Fragmentation functions using e$^+$e$-$ data from PETRA and LEP

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We investigate fragmentation of charged particles in e$^+$e$^-$ annihilation at 22, 35, and 44 GeV in terms of their polar angle and momentum distributions. From the angular distribution the ratio of longitudinal to total hadronic cross-section was determined at an average energy scale of 36.6 GeV to be $\sigma_L/\sigma_{\rm tot} = 0.076 \pm 0.013$. At next-to-leading order this yields $\alpha_S(36.6\ \rm GeV) = 0.150 \pm 0.025$. The $\xi \equiv \ln(1/x)$ distributions were used in conjunction with OPAL data from 91-208 GeV to study the scale dependence of the maximum position, ξ_0. We studied flavour dependence of ξ_0 as a possible explanation of this problem, using direct flavour dependent measurements of ξ_0 at 91 GeV by OPAL.

1. $\sigma_L/\sigma_{\rm tot}$ with JADE data

1.1. Introduction

The energy and the momentum spectrum of a hadron h produced in the annihilation process $\rm e^+e^- \to \gamma, Z^0 \to h + X$ is described by a fragmentation function $\mathcal{F}^h(x) \equiv (1/\sigma_{\rm tot}) \cdot ({\rm d}\sigma^{\rm h}/{\rm dx})$. With $x \equiv 2p/\sqrt{s}$, p is the momentum carried by the hadron h, and $\sqrt{s}$ is the centre-of-mass energy of the annihilation process with total hadronic cross-section $\sigma_{\rm tot}$. Ignoring polarisation effects the fragmentation function receives contributions from the transverse (T) and longitudinal (L) polarization states of the intermediate electroweak vector bosons, γ and $\rm Z^0$, and from their interference yielding an asymmetric contribution (A) [1]. At centre-of-mass energies much larger than the mass of the produced quark q, the longitudinal contribution is negligible [3]. A sizeable contribution to the longitudinal fragmentation function comes from gluon radiation from the $q\bar{q}$ system in the final state [1]. The asymmetric contribution vanishes, because we don't distinguisch quark and antiquark. The fragmentation functions are related to the perturbatively calculable ratios of the longitudinal, σ_L, and transverse, σ_T, cross-sections to the total cross-section. The contribution of gluon radiation to $\sigma_{T,L}/\sigma_{\rm tot}$ has been calculated in second order of α_S [4]. This allows tests of QCD and determinations of α_S from measurements of $\sigma_L/\sigma_{\rm tot}$.

1.2. Detector and data samples

We present in this paper a re-analysis of data recorded by the JADE detector [5] at the PETRA electron-positron collider. The data used for this study were recorded between 1979 and 1986 at centre-of-mass energies of $\sqrt{s}$ = 34-36 GeV and $\sqrt{s}$ = 43-45 GeV. Multihadronic events were selected according to the criteria described in [6]. For this analysis we used the JADE collaboration's original Monte Carlo samples of multihadronic events from the JETSET program version 6.3 [7] including a detailed simulation of the JADE detector.

1.3. Meaurement of $\cos\theta$ distribution

The distribution of $\cos\theta$ of all tracks of charged particles was measured, where θ is the polar angle between the direction of beam axis and the outgoing hadron h. The effects of limited acceptance and resolution of the detector were corrected using a bin-by-bin correction method. Fig. 1 shows the measured distribution of $q \cdot \cos\theta$ after application of the binwise correction factors which are shown in the upper part of the figure.

1.4. Determination of $\sigma_{\rm L}/\sigma_{\rm tot}$ and $\alpha_{\rm S}$

Assuming that at the hadron level the correction for neutral particles is identical for the longitudinal and transverse cross-sections, we can

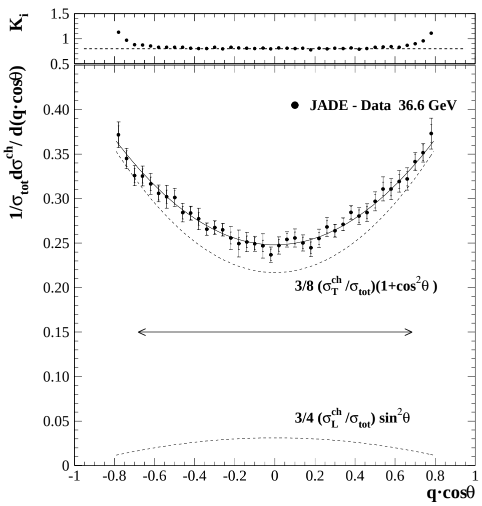

Figure 1. The distribution of $q \cdot \cos\theta$ is shown after correction for detector effects using the factors K_i presented in the upper part of the figure. The inner error bars are the statistical uncertainties and the outer bars are the total errors. The range considered for the fit is indicated by the arrow.

write :

$$\frac{1}{\sigma_{\rm tot}}\frac{{\rm d}\sigma^{\rm ch}}{{\rm d}({\rm q}\cdot\cos\theta)} = \frac{3}{8}\eta^{\rm ch}\left[\frac{\sigma_L}{\sigma_{\rm tot}}\left(1-3\cos^2\theta\right)+\left(1+\cos^2\theta\right)\right]. \quad (1)$$

The unknown parameters to be determined from a fit to the data are $\eta^{\rm ch}$, the correction factor for the total cross-section accounting for the neutral particles, and $\sigma_L/\sigma_{\rm tot}$. From substituting in Eq. (1) with the relation known in $\mathcal{O}(\alpha_S^2)$ [4].

$$\left(\frac{\sigma_L}{\sigma_{\rm tot}}\right)_{\rm PT} = \frac{\alpha_S}{\pi} + 8.444\left(\frac{\alpha_S}{\pi}\right)^2 \quad (2)$$

a formula can be obtained which allows for a direct determination of the strong coupling constant α_S at a renormalization scale $\mu = \sqrt{s}$. Values of

$$\frac{\sigma_L}{\sigma_{\rm tot}} = 0.067 \pm 0.011({\rm stat.}) \pm 0.007({\rm syst.})$$

were obtained for the longitudinal and transverse cross-sections relative to the total hadronic cross-section. Using the second order QCD prediction for the relative longitudinal cross-section, a value of

$$\alpha_S(36.6\ {\rm GeV}) = 0.150 \pm 0.020({\rm stat.}) \pm 0.013({\rm syst.}) \pm 0.008({\rm scal.})$$

was determined for the strong coupling.

2. $\ln(1/x)$ distribution between $\sqrt{s} = 22$ and 44 GeV

2.1. Introduction

Even though the shape of the momentum spectra cannot be calculated for the complete phase space, sound predictions have been made for the shape and the energy evolution of the $\xi \equiv \ln(1/x)$ distribution [11,12]. Destructive interference for soft gluon emission suppresses the production of particles with very low momentum thus turning the ξ distribution into an approximate gaussian shape at asymptotic energies [11]. The peak position, ξ_0, of this distribution is expected to depend in leading order linearly on $Y \equiv \ln(\sqrt{s}/2\Lambda_{\rm eff})$. Here $\Lambda_{\rm eff}$ is related to the Λ parameter of the running strong coupling constant but is not identical to it due to the approximations made in the calculation. Corrections in $\mathcal{O}(\alpha_S)$ to the asymptotic prediction yield a skewed gaussian shape for the ξ distribution next to its maximum [12]:[1]

$$F_q(\xi, Y) = \frac{N(Y)}{\sigma\sqrt{2\pi}} \cdot \exp\left(\frac{k}{8} - \frac{s\delta}{2} - \frac{(2+k)\delta^2}{4} + \frac{s\delta^3}{6} + \frac{k\delta^4}{24}\right), \quad (3)$$

where $N(Y)$ is a normalization related to the multiplicity of charged particles, $\delta \equiv (\xi - \langle\xi\rangle)/\sigma$ and [12]

$$\langle\xi\rangle \equiv \langle\xi(Y)\rangle = \frac{Y}{2}\left(1 + \frac{\rho}{24}\sqrt{\frac{48}{\beta Y}}\right)\cdot\left[1 - \frac{\omega}{6Y}\right] + \mathcal{O}(1) \quad (4)$$

with $\beta \equiv 11 - 2N_f/3$, $\rho \equiv 11 + 2N_f/27$, $\omega = 1 + N_f/27$, $C_A = 3$, $C_F = 4/3$, and N_f the number of active flavours, which is usually set to 3 since gluons predominantly split into a pair of the lightest quarks (u, d, s). The y-dependence of σ, s, k and $N(Y)$ can be found in references

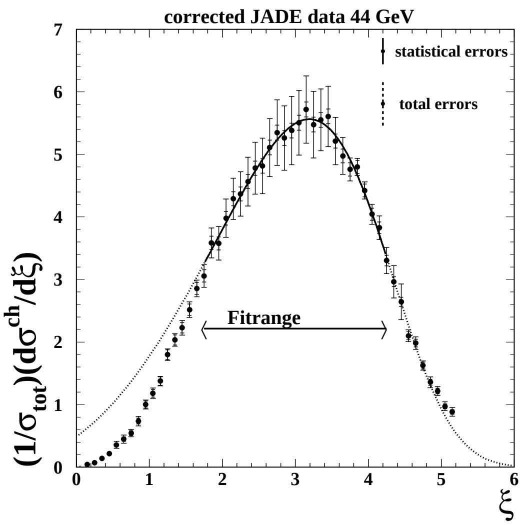

Figure 2. Measured ξ distribution for 44 GeV corrected for detector effects. The inner error bars are the statistical uncertainties and the outer bars are the total errors.

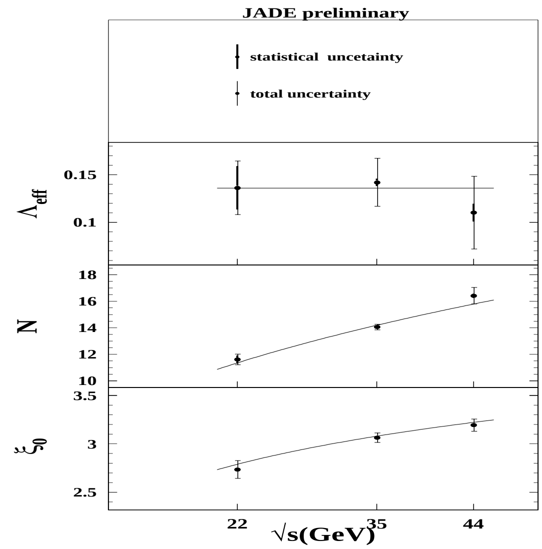

Figure 3. Results for $\Lambda_{\rm eff}$, N, ξ_0 versus the centre-of-mass energy $\sqrt{s}$. The curves from the NLLA calculations are overlaid.

[12]. The peak position $\xi_0(Y)$, is related to the mean value by the relation[1]

$$\xi_0 - \langle\xi\rangle \approx \frac{3\rho}{32C_A} \approx 0.35, \quad (5)$$

where the approximation is for large Y, and the numerical value is for $N_f = 3$ [12].

2.2. Detector and data samples

For the measurement of the ξ distribution, data recorded by JADE between 1979 and 1986 at centre-of-mass energies of $\sqrt{s} = 22$, 35 and 44 GeV are analyzed. Our investigation follows the same lines as the measurement of the longitudinal and transverse cross-sections. However we use here MC simulation data based on the PYTHIA 5.722 event generator [13] with parameters of [14].

2.3. Measurement of the ξ distribudion

All charged particles are used in the measurement of the ξ distribution. Fig. 2 shows the ξ distribution measured from tthe 44 GeV data set and corrected for the limited acceptance and resolution of the detector and for initial state radiation (ISR). The position of the maximum ξ_0 is determined by fitting Eq. (3) to the data. The fit considers three parameters N, $\Lambda_{\rm eff}$ and ξ_0. The remaining parameters of the skewed gaussian (3) are calculated using their y-dependance to be found in references [12]. When fitting ξ_0 the asymptotic relation (5) is employed to substitute $\langle\xi(Y)\rangle$ in Eq. (3). The final results of the fitted parameters [16] including statistical and systematic uncertainties are summarized in Fig. 3.
Values of
$\xi_0(22\text{GeV}) = 2.735 \pm 0.019(\text{stat.}) \pm 0.090(\text{syst.})$
$\xi_0(35\text{GeV}) = 3.064 \pm 0.003(\text{stat.}) \pm 0.049(\text{syst.})$
$\xi_0(44\text{GeV}) = 3.193 \pm 0.010(\text{stat.}) \pm 0.063(\text{syst.})$
were obtained at PETRA energies of 22, 35 and 44 GeV.

2.4. Investigation of flavour dependence of ξ distribution

ξ_0 for the inclusive ξ distribution can be written as a supperposition of ξ_0 for flavour sorted ξ distributions weighted with due to the different

[1]For simplicity the explicit Y dependence of δ, $\langle\xi\rangle$, ξ_0, σ, s, k, and k_5 is not exhibited.

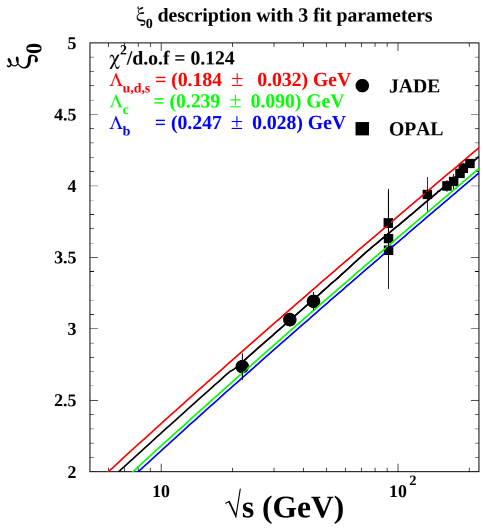

Figure 4. The measured position of the maximum ξ_0 is shown together with results at higher centre-of-mass energies. The curve is the expectation of QCD in NLLA, Eqs. (5) and (4) with a flavour dependent Λ_{eff}, this mean $\Lambda_{uds} \neq \Lambda_c \neq \Lambda_b$.

couplings of the intermediate photon and Z boson to the quarks center-of-mass energy dependent branching ratio for the corresponding flavour. We consider the meassured values of ξ_0 from JADE data between $\sqrt{s} = 22$ and 44 GeV and OPAL data between $\sqrt{s} = 91$ and 208 GeV. As constraints we consider the directly meassured values of ξ_0 for uds, c and b at 91 GeV by OPAL [15]. We assume that $\Lambda_{uds} \neq \Lambda_c \neq \Lambda_b$. With 3 fit parameters we described the energy evolution of ξ_0 for the inclusive ξ distribution as shown in figure 4. We meassured a differtent of 30% between Λ_b and Λ_{uds}.

3. Summary and conclusions

We measured with JADE data the longitutinal cross-section $\sigma_L/\sigma_{\rm tot}$ and the value of the strong copling constant α_S at 36.6 GeV and the peak position ξ_0 of the ξ distribution at 22, 35 and 44 GeV. Combining JADE and OPAL data we also investigated the flavour dependence of ξ_0.

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