<sup>3</sup> The results of these fits are shown in Fig. 1. We find that the mass-dependence of  $B_K$  is consistent with expectations from  $\chi$ PT, though results at lower masses and with better statistics would be necessary to confirm the coefficient of the logarithm. Until such results become available, we choose not to quote a number in the chiral limit. At the physical point  $M_K = M_K^{phys}$ , however, where the value of the  $B$ -parameter obtained is insensitive to the choice of functional form, these considerations are not important.<sup>4</sup>

### 4. Non-perturbative renormalization

We perform all renormalizations non-perturbatively in the RI/MOM scheme à la [8]. Thus we fix gluon configurations to Landau gauge and compute numerically appropriate, amputated forward quark Green's functions with legs of momentum  $p \equiv \sqrt{p^2}$ . Then we use the fact that with overlap fermions the renormalization constants  $Z_{JL} = Z_V$  and  $Z_{\Delta S=2} = Z_{VV+AA}$  to define

$$Z_{B_K}^{RI}(ap, g_0) = \frac{\Gamma_V(pa)^2}{\Gamma_{VV+AA}(pa)},$$

<sup>3</sup>Note that the definition of  $b$  changes with  $F$ .

<sup>4</sup>The result for  $B_K$  given below is obtained from the chiral fit with  $F = F_\pi^{phys}$  and  $b \neq 0$ .

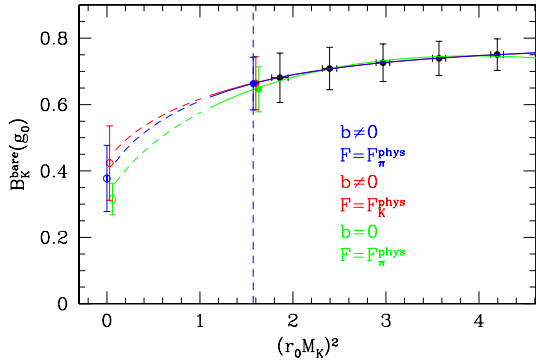


Figure 1. Chiral behavior of  $B_K$ . Fits are to Eq. (4), as described in the text. The vertical dashed line indicates the physical point.

where  $\Gamma_{\mathcal{O}}$  is the value of the non-perturbative, amputated Green's function of operator  $\mathcal{O}$  projected onto the spin-color structure of  $\mathcal{O}$ .

We find that in a range from approximately 1.5 to 2 GeV the  $p$ -dependence of  $Z_{B_K}^{\text{RI}}$  matches 2-loop running obtained by combining the results of [10] and [11], with  $N_f = 0$  and  $\alpha_s$  taken from [9]. This is shown in Fig. 2 where we plot, vs  $p$ ,  $Z_{B_K}^{\text{RGI}}$  which we define as the ratio of  $Z_{B_K}^{\text{RI}}$  to the 2-loop running expression. Also shown in Fig. 2 are the non-perturbative  $Z_{B_K}^{\text{RI}}(p)$  and its 1-loop counterpart  $Z_{B_K}^{\text{RI,PT}}(p)$ , obtained by expanding in  $\alpha_s$  the ratio of  $Z_{\Delta S=2}^{\text{RI,PT}}(p)$  to  $(Z_A^{\text{RI,PT}})^2$ . The latter are taken from [12] and are properly matched. The value of  $Z_{B_K}^{\text{RGI}}$  that we use is the result of fitting our data to a constant, in the momentum range shown in the plot. The renormalization constant in the  $\overline{\text{MS}}$  – NDR scheme is then computed through multiplying this value by the 2-loop running expression of [10] with  $N_f$  and  $\alpha_s$  chosen as above.

## 5. Conclusion

Though costly, it is entirely feasible to compute weak matrix elements with overlap fermions, at least in the quenched approximation. The advantages of this approach are the chiral symmetry and the  $\mathcal{O}(a)$ -improvement.

Our preliminary results are  $Z_{B_K}^{\text{RGI}} = 1.29(1)$  for the renormalization constant and, for the  $B$ -parameter:

$$B_K^{\text{RGI}} = 0.85(10), \quad B_K^{\text{NDR}}(2 \text{ GeV}) = 0.61(7),$$

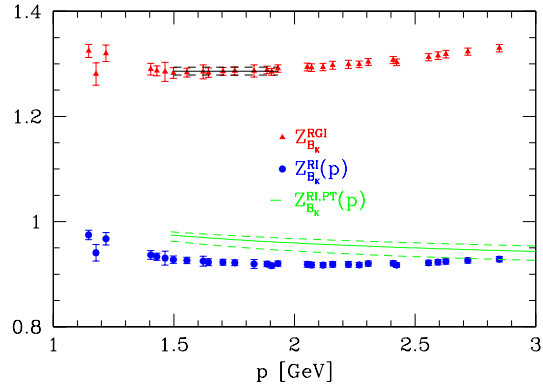


Figure 2. Scale dependence of the renormalization constants for  $B_K$  defined in the text. The three curves for  $Z_{B_K}^{\text{RI,PT}}(p)$  are obtained by allowing the scale parameter in the coupling of [9] to take the values  $\Lambda_{QCD}^{\overline{\text{MS}}}/2$ ,  $\Lambda_{QCD}^{\overline{\text{MS}}}$  and  $2\Lambda_{QCD}^{\overline{\text{MS}}}$ .

where the errors do not yet include an estimate of systematic uncertainties. For the  $B$ -parameter, agreement with continuum-limit world averages (see e.g. [13]) based on the staggered result of [14] is excellent. Agreement with the results of [15], obtained with a different formulation of overlap fermions, perturbative matching and larger quark masses is good. Finally, our result is respectively a half and one standard deviations above the results of [16] and [17], both obtained using domain-wall fermions with a finite fifth dimension.

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