

# Rotating D-branes and O-planes

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## Abstract

We review orientifold constructions in the presence of magnetic backgrounds both in the open and closed sectors. Generically, the resulting orientifold models have a nice geometric description in terms of rotated D-branes and/or O-planes. In the case of multiple magnetic backgrounds, some amount of supersymmetry is recovered if the magnetic fields are suitably chosen and part of the original D-branes and/or O-planes are transmuted into new ones.

## 1. Introduction

Homogeneous magnetic fields provide an interesting example of a non-trivial deformation compatible with two-dimensional conformal invariance. The study of their effect in String Theory is relatively simple for string coordinates are still described in terms of free fields and the magnetic deformations only affect the world-sheet dynamics by modifying boundary and periodicity conditions for open and closed strings.

Despite their simple rôle in world-sheet dynamics, magnetic fields lead to new interesting phenomena in D-branes and orientifold constructions. Considered at the very beginning as an interesting tool to break supersymmetry[1, 2], deeper investigations have shown how they can affect the geometry of branes and O-planes. As usual, vacuum expectations values of gauge fields, after appropriate T-dualities, admit a very elegant and simple geometrical interpretation: while Wilson lines correspond to brane displacements, magnetic backgrounds translate into brane/O-plane rotations[3, 4, 5, 6, 7]. As a result, the corresponding brane/O-plane configurations are no longer BPS and typically break supersymmetry. In lower dimensions, however, some amount of supersymmetry can be recovered if background fields, *i.e.* rotation angles, are chosen appropriately.

## 2. Open-string magnetic field

Let us start with open-string magnetic fields. As already stated, the study of their effect is relatively simple, for they interact only with the string ends [8].

For simplicity, let us consider the bosonic string in the presence of a uniform magnetic field  $H$  in a plane with coordinates  $(X^1, X^2)$ . The variational principle for the world-sheet action

$$\begin{aligned} \mathcal{S} = & \frac{1}{2\pi\alpha'} \int d\tau \int_0^\pi d\sigma \partial_\alpha X^a \partial^\alpha X^a \\ & + i q_L H \int d\tau \epsilon_{ab} X^a \partial_\tau X^b \Big|_{\sigma=0} + i q_R H \int d\tau \epsilon_{ab} X^a \partial_\tau X^b \Big|_{\sigma=\pi}, \end{aligned} \quad (1)$$

yields the wave equation

$$\left( \frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial \sigma^2} \right) X^a = 0, \quad (2)$$

together with the boundary conditions

$$\begin{aligned} \partial_\sigma X^a - 2\pi\alpha' q_L H \epsilon^{ab} \partial_\tau X_b &= 0, \\ \partial_\sigma X^a + 2\pi\alpha' q_R H \epsilon^{ab} \partial_\tau X_b &= 0. \end{aligned} \quad (3)$$

Eq. (3) admits an alternative geometric formulation in terms of rotated branes[3]. Indeed, after performing a T-duality along the  $X^2$  direction, so that  $\tau$  and  $\sigma$  derivatives

are interchanged,

$$\partial_\tau X^2 \rightarrow \partial_\sigma Y^2, \quad \partial_\sigma X^2 \rightarrow \partial_\tau Y^2, \quad (4)$$

the boundary conditions become standard Neumann and Dirichlet ones, say, at  $\sigma = 0$

$$\begin{aligned} \partial_\tau (\cos \theta X^1 - \sin \theta Y^2) &= 0, \\ \partial_\sigma (\sin \theta X^1 + \cos \theta Y^2) &= 0, \end{aligned} \quad (5)$$

for a rotated set of coordinates, where the angle  $\theta = \tan^{-1}(2\pi\alpha'q_L H)$  is determined by the magnetic field  $H$ .

The interesting feature of intersecting brane constructions that has triggered a massive interest is that, in general, chiral fermions emerge from strings with charged ends while supersymmetry is generically broken.

From eq. (5), however, it is not hard to see that something special can happen if one introduces a second magnetic field on a different two-plane, or, alternatively, if one further rotates the branes along another pair of directions. In this case, the choice  $\theta_1 = \theta_2 = \frac{1}{2}\pi$  yields a configuration of orthogonal D7-branes that preserve half of the supersymmetries. As a result, one can use magnetised, or rotated, branes to construct, for example, new  $T^4/\mathbb{Z}_2$  type I vacua as in [9].

As a remark, rotated branes are recently playing a pivotal role in brane world constructions, where the Standard Model degrees of freedom live at their intersections [10]

### 3. Compactification on Melvin spaces

We can now turn to the case of closed-string magnetic backgrounds, corresponding to non-vanishing vacuum expectation values for the graviphotons[11]. This background belongs to the class of Melvin metrics

$$ds^2 = G_{\mu\nu}(\gamma, R)dX^\mu dX^\nu = d\rho^2 + \rho^2 \left( d\phi + \frac{\gamma}{R} dy \right)^2 + dy^2 + dx_i dx^i, \quad (6)$$

where  $\gamma$  is related to the background magnetic field,  $(\rho, \phi)$  are polar coordinates on a two-plane,  $y$  is the compact coordinate on a circle of radius  $R$  and  $x^i$  are spectator coordinates.

According to eq. (6), the string dynamics in the Melvin space-time is governed by the world-sheet Action

$$\mathcal{S} = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau G_{\mu\nu}(\gamma, R) \partial_\alpha X^\mu \partial^\alpha X^\nu. \quad (7)$$

This model has a very simple equivalent description after introducing the complex coordinate

$$Z = \rho e^{i\varphi} = \rho e^{i(\phi + \frac{\gamma}{R}y)}, \quad (8)$$

that actually corresponds to a free boson

$$\left(\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\sigma^2}\right)Z = 0 \tag{9}$$

with the twisted boundary condition

$$Z(\tau, \sigma + 2\pi) = e^{2\pi i n \gamma} Z(\tau, \sigma), \tag{10}$$

where  $n$  is the winding number in the compact  $y$  direction.

As a result, we can use all the techniques of orbifold compactifications to solve exactly the model in eq. (7).

The background we are considering has the nice property of preserving the invariance of the IIB superstring under the world-sheet parity  $\Omega$ . In the spirit of [12], we can then proceed to construct the associated orientifolds, that, as we shall see, have interesting features [7].

Aside from the standard  $\Omega$  projection, that due to lack of space we shall not discuss here (see, however, [7]), one has actually the option of combining world-sheet parity with other geometrical symmetries as, for example, parities along the angular  $\varphi$  and compact  $y$  coordinates,  $\tilde{\Omega} = \Omega \Pi_\varphi \Pi_y$ . The resulting action on the world-sheet coordinates  $(y, Z)$  is

$$\tilde{\Omega}y(\tau, \sigma)\tilde{\Omega}^{-1} = -y(\tau, -\sigma), \quad \tilde{\Omega}Z(\tau, \sigma)\tilde{\Omega}^{-1} = Z^\dagger(\tau, -\sigma), \tag{11}$$

and, together with the identification (10), implies that

$$y = -y + 2\pi s R, \quad \varphi = -\varphi + 2\pi s \gamma + 2\pi m, \quad (s, m \in \mathbb{Z}) \tag{12}$$

whose fixed points

$$(y, \varphi)_1 = (s\pi R, s\pi\gamma), \quad (y, \varphi)_2 = (s\pi R, s\pi\gamma + \pi), \tag{13}$$

identify the positions of orientifold planes. Actually, we are now dealing with two infinite sets of rotated O-planes, the angle being proportional to the twist, or magnetic background,  $\gamma$ . Moreover, O-planes in different sets undergo a relative  $\pi$  rotation. As a result, they carry opposite R-R charge, a result that we shall soon recover from the corresponding Klein-bottle partition function.

In this orientifold construction, although the string partition function accounts for the whole spectrum of physical states, built from the vacuum with string oscillators and/or Kaluza-Klein modes, a proper derivation of the divergent contributions to the tree-channel amplitudes is quite involved. In fact, as we have seen, a key feature of Melvin compactifications is that the twist on the complex coordinate  $Z$  is coupled to the number of times

the closed string winds the compact coordinate  $y$ . This means that each winding state has a different corresponding “twisted sector”, and, as such, in the partition function the characteristics of the theta-functions are shifted accordingly. For example, the direct-channel Klein-bottle amplitude reads

$$\mathcal{K} \sim \frac{1}{\tau_2^{7/2} \eta^6} \sum_n \mathcal{K}(n) q^{\frac{1}{2\alpha'}(nR)^2}, \quad (14)$$

with

$$\begin{aligned} \mathcal{K}(0) &= \frac{1}{2} \frac{1}{\tau_2^{1/2} \eta^2} \sum_{\alpha, \beta} \frac{1}{2} \eta_{\alpha\beta} \frac{\vartheta^4 \left[ \begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right]}{\eta^4}, \\ \mathcal{K}(n) &= \sum_{\alpha, \beta} \frac{1}{2} \eta_{\alpha\beta} e^{-i\pi\gamma n(2\beta-1)} \frac{\vartheta^3 \left[ \begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right]}{\eta^3} \frac{\vartheta \left[ \begin{smallmatrix} \alpha+\gamma n \\ \beta \end{smallmatrix} \right]}{\vartheta \left[ \begin{smallmatrix} 1/2+\gamma n \\ 1/2 \end{smallmatrix} \right]}. \end{aligned} \quad (15)$$

(Here  $\eta_{\alpha\beta} = (-1)^{2\alpha+2\beta+4\alpha\beta}$  denotes the standard GSO projection.) Therefore, one can not perform explicitly the  $S$ -modular transformation, and the resulting amplitude does not manifest the usual physical interpretation.

However, divergent contributions originate only from massless states. Hence, to extract their tadpole it suffices to restrict ourselves to the massless Hamiltonian and to its Kaluza-Klein modification

$$H = \frac{\alpha'}{2} \left[ -\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) - \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} - \frac{\partial^2}{\partial y^2} + \frac{1}{R^2} \left( k + i\gamma \frac{\partial}{\partial \phi} \right)^2 \right]. \quad (16)$$

As a result, the leading contributions to the transverse-channel Klein-bottle amplitude can be extracted from

$$\langle c | e^{-\pi\ell H} | c \rangle = \sum_{\Psi} \langle c | \Psi \rangle e^{-\pi\ell E_{\Psi}} \langle \Psi | c \rangle, \quad (17)$$

where  $H|\Psi\rangle = E_{\Psi}|\Psi\rangle$ . Taking into account only the zero-mode contributions to the boundary state  $|c\rangle$ , one gets[7]

$$\langle c | e^{-\pi\ell H} | c \rangle_{\text{NS-NS}} \sim \sum_{m,k} [1 + (-1)^m] [1 + (-1)^k] e^{-\frac{\pi\alpha'\ell}{2R^2}(k-\gamma m)^2} \quad (18)$$

for the NS-NS sector, and

$$\langle c | e^{-\pi\ell H} | c \rangle_{\text{R-R}} \sim \sum_{m,k} [1 - (-1)^m] [1 + (-1)^k] e^{-\frac{\pi\alpha'\ell}{2R^2}(k-\gamma m)^2} \quad (19)$$

for the R-R sector, where  $k$  is the momentum quantum number and  $m$  is the quantised angular momentum on the two-plane. From eq. (18) we can then extract the non-vanishing dilaton tadpole (for  $m = 0$  and  $k = 0$ ), as well as informations on the geometry of the O-planes. The factor  $1 + (-1)^k$  suggests that in the  $y$  direction there are O-planes

sitting at 0 and  $\pi R$ , while the projector  $1 + (-1)^m$  implies that, in  $(\rho, \varphi)$  space, there are pairs of orientifold planes rotated by a  $\pi$  angle. Furthermore, from eq. (19) one can read that O-planes differing by a  $\pi$  rotation have opposite R-R charge, in agreement with (13). This phenomenon is reminiscent of the intersecting brane models discussed in the previous section. Although the overall R-R tadpole vanishes, cancellation of the NS-NS one calls for the introduction of D-branes, as in standard orientifold constructions [7].

Also in this case, one can generalise the construction considering multiple two-planes subject to independent twists. We have now the option to couple each twist to the same  $S^1$  or to different ones. While the latter choice corresponds to a trivial generalisation of the single twist previously reviewed, the former turns out to be quite interesting and leads to amusing phenomena.

To be more specific let us consider the case of two two-planes with twists  $\gamma_1$  and  $\gamma_2$ . A simple analysis of Killing spinors on this Melvin background[13] shows that whenever the  $\gamma_i$  are even integers all supersymmetry charges are preserved. However, one has the additional possibility of preserving only half of the original supersymmetries if

$$\gamma_1 \pm \gamma_2 \in 2\mathbb{Z}. \quad (20)$$

This is very reminiscent of the condition one gets for orbifold compactifications. Modding out by  $\Omega\Pi_y\Pi_{\varphi_1}\Pi_{\varphi_2}$  then introduces O-planes at the fixed points of the involution. Particularly interesting is the case  $\gamma_i = \frac{1}{2}$ . The resulting model is the traditional  $(\mathbb{C}^2 \times S^1)/\mathbb{Z}_2$  orbifold, where the  $\mathbb{Z}_2$  acts as a reflection on the  $\mathbb{C}^2$  coordinates accompanied by a momentum or a winding shift along the compact circle, and mutually orthogonal O6 planes are generated[7], as pertains to a  $(\mathbb{C}^2 \times S^1)/\mathbb{Z}_2$  orientifold.

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