QCD at high energy [1]

Stefano Frixione [a2]

[a]LAPTH, Annecy, France, and CERN, TH Division, Geneve, Switzerland

I review recent results in QCD at high energy, emphasizing the role of higher-order computations, power corrections, and Monte Carlo simulations in the study of a few discrepancies between data and perturbative predictions, and discussing future prospects.

1. INTRODUCTION

After more than 25 years of considerable theoretical and experimental efforts, it appears that QCD is the theory of strong interactions. Ideally, in high-energy QCD one needs one single piece of information from the experiments: the value of α_s. Starting from that measured value, every observable can be computed from first principles. In practice this is not feasible, since we don't know how to perform calculations in terms of the hadrons that experiments measure in their detectors. Perturbation theory offers a viable way out, since it allows to prove, at least formally, the so-called factorization theorems. These give explicit prescriptions to write physical observables as the convolution of short- and long-distance parts, up to terms suppressed by the power of some large scale. We can imagine this factorization to occur at an arbitrary scale μ; with a suitable choice of μ the short-distance pieces, which are entirely expressed in terms of quarks and gluons, are perturbatively calculable. The long-distance pieces (such as parton densities) cannot be computed in perturbation theory, but their dependence on μ can. Furthermore, they are universal, which means that they don't depend on short-distance physics, but solely on the nature of the hadrons involved, which is a key factor for perturbative QCD to have predictive power.

Pending a general solution of QCD, the computing framework based on perturbation theory may be regarded as a hypothesis, which needs to be supported, or disproved, by experimental observations. Countless tests have indeed been successful, convincing us of the correctness of this approach and of the capability of QCD to describe strong interactions; in many areas precise measurements, rather than tests, are being carried out. This success may give to the non-expert the impression that current efforts in theoretical QCD are perhaps technically appealing, but not compelling physics-wise. To counter this view, it is worth reminding that in the past decade the studies of several technically-involved problems, such as computations to next-to-leading order accuracy, resummation techniques, and Monte Carlo simulations, have been key factors to the outstanding achievements of LEP, SLC, HERA, and Tevatron. However, the solutions devised so far are not sufficient any longer. For an improvement of the accuracy in the extraction of α_s, for a deeper understanding of the interplay between perturbative and non-perturbative physics, and for a realistic modelling of Tevatron Run II and LHC physics, new ideas and computations are necessary. It must be clear that such investigations are not only relevant to the study of QCD itself, but also to a variety of other issues, from SM precision tests to searches of beyond-the-SM physics. Besides, a few unsatisfactory results remain in QCD, which deserve further studies.

Needless to say, this review cannot be complete, and I'll have to leave out several interesting results, such as new NLO calculations, progress in small-x physics, diffraction, fully numerical com-

[1]Plenary talk at XXXI International Conference on High Energy Physics (ICHEP), Amsterdam, July 2002.

[2]On leave of absence from INFN, Sez. di Genova, Italy.

putations, and spin physics. Also, I'll not present most of the recent experimental results in high-energy QCD, since they can be found in K. Long's writeup [1]. I'd rather use a few phenomenological examples to discuss some theoretical advancements, and open problems. Related topics can be found in Z. Bern's writeup [2]. I'll quote papers submitted to this conference as [S-NNN], S and NNN being the session and paper numbers respectively.

2. HEAVY FLAVOURS

Heavy flavour production is one of the most extensively studied topics in QCD. An impressive amount of data is available, for basically all kinds of colliding particles. The non-vanishing quark mass allows the definition of open-heavy-quark cross sections (whereas for light quarks one must convolute the short-distance cross sections with fragmentation functions, in order to cancel final-state collinear divergences). On the other hand, the presence of the mass makes the calculation of the matrix elements more involved. The breakthrough was the computation of total $Q\bar{Q}$ hadroproduction rates to NLO accuracy [3, 4], readily extended to other production processes and more exclusive final states [5, 6, 7, 8, 9, 10, 11, 12]. The resummation of various classes of large logarithms affecting these fixed-order computations, such as threshold, large-$p_{\rm T}$, and small-x logs, has also been accomplished, typically at the next-to-leading log (NLL) accuracy.

In those kinematical regions not affected by large logs, the mass of the heavy quark sets the hard scale. Furthermore, the impact of effects of non-perturbative origin (such as colour drag or intrinsic $k_{\rm T}$) is known to be larger the smaller the quark mass and CM energy. Thus, top physics is expected to be the ideal testing ground for perturbative computations. The agreement between NLO results [3] (dashed lines – the band is the spread of the prediction due to scale variation), and Tevatron Run I data [13, 14], shown in fig. 1 for total $t\bar{t}$ rates, appears in fact to be satisfactory. The inclusion of soft-gluon effects (solid lines), resummed to NLL accuracy according to

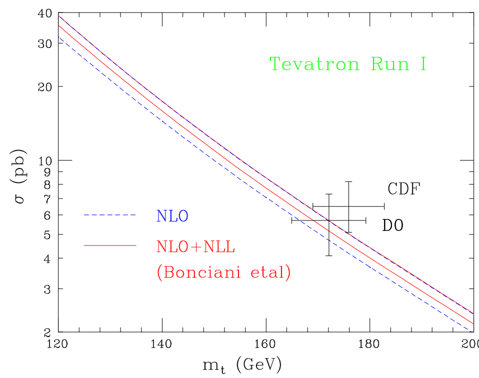

Figure 1. Total $t\bar{t}$ rate at the Tevatron.

the computation of ref. [15], is seen to increase only marginally the NLO prediction, while sizably reducing the scale uncertainty. Top production appears therefore under perturbative control. More stringent tests will be performed in Run II: the errors on mass and rate will be smaller, and measurements will be performed of more exclusive $t\bar{t}$ observables and of single-top cross section (for which fully differential NLO results are now available [16]).

Bottom quarks are copiously produced at colliders, and precise data for single-inclusive distributions have been available for a long time. It is well known (see ref. [17] for a review) that NLO predictions are about a factor of two smaller than data at the SpS and at the Tevatron (on the other hand, the shape of the $p_{\rm T}$ spectrum of the centrally-produced b is fairly well described by QCD). In a recently-published measurement [18] of the B^+ $p_{\rm T}$ spectrum, CDF find that the average data/theory ratio is $2.9 \pm 0.2 \pm 0.4$. However, this worrisome result is largely due to an improper computation of the NLO cross section. Let me remind that the spectrum of a b-flavoured meson B is computed as follows:

$$\frac{d\sigma_B}{dp_{\rm T}} = \int dz d\hat{p}_{\rm T} D(z;\epsilon) \frac{d\sigma_b}{d\hat{p}_{\rm T}} \delta(p_{\rm T} - z\hat{p}_{\rm T}), \tag{1}$$

where $\hat{p}_{\rm T}$ ($p_{\rm T}$) is the transverse momentum of b (B), $d\sigma_b$ is the cross section for open-b production, and $D(z;\epsilon)$ is the non-perturbative fragmentation function (NPFF), which describes the $b \to B$ fragmentation. NPFF is not calculable from first principles, and the free parameter it contains (ϵ) is fitted to data after assuming a functional form in z (such as Peterson [19], Kartvelishvili [20], etc). This fit is typically performed using eq. (1), identifying the l.h.s. with e^+e^- data. It follows that the value of ϵ is strictly correlated to the short-distance cross section $d\sigma_b$ used in the fitting procedure, and thus is *non-physical*. When eq. (1) is used to predict B-meson cross sections, it is therefore mandatory to make consistent choices for ϵ and $d\sigma_b$. This has not been done in ref. [18]: for $d\sigma_b$, the NLO result of ref. [8] is used, but the value of ϵ adopted (0.006) has been derived in the context of a LO, rather than NLO, computation. On the other hand, if a more appropriate value of ϵ is chosen ($\sim$0.002 [21]), the theoretical prediction increases by a mere 20% [22], still rather far from the data.

There are, however, a couple of observations which save the day. First, one has to remark that in the upper end of the $p_{\rm T}$ range probed by CDF ($p_{\rm T} \sim 20$ GeV), large-log $p_{\rm T}/m$ effects may start to show up. Therefore, FONLL computations [23] should be used rather than NLO ones. The FONLL formalism consistently combines (i.e., avoids overcounting) the NLO result with the cross section in which $p_{\rm T}/m$ logs are resummed to NLL accuracy (such resummed cross section is sometimes incorrectly referred to as "massless"). Thus, FONLL can describe both the small-$p_{\rm T}$ ($p_{\rm T} \sim m$, where resummed results don't make sense) and the large-$p_{\rm T}$ ($p_{\rm T} \gg m$, where NLO results are not reliable) regimes. The second observation concerns again the NPFF: $d\sigma_b/d\hat{p}_{\rm T}$ is a rather steeply falling function, and one can approximate it with $C/\hat{p}_{\rm T}^N$ in the whole $\hat{p}_{\rm T}$ range; then (from eq. (1)) $d\sigma_B/dp_{\rm T} = D_N C/p_{\rm T}^N$, where $D_N = \int dz z^{N-1} D(z)$ is the N^{th} Mellin moment of the NPFF. This fact has been noticed some time ago [17], and $D_N C/p_{\rm T}^N$ is seen to approximate the exact result fairly well [24]. Since $N = 3$–5 (at the Tevatron), it follows that, in order to have an accurate prediction for the $p_{\rm T}$ spectrum in hadroproduction, it is mandatory that the first few Mellin moments computed with $D(z)$ agreed with those measured. In ref. [22], it is pointed out that this is not the case, in spite of the fact that the prediction for the inclusive b cross section in e^+e^- collisions, obtained with *the same* $D(z)$, displays an excellent agreement with the data. There may seem to be a contradiction in this statement: if the shape is reproduced well, why this is not true for Mellin moments? The reason is that when fitting $D(z)$ one excludes the region of large z, since it is affected by Sudakov logs, and by complex non-perturbative effects which are unlikely to be described by the NPFF. On the other hand, the large-z region is important for the computation of D_N (because of the factor z^{N-1} in the integrand). Therefore, for the purpose of predicting B-meson spectra at colliders, ref. [22] advocates the procedure of fitting the NPFF directly in the N-space. A fit to the second moment (denoted as $N = 2$ fit henceforth) is found to fit well all the D_N's for N up to 10; and although Kartvelishvili's form is used, Peterson's gives comparable results. Using the FONLL computation, and a $N = 2$ fit for the NPFF, the average data/theory ratio reduces to $1.7 \pm 0.5 \pm 0.5$ [22]. Taking the scale uncertainty into account, B^+ data appear to be compatible with QCD predictions (see fig. 2).

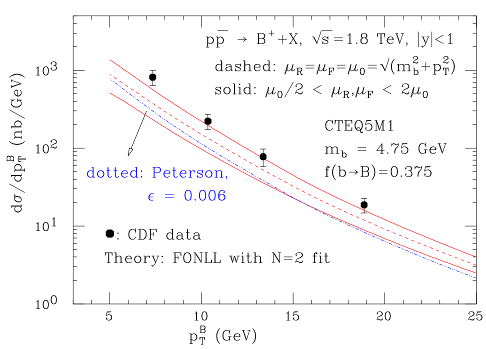

Figure 2. B^+ data vs theory [22].

If one wants to avoid the pitfalls of NPFF's, an alternative possibility consists in considering b-jets rather than B mesons, since in this case the NPFF simply doesn't enter the cross section. The comparison between NLO predictions for b-jets [25] and D0 measurements [26] is indeed satisfactory: data are consistent with theory in the range

$25 < E_{\mathrm{T}}^{b-jet} < 100$ GeV. Overall, one can conclude that b data at the Tevatron are reasonably described by NLO QCD. It is worth mentioning that some existing results, presented in terms of b-quark cross sections, are likely affected by the findings of ref. [22], and need to be reconsidered. Among the various mechanisms which can further increase the theoretical predictions, small-x [27, 28] and threshold resummations [15] will probably play a secondary role wrt NNLO contributions, which are expected to be large given the size of the K-factor at the NLO.

I now turn to the case of charm production. A thorough discussion on this topic is beyond the scope of this review, and I'll only give the briefest of the summaries (which will not do any justice to the field). LEP data for total rates are in agreement with NLO QCD predictions [7]; the shapes of single-inclusive D^* spectra in $\gamma\gamma$ collisions are as predicted by NLO QCD [29], whereas normalization is off by a factor 1.5–2, but still consistent with QCD when theoretical uncertainties are taken into account. The vast majority of fixed target hadro- and photoproduction data are well described by NLO computations, but only if predictions for single-inclusive distributions and correlations are supplemented by some parametrizations of non-perturbative phenomena (such as intrinsic k_{T}). At HERA, DIS data are in agreement with NLO QCD results [12]. In photoproduction, some concerns have arisen in the past because of the discrepancy between ZEUS and H1 measurements in the comparison with theory: H1 [30] appears to be in perfect agreement with QCD, whereas ZEUS [31] is at places (for intermediate p_{T}'s and large η's) incompatible with NLO predictions. ZEUS have submitted to this conference [5-786] data with unprecedented coverage at large p_{T}. The comparison to FONLL predictions [32, 33], shown in fig. 3, appears to be satisfactory, although data are marginally harder than theory. The agreement improves if a $N = 2$ fit for the NPFF is adopted (this result is still preliminary).

Let me finally mention the increasing amount of measurements for b rates from fixed-target [34], HERA [35, 36], [5-783,5-784,5-785,5-1013,5-1014], and LEP [37], [5-366,5-475] experiments.

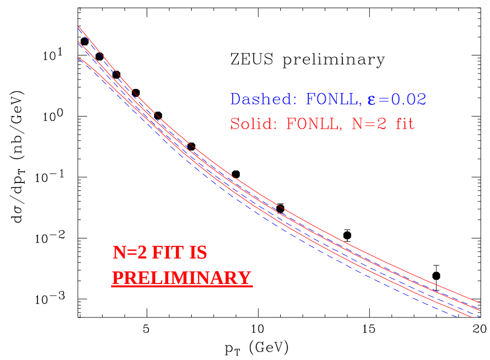

Figure 3. ZEUS D^* data in γp collisions vs FONLL predictions

A summary of the situation, in the form of ratios data/NLO QCD, is presented in fig. 4. While the fixed-target measurements are in overall agreement with QCD, HERA and LEP measurements are largely incompatible with theory. I find this hard to reconcile with the results presented so far, and the size of the discrepancy also makes it problematic to find an explanation in terms of beyond-the-SM physics, let alone higher orders in QCD. It is necessary to note that in many cases

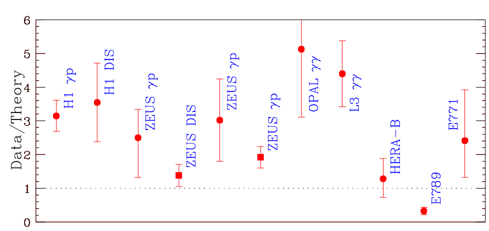

Figure 4. Ratios data/theory for b rates.

the experimental results are extrapolated to the full phase space from a rather narrow visible region. It is encouraging to note that in the cases of the recent ZEUS measurements in DIS [5-783] and photoproduction [5-785] (full boxes in fig. 4), results for single-inclusive distributions are given too, which are seen to be fully compatible with the corresponding NLO predictions.

3. POWER CORRECTIONS

The necessity of understanding long-distance effects in final-state measurements is not peculiar to b hadroproduction. The hadron-parton duality assumption states that there is a class of observables (such as jet variables or event shapes in e^+e^- collisions) whose description in terms of quarks and gluons is expected to reproduce the data, up to terms suppressed by some inverse power of the hard scale Q of the process (power corrections). These terms are usually estimated by comparing the parton- and hadron-level predictions of Monte Carlo (MC) generators. This procedure is not really satisfactory, since MC parameters are tuned to data (which creates a bias on the "predictions" for power-suppressed effects), and since the definition of parton- and hadron-level is far from being straightforward.

It is remarkable that we can get insight on non-perturbative physics from perturbative considerations. The perturbative series, being asymptotic, can be summed to all orders only after defining a summation procedure (in a rather arbitrary manner); one assumes that this technical manipulation mimics the role played in Nature by non-perturbative effects, which are necessary for QCD to be self-consistent. The summation procedure must eliminate the divergence of the perturbative series, but some finite quantities are left unconstrained. Thus, the idea is to use the ambiguities of the summation procedure to study non-perturbative effects. Although the regularization of the divergence can be technically very complicated, it can always be seen as a prescription to deal with the Landau pole of $\alpha_{\rm s}$. The idea of ref. [38, 39] (DMW from now on) is to bypass such a prescription by *defining* $\alpha_{\rm s}$ in the infrared (IR) region, assuming its universality; thus, $\alpha_{\rm s}$ should effectively measure confinement effects in inclusive quantities (in order to use such an $\alpha_{\rm s}$ in actual computations, it is also necessary to assume that the concepts of quarks and gluons still make sense in the IR). After giving the gluon a fake ("trigger") mass μ, the (fully inclusive) observable under study is computed in perturbation theory; the small-μ behaviour determines the power p of the leading power-suppressed term A_p/Q^p. The coefficient A_p cannot be computed, but can be expressed as an integral of $k^{p-1}\alpha_{\rm s}(k)$ over $0 < k < \mu_{\rm I}$, with $\mu_{\rm I} \sim \mathcal{O}(\text{GeV})$. Since all the (non computable) power-correction effects are contained in this coefficient, one effectively gets a factorization formula. A class of observables of great physical relevance is that of event shapes for which $p = 1$; their mean values can be computed with the DMW approach. The property of $\alpha_{\rm s}$ universality is *formally* even more far-reaching; for example, it implies that the non-perturbative effects it describes exponentiate for observables that do so [40, 41, 42]. This offers the possibility of studying not only mean values, but distributions. The results of DMW for mean values can then be recovered by an expansion of the (Sudakov) exponent and subsequent average. More precisely, if $\mathcal{T}$ denotes the observable, the Sudakov is expanded in the region $\mu_{\rm I}/Q \ll \mathcal{T} \ll 1$, and only the first non-trivial term in the expansion is kept.

In this context, factorization derives from the hypothesis of $\alpha_{\rm s}$ IR universality, which is an effective description of long-distance effects, and as such must be insensitive to the details of parton dynamics. For example, these details are irrelevant when one sums inclusively over the decay products of any parton branchings. It has been observed [43] that inclusiveness is lost to a certain extent when recoil effects (i.e., higher orders) are considered, and thus factorization breaks down in this case. One may still insist that factorization holds, and devise a procedure to systematically account for those effects which would break it in a naive treatment [44, 45, 46]. The results for mean values and distributions of event shapes can be written as follows:

$$\langle\mathcal{T}\rangle = \langle\mathcal{T}\rangle_{pert} + c_{\mathcal{T}}\mathcal{P}, \tag{2}$$

$$\frac{d\sigma}{d\mathcal{T}}(\mathcal{T}) = \left.\frac{d\sigma}{d\mathcal{T}}\right|_{pert}(\mathcal{T} - c_{\mathcal{T}}\mathcal{P}), \tag{3}$$

$$\mathcal{P} = \frac{4C_F}{\pi^2}\mathcal{M}\frac{\mu_{\rm I}}{Q}\Big[\alpha_0(\mu_{\rm I}) - \alpha_{\rm s}(Q) + \mathcal{O}(\alpha_{\rm s}^2(Q))\Big], \tag{4}$$

$$\mu_{\rm I}\alpha_0(\mu_{\rm I}) = \int_0^{\mu_{\rm I}} dk\,\alpha_{\rm s}(k). \tag{5}$$

Here, $c_{\mathcal{T}}$ is a computable coefficient, "pert" means perturbatively-computed, and $\mathcal{M}$ includes the two-loop results of refs. [44, 45, 46]; in principle, $\mathcal{M}$ can depend upon $\mathcal{T}$, but (accidentally) it does not. The $\alpha_s(Q)$ and $\alpha_s^2(Q)$ terms in eq. (4) are there to avoid double counting with the perturbatively-computed part, which shows again that long- and short-distance effects are correlated. Since no small parameter is involved in the two-loop computation of $\mathcal{M}$, one may wonder whether factorization could be spoiled beyond two loops. Although it is *argued* that this is not the case [44], a comparison with the data is mandatory. Also notice that the formulae above need to be modified when more complicated kinematical effects have to be described, as in the case of broadenings [47]. Eqs. (2)–(4) are used to fit the data in terms of $\alpha_s(Q)$ and $\alpha_0(\mu_I)$. Updated analyses relevant to e^+e^- collisions have been presented to this conference [5-228,5-229,5-389], [48] – see also [49]. Results obtained from mean values are satisfactory, with comparable α_0's obtained from different observables, and $\alpha_s(M_Z)$ values fairly consistent with the world average. The situation worsens in the case of distributions: α_0 universality holds at $\sim$25% level (1–2 σ), and $\alpha_s(M_Z)$ values are systematically lower than the world average, especially for observables such as wide broadening B_W and heavy jet mass M_H. This fact is disturbing since the same data for distributions, with hadronization effects described by MC's, return $\alpha_s(M_Z)$ values in much better agreement with the world average. These findings are summarized in fig. 5, where the results obtained with the DMW approach are shown as boxes and crosses for mean values and distributions respectively; the world average $\alpha_s(M_Z) = 0.1184 \pm 0.0031$ [50] is also shown. Data have been taken in the PETRA, LEP and LEP2 energy range, and analyses have been presented by Aleph [5-296], Delphi [5-228,5-229], Jade [5-389] [49], L3 [5-495], and Opal [5-368]. One of the two results presented in ref. [49], reported as the uppermost cross in fig. 5, is obtained by excluding B_W from the fit. Recently, the NLL resummation of many DIS event shapes has been achieved ([51], and references therein). Power-correction effects can then be studied similarly to what done for e^+e^- collisions, and the results are intriguing. Fig. 6 (taken from ref. [52]) presents the $\alpha_s(M_Z)$ and α_0 values obtained from fitting event shape *distributions* in DIS, and event shape *means* in e^+e^-. The DIS and e^+e^- results are largely consistent, which implies, in view of what shown in fig. 5, that event shape distributions in e^+e^- and DIS prefer different values for $\alpha_s(M_Z)$.

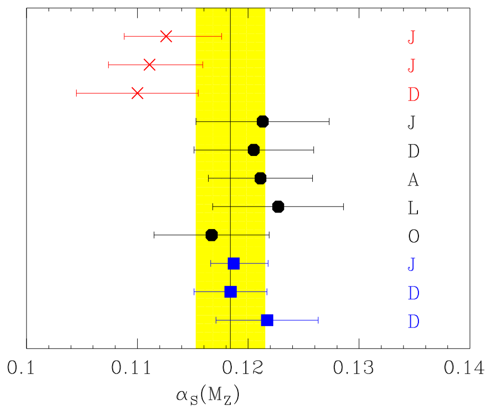

Figure 5. Results for $\alpha_s(M_Z)$ from event shapes in e^+e^- collisions.

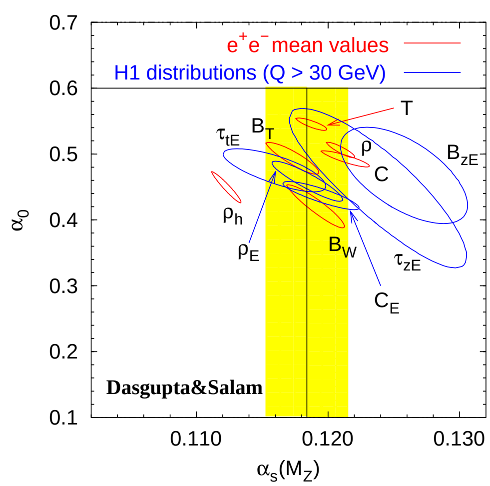

Figure 6. Results for $\alpha_s(M_Z)$ and α_0 from DIS and e^+e^- event shapes. From [52].

Although not compelling from the statistical point of view, these results for event shape distributions in the DMW approach may hint to the necessity of a more complete description of hadronization effects. As mentioned before, eq. (3) results from keeping the first non-trivial term in a Taylor expansion. If no expansion is made, from rather general factorization arguments in the two-jet limit $\mathcal{T} \to 0$ one gets the following formula [53]

$$\frac{d\sigma}{d\mathcal{T}}(\mathcal{T}) = \int_0^{\mathcal{T}Q} d\varepsilon f_{\mathcal{T}}(\varepsilon)\, \frac{d\sigma}{d\mathcal{T}}\bigg|_{pert} (\mathcal{T} - \varepsilon/Q)\,, \quad (6)$$

where $f_{\mathcal{T}}(\varepsilon)$ is known as *shape function*. DMW formulae are recovered with $f_{\mathcal{T}}(\varepsilon) = \delta(\varepsilon - Q c_{\mathcal{T}} \mathcal{P})$; in the general case, the first Mellin moment of $f_{\mathcal{T}}$ has the same meaning as α_0 of DMW. With eq. (6) it is not necessary to assume that $\mathcal{T}Q \gg \mu_{\rm I}$ (all terms $1/(\mathcal{T}Q)^n$ are now expected to be included), and therefore the fit ranges can be extended. Similarly to α_0, $f_{\mathcal{T}}$ cannot be computed from first principles; thus, in order not to lose predictive power, a functional form depending on a small number of parameters must be assumed [54, 55, 56], keeping in mind that QCD dynamics and Lorentz invariance considerations [57] can be used to severely constrain the form of a more general shape function, from which $f_{\mathcal{T}}$ is derived, independently of phenomenological arguments. In a different approach (DGE [55, 58]), whose final result has the same form as eq. (6), it is suggested to combine Sudakov and renormalon resummations in a single formalism. A renormalon ambiguity appears in the exponent, and the prescription necessary to resolve it can be naturally formulated in terms of a shape function, automatically constraining its functional form. Although the $f_{\mathcal{T}}$ which one gets in DGE is consistent with the one obtained in refs. [54, 57], it has to be stressed that the perturbative result $d\sigma/d\mathcal{T}|_{pert}$ in eq. (6) is different in the two formalisms, since DGE includes a class of subleading logs of renormalon origin. In e^+e^- physics, eq. (6) gives satisfactory results: a good fit to the second moments of 1-thrust, M_H, and C parameter is obtained in ref. [54], and the fits for thrust and M_H of ref. [56] are in better agreement in the $(\alpha_{\rm s}, \alpha_0)$ plane wrt those obtained with DMW. On the other hand, according to ref. [56], $\alpha_{\rm s}(M_Z) = 0.1086 \pm 0.0004(exp)$ (the theory error is estimated to be around 5%). Therefore, it seems that a more refined treatment of non-perturbative effects, which is helpful in other respects, is not what one needs in order to get larger $\alpha_{\rm s}$ values.

In a couple of interesting analyses, Delphi adopted rather unconventional methods to study event shapes. In ref. [59], event shape distributions were compared to fixed-order $\mathcal{O}(\alpha_{\rm s}^2)$ (NLO) results (i.e., resummation has not been included), using the renormalization scale as a free parameter in the fit, and correcting for hadronization effects with MC's. Although I'm not aware of any theoretical consideration which justifies such a procedure (called in ref. [59] "experimental optimization of the scale"), the $\alpha_{\rm s}$ values obtained from different observables display a remarkable consistency, and they are also consistent with those obtained by using NLL-resummed predictions. In [5-228] (see also ref. [60]) event shape means were shown to give mutually consistent $\alpha_{\rm s}$ values in the context of a renormalization group approach (RGI [61]), without needing any hadronization corrections. The final $\alpha_{\rm s}$ results of refs. [59] and [5-228] are in excellent agreement with the world average. I interpret these findings as the indication that (at least in a given CM energy range) the uncertainties affecting theoretical predictions at the NLO are larger than or of the same order as the power-suppressed effects that one aims to study. It thus appears that the computation of event shapes at the NNLO is necessary for a deeper understanding of this matter.

In summary, in the past few years a solid progress has been achieved in the understanding of power-suppressed effects in e^+e^- collisions and in DIS. Although models such as DMW can successfully describe many features of the data, some aspects deserve further studies. In some cases, improvements are obtained within approaches which refine the treatment of the non-perturbative part, using a shape function, but the computation of the next order in perturbation theory will likely be necessary in order to obtain a more consistent overall picture. It is worth recalling that the study of hadron-mass effects has been found [62] to induce further power-suppressed terms, some of which can be eliminated by adopting a suitable definition for the observables (E-scheme). Furthermore, more stringent tests of the models for power-suppressed effects should be performed using observables with more complicated kinematic structure and/or gluons at the LO (see refs. [63, 64] and references therein). Finally, it appears to be mandatory to extend the studies of such models to the case of hadronic collisions (for jet observables in particular), where little work has been done so far.

4. NNLO COMPUTATIONS

Bottom production at the Tevatron and event shapes in e^+e^- collisions are a couple of examples which provide physical motivations to increase the precision of the perturbative computations. If tree n-point functions contribute to a given reaction at the LO, the N^{k}LO result (i.e., of relative order α_s^k wrt to the LO) will get contributions from the l-loop, $(n+p)$-point functions, with $l + p \leq k$. There are basically three major steps to make in order to get physical predictions: *i*) explicit computation of all the l-loop, $(n+p)$-point functions; *ii*) cancellation of soft and collinear divergences (which I'll denote – improperly – as IR cancellation henceforth); *iii*) numerical integration of the *finite* result obtained from *i*) and *ii*), with MC techniques to allow more flexibility. One should also mention that UV renormalization has in general to be carried out; however, this is basically textbook matter by now, and thus I'll not deal with it in the following. For NLO computations ($k = 1$), steps *i)*–*iii)* appear to be understood. One-loop integrals have been computed up to five external legs [65]; the case of $n \geq 6$ cannot probably be handled with Feynman-diagram techniques only, and still awaits for a general solution (see [2]). Subtraction [66] and slicing [67] methods, to achieve IR cancellation with semi-analytical techniques, have been available for a long time. In their modern versions [68, 69, 70, 71, 72, 73, 74] they are formulated in an universal (i.e., process- and n-independent) way, which simplifies step *iii)* considerably, and allows the computation of any IR safe observable (no matter how exclusive). No modification is needed in order to incorporate new one-loop results. Attempts to achieve IR cancellation through full numerical computations [75, 76, 77] are still in a preliminary stage, reproducing known results for three-jet production in e^+e^- collisions.

Only a handful of production processes have been computed to NNLO accuracy and beyond: results are available for DIS coefficient functions [78, 79] and for the Drell-Yan K-factor [80, 81] at $\mathcal{O}(\alpha_s^2)$, and for the rate $e^+e^- \to$ hadrons [82] at $\mathcal{O}(\alpha_s^3)$ (I'll later deal with inclusive Higgs production in some details). All these computations are inclusive enough to allow a complete analytical integration over the phase-space of final-state partons. Such an integration is not possible in general, and an NNLO process-independent and exclusive formulation of IR cancellation will likely proceed through semi-analytical techniques, similar to those adopted at the NLO. Therefore, one needs to know the IR-divergent pieces of all the quantities contributing to the cross section at the NNLO. These are (I still assume that the LO gets contribution from the tree n-point functions): *a)* tree-level $(n+2)$-parton amplitudes squared $|T_{n+2}|^2$; *b)* the interference between tree-level $(n+1)$-parton amplitudes and one-loop $(n+1)$-parton amplitudes $\Re(T^\star_{n+1} L^{(1)}_{n+1})$; *c)* the interference between tree-level n-parton amplitudes and two-loop n-parton amplitudes $\Re(T^\star_n L^{(2)}_n)$; *d)* one-loop n-parton amplitudes squared $|L^{(1)}_n|^2$. The IR divergences of $|T_{n+2}|^2$ result from having two soft partons, or three collinear partons, or one soft parton plus two other collinear partons, or two pairs of collinear partons. Only the latter configuration is trivial, in the sense that the corresponding singular behaviour of $|T_{n+2}|^2$ can be obtained from known NLO results; the other limits have been studied in refs. [83, 84, 85, 86, 87], and can be generally cast in the form of a reduced n-resolved-parton matrix element, times a suitable kernel. The problem of combining these singular pieces into local IR counterterms, and of integrating these counterterms over the appropriate region of the phase space, is still unsolved. One also needs to know the IR divergences of $L^{(1)}_{n+1}$ when one parton is soft or two partons are collinear: this is a new feature of NNLO computations, since at the NLO all partons in a virtual contribution are resolved. These limits are also known [88, 89, 90, 91, 92, 93, 94]. Finally, the general form of the residues of the poles $1/\varepsilon^4$, $1/\varepsilon^3$, and $1/\varepsilon^2$ appearing in $L_n^{(2)}$ has been given in ref. [95], without computing any two-loop integrals (see also [96]).

The pole terms found in ref. [95] must precisely match those resulting from the explicit computations of two-loop integrals. A lot of progress has been made in the past couple of years in such computations, and now all the two-loop $2 \to 2$ and $1^* \to 3$ amplitudes are available. First, all the (very many) tensor integrals are reduced to a much smaller number of master scalar integrals, thanks to integration-by-part identities [97, 98] (previously used for two-point functions), and Lorentz-invariance identities [99]; integration-by-part identities for n-leg, l-loop integrals were also shown [100] to be equivalent to those for $(n-m)$-leg, $(l+m)$-loop integrals. The problem of actually computing the master integrals is of a different nature. The breakthrough [101, 102] was the use of a Mellin-Barnes representation for the propagators in the computation of planar and non-planar massless double box integrals (expanded in the dimensional-regularization parameter ε). Negative space-dimension techniques [103] can also be applied to simpler topologies [104, 105]. The computation of master integrals is mapped onto the problem of solving differential equations in the approach of refs. [106, 107]. This approach has been adopted to compute all of the double box master integrals with one off-shell leg [108, 109], not all of which had been computed with Mellin-Barnes techniques [110, 111]. The master integrals so far computed, together with the reduction-to-master-integral techniques, *would* allow the computation of $e^+e^- \to$ 3 jets, and of two-jet production in hadronic collisions (and a few other processes: see for example ref. [112] for a discussion – unfortunately, these processes don't include heavy flavour hadroproduction) if one knew how to achieve IR cancellation for a generic observable at the NNLO.

In hadronic physics, the computation of NNLO short-distance cross sections is not sufficient to get NNLO-accurate predictions, since NNLO-evolved PDF's are also necessary. NNLO-evolved PDF's require the computation of Altarelli-Parisi splitting functions to three loops. It turns out to be convenient to perform such a computation in Mellin space; the results for the first few Mellin moments [113, 114, 115] (together with constraints on the small-x behaviour) have been used [116, 117, 118] to obtain *approximate* expressions for the splitting functions in the x space. Very recently, the complete three-loop computation of the $n_{\rm F}$ part of the non-singlet structure function in DIS has become available [119], from which the corresponding coefficient functions (relevant to N^{3}LO computations) and splitting function can be extracted; the latter has been cross-checked against the approximate results mentioned above, and full agreement has been found. Although the complete expressions for the splitting functions will not appear soon [119], this fairly impressive result gives confidence on the accuracy of the approximate solution of ref. [118]. One has to keep in mind that, in order for any PDF set (such as NNLO-MRST [120]) to actually be of NNLO accuracy, not only the three-loop splitting functions are needed, but also *all* the short-distance cross sections used in the fits must be computed to NNLO. At present, this is the case for the DIS coefficient functions only (which implies that the approximation is generally good, the bulk of the data being from DIS); for example, in the case of Drell-Yan the $x_{\rm F}$ distribution needed for the fits has only NLO accuracy (the Drell-Yan NNLO K-factor is used for normalization purposes [120]). This is another hint of the necessity of solving the problem of IR cancellation at the NNLO in a general way.

In view of enormous amount of work done and yet to be done, one may ask whether the final outcome will justify such an effort. The answer is certainly positive, but one must keep in mind that NNLO computations will not automatically mean *precision* physics. As shown in the case of e^+e^- event shapes, short- and large-distance effects are always correlated, and any advancement in perturbation theory should be complemented by a deeper understanding of this correlation. Furthermore, no precision study in hadronic collisions can be made without an accurate assessment of uncertainties due to PDFs. This matter is now receiving considerable attention: see for example ref. [112] for a review. NNLO computations will certainly play a major role in those cases in which NLO results still give an unsatisfactory description of data: examples are the processes with large K-factors (such as b production), or the observables which require a better description in terms of kinematics (such as jet profiles).

A large-K-factor process is the direct production of SM Higgs at hadron colliders. At the NLO, the exact computation [121] for the gg channel is found to be in excellent agreement with approximate results [122, 123] based on keeping the leading term of an expansion in $m_{\rm H}^2/m_{top}^2$ (the agreement further improves if the full dependence on $m_{\rm H}^2/m_{top}^2$ is kept in the LO term). Therefore, one assumes the same to hold at the NNLO, and computes the NNLO contribution in the $m_{top} \to \infty$ limit. This is feasible since the effective Lagrangian ggH has been obtained [124, 125] to $\mathcal{O}(\alpha_{\rm s}^4)$ (actually, one order larger than necessary here). Exploiting the technique of ref. [100], the two-loop virtual correction to $gg \to H$ has been obtained in ref. [126]. The missing contributions to the physical cross section (including qg and $q\bar{q}$ channels) have been presented in ref. [127], where an expansion for $x_{\rm H} \simeq 1$ ($x_{\rm H} = m_{\rm H}^2/\hat{s}$) has been used to compute the double-real contribution. Terms up to $(1-x_{\rm H})^{16}$ have been included. The $x_{\rm H} \simeq 1$ expansion is expected to work well, since the gluon density is rather peaked towards small x's, and thus the CM energy available at the partonic level is never too far from threshold. As argued in ref. [128], collinear radiation also gives a sizable effect. Methods similar to [128] have been used in refs. [129, 130], with the result of ref. [126], to give early estimates of the complete NNLO rate. These estimates are seen to agree well with the result of ref. [127], which confirms the soft-collinear dominance in inclusive Higgs production at colliders. Finally, the result of ref. [127] has been found to agree to 1% level or better with the computation of ref. [131], where the double-real contribution is also evaluated exactly (it is interesting to notice that in ref. [131] the phase-space integrals relevant to real-emission terms are computed with techniques used so far only for loop diagrams – it remains to be seen whether this method can be generalized to more exclusive observables). As shown in fig. 7, the

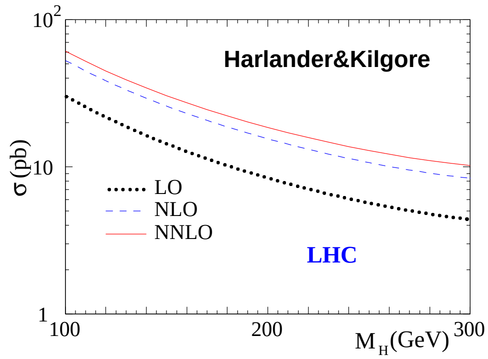

Figure 7. SM Higgs total rate at the LHC.

inclusion of NNLO corrections seems to suggest that effects beyond this order are negligible. The scale dependence is reduced wrt the one observed at the NLO (see ref. [131]). The dominance of the region $x_{\rm H} \sim 1$ implies the potential relevance of soft-gluon resummation. Preliminary results [132, 112] indicate that NNLL resummation enhances the NNLO rate by 5–6% (12–15%) at the LHC (Tevatron), for $100 < m_{\rm H} < 200$ GeV.

The result for the fully-inclusive rate also serves to compute a slightly less inclusive observable, namely the rate for Higgs+jets, with the $p_{\rm T}$ of any jets imposed to be smaller than a fixed quantity $p_{\rm T}^{(veto)}$ (jet veto). Such an observable, which should help in reducing the background due to the decay channel $H \to W^* W^*$, has been computed in ref. [133] by subtracting the anti-vetoed jet cross section $p_{\rm T} > p_{\rm T}^{(veto)}$ (obtained in ref. [134]) from the inclusive NNLO result discussed so far. The study of more exclusive observables, which implies the understanding of IR cancellation at NNLO, will certainly prove useful in the future. In this case, the dominance of the region $x_{\rm H} \sim 1$ might not be as strong as in the case of inclusive rates. Although it is unlikely that the region $x_{\rm H} \sim 0$ will play any role in phenomenological studies, it is worth recalling that in this region the approximation $m_{top} \to \infty$ is not expected to work well: in the full theory the dominant contribution for $x_{\rm H} \to 0$ is single-logarithmic, whereas double logs are also found in the large-m_{top} theory. The latter terms have been identified explicitly in ref. [135] with $k_{\rm T}$-factorization arguments; at the NNLO, they are seen to coincide with those resulting from the explicit computation of ref. [131], thus providing a cross check impossible to achieve in the comparison of ref [131] with ref. [127].

5. MONTE CARLO SIMULATIONS

Monte Carlo (MC) programs are essential tools in experimental physics, giving fully-fledged descriptions of hadronic final states which cannot be obtained in fixed-order computations. Schematically, an MC works as follows: for a given process, which at the LO receives contribution from $2 \to n_0$ reactions, $(2+n_0)$-particle configurations are generated, according to exact tree-level matrix element (ME) computations. The quarks and gluons (partons henceforth) among these primary particles are then allowed to emit more quarks and gluons, which are obtained from a parton shower or dipole cascade *approximation* to QCD dynamics. This implies that MC's cannot simulate the emission of final-state hard (i.e., with large relative transverse momenta; thus, hard is synonymous of resolved here) partons other than the primary ones obtained from ME computations. Furthermore, total rates are accurate to LO.

Although these problems are always present in MC simulations, they become acute when CM energies grow large, since in this case channels with large numbers of well-separated jets are phenomenologically very important and, correspondingly, total rates need to be computed to an accuracy better than LO. Two strategies can be devised in order to improve MC's. The first aims at having $n_{\rm E}$ extra hard partons in the final state; thus, in the example given above, the number of final-state hard particles would increase from n_0 to $n_0 + n_{\rm E}$. This approach is usually referred to as *matrix element corrections*, since the MC must use the $(2+n_0+n_{\rm E})$-particle ME's to generate the correct hard kinematics; more details are given in sect. 5.1. The second strategy also aims at simulating the production of $n_0 + n_{\rm E}$ hard particles, but improves the computation of rates as well, to $\rm N^{n_E}LO$ accuracy. A discussion is given in sect. 5.2.

5.1. Matrix element corrections

There are basically two major problems in the implementation of ME corrections. The first problem is that of achieving a fast computation of the ME's themselves for the largest possible $n_0 + n_{\rm E}$, and an efficient phase-space generation. The second problem stems from the fact that multi-parton ME's are IR divergent. Clearly, in hard-particle configurations IR divergences don't appear; however, the definition of what hard means is, to a large extent, arbitrary. In practice, hardness is achieved by imposing some cuts on suitable partonic variables, such as $p_{\rm T}$'s and (η, φ)-distances dR in hadronic collisions. I collectively denote these cuts by δ_{sep}. One assumes that n hard partons will result (after the shower) into n jets; but, with a probability depending on δ_{sep}, a given n-jet event could also result from $n+m$ hard partons. This means that, when generating events at a fixed $n_0 + n_{\rm E}$ number of primary particles, physical observables in general depend upon δ_{sep}; I refer to this as the δ_{sep}-bias problem. Any solution to the δ_{sep}-bias problem implies a procedure to combine consistently the treatment of ME's with different $n_0 + n_{\rm E}$'s. Here, the difficulty is that of avoiding double counting, that is, the generation of the same kinematical configuration more often than prescribed by QCD.

The vast majority of recent approaches to ME corrections address only the first of the two problems mentioned above. A considerable amount of work has been devoted to the coding of hadronic processes with vector bosons/Higgs plus heavy quarks in the final state, which cannot be found (regardless of the number of extra partons) in standard MC's. The complexity of hard-process generation for large $n_0 + n_{\rm E}$ suggests to implement it in a package (which I call ME generator) distinct from the shower MC. The ME generator stores a set of hard configurations in a file (event file); the event file is eventually read by the MC, which uses the hard configurations as initial conditions for the showers. The advantage of this procedure is that it is completely modular: one given event file can be read by different MC's, and conversely one MC can read event files produced by different ME generators. It is clearly convenient to reach an agreement on the format of such event records: this is now available [136] (Les Houches accord #1). Ready-to-use ME generators (with different numbers of hard processes implemented) are AcerMC [137], ALPGEN [138, 139, 140], and MadGraph/MadEvent [145, 146]. Related work, at present set up to function only with Pythia, has been presented by the CompHEP [141, 142] and Grace [143, 144] groups. All of these ME generators use Feynman-diagram techniques in the computation of ME's, except ALPGEN, which uses the iterative algorithm Alpha [147] (see also [148]).

When using an ME generator, the cuts δ_{sep} must be looser than those used to define the observables. For example, the p_{T}-cut imposed at the parton level must be smaller than the minimum p_{T} of any jets. On the other hand, the cuts should not be too loose: the looser the cuts, the larger the probability of getting a n-jet event starting from $n+m$ hard partons. Thus, δ_{sep} must be chosen in a range which is somewhat dependent upon the observables that one wants to study. This implies that, strictly speaking, the combination of an ME generator with a shower MC is *not* an event generator, since the event record depends upon the observables. This happens precisely because such a combination is affected by the δ_{sep}-bias problem: it is therefore necessary to assess its impact on physical observables. An example is given in fig. 8, obtained using ALPGEN+Herwig (any other ME generator and shower MC would give equivalent results for the same observable). The plot presents jet rates, integrated over $E_{\mathrm{T}}^{(jet)} > E_{\mathrm{T}0}$, for the hardest jet in W+3-jet events at the Tevatron, versus the parton separation dR_{part} imposed at the level of ME generation. Jets are reconstructed with the cone algorithm, with $R = 0.7$. Rates are normalized to the result obtained with $dR_{part} = 0.7$. The conclusion is that, in the "reasonable" range $0.3 < dR_{part} < 0.7$, the physical prediction has a $\mathcal{O}(20\%)$ dependence on dR_{part}. The existence of a δ_{sep} dependence should always be kept in mind when using an ME generator + shower MC combination, because it affects the precision of the prediction. On the other hand, this is a rather modest price to pay: for multi-jet observables, ordinary MC's can underestimate the cross section by orders of magnitude, and the use of ME generators is mandatory.

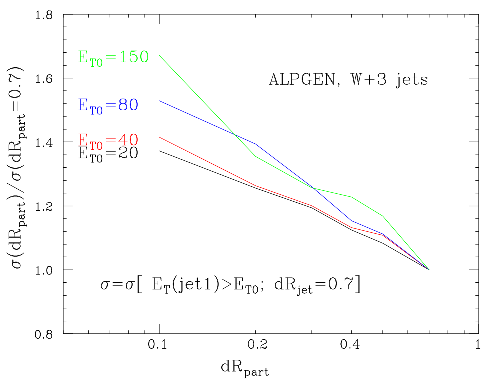

Figure 8. Dependence of (normalized) jet rates on interparton separation.

As mentioned before, the δ_{sep} bias can be avoided by suitably combining the generation of ME's with different $n_0 + n_{\mathrm{E}}$. Early proposal for ME corrections [149, 150, 151, 152, 153] achieved this in the case $n_{\mathrm{E}} = 0, 1$. The solution for arbitrary n_{E} appears to be more complicated; it has been fully implemented for shower MC's in e^+e^- collisions [154, 155] in the case of jet production; along similar lines, proposal for colour dipole MC's [156] and shower MC's in hadronic collisions [157] have also been made. The idea of ref. [154] is the following. *a)* Integrate all the $\gamma^* \to 2 + n_{\mathrm{E}}$ ME's by imposing $y_{ij} > y_{\mathrm{INI}}$ for any pairs of partons i, j, with y_{INI} a fixed parameter and $y_{ij} = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})/Q^2$ the interparton distance defined according to the k_{T}-algorithm [158]. *b)* Choose statistically an n_{E}, using the rates computed in *a)*. *c)* Generate a $(2+n_{\rm E})$-parton configuration using the exact $\gamma^* \to 2+n_{\rm E}$ ME, and reweight it with a suitable combination of Sudakov form factors (corresponding to the probability of no other branchings). *d)* Use the configuration generated in *c)* as initial condition for a *vetoed* shower. A vetoed shower proceeds as the usual one, except that it forbids all branchings $i \to jk$ with $y_{jk} > y_{\rm INI}$ without stopping the scale evolution. In ref [154], $y_{\rm INI}$ plays the role of δ_{sep}. Although the selection of an $n_{\rm E}$ value has a leading-log dependence on $y_{\rm INI}$, it can be proved that this dependence is cancelled up to next-to-next-to-leading logs in physical observables [154]. This is illustrated in

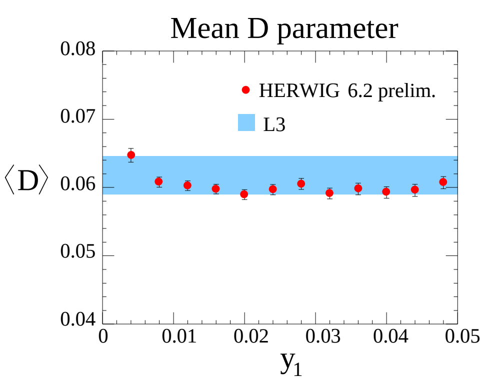

Figure 9. Dependence of mean D parameter on $y_1 \equiv y_{\rm INI}$.

fig. 9, where the mean value of the D parameter [159, 66] (full circles) is plotted versus the value of $y_{\rm INI}$, and compared to the measurement of ref. [160] (band). Figure 9 clearly documents that the consistent combination of ME's with different $n_0+n_{\rm E}$ largely reduces the dependence of observables on δ_{sep}. This, however, comes at the price of modifying the shower algorithm. Furthermore, the procedure is more computing-intensive, since *all* $n_{\rm E}$'s need to be considered (this being impossible in practice, in e^+e^- collisions the procedure has an error of $\mathcal{O}(\alpha_{\rm s}^{N-1})$ if only ME's with $2+n_{\rm E} \leq N$ partons are considered; in ref. [155], $N=5$).

5.2. Complete matching of Monte Carlos and perturbative computations

The problem of fully matching MC's with higher-order computations can be seen as an upgrade of ME corrections: not only one wants to describe the kinematics of $n_0+n_{\rm E}$ hard particles correctly, but the information on N$^{n_{\rm E}}$LO rates must also be included. First attempts at solving this problem have only recently become available, and only for the case $n_{\rm E}=1$. Following ref. [161], let me denote by MC@NLO the improved MC we aim at constructing. The naive idea, of defining an MC@NLO by multiplying an MC with ME corrections by the NLO K-factor, is simply not acceptable: the inclusiveness of the K-factor does not really fit well into the exclusive framework of an MC. Thus, any MC@NLO must involve the computation of virtual ME's. This is the reason why the construction of an MC@NLO is conceptually more complicated than ME corrections: the IR divergences of the virtual ME's can only be cancelled by computing real-emission ME's with one soft parton or two collinear partons. These soft and collinear configurations never occur in ordinary MC's; in the case of ME corrections, the cuts δ_{sep} are specifically introduced to avoid them. The presence of virtual ME's also requires a less-intuitive definition of double counting [161]: in the context of MC@NLO's, double counting may correspond to either an excess or a deficit in the prediction. This generalization is necessary, since the ME's used in MC@NLO's are not positive-definite.

In order to describe in more details current approaches to MC@NLO, I adopt the toy model of ref. [161], in which a system S (say, a quark) can emit "photons", massless particles with only one degree of freedom (say, the energy). The initial energy of S is 1, which becomes $1-x$ after the emission of one photon of energy $0<x\leq 1$. The LO, virtual, and real contributions to the NLO cross section are (a is the coupling constant):

$$\left(\frac{d\sigma}{dx}\right)_{\rm B} = B\delta(x), \tag{7}$$

$$\left(\frac{d\sigma}{dx}\right)_{\rm V} = a\left(\frac{B}{2\varepsilon}+V\right)\delta(x), \tag{8}$$

$$\left(\frac{d\sigma}{dx}\right)_{\rm R} \;=\; a\frac{R(x)}{x}, \tag{9}$$

where $\delta(x)$ in eqs. (7) and (8) reminds that there's no emission of real photons, and the IR divergence $1/\varepsilon$ in eq. (8) results from the loop integration over virtual photon momentum in $4-2\varepsilon$ dimensions. I denote by (S,z) the configuration of the system plus up to one photon, with $z=0$ in the case of the LO or virtual contributions (eqs. (7) and (8) respectively), and $z=x\neq 0$ in the case of the real contribution (eq. (9)). Energy conservation is understood, and therefore in the configuration (S,z) the system has energy $1-z$. The function $R(x)$ characterizes real emissions; its specific form is irrelevant, except that, for IR cancellation to occur, it must fulfil $R(x)\to B$ for $x\to 0$. With one photon emission at most, any observable O can be represented by a function $O(S,z)$; the computation of its expectation value $\langle O\rangle$ can be achieved through standard techniques for IR cancellation:

$$\begin{aligned}\langle O\rangle \;&=\; BO(S,0)+a\Bigg[\left(B\log\delta+V\right)O(S,0)\\ &+\;\int_\delta^1 dx\,\frac{O(S,x)R(x)}{x}\Bigg]\end{aligned} \tag{10}$$

in the slicing method [67], and

$$\begin{aligned}\langle O\rangle \;&=\; \int_0^1 dx\Bigg[O(S,x)\frac{aR(x)}{x}\\ &+\;O(S,0)\left(B+aV-\frac{aB}{x}\right)\Bigg]\end{aligned} \tag{11}$$

in the subtraction method [66].

In an MC approach, the system can undergo an arbitrary number of photon emissions. I denote by $BI_{\rm MC}(O;S,0)$ the distribution in the observable O obtained with MC methods; this notation reminds that in a standard MC the initial condition for the shower is $(S,0)$ (the LO kinematics), and that the total rate is B (the LO rate, see eq. (7)). The most straightforward implementation of an MC@NLO can then be done by analogy: since two kinematical configurations, (S,x) and $(S,0)$, appear in the NLO cross section, one can use both of them as initial conditions for the showers. In order to recover the correct total rate, each event resulting from a shower with initial condition (S,z) will be weighted with the coefficient of $O(S,z)$ which appears in eq. (10) or in eq. (11). Using eq. (11), one gets

$$\begin{aligned}\left(\frac{d\sigma}{dO}\right) \;&=\; \int_0^1 dx\Bigg[I_{\rm MC}(O;S,x)\frac{aR(x)}{x}\\ &+\;I_{\rm MC}(O;S,0)\left(B+aV-\frac{aB}{x}\right)\Bigg].\end{aligned} \tag{12}$$

Unfortunately, this naive approach does not work. The weights $aR(x)/x$ and $B+aV-aB/x$ are IR-divergent at $x=0$; since the corresponding showers have different initial conditions (S,x) and $(S,0)$, it would take an infinite amount of time to cancel the divergences (in other words, unweighting is impossible). This is not a practical problem, is a fundamental one: the cancellation works for inclusive quantities, and the shower is exclusive. So the main problem in the construction of an MC@NLO can be reformulated as follows: how to achieve IR cancellation, without giving up the exclusive properties of the showers. Besides, eq. (12) also suffers from double counting.

In the context of the slicing method, an approach has been proposed [162, 163, 164, 165] which exploits an idea of refs. [166, 167] (see also [168]). The slicing parameter δ in eq. (10) is fixed to the value δ_0, by imposing that no $(S,0)$ contribution be present in the NLO cross section:

$$B+a\left(B\log\delta_0+V\right)=0\,. \tag{13}$$

This effectively restricts the energy of the real photons emitted to the range $\delta_0<x\leq 1$ (see eq. (10)). This range is further partitioned by means of an arbitrary parameter $\delta_{\rm PS}$. One starts by generating the emission of a real photon with energy x distributed according to $aR(x)/x$. Then, if $\delta_0<x\leq\delta_{\rm PS}$, the real-emission kinematics (S,x) is mapped onto the LO kinematics $(S,0)$ (in other words, the photon with energy x is thrown away). The configuration $(S,0)$ is used as initial condition for the shower, requiring the shower to forbid photon energies larger than $\delta_{\rm PS}$. If $x>\delta_{\rm PS}$, the real emission is kept, and (S,x) is used as initial condition for the shower. The corresponding formula is:

$$\left(\frac{d\sigma}{dO}\right) = a\int_0^1 dx \Bigg[I_{\rm MC}(O;S,x)\frac{R(x)}{x}\Theta(x-\delta_{\rm PS}) + I_{\rm MC}(O;S,0)\frac{R(x)}{x}\Theta(x-\delta_0)\Theta(\delta_{\rm PS}-x)\Bigg]. \quad (14)$$

The advantage of eq. (14) is that it is manifestly positive-definite, and that its implementation can be carried out with little or no knowledge of the structure of the MC. On the other hand, it can be shown [161] that eq. (14) still has double counting, and does not have a formal perturbative expansion in a. These problems arise since the technique adopted to deal with the IR cancellation is not exclusive enough: eq. (13) is an integral equation (the $\log\delta_0$ term is due to an integral over soft-photon configurations in the real-emission contribution). Although the absence of a perturbative expansion can be seen as a minor drawback from a practical point of view, the impact of the double counting should be assessed for each observable studied, by considering the dependence of physical predictions upon $\delta_{\rm PS}$.

Other approaches [161, 169, 170, 171, 172, 173] are based on the subtraction method. One can observe that ordinary MC's do contain the information on the leading IR singular behaviour of NLO ME's. Formally, this implies that the $\mathcal{O}(\alpha_{\rm s})$ term in the perturbative expansion of the MC result can act as a *local* counterterm to the IR divergences at the NLO (strictly speaking, this is not exactly true in the case of large-angle soft gluon emission, and a few technical complications arise – see ref. [161]). Furthermore, the form of the counterterm does not depend upon the observable studied. Thus, IR cancellation is achieved locally, but without any reference to a specific observable: this allows to implement it at the level of event generation, without giving up the exclusive treatment of the branchings. The prescription of ref. [161] is

$$\left(\frac{d\sigma}{dO}\right) = \int_0^1 dx \Bigg[I_{\rm MC}(O;S,x)\frac{a[R(x)-BQ(x)]}{x} + I_{\rm MC}(O;S,0)\left(B+aV+\frac{aB[Q(x)-1]}{x}\right)\Bigg], \quad (15)$$

where the $Q(x)$-dependent quantities are the $\mathcal{O}(a)$ term of the MC result. The local IR cancellation mentioned before shows in the fact that the coefficients of $I_{\rm MC}(O;S,x)$ and $I_{\rm MC}(O;S,0)$ in eq. (15) are finite, since the condition $Q(x)\to 1$ for $x\to 0$ always holds, regardless of the specific MC used. Therefore, these coefficients can be given as weights to the showers with (S,x) and $(S,0)$ initial conditions respectively. Eq. (15) does not have double counting, and features a smooth matching between the soft- and hard-emission regions of the phase space, without the need to introduce any extra parameter such as $\delta_{\rm PS}$. The price to pay for this is the presence of negative weights (which however do not spoil the probabilistic interpretation of the results). Furthermore, one needs to know details of the MC ($Q(x)$) in order to implement eq. (15): this seems unavoidable, since one should expect different MC's to match differently with a given NLO computation. The first QCD implementation of eq. (15) has been presented in ref. [174].

The approach of refs. [169, 170, 171, 172] uses a technique similar to that of ref. [161], based on the definition of a local IR counterterm. At variance with ref. [161], where the resummation of large logs is performed to LL accuracy, refs. [169, 170, 171, 172] advocate an NLL (or beyond) resummation; for this to happen, it is argued that the standard formulation of collinear factorization must be extended. This approach has been fully formulated [171] only in the unphysical $\phi^3_{d=6}$ theory so far. Current QCD implementations do not include gluon emission.

6. CONCLUDING REMARKS

It seems appropriate to start this section by mentioning a couple of phenomenological issues which would have deserved more attention. One is the problem affecting the single-inclusive, isolated-photon measurements at the Tevatron. If D0 data are considered [175, 176], a moderate disagreement with NLO QCD is present in the low-$p_{\rm T}$, central-η region, which disappears when the ratio $R=\sigma(\sqrt{S}=1800$ GeV$)/\sigma(\sqrt{S}=630$ GeV$)$ is considered. The situation worsens in the case of CDF data [177]: not only the discrepancy with QCD is statistically more significant for cross sections, but the measured ratio R also disagrees with theory. The consistent inclusion of recoil effects, along the lines of refs. [178, 179], might increase the QCD prediction at small p_{T}'s, and thus reduce the discrepancy. The second problem affects the single-inclusive jet cross section as measured by D0 [180], the jets being reconstructed with a k_{T}-algorithm. The data display a rather poor agreement with theory for $p_{\mathrm{T}} < 100$ GeV; this is disappointing, given the excellent results obtained with the cone algorithm (as far as the cone algorithm is concerned, I should also mention here that the previously reported excess of data over theory at large p_{T} has now completely disappeared: NLO QCD perfectly reproduces the data, if updated PDF sets are used. An explanation in terms of PDFs has been already given in the past, but the PDF set used at that time resulted from an *ad hoc* fit – named "HJ" by the CTEQ collaboration. This is not necessary any longer, since in the newest PDF releases the gluon density of the *best* fit naturally results to be HJ-like. See ref. [181] for a discussion on this point). It is hard to believe that the discrepancy in the case of the k_{T}-algorithm is the signal of a serious problem in QCD (since it would probably affect the cone algorithm as well); however, it may indicate a deficiency in current MC simulations, or in the understanding of hadronization and/or detector effects, which would surely worsen at the LHC energies. It has to be remarked that the k_{T}-algorithm appears to work well at HERA.

In general, the capability of QCD to describe hard production processes is quite remarkable (with the exception of b-production at LEP and HERA). The definition of a formalism for the computation of exclusive observables at the NNLO is one of the most challenging and hot topics in perturbative QCD, which will have an important impact on phenomenological studies and will be crucial in improving the precision of α_{s} (and other fundamental parameters) measurements – at colliders, exclusive Drell-Yan production will necessarily have to be calculated to NNLO, in order to match the experimental precision. The substantial progress made in the past couple of years in the computation of two-loop integrals and three-loop splitting functions, and the NNLO results for direct inclusive SM-Higgs production, are certainly very encouraging. NNLO results will also help to understand better the interplay between soft and hard physics; the e^+e^- and DIS environments will serve as a laboratory for the more involved case of hadronic collisions, where most of the work remains to be done. Monte Carlos are much better equipped to face the challenges of Tevatron Run II and LHC. The implementation of new processes in matrix element generators proceeds steadily and, although there is still work to be done on the general structure of matrix element corrections, these techniques are rather well established by now. Formalisms for MC@NLO's have received considerable attention in the recent past; the field has still to reach a mature stage, and new ideas will certainly be presented in the near future. A rather obvious but quite challenging step is that of merging matrix element corrections and MC@NLO approaches.

It is a pleasure to thank my friends and colleagues, unfortunately too numerous to mention, who helped me so much in the preparation of the talk and of this manuscript.

REFERENCES

1. K. Long, these proceedings.
2. Z. Bern, these proceedings.
3. P. Nason, S. Dawson and R. K. Ellis, Nucl. Phys. B **303** (1988) 607.
4. W. Beenakker, W. L. van Neerven, R. Meng, G. A. Schuler and J. Smith, Nucl. Phys. B **351** (1991) 507.
5. R. K. Ellis and P. Nason, Nucl. Phys. B **312** (1989) 551.
6. J. Smith and W. L. van Neerven, Nucl. Phys. B **374** (1992) 36.
7. M. Drees, M. Kramer, J. Zunft and P. M. Zerwas, Phys. Lett. B **306** (1993) 371.
8. P. Nason, S. Dawson and R. K. Ellis, Nucl. Phys. B **327** (1989) 49 [Erratum-ibid. B **335** (1990) 260].
9. M. L. Mangano, P. Nason and G. Ridolfi, Nucl. Phys. B **373** (1992) 295.

10. S. Frixione, M. L. Mangano, P. Nason and G. Ridolfi, Nucl. Phys. B **412** (1994) 225 [arXiv:hep-ph/9306337].
11. E. Laenen, S. Riemersma, J. Smith and W. L. van Neerven, Nucl. Phys. B **392** (1993) 162.
12. E. Laenen, S. Riemersma, J. Smith and W. L. van Neerven, Nucl. Phys. B **392** (1993) 229.
13. T. Affolder *et al.* [CDF Collaboration], Phys. Rev. D **64** (2001) 032002 [arXiv:hep-ex/0101036].
14. V. M. Abazov *et al.* [D0 Collaboration], arXiv:hep-ex/0205019.
15. R. Bonciani, S. Catani, M. L. Mangano and P. Nason, Nucl. Phys. B **529** (1998) 424 [arXiv:hep-ph/9801375].
16. B. W. Harris, E. Laenen, L. Phaf, Z. Sullivan and S. Weinzierl, Phys. Rev. D **66** (2002) 054024 [arXiv:hep-ph/0207055].
17. S. Frixione, M. L. Mangano, P. Nason and G. Ridolfi, Adv. Ser. Direct. High Energy Phys. **15** (1998) 609 [arXiv:hep-ph/9702287].
18. D. Acosta *et al.* [CDF Collaboration], Phys. Rev. D **65** (2002) 052005 [arXiv:hep-ph/0111359].
19. C. Peterson, D. Schlatter, I. Schmitt and P. M. Zerwas, Phys. Rev. D **27** (1983) 105.
20. V. G. Kartvelishvili, A. K. Likhoded and V. A. Petrov, Phys. Lett. B **78** (1978) 615.
21. P. Nason and C. Oleari, Nucl. Phys. B **565** (2000) 245 [arXiv:hep-ph/9903541].
22. M. Cacciari and P. Nason, Phys. Rev. Lett. **89** (2002) 122003 [arXiv:hep-ph/0204025].
23. M. Cacciari, M. Greco and P. Nason, JHEP **9805** (1998) 007 [arXiv:hep-ph/9803400].
24. P. Nason *et al.*, arXiv:hep-ph/0003142.
25. S. Frixione and M. L. Mangano, Nucl. Phys. B **483** (1997) 321 [arXiv:hep-ph/9605270].
26. B. Abbott *et al.* [D0 Collaboration], Phys. Rev. Lett. **85** (2000) 5068 [arXiv:hep-ex/0008021].
27. S. Catani, M. Ciafaloni and F. Hautmann, Nucl. Phys. B **366** (1991) 135.
28. J. C. Collins and R. K. Ellis, Nucl. Phys. B **360** (1991) 3.
29. S. Frixione, M. Kramer and E. Laenen, Nucl. Phys. B **571** (2000) 169 [arXiv:hep-ph/9908483].
30. C. Adloff *et al.* [H1 Collaboration], Nucl. Phys. B **545** (1999) 21 [arXiv:hep-ex/9812023].
31. J. Breitweg *et al.* [ZEUS Collaboration], Eur. Phys. J. C **6** (1999) 67 [arXiv:hep-ex/9807008].
32. M. Cacciari, S. Frixione and P. Nason, JHEP **0103** (2001) 006 [arXiv:hep-ph/0102134].
33. S. Frixione and P. Nason, JHEP **0203** (2002) 053 [arXiv:hep-ph/0201281].
34. I. Abt *et al.* [HERA-B Collaboration], arXiv:hep-ex/0205106.
35. C. Adloff *et al.* [H1 Collaboration], Phys. Lett. B **467** (1999) 156 [Erratum-ibid. B **518** (2001) 331] [arXiv:hep-ex/9909029].
36. J. Breitweg *et al.* [ZEUS Collaboration], Eur. Phys. J. C **18** (2001) 625 [arXiv:hep-ex/0011081].
37. M. Acciarri *et al.* [L3 Collaboration], Phys. Lett. B **503** (2001) 10 [arXiv:hep-ex/0011070].
38. Y. L. Dokshitzer and B. R. Webber, Phys. Lett. B **352** (1995) 451 [arXiv:hep-ph/9504219].
39. Y. L. Dokshitzer, G. Marchesini and B. R. Webber, Nucl. Phys. B **469** (1996) 93 [arXiv:hep-ph/9512336].
40. G. P. Korchemsky and G. Sterman, Nucl. Phys. B **437** (1995) 415 [arXiv:hep-ph/9411211].
41. G. P. Korchemsky and G. Sterman, arXiv:hep-ph/9505391.
42. Y. L. Dokshitzer and B. R. Webber, Phys. Lett. B **404** (1997) 321 [arXiv:hep-ph/9704298].
43. P. Nason and M. H. Seymour, Nucl. Phys. B **454** (1995) 291 [arXiv:hep-ph/9506317].
44. Y. L. Dokshitzer, A. Lucenti, G. Marchesini and G. P. Salam, Nucl. Phys. B **511** (1998) 396 [Erratum-ibid. B **593** (2001) 729] [arXiv:hep-ph/9707532].
45. Y. L. Dokshitzer, A. Lucenti, G. Marchesini and G. P. Salam, JHEP **9805** (1998) 003 [arXiv:hep-ph/9802381].
46. M. Dasgupta and B. R. Webber, JHEP **9810** (1998) 001 [arXiv:hep-ph/9809247].
47. Y. L. Dokshitzer, G. Marchesini and

G. P. Salam, Eur. Phys. J. directC **3** (1999) 1 [arXiv:hep-ph/9812487].
48. P. A. Movilla Fernandez, arXiv:hep-ex/0209022.
49. P. A. Movilla Fernandez, S. Bethke, O. Biebel and S. Kluth, Eur. Phys. J. C **22** (2001) 1 [arXiv:hep-ex/0105059].
50. S. Bethke, Eur. Phys. J. directC **1** (2002) 1 [arXiv:hep-ex/0112032].
51. M. Dasgupta and G. P. Salam, JHEP **0208** (2002) 032 [arXiv:hep-ph/0208073].
52. G. P. Salam, arXiv:hep-ph/0207147.
53. G. P. Korchemsky and G. Sterman, Nucl. Phys. B **555** (1999) 335 [arXiv:hep-ph/9902341].
54. G. P. Korchemsky and S. Tafat, JHEP **0010** (2000) 010 [arXiv:hep-ph/0007005].
55. E. Gardi and J. Rathsman, Nucl. Phys. B **609** (2001) 123 [arXiv:hep-ph/0103217].
56. E. Gardi and J. Rathsman, Nucl. Phys. B **638** (2002) 243 [arXiv:hep-ph/0201019].
57. A. V. Belitsky, G. P. Korchemsky and G. Sterman, Phys. Lett. B **515** (2001) 297 [arXiv:hep-ph/0106308].
58. E. Gardi, Nucl. Phys. B **622** (2002) 365 [arXiv:hep-ph/0108222].
59. P. Abreu *et al.* [DELPHI Collaboration], Eur. Phys. J. C **14** (2000) 557 [arXiv:hep-ex/0002026].
60. D. Wicke, arXiv:hep-ph/0205299.
61. A. Dhar and V. Gupta, Phys. Rev. D **29** (1984) 2822.
62. G. P. Salam and D. Wicke, JHEP **0105** (2001) 061 [arXiv:hep-ph/0102343].
63. A. Banfi, Y. L. Dokshitzer, G. Marchesini and G. Zanderighi, JHEP **0105** (2001) 040 [arXiv:hep-ph/0104162].
64. A. Banfi, G. Marchesini, G. Smye and G. Zanderighi, JHEP **0108** (2001) 047 [arXiv:hep-ph/0106278].
65. Z. Bern, L. J. Dixon and D. A. Kosower, Nucl. Phys. B **412** (1994) 751 [arXiv:hep-ph/9306240].
66. R. K. Ellis, D. A. Ross and A. E. Terrano, Nucl. Phys. B **178** (1981) 421.
67. K. Fabricius, I. Schmitt, G. Kramer and G. Schierholz, Z. Phys. C **11** (1981) 315.
68. W. T. Giele and E. W. Glover, Phys. Rev. D **46** (1992) 1980.
69. W. T. Giele, E. W. Glover and D. A. Kosower, Nucl. Phys. B **403** (1993) 633 [arXiv:hep-ph/9302225].
70. S. Frixione, Z. Kunszt and A. Signer, Nucl. Phys. B **467** (1996) 399 [arXiv:hep-ph/9512328].
71. S. Catani and M. H. Seymour, Phys. Lett. B **378** (1996) 287 [arXiv:hep-ph/9602277].
72. S. Catani and M. H. Seymour, Nucl. Phys. B **485** (1997) 291 [Erratum-ibid. B **510** (1997) 503] [arXiv:hep-ph/9605323].
73. S. Frixione, Nucl. Phys. B **507** (1997) 295 [arXiv:hep-ph/9706545].
74. Z. Nagy and Z. Trocsanyi, Nucl. Phys. B **486** (1997) 189 [arXiv:hep-ph/9610498].
75. D. E. Soper, Phys. Rev. Lett. **81** (1998) 2638 [arXiv:hep-ph/9804454].
76. D. E. Soper, Phys. Rev. D **62** (2000) 014009 [arXiv:hep-ph/9910292].
77. M. Kramer and D. E. Soper, Phys. Rev. D **66** (2002) 054017 [arXiv:hep-ph/0204113].
78. E. B. Zijlstra and W. L. van Neerven, Nucl. Phys. B **383** (1992) 525.
79. E. B. Zijlstra and W. L. van Neerven, Phys. Lett. B **297** (1992) 377.
80. R. Hamberg, W. L. van Neerven and T. Matsuura, Nucl. Phys. B **359** (1991) 343.
81. W. L. van Neerven and E. B. Zijlstra, Nucl. Phys. B **382** (1992) 11.
82. S. G. Gorishnii, A. L. Kataev and S. A. Larin, Phys. Lett. B **259** (1991) 144.
83. F. A. Berends and W. T. Giele, Nucl. Phys. B **313** (1989) 595.
84. J. M. Campbell and E. W. Glover, Nucl. Phys. B **527** (1998) 264 [arXiv:hep-ph/9710255].
85. S. Catani and M. Grazzini, Phys. Lett. B **446** (1999) 143 [arXiv:hep-ph/9810389].
86. V. Del Duca, A. Frizzo and F. Maltoni, Nucl. Phys. B **568** (2000) 211 [arXiv:hep-ph/9909464].
87. S. Catani and M. Grazzini, Nucl. Phys. B **570** (2000) 287 [arXiv:hep-ph/9908523].
88. Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower, Nucl. Phys. B **425** (1994) 217 [arXiv:hep-ph/9403226].
89. Z. Bern and G. Chalmers, Nucl. Phys. B **447**

(1995) 465 [arXiv:hep-ph/9503236].
90. Z. Bern, V. Del Duca and C. R. Schmidt, Phys. Lett. B **445** (1998) 168 [arXiv:hep-ph/9810409].
91. D. A. Kosower, Nucl. Phys. B **552** (1999) 319 [arXiv:hep-ph/9901201].
92. D. A. Kosower and P. Uwer, Nucl. Phys. B **563** (1999) 477 [arXiv:hep-ph/9903515].
93. Z. Bern, V. Del Duca, W. B. Kilgore and C. R. Schmidt, Phys. Rev. D **60** (1999) 116001 [arXiv:hep-ph/9903516].
94. S. Catani and M. Grazzini, Nucl. Phys. B **591** (2000) 435 [arXiv:hep-ph/0007142].
95. S. Catani, Phys. Lett. B **427** (1998) 161 [arXiv:hep-ph/9802439].
96. G. Sterman and M. E. Tejeda-Yeomans, arXiv:hep-ph/0210130.
97. F. V. Tkachov, Phys. Lett. B **100** (1981) 65.
98. K. G. Chetyrkin and F. V. Tkachov, Nucl. Phys. B **192** (1981) 159.
99. T. Gehrmann and E. Remiddi, Nucl. Phys. B **580** (2000) 485 [arXiv:hep-ph/9912329].
100. P. A. Baikov and V. A. Smirnov, Phys. Lett. B **477** (2000) 367 [arXiv:hep-ph/0001192].
101. V. A. Smirnov, Phys. Lett. B **460** (1999) 397 [arXiv:hep-ph/9905323].
102. J. B. Tausk, Phys. Lett. B **469** (1999) 225 [arXiv:hep-ph/9909506].
103. I. G. Halliday and R. M. Ricotta, Phys. Lett. B **193** (1987) 241.
104. C. Anastasiou, E. W. Glover and C. Oleari, Nucl. Phys. B **565** (2000) 445 [arXiv:hep-ph/9907523].
105. A. T. Suzuki and A. G. Schmidt, J. Phys. A **35** (2002) 151 [arXiv:hep-th/0110047].
106. A. V. Kotikov, Phys. Lett. B **267** (1991) 123.
107. E. Remiddi, Nuovo Cim. A **110** (1997) 1435 [arXiv:hep-th/9711188].
108. T. Gehrmann and E. Remiddi, Nucl. Phys. B **601** (2001) 248 [arXiv:hep-ph/0008287].
109. T. Gehrmann and E. Remiddi, Nucl. Phys. B **601** (2001) 287 [arXiv:hep-ph/0101124].
110. V. A. Smirnov, Phys. Lett. B **491** (2000) 130 [arXiv:hep-ph/0007032].
111. V. A. Smirnov, Phys. Lett. B **500** (2001) 330 [arXiv:hep-ph/0011056].
112. W. Giele *et al.*, arXiv:hep-ph/0204316.
113. S. A. Larin, T. van Ritbergen and J. A. Vermaseren, Nucl. Phys. B **427** (1994) 41.
114. S. A. Larin, P. Nogueira, T. van Ritbergen and J. A. Vermaseren, Nucl. Phys. B **492** (1997) 338 [arXiv:hep-ph/9605317].
115. A. Retey and J. A. Vermaseren, Nucl. Phys. B **604** (2001) 281 [arXiv:hep-ph/0007294].
116. W. L. van Neerven and A. Vogt, Nucl. Phys. B **568** (2000) 263 [arXiv:hep-ph/9907472].
117. W. L. van Neerven and A. Vogt, Nucl. Phys. B **588** (2000) 345 [arXiv:hep-ph/0006154].
118. W. L. van Neerven and A. Vogt, Phys. Lett. B **490** (2000) 111 [arXiv:hep-ph/0007362].
119. S. Moch, J. A. Vermaseren and A. Vogt, arXiv:hep-ph/0209100.
120. A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Phys. Lett. B **531** (2002) 216 [arXiv:hep-ph/0201127].
121. M. Spira, A. Djouadi, D. Graudenz and P. M. Zerwas, Nucl. Phys. B **453** (1995) 17 [arXiv:hep-ph/9504378].
122. S. Dawson, Nucl. Phys. B **359** (1991) 283.
123. A. Djouadi, M. Spira and P. M. Zerwas, Phys. Lett. B **264** (1991) 440.
124. K. G. Chetyrkin, B. A. Kniehl and M. Steinhauser, Phys. Rev. Lett. **79** (1997) 353 [arXiv:hep-ph/9705240].
125. K. G. Chetyrkin, B. A. Kniehl and M. Steinhauser, Nucl. Phys. B **510** (1998) 61 [arXiv:hep-ph/9708255].
126. R. V. Harlander, Phys. Lett. B **492** (2000) 74 [arXiv:hep-ph/0007289].
127. R. V. Harlander and W. B. Kilgore, Phys. Rev. Lett. **88** (2002) 201801 [arXiv:hep-ph/0201206].
128. M. Kramer, E. Laenen and M. Spira, Nucl. Phys. B **511** (1998) 523 [arXiv:hep-ph/9611272].
129. S. Catani, D. de Florian and M. Grazzini, JHEP **0105** (2001) 025 [arXiv:hep-ph/0102227].
130. R. V. Harlander and W. B. Kilgore, Phys. Rev. D **64** (2001) 013015 [arXiv:hep-ph/0102241].
131. C. Anastasiou and K. Melnikov, arXiv:hep-ph/0207004.
132. M. Grazzini, arXiv:hep-ph/0209302.
133. S. Catani, D. de Florian and M. Grazzini, JHEP **0201** (2002) 015 [arXiv:hep-

ph/0111164].
134.D. de Florian, M. Grazzini and Z. Kunszt, Phys. Rev. Lett. **82** (1999) 5209 [arXiv:hep-ph/9902483].
135.F. Hautmann, Phys. Lett. B **535** (2002) 159 [arXiv:hep-ph/0203140].
136.E. Boos *et al.*, arXiv:hep-ph/0109068.
137.B. P. Kersevan and E. Richter-Was, arXiv:hep-ph/0201302.
138.F. Caravaglios, M. L. Mangano, M. Moretti and R. Pittau, Nucl. Phys. B **539** (1999) 215 [arXiv:hep-ph/9807570].
139.M. L. Mangano, M. Moretti and R. Pittau, Nucl. Phys. B **632** (2002) 343 [arXiv:hep-ph/0108069].
140.M. L. Mangano, M. Moretti, F. Piccinini, R. Pittau and A. D. Polosa, arXiv:hep-ph/0206293.
141.A. Pukhov *et al.*, arXiv:hep-ph/9908288.
142.A. S. Belyaev *et al.*, arXiv:hep-ph/0101232.
143.K. Sato, S. Tsuno, J. Fujimoto, T. Ishikawa, Y. Kurihara and S. Odaka, arXiv:hep-ph/0104237.
144.S. Tsuno, K. Sato, J. Fujimoto, T. Ishikawa, Y. Kurihara, S. Odaka and T. Abe, arXiv:hep-ph/0204222.
145.T. Stelzer and W. F. Long, Comput. Phys. Commun. **81** (1994) 357 [arXiv:hep-ph/9401258].
146.F. Maltoni and T. Stelzer, arXiv:hep-ph/0208156.
147.F. Caravaglios and M. Moretti, Phys. Lett. B **358** (1995) 332 [arXiv:hep-ph/9507237].
148.P. D. Draggiotis, R. H. Kleiss and C. G. Papadopoulos, Eur. Phys. J. C **24** (2002) 447 [arXiv:hep-ph/0202201].
149.G. Gustafson, Phys. Lett. B **175** (1986) 453.
150.M. Bengtsson and T. Sjostrand, Phys. Lett. B **185** (1987) 435.
151.M. Bengtsson and T. Sjostrand, Nucl. Phys. B **289** (1987) 810.
152.G. Gustafson and U. Pettersson, Nucl. Phys. B **306** (1988) 746.
153.M. H. Seymour, Comput. Phys. Commun. **90** (1995) 95 [arXiv:hep-ph/9410414].
154.S. Catani, F. Krauss, R. Kuhn and B. R. Webber, JHEP **0111** (2001) 063 [arXiv:hep-ph/0109231].
155.R. Kuhn, F. Krauss, B. Ivanyi and G. Soff, Comput. Phys. Commun. **134** (2001) 223 [arXiv:hep-ph/0004270].
156.L. Lonnblad, JHEP **0205** (2002) 046 [arXiv:hep-ph/0112284].
157.F. Krauss, JHEP **0208** (2002) 015 [arXiv:hep-ph/0205283].
158.S. Catani, Y. L. Dokshitzer, M. Olsson, G. Turnock and B. R. Webber, Phys. Lett. B **269** (1991) 432.
159.G. Parisi, Phys. Lett. B **74** (1978) 65.
160.B. Adeva *et al.* [L3 Collaboration], Z. Phys. C **55** (1992) 39.
161.S. Frixione and B. R. Webber, JHEP **0206** (2002) 029 [arXiv:hep-ph/0204244].
162.B. Potter, Phys. Rev. D **63** (2001) 114017 [hep-ph/0007172];
163.B. Potter and T. Schorner, Phys. Lett. B **517** (2001) 86 [hep-ph/0104261].
164.M. Dobbs, Phys. Rev. D **64** (2001) 034016 [hep-ph/0103174];
165.M. Dobbs, Phys. Rev. D **65** (2002) 094011 [hep-ph/0111234].
166.H. Baer and M. H. Reno, Phys. Rev. D **44** (1991) 3375.
167.H. Baer and M. H. Reno, Phys. Rev. D **45** (1992) 1503;
168.H. Baer and M. H. Reno, Phys. Rev. D **54** (1996) 2017 [hep-ph/9603209].
169.J. Collins, JHEP **0005** (2000) 004 [hep-ph/0001040].
170.Y. Chen, J. C. Collins and N. Tkachuk, JHEP **0106** (2001) 015 [hep-ph/0105291].
171.J. Collins, Phys. Rev. D **65** (2002) 094016 [hep-ph/0110113].
172.Y. Chen, J. Collins and X. Zu, hep-ph/0110257;
173.Y. Kurihara, arXiv:hep-ph/0207214.
174.S. Frixione and B. R. Webber, arXiv:hep-ph/0207182.
175.B. Abbott *et al.* [D0 Collaboration], Phys. Rev. Lett. **84** (2000) 2786 [arXiv:hep-ex/9912017].
176.V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **87** (2001) 251805 [arXiv:hep-ex/0106026].
177.D. Acosta *et al.* [CDF Collaboration], Phys. Rev. D **65** (2002) 112003 [arXiv:hep-

ex/0201004].
178. E. Laenen, G. Sterman and W. Vogelsang, Phys. Rev. D **63** (2001) 114018 [arXiv:hep-ph/0010080].
179. A. Kulesza, G. Sterman and W. Vogelsang, Phys. Rev. D **66** (2002) 014011 [arXiv:hep-ph/0202251].
180. V. M. Abazov *et al.* [D0 Collaboration], Phys. Lett. B **525** (2002) 211 [arXiv:hep-ex/0109041].
181. J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. Nadolsky and W. K. Tung, JHEP **0207** (2002) 012 [arXiv:hep-ph/0201195].