Prospects for the B_c Studies at LHCb

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Abstract

We discuss the motivations and perspectives for the studies of the mesons of the (bc) family at LHCb. The description of production and decays at LHC energies is given in details. The event yields, detection efficiencies, and background conditions for several B_c decay modes at LHCb are estimated.

1 Introduction

The B_c meson is the ground state of $\bar{b}c$ system which in many respects is an intermediate between charmonium and bottonium systems. However because the B_c mesons carry flavour, they provide a window for studying the heavy-quark dynamics that is very different from those provided by $c\bar{c}$- and $b\bar{b}$-quarkonia.

The first observation of approximately 20 B_c events in the $B_c \to J\Psi l\nu$ decay mode by the CDF collaboration [1] demonstrates the possibility of the experimental study of the B_c meson.

The $\bar{b}c$ states have rich spectroscopy of the orbital and angular-momentum exitations. Below the threshold of the decay into $B - D$ pair, one can expect 16 extremely narrow states which cascadely decay into the ground pseudoscalar state with mass of about 6.3 GeV by radiating photons and pion pairs. The annihilation decays can occure due to weak interactions only and, hence, are suppressed for excited levels.

The production mechanism for $\bar{b}c$ system differs in an essential way from that for $\bar{b}b$ system, because two heavy quark-antiquark pairs must be created in a collision. While the $\bar{b}b$ pair can be created in the parton processes $q\bar{q}$, $gg \to b\bar{b}$ at the order of α_S^2, the lowest order mechanism for the creating of $\bar{b}c$ system is at least of α_S^4: $q\bar{q}$, $gg \to (\bar{b}c)b\bar{c}$, and gluon-gluon contribution dominates at Tevatron and LHC energies. At LHC with the luminosity of about $\mathcal{L} = 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ one could expect around 5×10^{10} B_c events per year. At Tevatron energy the expected yield should be at least one order smaller.

The weak decays of B_c mesons are attractive due to presence of both channels: a) the b-quark decay with the c quark as a spectator, and b) the c-constituent decay with b as a spectator. In addition the weak annihilation contribution into decay channels is quite visible (around 10%).

The dominant contribution into B_c life-time ($\tau_{B_c} \sim 0.5$ ps) comes from the c-quark decays ($\sim 70\%$) while the b-quark decays and weak annihilation add about 20% and 10% respectively.

The accurate measurement of the B_c life-time can provide us with the information on both the masses of charm and beauty quarks and the normalization point of non-leptonic weak Lagrangian in the B_c decays.

The experimental study of the semileptonic decays and the extraction of the decay formfactors can test the spin symmetry derived in NRQCD and HQET approaches. The measurement of the branching fractions for semileptonic and hadronic modes can provide an information about the parameters of weak Lagrangian and hadronic matrix elements determined by the non-perturbative effects due to quark confinement.

2 The mass spectrum of the $(\bar{b}c)$ family

The most accurate estimates of $(\bar{b}c)$ masses [2, 3] can be obtained in the framework of non-relativistic potential models based on the NRQCD expansion over both $1/m_Q$ and $v_{rel} \to 0$ [4].

The uncertainty of evaluation is about 30 MeV. The reason is the following. The potential models [5] were justified for the well measured masses of charmonium and bottomonium. So, the potentials with various global behaviour, i.e. with the different $r \to \infty$ and $r \to 0$ asymptotics, have the same form in the range of mean distances between the quarks in the heavy quarkonia at $0.2 < r < 1$ fm [6]. The observed regularity in the distances between the

excitation levels are approximately flavor-independent. The latter is exact for the logarithmic potential (the Feynman–Hell-Mann theorem), where the average kinetic energy of quarks T is a constant value independent of the excitation numbers (the virial theorem) [7]. A slow dependence of the level distances on the reduced mass can be taken into account by the use of the Martin potential (power law: $V(r) = A(r/r_0)^a + C$, $a \ll 1$) [8], wherein the predictions are in agreement with the QCD-motivated Buchmüller-Tye potential with the account for the two-loop evolution of the coupling constant at short distances [9].

So, one gets the picture of $(\bar{b}c)$ levels which is very close to the texture of charmonium and bottomonium. The difference is the jj-binding instead of the LS one.

The spin-dependent perturbation of the potential includes the contribution of the effective one-gluon exchange (the vector part) as well as the scalar confining term [10].

$$
\begin{aligned}
V_{SD}(\vec{r}) \;=\; & \left(\frac{\vec{L} \cdot \vec{S}_c}{2m_c^2} + \frac{\vec{L} \cdot \vec{S}_b}{2m_b^2} \right) \left(-\frac{dV(r)}{rdr} + \frac{8}{3}\, \alpha_s\, \frac{1}{r^3} \right) + \\
& + \frac{4}{3}\, \alpha_s\, \frac{1}{m_c m_b}\, \frac{\vec{L} \cdot \vec{S}}{r^3} + \frac{4}{3}\, \alpha_s\, \frac{2}{3 m_c m_b}\, \vec{S}_c \cdot \vec{S}_b\, 4\pi\, \delta(\vec{r}) \\
& + \frac{4}{3}\, \alpha_s\, \frac{1}{m_c m_b}\, \left(3(\vec{S}_c \cdot \vec{n})\,(\vec{S}_b \cdot \vec{n}) - \vec{S}_c \cdot \vec{S}_b \right) \frac{1}{r^3}\,, \quad \vec{n} = \frac{\vec{r}}{r}\,.
\end{aligned}
\tag{1}
$$

The model-dependent value of effective α_s [3] can be extracted from the data on the splitting in the charmonium

$$
M(J/\Psi) - M(\eta_c) = \alpha_s \frac{8}{9m_c^2} |R(0)|^2 \approx 117 \text{ MeV}.
$$

We take into account the renormalization-group dependence of α_s at the one-loop accuracy by means of introduction of the quarkonium scale [2]

$$
\mu^2 = \langle \mathbf{p}^2 \rangle = 2\langle T \rangle m_{red}.
$$

The estimated difference between the masses of basic pseudoscalar state and its vector excitation [2] is equal to

$$
M(B_c^{*+}) - M(B_c^{+}) = 65 \pm 15 \text{ MeV}.
$$

The mass of the ground state [2] equals

$$
M(B_c^{+}) = 6.25 \pm 0.03 \text{ GeV},
\tag{2}
$$

which is in agreement with the CDF measurements $M(B_c) = 6.4 \pm 0.19$ GeV [1].

2.1 Radiative transitions

The bright feature of the $(\bar{b}c)$ family is that there are no annihilation decay modes due to the strong interaction. So, the excitations, in a cascade way, decay into the ground state with the emission of photons and pion-pion pairs.

The formulae for the E1-transitions are slightly modified.

$$
\Gamma(\bar{n}P_J \to n^1 S_1 + \gamma) \;=\; \frac{4}{9}\, \alpha_{\text{em}}\, Q_{\text{eff}}^2\, \omega^3\, I^2(\bar{n}P; nS)\, w_J(\bar{n}P)\,,
$$
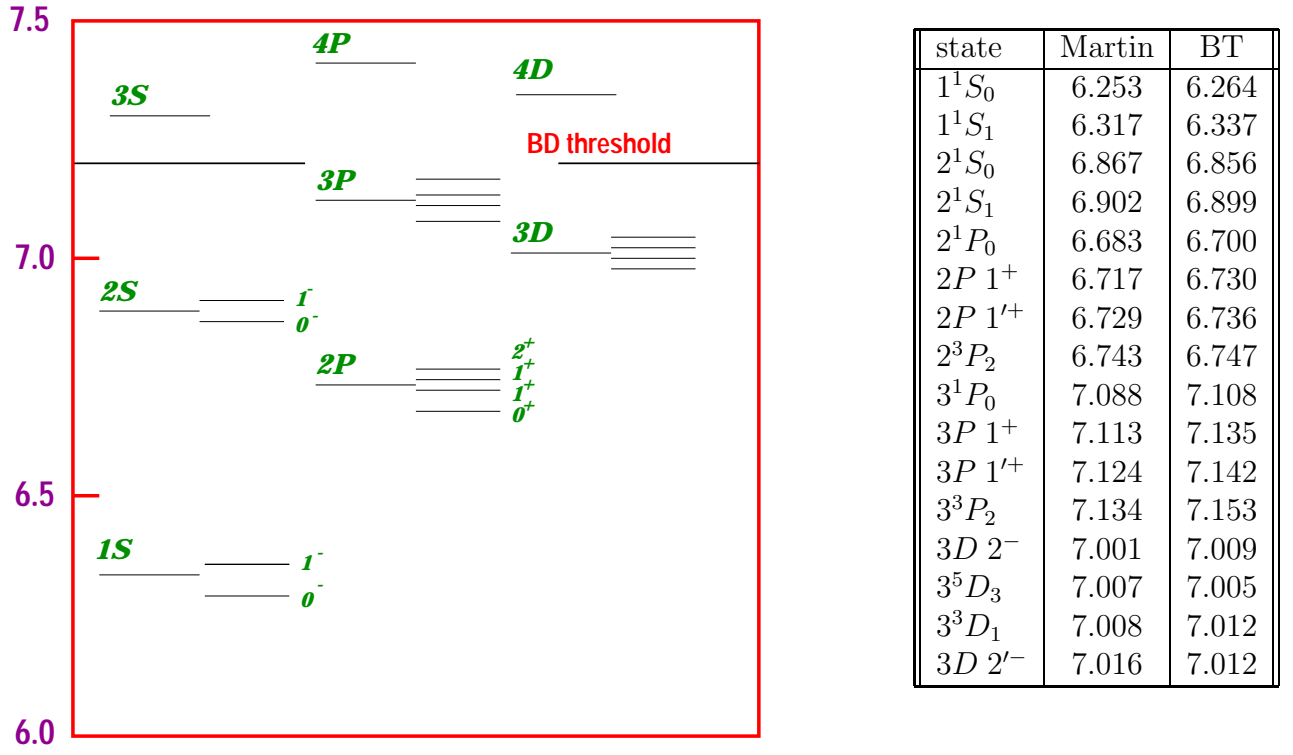
Figure 1: The mass spectrum of $(\bar{b}c)$ with account for the spin-dependent splittings.

state	Martin	BT
1^1S_0	6.253	6.264
1^1S_1	6.317	6.337
2^1S_0	6.867	6.856
2^1S_1	6.902	6.899
2^1P_0	6.683	6.700
$2P\ 1^+$	6.717	6.730
$2P\ 1'^+$	6.729	6.736
2^3P_2	6.743	6.747
3^1P_0	7.088	7.108
$3P\ 1^+$	7.113	7.135
$3P\ 1'^+$	7.124	7.142
3^3P_2	7.134	7.153
$3D\ 2^-$	7.001	7.009
3^5D_3	7.007	7.005
3^3D_1	7.008	7.012
$3D\ 2'^-$	7.016	7.012

$$
\begin{aligned}
\Gamma(\bar{n}P_J \to n^1S_0 + \gamma) &= \frac{4}{9}\,\alpha_{\rm em}\,Q_{\rm eff}^2\,\omega^3\,I^2(\bar{n}P; nS)\,(1 - w_J(\bar{n}P))\,, \\
\Gamma(n^1S_1 \to \bar{n}P_J + \gamma) &= \frac{4}{27}\,\alpha_{\rm em}\,Q_{\rm eff}^2\,\omega^3\,I^2(nS; \bar{n}P)\,(2J+1)\,w_J(\bar{n}P)\,, \\
\Gamma(n^1S_0 \to \bar{n}P_J + \gamma) &= \frac{4}{9}\,\alpha_{\rm em}\,Q_{\rm eff}^2\,\omega^3\,I^2(nS; \bar{n}P)\,(2J+1)\,(1 - w_J(\bar{n}P))\,, \\
\Gamma(\bar{n}P_J \to nD_{J'} + \gamma) &= \frac{4}{27}\,\alpha_{\rm em}\,Q_{\rm eff}^2\,\omega^3\,I^2(nD; \bar{n}P)\,(2J'+1) \\
&\quad w_J(\bar{n}P))w_{J'}(nD)S_{JJ'}\,, \\
\Gamma(nD_J \to \bar{n}P_{J'} + \gamma) &= \frac{4}{27}\,\alpha_{\rm em}\,Q_{\rm eff}^2\,\omega^3\,I^2(nD; \bar{n}P)\,(2J'+1) \\
&\quad w_{J'}(\bar{n}P))w_J(nD)S_{J'J}\,,
\end{aligned}
\tag{3}
$$

where ω is the photon energy, $\alpha_{\rm em}$ is the electromagnetic fine structure constant. In eq.(3) one uses

$$
Q_{\rm eff} = \frac{m_c Q_{\bar{b}} - m_b Q_c}{m_c + m_b}\,,
\tag{4}
$$

where $Q_{c,b}$ are the electric charges of the quarks. For the B_c meson with the parameters from the Martin potential, one gets $Q_{\rm eff} = 0.41$. $w_J(nL)$ is the probability that the spin $S = 1$ in the nL state. $S_{JJ'}$ are the statistical factors. The $I(\bar{n}L; nL')$ value is expressed through the radial wave functions,

$$
I(\bar{n}L; nL') = |\int R_{\bar{n}L}(r)R_{nL'}(r)r^3 {\rm d}r|\,.
\tag{5}
$$

4

For the dipole magnetic M1-transitions one has

$$\Gamma(\bar{n}^1 S_i \to n^1 S_f + \gamma) = \frac{16}{3} \mu_{\text{eff}}^2 \, \omega^3 \, (2f+1) \, A_{if}^2 \,, \tag{6}$$

where

$$A_{if} = \int R_{\bar{n}S}(r) R_{nS}(r) j_0(\omega r/2) r^2 \, \mathrm{d}r \,,$$

and

$$\mu_{\text{eff}} = \frac{1}{2} \frac{\sqrt{\alpha_{\text{em}}}}{2m_c m_b} \left(Q_c m_b - Q_{\bar{b}} m_c \right) \,. \tag{7}$$

Note, that in contrast to the ψ and Υ particles, the total width of the B_c^* meson is equal to the width of its radiative decay into the $B_c(0^-)$ state.

Thus, below the threshold of decay into the BD-pair the theory predicts the existence of 16 narrow $(\bar{b}c)$ states, which do not annihilate due to the strong interactions, but they have the cascade radiative transitions into the ground long-lived pseudoscalar state, the B_c^+ meson.

state	Γ_{tot}, KeV	dominant decay mode	BR, %
$1^1 S_1$	0.06	$1^1 S_0 + \gamma$	100
$2^1 S_0$	67.8	$1^1 S_0 + \pi\pi$	74
$2^1 S_1$	86.3	$1^1 S_1 + \pi\pi$	58
$2^1 P_0$	65.3	$1^1 S_1 + \gamma$	100
$2P\ 1^+$	89.4	$1^1 S_1 + \gamma$	87
$2P\ 1'^+$	139.2	$1^1 S_0 + \gamma$	94
$2^3 P_2$	102.9	$1^1 S_1 + \gamma$	100
$3^1 P_0$	44.8	$2^1 S_1 + \gamma$	57
$3P\ 1^+$	65.3	$2^1 S_1 + \gamma$	49
$3P\ 1'^+$	92.8	$2^1 S_0 + \gamma$	63
$3^3 P_2$	71.6	$2^1 S_1 + \gamma$	69
$3D\ 2^-$	95.0	$2P\ 1^+ + \gamma$	47
$3^5 D_3$	107.9	$2^3 P_2 + \gamma$	71
$3^3 D_1$	155.4	$2^1 P_0 + \gamma$	51
$3D\ 2'^-$	122.0	$2P\ 1'^+ + \gamma$	38

Table 1: The total widths of $(\bar{b}c)$-states with Martin potential

3 B_c lifetime and inclusive decay rates

The B_c-meson decay processes can be subdivided into three classes:
   1) the $\bar{b}$-quark decay with the spectator c-quark,
   2) the c-quark decay with the spectator $\bar{b}$-quark and
   3) the annihilation channel $B_c^+ \to l^+ \nu_l (c\bar{s}, u\bar{s})$, where $l = e,\ \mu,\ \tau$.

In the $\bar{b} \to \bar{c}c\bar{s}$ decays one separates also the Pauli interference with the c-quark from the initial state. In accordance with the given classification, the total width is the sum over the partial widths

$$\Gamma(B_c \to X) = \Gamma(b \to X) + \Gamma(c \to X) + \Gamma(\text{ann.}) + \Gamma(\text{PI}).$$

For the annihilation channel the $\Gamma(\text{ann.})$ width can be reliably estimated in the framework of inclusive approach, where one takes the sum of the leptonic and quark decay modes with account for the hard gluon corrections to the effective four-quark interaction of weak currents. These corrections result in the factor of $a_1 = 1.22 \pm 0.04$. The width is expressed through the leptonic constant of $f_{B_c} \approx 400$ MeV. This estimate of the quark-contribution does not depend on a hadronization model, since a large energy release of the order of the meson mass takes place. From the following expression, one can see that the contribution by light leptons and quarks can be neglected,

$$\Gamma(\text{ann.}) = \sum_{i=\tau,c} \frac{G_F^2}{8\pi} |V_{bc}|^2 f_{B_c}^2 M m_i^2 (1 - m_i^2/m_{B_c}^2)^2 \cdot C_i \,,$$

where $C_\tau = 1$ for the $\tau^+\nu_\tau$-channel and $C_c = 3|V_{cs}|^2 a_1^2$ for the $c\bar{s}$-channel.

As for the non-annihilation decays, in the approach of the Operator Product Expansion for the quark currents of weak decays [21], one takes into account the α_s-corrections to the free quark decays and uses the quark-hadron duality for the final states. Then one considers the matrix element for the transition operator over the bound meson state. The latter allows one also to take into account the effects caused by the motion and virtuality of decaying quark inside the meson because of the interaction with the spectator. In this way the $\bar{b} \to \bar{c}c\bar{s}$ decay mode turns out to be suppressed almost completely due to the Pauli interference with the charm quark from the initial state. Besides, the c-quark decays with the spectator $\bar{b}$-quark are essentially suppressed in comparison with the free quark decays because of a large bound energy in the initial state.

B_c decay mode	OPE, %	PM, %	SR, %
$b \to \bar{c}l^+\nu_l$	3.9 ± 1.0	3.7 ± 0.9	2.9 ± 0.3
$\bar{b} \to \bar{c}u\bar{d}$	16.2 ± 4.1	16.7 ± 4.2	13.1 ± 1.3
$\sum \bar{b} \to \bar{c}$	25.0 ± 6.2	25.0 ± 6.2	19.6 ± 1.9
$c \to sl^+\nu_l$	8.5 ± 2.1	10.1 ± 2.5	9.0 ± 0.9
$c \to su\bar{d}$	47.3 ± 11.8	45.4 ± 11.4	54.0 ± 5.4
$\sum c \to s$	64.3 ± 16.1	65.6 ± 16.4	72.0 ± 7.2
$B_c^+ \to \tau^+\nu_\tau$	2.9 ± 0.7	2.0 ± 0.5	1.8 ± 0.2
$B_c^+ \to c\bar{s}$	7.2 ± 1.8	7.2 ± 1.8	6.6 ± 0.7

Table 2: The branching ratios of the B_c decay modes calculated in the framework of inclusive OPE approach, by summing up the exclusive modes in the potential model [13, 14] and according to the semi-inclusive estimates in the sum rules of QCD and NRQCD [24, 25].

In the framework of exclusive approach, it is necessary to sum up widths of different decay modes calculated in the potential models. While considering the semileptonic decays due to

the $\bar{b} \to \bar{c}l^+\nu_l$ and $c \to sl^+\nu_l$ transitions, one finds that the hadronic final states are practically saturated by the lightest bound $1S$-state in the $(\bar{c}c)$-system, i.e. by the η_c and J/ψ particles, and the $1S$-states in the $(\bar{b}s)$-system, i.e. B_s and B_s^*, which can only enter the accessible energetic gap.

Further, the $\bar{b} \to \bar{c}u\bar{d}$ channel, for example, can be calculated through the given decay width of $\bar{b} \to \bar{c}l^+\nu_l$ with account for the color factor and hard gluon corrections to the four-quark interaction. It can be also obtained as a sum over the widths of decays with the $(u\bar{d})$-system bound states.

The results of calculation for the total B_c width in the inclusive OPE and exclusive PM approaches give the values consistent with each other, if one takes into account the most significant uncertainty related to the choice of quark masses (especially for the charm quark), so that finally, we have

$$\tau[B_c^+]_{\text{OPE, PM}} = 0.55 \pm 0.15 \text{ ps}, \tag{8}$$

which agrees with the measured value of B_c lifetime.

The OPE estimates of inclusive decay rates agree with recent semi-inclusive calculations in the sum rules of QCD and NRQCD [24, 25], where one assumed the saturation of hadronic final states by the ground levels in the $c\bar{c}$ and $\bar{b}s$ systems as well as the factorization allowing one to relate the semileptonic and hadronic decay modes. The coulomb-like corrections in the heavy quarkonia states play an essential role in the B_c decays and allow one to remove the disagreement between the estimates in sum rules and OPE. In contrast to OPE, where the basic uncertainty is given by the variation of heavy quark masses, these parameters are fixed by the two-point sum rules for bottomonia and charmonia, so that the accuracy of SR calculations for the total width of B_c is determined by the choice of scale μ for the hadronic weak lagrangian in decays of charmed quark. We show this dependence in Fig. 2, where $\frac{m_c}{2} < \mu < m_c$ and the dark shaded region corresponds to the scales preferred by data on the charmed meson lifetimes.

Supposing the preferable choice of scale in the $c \to s$ decays of B_c to be equal to $\mu_{B_c}^2 \approx (0.85 \text{ GeV})^2$, putting $a_1(\mu_{B_c}) = 1.20$ and neglecting the contributions caused by nonzero a_2 in the charmed quark decays [25], in the framework of semi-inclusive sum-rule calculations we predict

$$\tau[B_c]_{\text{SR}} = 0.48 \pm 0.05 \text{ ps}, \tag{9}$$

which agrees with the direct summation of exclusive channels calculated in the next sections. In Fig. 2 we show the exclusive estimate of lifetime, too.

4 Semileptonic and leptonic modes

4.1 Semileptonic decays

The semileptonic decay rates are underestimated in the QCD SR approach of ref. [22], because large coulomb-like corrections were not taken into account. The recent analysis of SR in [23, 24, 25] decreased the uncertainty, so that the estimates agree with the calculations in the potential models.

The absolute values of semileptonic widths are presented in Table 3 in comparison with the estimates obtained in potential models.
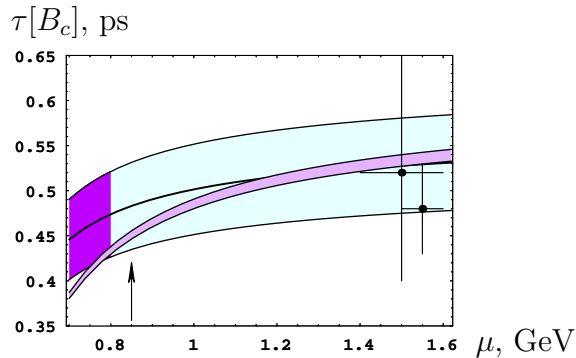
$\tau[B_c]$, ps

μ, GeV

Figure 2: The B_c lifetime calculated in QCD sum rules versus the scale of hadronic weak lagrangian in the decays of charmed quark. The wide shaded region shows the uncertainty of semi-inclusive estimates, the dark shaded region is the preferable choice as given by the lifetimes of charmed mesons. The dots represent the values in OPE approach taken from ref. [21]. The narrow shaded region represents the result obtained by summing up the exclusive channels with the variation of hadronic scale in the decays of beauty anti-quark in the range of $1 < \mu_b < 5$ GeV. The arrow points to the preferable prescription of $\mu = 0.85$ GeV as discussed in [25].

In practice, the most constructive information is given by the J/Ψ mode, since this charmonium is clearly detected in experiments due to the pure leptonic decays [1]. In addition to the investigation of various form factors and their dependence on the transfer squared, we would like to stress that the measurement of decay to the excited state of charmonium, i.e. ψ', could answer the question on the reliability of QCD predictions for the decays to the excited states. We see that to the moment the finite energy sum rules predict the width of $B_c^+ \to \psi' l^+ \nu$ decays in a reasonable agreement with the potential models.

4.2 Leptonic decays

The dominant leptonic decay of B_c is given by the $\tau\nu_\tau$ mode (see Table 2). However, it has a low experimental efficiency of detection because of hadronic background in the τ decays or a missing energy. Recently, in refs. [26] the enhancement of muon and electron channels in the radiative modes was studied. The additional photon allows one to remove the helicity suppression for the leptonic decay of pseudoscalar particle, which leads, say, to the double increase of muonic mode.

5 Non-leptonic modes

In comparison with the inclusive non-leptonic widths, which can be estimated in the framework of quark-hadron duality (see Table 2), the calculations of exclusive modes usually involves the approximation of factorization [11], which, as expected, can be quite accurate for the B_c, since the quark-gluon sea is suppressed in the heavy quarkonium. Thus, the important parameters are the factors a_1 and a_2 in the non-leptonic weak lagrangian, which depend on

8

Mode	Γ [16]	Γ [15]	Γ [17]	Γ [18]	Γ [19]	Γ [20]
$B_c^+ \to \eta_c e^+ \nu$	11	11.1	14.2	14	10.4	8.6
$B_c^+ \to \eta_c \tau^+ \nu$	3.3			3.8		2.9
$B_c^+ \to \eta_c' e^+ \nu$	0.60		0.73		0.74	
$B_c^+ \to \eta_c' \tau^+ \nu$	0.050					
$B_c^+ \to J/\psi e^+ \nu$	28	30.2	34.4	33	16.5	18
$B_c^+ \to J/\psi \tau^+ \nu$	7.0			8.4		5.0
$B_c^+ \to \psi' e^+ \nu$	1.94		1.45		3.1	
$B_c^+ \to \psi' \tau^+ \nu$	0.17					
$B_c^+ \to D^0 e^+ \nu$	0.059	0.049	0.094	0.26	0.026	
$B_c^+ \to D^0 \tau^+ \nu$	0.032			0.14		
$B_c^+ \to D^{*0} e^+ \nu$	0.27	0.192	0.269	0.49	0.053	
$B_c^+ \to D^{*0} \tau^+ \nu$	0.12			0.27		
$B_c^+ \to B_s^0 e^+ \nu$	59	14.3	26.6	29	13.8	15
$B_c^+ \to B_s^{*0} e^+ \nu$	65	50.4	44.0	37	16.9	34
$B_c^+ \to B^0 e^+ \nu$	4.9	1.14	2.30	2.1		
$B_c^+ \to B^{*0} e^+ \nu$	8.5	3.53	3.32	2.3		

Table 3: Exclusive widths of semileptonic B_c^+ decays, Γ in 10^{-15} GeV.

the normalization point suitable for the B_c decays.

The QCD SR estimates for the non-leptonic decays of charmed quark in B_c give the agreement of results with the values predicted by the potential models is rather good for the direct transitions with no permutation of colour lines, i.e. the class I processes with the factor of a_1 in the non-leptonic amplitude determined by the effective lagrangian. In contrast, the sum rule predictions are significantly enhanced in comparison with the values calculated in the potential models for the transitions with the colour permutation, i.e. for the class II processes with the factor of a_2.

Further, for the transitions, wherein the Pauli interference is significantly involved, the class III processes, we find that the absolute values of different terms given by the squares of a_1 and a_2 calculated in the sum rules are in agreement with the estimates of potential models. However, we stress that we have found that due to the Pauli interference determining the negative sign of two amplitudes with a_1 and a_2 the overall sign in some modes should be different from those obtained in the potential models. Taking into account the negative value of a_2 with respect to a_1, we see that half of decays should be enhanced in comparison with the case of Pauli interference switched off, while the other half is suppressed. The characteristic values of effects caused by the Pauli interference is presented in Table 4, where we put the widths in the form

$$\Gamma = \Gamma_0 + \Delta\Gamma, \qquad \Gamma_0 = x_1\, a_1^2 + x_2\, a_2^2, \quad \Delta\Gamma = z a_1\, a_2.$$

Then, we conclude that the Pauli interference can be straightforwardly tested in the listed decays, wherein its significance reaches about 50%.

At large recoils as in $B_c^+ \to J/\Psi \pi^+(\rho^+)$, the spectator picture of transition can be broken by the hard gluon exchanges [27]. The spin effects in such decays were studied in [28]. However,

Mode	$\Delta\Gamma/\Gamma_0$, %
$B_c^+ \to \eta_c D_s^+$	59
$B_c^+ \to \eta_c D_s^{*+}$	-41
$B_c^+ \to J/\psi D_s^+$	-55
$B_c^+ \to J/\psi D_s^{*+}$	53
$B_c^+ \to \eta_c D^+$	43
$B_c^+ \to \eta_c D^{*+}$	-47
$B_c^+ \to J/\psi D^+$	-46
$B_c^+ \to J/\psi D^{*+}$	48

Table 4: The effect of Pauli interference in the exclusive non-leptonic decay widths of the B_c meson with the c-quark as spectator at $a_1 = 1.18$, $a_2 = -0.22$.

we emphasize that the significant rates of B_c decays to the P- and D-wave charmonium states point out that the corrections in the second order of the heavy-quark velocity in the heavy quarkonia under study could be quite essential and suppress the corresponding decay rates, since the relative momentum of heavy quarks inside the quarkonium if different from zero should enhance the virtuality of gluon exchange, which suppresses the decay amplitudes.

The widths of non-leptonic c-quark decays in the framework of the sum rule are greater than those of potential models. In this respect we check that our calculations are consistent with the inclusive ones. So, we sum up the calculated exclusive widths and estimate the total width of B_c meson as shown in Fig. 2, which points to a good agreement of our calculations with those of OPE and semi-inclusive estimates.

Another interesting point is the possibility to extract the factorization parameters a_1 and a_2 in the c-quark decays by measuring the branching ratios

$$
\frac{\Gamma[B_c^+ \to B^+ \bar{K}^0]}{\Gamma[B_c^+ \to B^0 K^+]} = \frac{\Gamma[B_c^+ \to B^+ \bar{K}^{*0}]}{\Gamma[B_c^+ \to B^0 K^{*+}]} =
$$

$$
\frac{\Gamma[B_c^+ \to B^{*+} \bar{K}^0]}{\Gamma[B_c^+ \to B^{*0} K^+]} = \frac{\Gamma[B_c^+ \to B^{*+} \bar{K}^{*0}]}{\Gamma[B_c^+ \to B^{*0} K^{*+}]} = \tag{10}
$$

$$
= \left| \frac{V_{cs}}{V_{cd}^2} \right|^2 \left(\frac{a_2}{a_1} \right)^2 .
$$

This procedure can give the test for the factorization approach itself.

The suppressed decays caused by the flavor changing neutral currents were studied in [29].

The CP-violation in the B_c decays can be investigated in the same manner as made in the charged B decays. The expected CP-asymmetry of $\mathcal{A}(B_c^\pm \to J/\psi D^\pm)$ is about $4 \cdot 10^{-3}$, when the corresponding branching ratio is suppressed as 10^{-4} [30]. The reference-triangle ideology can by applied for the model-independent extraction of CKM-matrix angle γ. However, the corresponding branchings are suppressed, e.g. $\mathrm{Br}(B_c^+ \to D_s^+ D^0) \sim 10^{-5}$. Thus, the direct study of CP-violation in the B_c decays is practically difficult because of low relative yield of B_c with respect to ordinary B mesons: $\sigma(B_c)/\sigma(B) \sim 10^{-3}$.

Another possibility is the lepton tagging of B_s in the $B_c^\pm \to B_s^{(*)} l^\pm \nu$ decays for the study of mixing and CP-violation in the B_s sector [31].

Table 5: Branching ratios [16] of exclusive B_c^+ decays at the fixed choice of factors: $a_1^c = 1.20$ and $a_2^c = -0.317$ in the non-leptonic decays of c quark, and $a_1^b = 1.14$ and $a_2^b = -0.20$ in the non-leptonic decays of $\bar{b}$ quark. The lifetime of B_c is appropriately normalized by $\tau[B_c] \approx 0.45$ ps.

Mode	BR, %	Mode	BR, %	Mode	BR, %
$B_c^+ \to \eta_c e^+ \nu$	0.75	$B_c^+ \to J/\psi K^+$	0.011	$B_c^+ \to B_s^0 K^+$	1.06
$B_c^+ \to \eta_c \tau^+ \nu$	0.23	$B_c \to J/\psi K^{*+}$	0.022	$B_c^+ \to B_s^{*0} K^+$	0.37
$B_c^+ \to \eta_c' e^+ \nu$	0.041	$B_c^+ \to D^+ \overline{D}^0$	0.0053	$B_c^+ \to B_s^0 K^{*+}$	–
$B_c^+ \to \eta_c' \tau^+ \nu$	0.0034	$B_c^+ \to D^+ \overline{D}^{*0}$	0.0075	$B_c^+ \to B_s^{*0} K^{*+}$	–
$B_c^+ \to J/\psi e^+ \nu$	1.9	$B_c^+ \to D^{*+} \overline{D}^0$	0.0049	$B_c^+ \to B^0 \pi^+$	1.06
$B_c^+ \to J/\psi \tau^+ \nu$	0.48	$B_c^+ \to D^{*+} \overline{D}^{*0}$	0.033	$B_c^+ \to B^0 \rho^+$	0.96
$B_c^+ \to \psi' e^+ \nu$	0.132	$B_c^+ \to D_s^+ \overline{D}^0$	0.00048	$B_c^+ \to B^{*0} \pi^+$	0.95
$B_c^+ \to \psi' \tau^+ \nu$	0.011	$B_c^+ \to D_s^+ \overline{D}^{*0}$	0.00071	$B_c^+ \to B^{*0} \rho^+$	2.57
$B_c^+ \to D^0 e^+ \nu$	0.004	$B_c^+ \to D_s^{*+} \overline{D}^0$	0.00045	$B_c^+ \to B^0 K^+$	0.07
$B_c^+ \to D^0 \tau^+ \nu$	0.002	$B_c^+ \to D_s^{*+} \overline{D}^{*0}$	0.0026	$B_c^+ \to B^0 K^{*+}$	0.015
$B_c^+ \to D^{*0} e^+ \nu$	0.018	$B_c^+ \to \eta_c D_s^+$	0.86	$B_c^+ \to B^{*0} K^+$	0.055
$B_c^+ \to D^{*0} \tau^+ \nu$	0.008	$B_c^+ \to \eta_c D_s^{*+}$	0.26	$B_c^+ \to B^{*0} K^{*+}$	0.058
$B_c^+ \to B_s^0 e^+ \nu$	4.03	$B_c^+ \to J/\psi D_s^+$	0.17	$B_c^+ \to B^+ \overline{K^0}$	1.98
$B_c^+ \to B_s^{*0} e^+ \nu$	5.06	$B_c^+ \to J/\psi D_s^{*+}$	1.97	$B_c^+ \to B^+ \overline{K^{*0}}$	0.43
$B_c^+ \to B^0 e^+ \nu$	0.34	$B_c^+ \to \eta_c D^+$	0.032	$B_c^+ \to B^{*+} \overline{K^0}$	1.60
$B_c^+ \to B^{*0} e^+ \nu$	0.58	$B_c^+ \to \eta_c D^{*+}$	0.010	$B_c^+ \to B^{*+} \overline{K^{*0}}$	1.67
$B_c^+ \to \eta_c \pi^+$	0.20	$B_c^+ \to J/\psi D^+$	0.009	$B_c^+ \to B^+ \pi^0$	0.037
$B_c^+ \to \eta_c \rho^+$	0.42	$B_c^+ \to J/\psi D^{*+}$	0.074	$B_c^+ \to B^+ \rho^0$	0.034
$B_c^+ \to J/\psi \pi^+$	0.13	$B_c^+ \to B_s^0 \pi^+$	16.4	$B_c^+ \to B^{*+} \pi^0$	0.033
$B_c^+ \to J/\psi \rho^+$	0.40	$B_c^+ \to B_s^0 \rho^+$	7.2	$B_c^+ \to B^{*+} \rho^0$	0.09
$B_c^+ \to \eta_c K^+$	0.013	$B_c^+ \to B_s^{*0} \pi^+$	6.5	$B_c^+ \to \tau^+ \nu_\tau$	1.6
$B_c^+ \to \eta_c K^{*+}$	0.020	$B_c^+ \to B_s^{*0} \rho^+$	20.2	$B_c^+ \to c\bar{s}$	4.9

We present here the current status of the B_c meson decays. We have found that the various approaches: OPE, Potential models and QCD sum rules, result in the close estimates, while the SR as explored for the various heavy quark systems, lead to a smaller uncertainty due to quite an accurate knowledge of the heavy quark masses. So, summarizing, we expect that the dominant contribution to the B_c lifetime is given by the charmed quark decays ($\sim 70\%$), while the b-quark decays and the weak annihilation add about 20% and 10%, respectively. The coulomb-like α_s/v-corrections play an essential role in the determination of exclusive form factors in the QCD SR. The form factors obey the relations dictated by the spin symmetry of NRQCD and HQET with quite a good accuracy expected.

The predictions of QCD sum rules for the exclusive decays of B_c are summarized in Table 5 at the fixed values of factors $a_{1,2}$ and lifetime. In addition to the decay channels with the heavy charmonium J/ψ well detectable through its leptonic mode, one could expect a significant information on the dynamics of B_c decays from the channels with single heavy mesons, if an experimental efficiency allows one to extract a signal from the cascade decays. An interesting opportunity is presented by the relations for the ratios in (10), which can shed

light to characteristics of the non-leptonic decays in the explicit form.

We have found that the $\bar{b}$ decay to the doubly charmed states gives

$$\text{Br}[B_c^+ \to \bar{c}c\,c\bar{s}] \approx 3.26\%,$$

so that in the absolute value of width it can be compared with the estimate of spectator decay [21],

$$\Gamma[B_c^+ \to \bar{c}c\,c\bar{s}]\Big|_{\text{SR}} \approx 48 \cdot 10^{-15}\,\text{GeV},$$

$$\Gamma[B_c^+ \to \bar{c}c\,c\bar{s}]\Big|_{\text{spect.}} \approx 90 \cdot 10^{-15}\,\text{GeV},$$

and we find the suppression factor of about $1/2$. This result is in agreement with the estimate in OPE [21], where a strong dependence of negative term caused by the Pauli interference on the normalization scale of non-leptonic weak lagrangian was emphasized, so that at large scales one gets approximately the same suppression factor of $1/2$, too.

To the moment we certainly state that the accurate direct measurement of B_c lifetime can provide us with the information on both the masses of charmed and beauty quarks and the normalization point of non-leptonic weak lagrangian in the B_c decays (the a_1 and a_2 factors). The experimental study of semileptonic decays and the extraction of ratios for the form factors can test the spin symmetry derived in the NRQCD and HQET approaches and decrease the theoretical uncertainties in the corresponding theoretical evaluation of quark parameters, as well as the hadronic matrix elements, determined by the nonperturbative effects caused by the quark confinement. The measurement of branching fractions for the semileptonic and non-leptonic modes and their ratios can give information on the values of factorization parameters, which depend again on the normalization of non-leptonic weak lagrangian. The charmed quark counting in the B_c decays is related to the overall contribution of b quark decays, as well as with the suppression of $\bar{b} \to c\bar{c}\bar{s}$ transition because of the destructive Pauli interference, which value depends on the nonperturbative parameters (roughly estimated, the leptonic constant) and non-leptonic weak lagrangian.

Thus, the progress in measuring the B_c lifetime and decays could improve the theoretical understanding of what really happens in the heavy quark decays at all.

We point also to the papers, wherein some aspects of B_c decays and spectroscopy were studied: non-leptonic decays in [32], polarization effects in the radiative leptonic decays [33], relativistic effects [34], spectroscopy in the systematic approach of potential nonrelativistic QCD in [35], nonperturbative effects in the semileptonic decays [36], exclusive and inclusive decays of B_c states into the lepton pair and hadrons [37], rare decays in [38], the spectroscopy and radiative decays in [39].

6 B_c production

The $(\bar{b}c)$ system is a heavy quarkonium, i.e. it contains two heavy quarks. This determines the general features for the B_c meson production in various interactions:

1. Perturbative calculations for the hard associative production of two heavy pairs of $\bar{c}c$ and $\bar{b}b$ and

2. A soft nonperturbative binding of nonrelativistic quarks in the color-singlet state can be described in the framework of potential models.

The two above conditions result in the suppression of the B_c yield of the order of 10^{-3} with respect to beauty hadrons.

As was mentioned above, the consideration of mechanisms for the hadronic production of different spin B_c-states is based on the factorization of hard parton production of heavy quarks $(\bar{b}b\bar{c}c)$ and soft coupling of $(\bar{b}c)$ bound state [12]. In the first stage of description, the hard subprocess can be reliably calculated in the framework of QCD perturbation theory, while in the second stage the quark binding in the heavy quarkonium can be described in the nonrelativistic potential model assigned to the $(\bar{b}c)$-pair rest system. The latter means that one performs the integration of the final quark state over the quarkonium wave function in the momentum space. Since the relative quark velocity inside the meson is close to zero, the perturbative matrix element can be expanded in series over the relative quark momentum, which is low in comparison with the quark masses determining the scale of virtualities and energies in the matrix element. In the leading approximation one considers only the first nonzero term of such expansion, so that for the S-wave states the matrix element of the parton subprocess for the B_c production is expressed through the perturbative matrix element for the production of four heavy quarks $(gg \to \bar{b}b\bar{c}c)$ with the corresponding projection to the vector or pseudoscalar spin state of $(\bar{b}c)$-system, which is the color singlet, and through the factor of radial wave function at the origin, $R_{nS}(0)$, for the given quarkonium. The perturbative matrix element is calculated for the $(\bar{b}c)$ state, where the quarks move with the same velocity, i.e. one neglects the relative motion of $\bar{b}$ and c.

For the P-wave states, the potential model gives the factor in the form of first derivative of the radial wave function at the origin, $R'_{nL}(0)$. In the perturbative part, one has to calculate the first derivative of the matrix element over the relative quark momentum at the point, where the velocities of quarks, entering the quarkonium, equal each to other.

Thus, in addition to the heavy quark masses, the values of $R_{nS}(0)$, $R'_{nL}(0)$ and α_s are the parameters of calculation for the partonic production of B_c meson. In calculations we use the wave function parameters equal to the values shown in table X and $R'_{2P}(0) = 0.50$ GeV$^{3/2}$. The value of $R(0)$ can be related with the leptonic constant, $\tilde{f}$, so that we have

$$\tilde{f}_{1S} = 0.47 \text{ GeV}, \quad \tilde{f}_{2S} = 0.32 \text{ GeV}. \quad \text{and} \quad \tilde{f}_n = \sqrt{\frac{3}{\pi M_n}} R_{nS}(0) \ .$$

In the approximation of the weak quark binding inside the meson one has $M_{B_c} = m_b + m_c$, so that the performable phase space in calculations is close to physical one at the choices of $m_b = 4.8$ GeV, $m_c = 1.5$ GeV for the $1S$-state, $m_b = 5.1$ GeV, $m_c = 1.8$ GeV for the $2S$-state, $m_b = 5.0$ GeV, $m_c = 1.7$ GeV for the $2P$-state.

At large transverse momenta of the B_c meson, $p_T \gg M_{B_c}$, the production mechanism enters the regime of $\bar{b}$-quark fragmentation (see fig. 1), so that the scale determining the QCD coupling constant in hard $\bar{b}b$ production is given by $\mu_{\bar{b}b}^2 \sim M_{B_c}^2 + p_T^2$, and in the hard fragmentation production of the additional pair of heavy quarks $\bar{c}c$ we get $\mu_{\bar{c}c} \sim m_c$. This scale choice is caused by the high order corrections of perturbation theory to the hard gluon propagators, where the summing of logarithms over the virtualities leads to the pointed μ values. Therefore, the normalization of matrix element is determined by the value of $\alpha_s(\mu_{\bar{b}b})\alpha_s(\mu_{\bar{c}c}) \approx 0.18 \cdot 0.28$. In calculations we use the single combined value of $\alpha_s = 0.22$.

The parton subprocess of gluon-gluon fusion $gg \to B_c^+ + b + \bar{c}$ dominates in the hadron-hadron production of B_c mesons. In the leading approximation of QCD perturbation theory

it requires the calculation of 36 diagrams in the fourth order over the α_s coupling constant.

By the general theorem on factorization, it is clear that at high transverse momenta the fragmentation of the heavier quark $Q \to (Q\bar{q}) + q$ must dominate. It is described by the factorized formula

$$\frac{d\sigma}{dp_T} = \int \frac{d\hat{\sigma}(\mu; gg \to Q\bar{Q})}{dk_T}\Big|_{k_T=p_T/x} \cdot D^{Q\to(Q\bar{q})}(x;\mu)\, \frac{dx}{x}\ , \qquad (11)$$

where μ is the factorization scale, $d\hat{\sigma}/dk_T$ is the cross-section for the gluon-gluon production of quarks $Q + \bar{Q}$, D is the fragmentation function.

The calculation for the complete set of diagrams of the $O(\alpha_s^4)$-contribution [12] allows one to determine a value of the transverse momentum p_T^{min}, which is the low boundary of the region where the subprocess of gluon-gluon B_c-meson production enters the regime of factorization for the hard production of $b\bar{b}$-pair and the subsequent fragmentation of $\bar{b}$-quark into the bound $(\bar{b}c)$-state, as it follows from the theorem on the factorization of the hard processes in the perturbative QCD.

The p_T^{min} value turns out to be much greater than the M_{B_c} mass, so that the dominant contribution into the total cross-section of gluon-gluon B_c-production is given by the diagrams of nonfragmentational type, i.e. by the recombination of heavy quarks. Furthermore, the convolution of the parton cross-section with the gluon distributions inside the initial hadrons leads to the suppression of contributions at large transverse momenta, as well as the subprocesses with large energy in the system of parton mass centre, so that the main contribution into the total cross-section of hadronic B_c-production is given by the region of energies less or comparable to the B_c-meson mass, where the fragmentation model can not be applied by its construction. Therefore, one must perform the calculations with the account for all contributions in the given order under consideration in the region near the threshold.

The large numeric value of p_T^{min} means that the majority of events of the hadronic $B_c^{(*)}$-production does not certainly allow the description in the framework of the fragmentation model. This conclusion looks more evident, if one considers the B_c-meson spectrum over the energy.

The basic part of events for the gluon-gluon production of B_c is accumulated in the region of low z close to 0, where the recombination dominates. One can draw the conclusion on the essential destructive interference in the region of z close to 1 and $p_T < p_T^{min}$, for the pseudoscalar state.

We have considered in detail the contributions of each diagram in the region of $z \to 1$. In the covariant Feynman gauge the diagrams of the gluon-gluon production of $Q + \bar{Q}$ with the subsequent $Q \to (Q\bar{q})$ fragmentation dominate, as well as the diagrams when the $q\bar{q}$ pair is produced in the region of the initial gluon splitting. However, the contribution of the latter diagrams leads to the destructive interference with the fragmentation amplitude, and this results in the "reduction" of the production cross-section in the region of z close to 1. In the axial gauge with the vector $n^\mu = p_{\bar{Q}}^\mu$ this effect of the interference still manifests itself brighter, since the diagrams like the splitting of gluons dominate by several orders of magnitude over the fragmentation, but the destructive interference results in the cancellation of such extremely large contributions. This interference is caused by the nonabelian nature of QCD, i.e., by the presence of the gluon self-action vertices.

The using of CTEQ5L parameterization for the structure functions of nucleon [40] leads to the total hadronic cross-sections for the B_c mesons of about 0.8 μb that accepts contributions

14
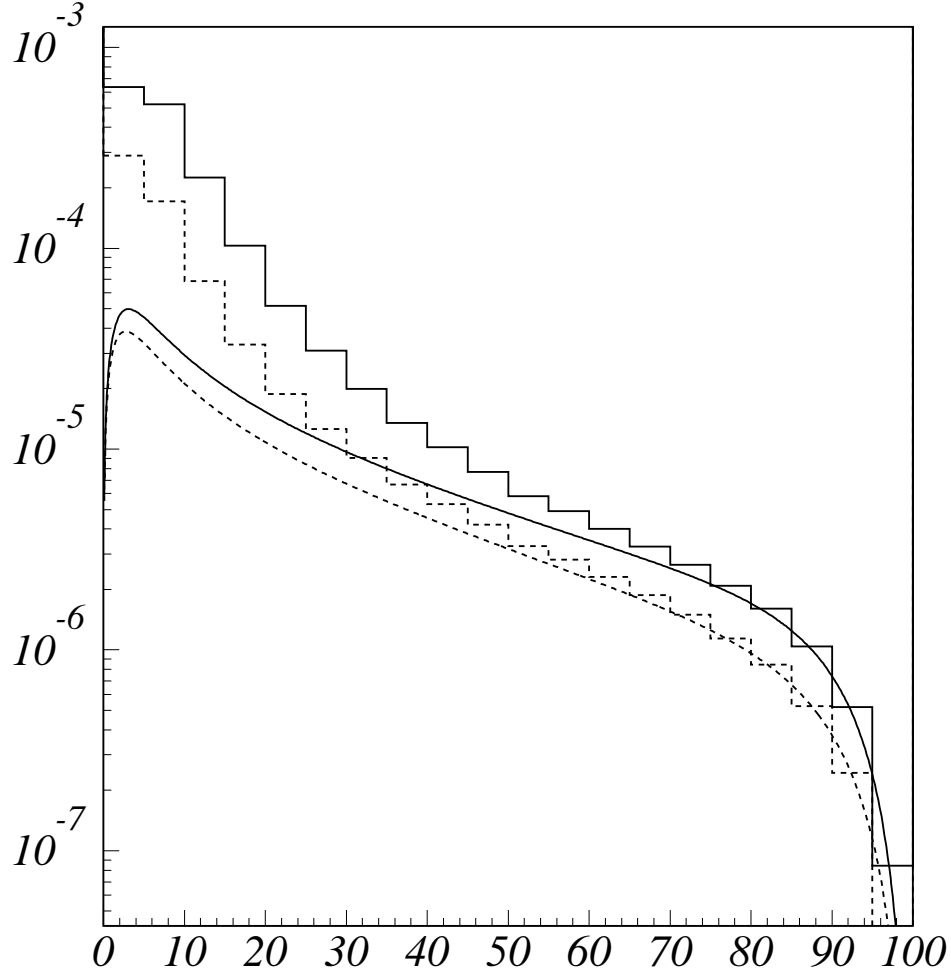
Figure 3: The differential cross-section for the $B_c^{(*)}$ meson production in gluon-gluon collisions as calculated in the perturbative QCD over the complete set of diagrams in the $O(\alpha_s^4)$ order at 200 GeV. The dashed and solid histograms present the pseudoscalar and vector states, respectively, in comparison with the corresponding results of fragmentation model shown by the smooth curves.

15

from:

$$1S_0 \qquad 1S_1 \qquad 2S_0 \qquad 2S_1$$
$$0.19 \ \mu b \quad 0.47 \ \mu b \quad 0.05 \ \mu b \quad 0.11 \ \mu b$$

After the summing over the different spin states, the total cross-sections for the production of P-wave levels is equal to 7% of the S-state cross-section.

At LHC with the luminosity $\mathcal{L} = 10^{34}$ cm^{-2}s^{-1} and $\sqrt{s} = 14$ TeV one could expect $4.5 \cdot 10^{10}$ B_c^+ events per year.

Nevertheless, the P-wave states could be of a particular interest due to their radiative decays with relatively energetic photons (around 500 MeV in the B_c rest frame). For P-wave states, the leading color-singlet matrix element and the leading color-octet matrix elements are both suppressed by a factor of v^2 (relative velocity of the charmed quark) relative to the color-singlet matrix element for S-wave that can enhance the P-vawe contribution.

In the Figure 4 the $d\sigma/dy$ distribution is presented. The y distribution shows the maximum in the central region however, considereing the experimental observability of the B_c states, one should care about the momentum of the meson to ensure the reasonable γ-factor for B_c and visible separation of B_c decay vertex from the primary one.

The right part of the same Figure shows the dependence of the B_c momentum on the angle. One can see that the central region is dominated by low-momentum mesons, which makes the observation of these states in the central region quite difficult task. On the other hand, the forward-backward (LHCb acceptance) regions are dominated by very energetic mesons.

In the region of the LHCb acceptance ($\Theta < 17^o$) the expected number of events with the nominal luminosity of 2×10^{32} is about 10^9 per year. Taking the value of ~ 0.1 as an approximation for the reconstruction efficiency of the decay $B_c \rightarrow J/\Psi\pi \rightarrow \mu\mu\pi$, for example, one gets the total amount of 2×10^4 reconstructed events per year.
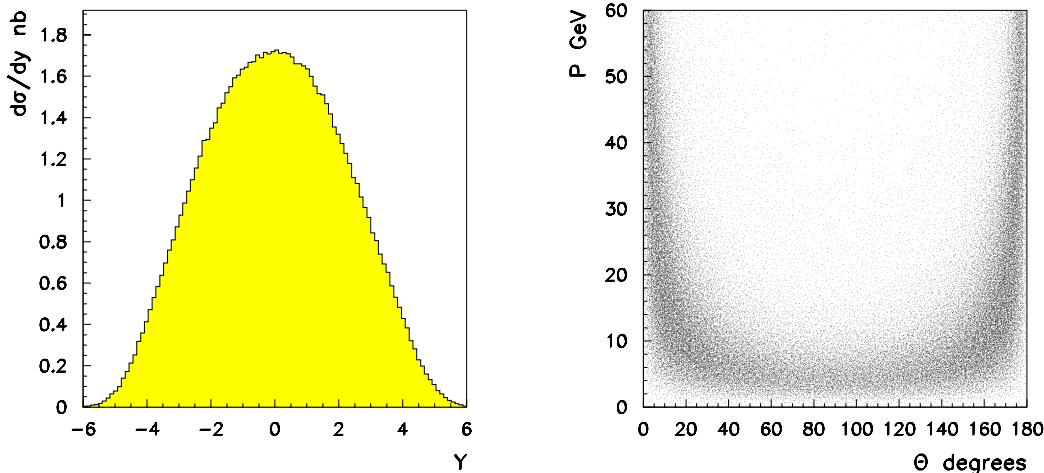

Figure 4: $d\sigma/dy$ (left) and B_c momentum vs. angle (right) for the $B_c^+(1S_0)$ at LHC energy.

The topology of the events with B_c-mesons production is somewhat specific due to extreme kinematics which, particularly, is responsible for the enhancement of the forward-backward regions. The main feature of these events topology consists in the strong corellation in the

direction of B_c and associated D- and b-mesons momenta. One should expect an associated production of all three heavy mesons in the same hemisphere and, moreover, in sufficiently narrow cone.

7 Conclusions

The family of $(\bar{b}c)$ mesons contains 16 narrow states. The S-wave ones will be produced in pp collisions at LHC energies with relatively large cross-sections, ~ 0.1 μb. The total cross-section of the B_c production, taking into accout the cascade decays of the narrow excited states, can be as high as ~ 1 μb. This value is more than order of magnitude larger than at Tevatron energy.

With the total luminosity of about $\mathcal{L} = 10^{34}$ cm^{-2}sec^{-1}, one could expect the total amount of B_c mesons produced of the order of $N_{B_c} \sim 5 \cdot 10^{10}$ per year.

The forward-backward regions of the B_c-mesons production are more favourable in the view of experimental observation due to the strong Lorentz boost of the initial parton system. One can expect $\sim 10^9$ B_c events per year inside LHCb acceptance. This amount is quite sufficient to study the spectroscopy and various decay modes, as well as the life-time of the ground state.

The inclusive decay mode $B_c \to J/\Psi X$ has a branching of about 17%, in comparison with $\sim$1% in $B_{d,s}$-mesons decays. These channels produce a very well visible signatures and one could expect $\sim (2-4) \cdot 10^4$ events in the decay mode $B_c \to J/\Psi\pi \to \mu\mu\pi$ and $\sim 8 \cdot 10^4$ events in the $B_c \to J/\Psi\mu\nu \to \mu\mu\mu\nu$ mode. The Cabbibo-suppressed mode $B_c \to J/\Psi K \to \mu\mu K$ can be observed at the level of $\sim 10^3$ events.

The most probable b-spectator decay modes are saturated by the two-body decays $B_c \to B_s^{(*)}\pi^\pm$ and $B_c \to B_s^{(*)}\rho^\pm$, that makes these channels quite interesting for the study of the c-quark decays in the B_c-meson. The estimated yield of the reconstructed events could be $\sim 10^3$ events per year in the $B_c \to B_s\pi^\pm$ mode. Approximately the same amount of events can be observed in other modes with B_s in the final state, although with worse background conditions.

Rare decay modes, e.g. $B_c \to D^\pm D^0$, could be interesting in view of CP-violation studies, however the preliminary estimations of the detection efficiencies and branchings involved are not too optimistic and additional studies of the reconstruction and selection procedures are required.

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