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# Remarks on Black Hole Instabilities and Closed String Tachyons

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## ABSTRACT

Physical arguments stemming from the theory of black-hole thermodynamics are used to put constraints on the dynamics of closed-string tachyon condensation in Scherk–Schwarz compactifications. A geometrical interpretation of the tachyon condensation involves an effective capping of a noncontractible cycle, thus removing the very topology that supports the tachyons. A semiclassical regime is identified in which the matching between the tachyon condensation and the black-hole instability flow is possible. We formulate a generalized correspondence principle and illustrate it in several different circumstances: an Euclidean interpretation of the transition from strings to black holes across the Hagedorn temperature and instabilities in the brane-antibrane system. <sup>2</sup>

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<sup>2</sup> Contribution to Jacob Bekenstein's Festschrift.

## 1. Introduction

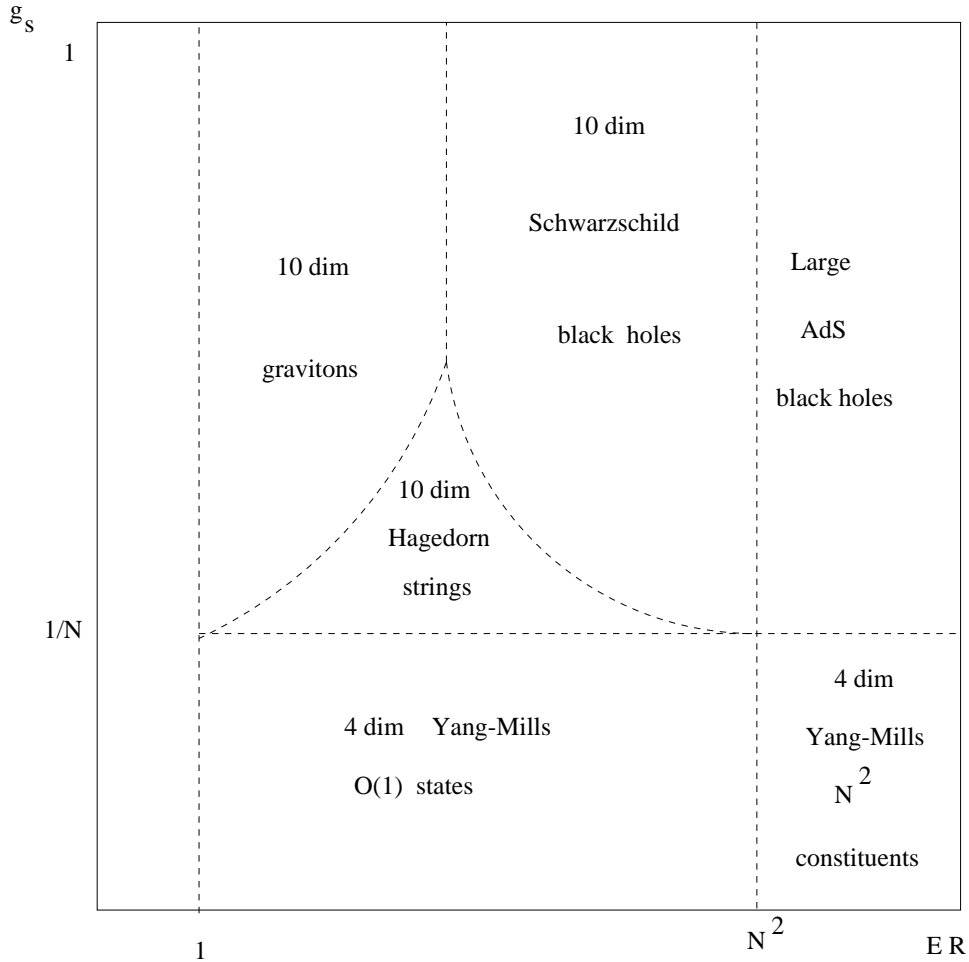
The relation between string theory and gravity was not one of love at first sight. String theory was born in an effort to explain the strong interactions and still treats them as an old flame. In the original open string formulation the effective low energy theory was that of a gauge theory. Unitarity however, required the existence of a closed string sector as well. The low energy theory of closed strings included gravity [1]. Once it was realized that gravity is involved string theory attempted to generalize and replace general relativity, this did not serve to endear string theory to some practitioners of general relativity.

In some sense, a full circle was closed with the nonperturbative formulation of some closed string theories in terms of large- $N$  gauge theories living on the boundary of spacetime [2]. In these developments, one often finds string theory looking for guidance into, general relativity, the very framework it would wish to generalize.

In this contribution we will describe several instances of such “retarded impact” of general relativity in string theory. One such case involves attempts to construct phase diagrams of gravity. String theory is a theory in which, at least it seems, the basic constituents are extended objects. This has many ramifications on the symmetries and the dynamics of the theory. One of the very striking new features of strings is their very large entropy; for free strings it is a linear function of the energy. For a regular field theory in  $d$  spatial dimensions the entropy only increases as  $E^{d/d+1}$ . One consequence of the large string entropy, if it persists for all energies and for finite couplings, is that the system of strings has a maximal temperature called the Hagedorn temperature. The Hagedorn temperature is one of the stop signs appearing in string theory, which may suggest for example an upper bound to the temperature of the universe at short time scales. In the strong interactions setting, it was suggested that this temperature, rather than being limiting, was signaling the limits of the effective description of the theory at that temperature. The constituents of QCD field theory are quarks and gluons, not hadrons. They manifest themselves clearly at high energies and temperatures.

One can address this question also in closed string theory. Can one cross the Hagedorn temperature? Will one uncover more basic constituents of strings in the process? Here general relativity offered some guidance and some resolutions. The entropy of black holes [3,4] and the property of holography [5], both uncovered by Bekenstein, play a key role in the attempts to understand the significance of the Hagedorn temperature. It turns out that when there is a more basic picture of constituents of strings such as in the AdS/CFT correspondence case, the Hagedorn temperature can be surpassed; in other cases the Hagedorn temperature remains the maximal temperature even after black holes are formed. In both cases black holes and their entropy dominate at the high energy scale.

One may draw phase diagrams of string/gravitational systems by constructing a cocktail of, among others, massless fields, strings, branes, black branes and various types of black holes. Such phase diagrams, constructed along the blueprints of [6] and [7], generally are consistent and complete. We present the simplest example in Figure 1 (see, for instance [8,9]). This is the phase diagram of type IIB string theory on  $\text{AdS}_5 \times \mathbf{S}^5$  with  $N$  units of RR flux. Alternatively, it is also the phase diagram of  $\mathcal{N} = 4$  Super Yang–Mills theory on  $\mathbf{S}^3$ . The different regions in this picture are labelled by the degrees of freedom that dominate the density of states.



**Fig. 1:** The phase diagram of type IIB strings on  $\text{AdS}_5 \times \mathbf{S}^5$  with  $N$  units of RR flux, as a function of the string coupling  $g_s$  and the energy, in units of the curvature radius  $R$ . The phase diagram can be continued past  $g_s = 1$  using the self-duality of the theory under  $g_s \rightarrow 1/g_s$ .

If the phase diagram is plotted as a function of temperature rather than energy (i.e. going to the canonical rather than the microcanonical ensemble) one finds that all phases

dominated by *localized* degrees of freedom in ten dimensions disappear (gravitons, strings and Schwarzschild black holes). This shows that these configurations are unstable at fixed temperature. The canonical instability of Schwarzschild black holes is well known on account of their negative specific heat. A thermal gas of gravitons is also unstable to gravitational collapse through the Jeans instability. Both processes end up with a large Anti-de Sitter black hole, which is the only stable configuration at large temperatures. Since such black holes are larger than the curvature radius of AdS, all scales that look approximately like flat ten-dimensional space disappear behind the horizon.

Thus, it is natural to connect the remaining unstable phase, that of Hagedorn strings in ten dimensions, to an instability that ends in the large AdS black hole. At a technical level, the Hagedorn instability is related to the condensation of tachyonic winding modes, when studied in the canonical Euclidean formalism [10,11]. Hence, Euclidean black-hole dynamics is bound to teach us some general lessons about tachyon condensation processes in string theory. This will be the main theme of this contribution.

String theory has been known from its inception to generically contain perturbative instabilities, that is tachyons. In special circumstances these tachyons are absent in the physical sector. Both bulk and boundary tachyons were studied as relevant operators in the CFT worldsheet theory. Following their flow one needs to understand how to maintain fixed the value of the Virasoro central charge. For boundary tachyons the value remains unchanged during the flow, whereas for bulk tachyons it seems that large parts of spacetime disappear during the flow (for boundary tachyons only small parts may disappear). In most cases it is very difficult to follow the flow to a new vacuum.

Euclidean black holes also tend to gobble-up chunks of spacetime as they evolve. Moreover, their topology can be related to their unique thermodynamical properties. We observe that certain instabilities of near-extremal black holes should be identified with tachyonic instabilities in Scherk–Schwarz compactifications. We suggest to use general thermodynamical properties of black holes to identify and constrain the possible flows of stringy tachyon instabilities. This should be done in the regions of the flow which are reliably treated semiclassically. The common feature of the cases treated is that the tachyons can form in the stringy regime because a nontrivial topology supports them. We suggest that, upon condensation, a topology change occurs that stabilizes the system, in such a way that the tachyons undo the very topology that enabled their appearance. The overall geometrical picture bears a resemblance to recent discussions of tachyon condensation in orbifolds (see for example [12] for a recent review).

The paper is organized as follows. In Section 2 we review some standard results and some less known subtleties on the relation between black-hole entropy and spacetime topology. In Section 3 we consider warped Scherk–Schwarz compactifications as our model of

tachyon instability and we formulate our hypothesis about the geometrical interpretation of the corresponding flows. In Section 4 we consider two examples of the same phenomenon in which the physics is under better control. This is the example of the Hagedorn transition in AdS space, and the semiclassical picture of D-brane/antiD-brane annihilation. In Section 5 we abstract from these examples a generalization of the string/black-hole correspondence principle [6,13] that applies to off-shell effective actions.

## 2. Black Hole Entropy and Spacetime Topology

One of the most significant aspects of the Bekenstein–Hawking entropy formula [3,4] is its mysterious relation to spacetime topology [14]. In the semiclassical approximation to the Euclidean functional integral, asymptotic boundary conditions for a thermal ensemble at temperature  $T = 1/\beta$  are such that the metric at infinity must approach  $\mathbf{R}^3 \times \mathbf{S}^1$  with  $\text{Length}(\mathbf{S}^1) = \beta$ . A Schwarzschild black hole in (unstable) equilibrium with the thermal ensemble is described by the classical geometry

$$ds^2 = \left(1 - \frac{2GM}{r}\right) d\tau^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (2.1)$$

with the identification  $\tau \equiv \tau + \beta$ . The Hawking temperature  $1/\beta$  is related to the ADM mass by  $\beta = 8\pi GM$ , in order to avoid the conical singularity at  $r = 2GM$ .

The Euclidean gravitational action of a manifold  $X$  is given by

$$I(X) = -\frac{1}{16\pi G} \int_X (\mathcal{R} - 2\Lambda) - \frac{1}{8\pi G} \oint_{\partial X} \mathcal{K} + C_\infty, \quad (2.2)$$

with  $C_\infty$  an appropriate set of counterterms at non-compact boundaries. When this action is evaluated on the Euclidean Schwarzschild solution,  $I(X)$  is interpreted as the canonical free energy. Now, we would expect any internal degrees of freedom of the black hole to show up at the one-loop level. At the classical level we may imagine that the result should be that of a heavy static particle of mass  $M$ , that is  $I = \beta M$ . However, explicit calculation yields half of this result, i.e. one finds  $I = \beta M/2$ . This we can rewrite as  $I = \beta F = \beta M - S$ , with  $S$  representing an effective “classical” entropy. The result is

$$S = \frac{A_H}{4G} = 4\pi G M^2, \quad (2.3)$$

the Bekenstein–Hawking entropy. Therefore, the formalism “mimics” the fact that black holes have the microstate degeneracy envisaged by Bekenstein, even if the entropy appears as a purely classical feature in this geometry.

In fact, this classical contribution to the entropy has a topological interpretation. Notice that the Schwarzschild manifold (2.1) is simply connected, with topology  $\mathbf{R}^2 \times \mathbf{S}^2$ . This is unlike the standard topology associated to thermal boundary conditions,  $\mathbf{S}^1 \times \mathbf{R}^3$ , which has a non-contractible circle. We can decompose the Euclidean Schwarzschild section as  $X_\epsilon \cup (X - X_\epsilon)$ , where  $X_\epsilon = D_\epsilon \times \mathbf{S}^2$  and  $D_\epsilon$  is a microscopic disc of radius  $\epsilon$  cut out around  $r = 2GM$ . The piece  $X - X_\epsilon$  has cylindrical topology and gives the standard Hamiltonian result:

$$\lim_{\epsilon \rightarrow 0} I(X - X_\epsilon) = \beta M ,$$

whereas the non-Hamiltonian part,  $X_\epsilon$ , gives the entropy:

$$\lim_{\epsilon \rightarrow 0} I(X_\epsilon) = -S .$$

The *ab initio* statistical interpretation of this “classical” entropy has been a long-standing problem that was finally solved for models with holographic dual interpretation [2]. In these cases the classical gravity approximation emerges as the large- $N$  master field of some dual gauge theory living on the boundary of spacetime. In this sense, the classical gravitational entropy computes the planar approximation to the *quantum* entropy of the dual theory.

This topological avatar of the black-hole entropy leads to an interesting puzzle in the case of extremal black holes. Consider a Reissner–Nordstrom (RN) black hole with charge  $Q < M$  and Euclidean metric

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) d\tau^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2 , \quad (2.4)$$

where we have switched to  $G = 1$  units. Now the Hawking inverse temperature is related to  $M$  and  $Q$  by

$$\beta = \frac{4\pi r_0^3}{r_0^2 - Q^2} , \quad (2.5)$$

where the horizon radius is

$$r_0(M) = M + \sqrt{M^2 - Q^2} . \quad (2.6)$$

Notice that the function  $\beta(M, Q)$  diverges as  $(M - Q)^{-1/2}$  in the extremal limit  $M \rightarrow Q$ , i.e. the extremal RN black hole has zero temperature. On the other hand, we recover the Schwarzschild form for  $M \gg Q$ . For a given fixed charge  $Q$ , there is a minimum  $\beta_{\min} = 6\pi\sqrt{3}Q$  occurring at  $M_c = Q/\sqrt{3}$ . This means that, for a fixed  $Q$ , RN black holes have a maximal temperature  $T_{\max} = 1/\beta_{\min}$ . For temperatures below the maximal, there are two RN black holes with the same temperature, one with  $M = M_+(\beta) > M_c$  that has

negative specific heat, and another one with  $M = M_-(\beta) < M_c$  that is thermodynamically stable (it has positive specific heat).

For extremal black holes with  $Q = M$  the horizon at  $r = Q$  degenerates and sits at the end of a “throat” with infinite proper length. The resulting Euclidean geometry has cylindrical topology  $\mathbf{S}^1 \times \mathbf{R}^3$ , i.e. it is a “Hamiltonian” thermal manifold. Therefore, despite the fact that the extremal black hole has non-zero horizon area, one could imagine assigning to it a zero entropy since  $I = \beta M$  in this case [15]. This is at odds with modern microscopic determinations of the entropy for extremal supersymmetric black holes that can be embedded in string theory [16]. In these cases, a non-zero horizon area is always associated to a true degeneracy of supersymmetric ground states of the system. Hence the puzzle.

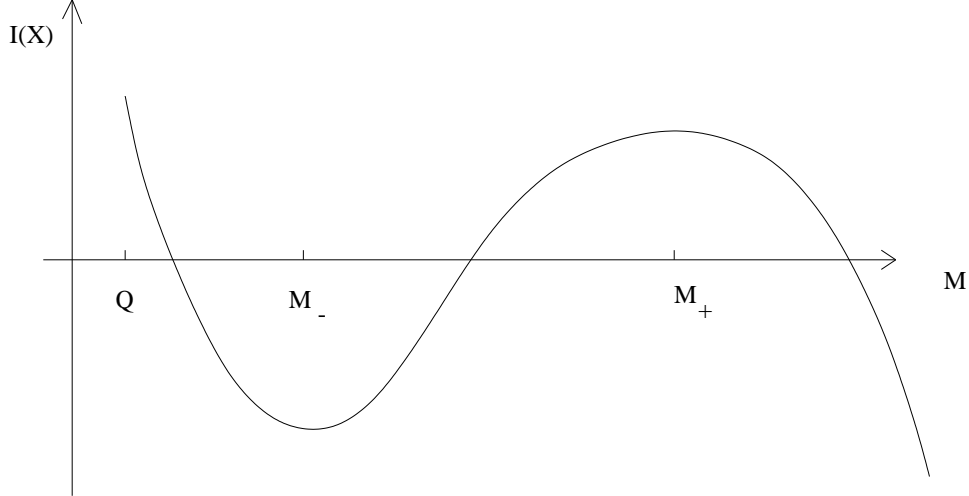
In fact, the definition that assigns non-zero entropy to the extremal black hole turns out to be more physical. The asymptotic boundary conditions of the  $M = Q$  Euclidean metric imply a non-vanishing temperature, whereas the intrinsic temperature of the extremal black hole is zero. Physically, this situation corresponds to introducing a zero-temperature extremal black hole into a thermal bath. The black hole will accrete energy from the bath, thereby increasing its mass, until it matches that of the stable black hole with temperature  $1/\beta$  and mass  $M_-(\beta)$ . Therefore, stable configurations have a non-vanishing limiting entropy as  $M \rightarrow Q$ .

In the Euclidean formalism, we can approximate the black-hole growth process by a set of interpolating metrics,  $X_M$ , with effective mass in the range  $Q < M < M_-(\beta)$ , and with fixed thermal boundary conditions (temperature) at infinity. Such metrics have the form (2.4) with a conical singularity at  $r = r_0$ , which is still given by (2.6). However, the asymptotic temperature does not satisfy (2.5) and the interpolating metrics are not solutions of Einstein’s equations (due to the conical singularity). Along the interpolating family, the Euclidean gravitational action

$$I(X_M) = \beta M - S = \beta M - \pi r_0^2(M)$$

decreases monotonically between the initial and final true solutions (c.f. Fig 2). This is exactly the expected behaviour for a thermal effective potential.

If the RN metric is embedded in string theory, what possible interpretation could the off-shell interpolating metrics have? One context in which off-shell spacetime manifolds have a meaning is in terms of world-sheet renormalization-group (RG) flows. World-sheet conformal fixed points correspond to classical solutions to the background equations of motion, while departures from conformality can be viewed as off-shell deformations. With this analogy in mind, we will refer to the set of interpolating metrics as a “semiclassical flow”.



**Fig. 2:** *The Euclidean action of the interpolating black-hole metrics with fixed  $\beta > \beta_{\min}$ . There is a local minimum at the stable near-extremal black hole  $M = M_-(\beta)$  and a local maximum at the unstable RN Schwarzschild-like black hole  $M_+(\beta)$ .*

### 3. Closed-String Tachyons and Off-Shell Topology Change

Closed-string tachyon condensation is a difficult dynamical problem, partly because it involves background dynamics in the infrared, i.e. the spacetime at infinity is unstable, even at string tree level. For tachyons that are localized at defects or “impurities”, the analysis of the dynamics is more successful. Such is the flow in open string theories [17], the decay of D-branes [18], or tachyonic orbifolds [19]. This suggests an attempt at “quasilocalizing” the closed-string tachyons. One possibility is to use tachyonic winding modes from a *warped* Scherk–Schwarz compactification. Consider backgrounds with topology  $X = \mathbf{S}^1 \times Y$  where the proper length of the circle  $L(\mathbf{S}^1)$  varies monotonically as a function of some “radial” direction of  $Y$ . Let us choose the radial coordinate so that  $L(\mathbf{S}^1)_r$  increases with  $r$  and define a “correspondence radius”  $r_s$  by

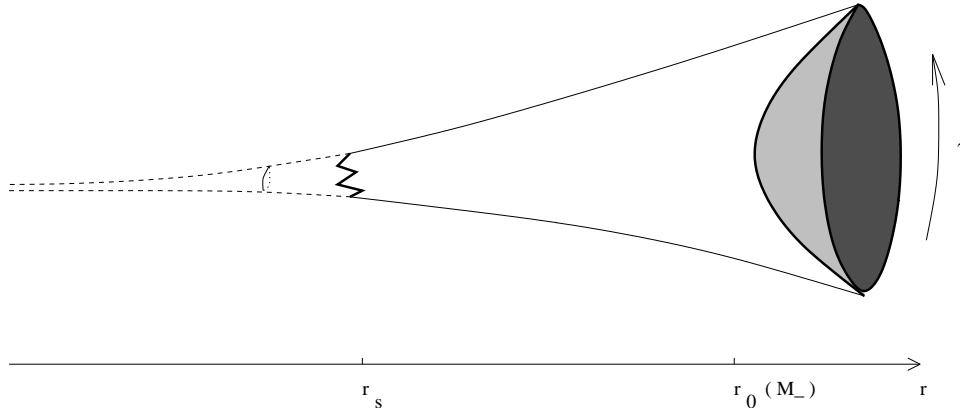
$$L(\mathbf{S}^1)_{r=r_s} = \ell_s, \quad (3.1)$$

where  $\ell_s \sim \sqrt{\alpha'}$  is the fundamental string length scale. Assuming adiabaticity in  $r$ , light winding modes will be supported for  $r < r_s$ , since  $L(\mathbf{S}^1) < \ell_s$  in that region. Moreover, if  $\mathbf{S}^1$  is endowed with a supersymmetry-breaking spin structure, these light modes will be generically tachyonic, due to the negative contribution of the world-sheet Casimir energy.

As a result, the region  $r < r_s$  acts like a “box” that localizes the tachyons, providing an infrared cutoff. This is analogous to the twisted tachyons in orbifolds, which are nothing but winding modes around the tip of the cone. The orbifold example also shows that



tachyons are generic when supersymmetry is broken, even if the adiabatic approximation is violated. On the other hand, depending on the geometry of  $Y$ , we might be able to tune the size of the box and make it macroscopic in string units. In this way we can study the condensation of “quasilocalized” tachyons.



**Fig. 3:** *Adiabatic picture of the tachyonic Scherk–Schwarz compactification. Tachyonic winding modes are confined to  $r < r_s$ . The RG flow ends with the “capping of the throat” at  $r = r_0(M_-)$  and the removal of the nontrivial topology that supported the tachyons.*

If the dilaton can be kept small throughout the background  $X$ , the process of closed-string tachyon condensation can be viewed as some RG flow on the world-sheet, or as a flow in the space of off-shell string backgrounds. This set up is qualitatively similar to our discussion of off-shell RN metrics in the previous section. If the RN black hole admits an embedding in string theory (perhaps completed with other factors), the throat region of the extremal metric with  $M = Q$  is a warped Scherk–Schwarz compactification with

$$L(\mathbf{S}^1)_r = \beta \left( 1 - \frac{Q}{r} \right).$$

Then, the correspondence radius  $r_s$ , defined in (3.1), is nothing but the “stretched horizon” of Refs. [20].

This analogy suggests the identification of the stringy tachyonic instability with the black-hole instability, and the corresponding flow with the process of black-hole growth. The evidence from orbifold decay [19] supports this hypothesis, i.e. the “ironing” of the orbifold into flat space is topologically analogous to the “capping” of the throat by an Euclidean black-hole horizon [21,22] (c.f. Fig. 3).

Based on this intuition, we can derive a boundary condition on the stringy RG flows, giving the rough features of the exit to the semiclassical regime. The proposal is that the

condensation amounts to an effective capping of the non-contractible circle that supports the tachyons [21,22]. When the capping radius  $r_0$  exceeds the correspondence radius  $r_s$ , the flow can be approximated by a semiclassical flow of the type described in the previous section. The final stable geometry does not support winding modes.

What we propose is an off-shell version of the string/black-hole correspondence principle. The general feature is the removal of that part of spacetime that supports the tachyons topologically. In the following, we qualify our hypothesis with a series of remarks.

### 3.1. Remarks

(i)

The particular set of interpolating metrics with a conical singularity at  $r = r_0$  is just one of many possible semiclassical flow trajectories that connect the unstable solution of string theory at  $r_0 = Q$  and the final non-extremal black-hole background with Hawking temperature  $1/\beta$ . Along this particular trajectory, the world-sheet beta function receives contributions localized at the conical defect. One can imagine stringy effects that smooth out the tip of the cone, perhaps along the lines of [19], as well as flows that excite the dilaton. Still, the metrics with conical defect give a good approximation to the thermodynamic quantities (free energy, energy and entropy) for  $r_0 > r_s$ .

(ii)

The flow takes place in RG time, not in real time. The Euclidean action (free energy) is minimized along the flow and thus it is similar to a thermal effective potential. Just as a thermal effective potential compares the free energy of different *static* configurations and has no direct relevance to the discussion of real time evolution of the thermal instability, so the RG flow here must be distinguished from real-time evolution (although we expect RG flow to give an approximation of a sufficiently adiabatic real-time flow). In principle, real-time processes can be realized in some cases as semiclassical “bubbles of nothing”. For example, one can consider asymptotic geometries of the form  $\mathbf{S}^1 \times \mathbf{R}^{n+1}$  with Minkowski signature and Scherk–Schwarz boundary conditions on the circle. Then the analytic continuation of the Schwarzschild Euclidean section yields the “bubble of nothing”, which is then interpreted as a *vacuum* instability, rather than a thermal instability [23,24,25]. Unlike the thermal Euclidean flows discussed in this paper, the real-time “bubbles of nothing” do not stabilize at finite distance and run away to infinity.

(iii)

The decrease of the Euclidean effective action  $I(X)$  along the semiclassical flows is reminiscent of Zamolodchikov’s monotonicity in the central charge [26,27]. However, in the flows discussed above, the overall dimensionality of spacetime *does not* change. Rather, it

is similar to the case of boundary D-brane flows or tachyonic orbifold flows, i.e. the background dynamics leaves a non-compact piece of spacetime essentially untouched. Thus, if the final background is semiclassical, its dimension must coincide with the asymptotic dimension of the original unstable background.

Following Ref. [28], one may try to define a local notion of spacetime central charge as  $C(x) = \mathcal{L}_{\text{eff}}(x)$ , where  $\mathcal{L}_{\text{eff}}$  denotes the integrand of  $I(X)$ . We assume an additive normalization so that  $C(x) = D$  if the initial unstable manifold had dimension  $D$  in the low-curvature limit. Then, in terms of the coordinates  $x^\mu$  that parametrized the complete unstable manifold  $X$ , one can say that  $C(x) \rightarrow 0$  on the region that has fallen behind the black-hole horizon.

This concept of local central charge has some limitations. For example, it misses the importance of boundary contributions to  $I(X)$ , the really relevant quantity. One indication of the primary relevance of the integrated action  $I(X)$  comes from holography. If  $X$  has a holographic dual defined on  $\partial X$ , the classical action  $I(X)$  coincides with the quantum free energy of the dual in the planar approximation. In this sense, it is directly related to a counting of degrees of freedom.

Notice that the flow can be marginal at the classical level. This is the case of orbifolds that are products of two-dimensional cones, as in [19]. In these cases the Einstein action (2.2) is topological and invariant under the “ironing” of the orbifold. One must go to one-loop order to see the depletion of degrees of freedom through the effect of the “excluded volume”. The corresponding one-loop density of states was defined in [29].

**(iv)**

Our proposal should be understood as a boundary condition for the transition between stringy and semiclassical parts of a given flow. Of course, we cannot exclude the existence of other endpoints (possibly metastable) lying entirely within the region of strong worldsheet coupling.

A natural guess for such endpoints comes from considerations of T-duality. Since it is the winding modes that condense, this corresponds to condensation of non-zero momentum modes in T-dual language, a sort of “spontaneous localization”, such as that described in [30,31]. Somewhat similar localization effects can be seen in the case of flat orbifolds (c.f. [29]). All these examples require special arrangements where symmetries highly constrain the gravitational dynamics. This fact, together with the AdS/CFT considerations of the next section, suggest that the black-hole like endpoint would be more generic.

## 4. Other Examples

In this section we consider two other examples of the main idea: off-shell topology change as the semiclassical imprint of tachyon condensation processes in string theory.

The first example is a generalization of the RN set up to the case of D3-branes at finite temperature. In this way we are able to present a regularization of the Hagedorn behaviour of type IIB strings in flat ten-dimensional spacetime. The second example is a semiclassical picture of the problem of D-brane-antiD-brane annihilation that illustrates the same general phenomenon.

### 4.1. Hagedorn Behaviour in AdS Space

One example where the decoupling of the throat from the asymptotically flat spacetime is well understood is that of the AdS/CFT duality. The near-horizon region of  $N$  D3-branes is  $\text{AdS}_5 \times \mathbf{S}^5$  with radius  $R \sim \ell_s (g_s N)^{1/4}$ . The same story that was told for the RN black hole can be repeated here, with the advantage that one has a holographic interpretation of the various actors. For example, the classical action  $I(X)$  is here interpreted as the true quantum free energy of the  $\mathcal{N} = 4$  SYM theory in the planar approximation.

A further advantage of the AdS/CFT background is that it serves as an example of a good adiabatic approximation. Scherk–Schwarz compactifications with vanishing circles can develop strong-coupling behaviour at the singular locus. One argument for this is as follows: if  $L(\mathbf{S}^1) \rightarrow 0$  is approached adiabatically, we may apply T-duality for each circle at fixed  $r$ , resulting in a “trumpet” like geometry with diverging dilaton. Experience with orbifolds of large deficit angle [19], together with the robustness of black-hole thermodynamics, suggests that these effects will not alter the general features of the endpoint proposed here.

In the AdS/CFT example we can actually circumvent these subtleties by considering the AdS space in global coordinates,

$$ds^2 = \left(1 + \frac{r^2}{R^2}\right) d\tau^2 + \left(1 + \frac{r^2}{R^2}\right)^{-1} dr^2 + r^2 d\Omega^2. \quad (4.1)$$

Picking thermal boundary conditions in Euclidean time  $\tau \equiv \tau + \beta$ , this background characterizes the finite-temperature dual CFT theory on a three-sphere  $\mathbf{S}^3$  of finite radius  $R$ .

We have a warped compact circle with

$$L(\mathbf{S}^1)_r = \beta \sqrt{1 + r^2/R^2} \geq \beta.$$

Therefore, there is a lower bound on  $L(\mathbf{S}^1)$ , attained at  $r = 0$ , and no singularity whatsoever. For  $R \gg \ell_s$ , the region  $r < R$  is well approximated by the flat  $\mathbf{S}^1 \times \mathbf{R}^9$  background of type IIB string theory with thermal boundary conditions on the circle. Hagedorn tachyons are “quasilocalized” in this region for  $\beta < \ell_s$  and the adiabatic approximation is controlled by the  $\alpha'$ -corrections effective parameter,  $(g_s N)^{-1/2}$ .

According to the AdS/CFT dictionary, the thermal AdS metric (4.1) in the Hagedorn regime,  $\beta < \ell_s \ll R$ , would represent a highly superheated state of the dual CFT on  $\mathbf{S}^3$  of radius  $R$ , since the temperature is well above the Hawking–Page transition [32,33]. One concludes that this state is highly unstable and will thermalize back into the canonical equilibrium state. The instability is described in the gravity side as the condensation of Hagedorn tachyons. The endpoint, on the other hand, corresponds to the canonical equilibrium state of the high temperature CFT. In the gravity dual it is described by the large AdS black hole with horizon radius  $r_0 \sim R^2/\beta$  and Euclidean metric

$$ds^2 = \left(1 + \frac{r^2}{R^2} - c \frac{M}{r^2}\right) d\tau^2 + \left(1 + \frac{r^2}{R^2} - c \frac{M}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (4.2)$$

with  $c = 16\pi/3\text{Vol}(\mathbf{S}^3)$  and

$$M = \frac{3\text{Vol}(\mathbf{S}^3)}{16\pi} \left(\frac{r_0^4}{R^2} + r_0^2\right). \quad (4.3)$$

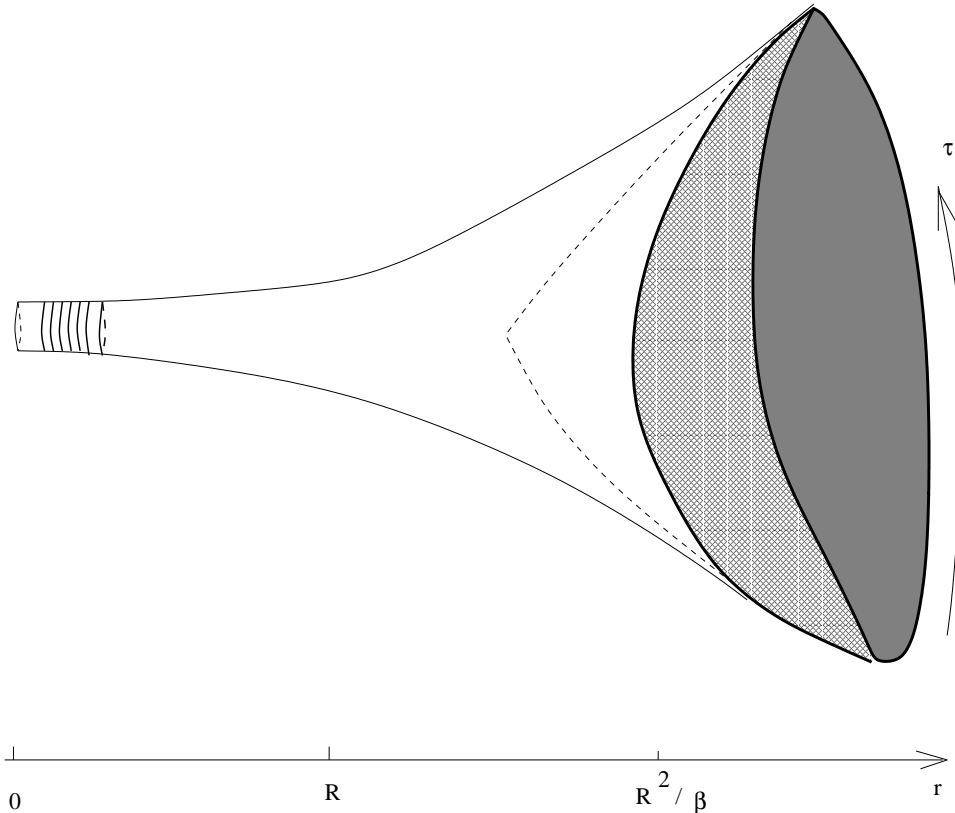
Hence, AdS/CFT gives strong arguments in favour of our geometrical interpretation of the flow [22].

Just as before, we can define a matching radius  $r_s$  by the requirement that the thermal circle be stringy:

$$\beta \sqrt{1 + r_s^2/R^2} = \ell_s. \quad (4.4)$$

In this case, we see that  $r_s \sim R$  for  $\beta < \ell_s$  unless  $\beta/\ell_s$  is fine-tuned to unity. The conjecture is that the stringy part of the tachyon condensation on  $\mathbf{S}^1 \times \mathbf{R}^9$  ends up with the depletion of the whole flat spacetime and one has an  $\text{AdS}_5 \times \mathbf{S}^5$  metric capped at  $r \sim r_s \sim R$ . After this comes the semiclassical part of the flow. It corresponds to black holes that are smaller than the stable AdS black-hole and grow because they are cooler than the asymptotic thermal boundary conditions. Such off-shell black holes can be modelled by the metric (4.2) with fixed  $\beta$  and varying  $r_s < r_0 < R^2/\beta$ , and a conical singularity at the horizon.

The local curvature of the final large AdS black-hole geometry is of order  $(g_s N)^{-1/2}$  in string units. Hence, it vanishes in the flat-space limit  $R/\ell_s \rightarrow \infty$  in which we remove the infrared AdS regulator of the  $\mathbf{S}^1 \times \mathbf{R}^9$  background. In this sense, we can say that the endpoint of tachyon condensation of  $\mathbf{S}^1 \times \mathbf{R}^9$  is the flat  $\mathbf{R}^{10}$  Euclidean background of type IIB string theory. Thus, we can make contact with similar proposals in the literature



**Fig. 4:** *The  $d\Omega = 0$  section of the thermal AdS manifold. It has a cylindrical part that is unstable under condensation of string winding modes, as shown here. The endpoint of the decay process must be the Euclidean large AdS black hole (in dark), which does not support string winding modes. In the  $R/\beta \rightarrow \infty$  limit the unstable cylinder approximates  $\mathbf{S}^1 \times \mathbf{R}^9$  whereas the stable endpoint metric recedes to infinity and approximates locally  $\mathbf{R}^{10}$ . We also show in dashed lines an interpolating off-shell black-hole metric with a conical singularity at the horizon.*

that connect various processes of tachyon condensation with supersymmetric endpoints [34,19,35]. Our picture gives a nice infrared regularization of the process in which the flat endpoint background sits on a “transverse” plane in ten dimensions (see [22] for details). This is entirely analogous to the decay of nonsupersymmetric cones into flat space as described in [19].

There is an interesting connection between this scenario of Hagedorn behaviour and that of Ref. [11], where it was argued that the effective potential for the thermal tachyon on  $\mathbf{S}^1 \times \mathbf{R}^9$  would naturally yield a first-order phase transition at a temperature slightly below the nominal Hagedorn temperature. In our picture, we identify the expectation value of the thermal tachyon with the extent of the  $\mathbf{S}^1$  capping at the correspondence point. Therefore, it is natural to identify the Atick–Witten effective potential itself with

the Euclidean gravitational action,  $I(r_0)$ , as a function of the capping radius  $r_0$

$$\beta \text{Vol}(\mathbf{R}^9) V_{\text{eff}} \approx I(r_0).$$

In this equation, the volume of  $\mathbf{R}^9$  should be regularized to be of order  $R^9$ .

To see the first-order phase transition, notice that the action of AdS black holes,

$$I(r_0) = \beta M(r_0) - S(r_0) = \frac{3 \text{Vol}(\mathbf{S}^3)}{16\pi} \left[ \frac{\beta r_0^4}{R^2} + \beta r_0^2 - \frac{4r_0^3}{3} \right],$$

has two extrema for  $\beta \gg \ell_s$ . One is the large stable AdS black hole with  $r_0 \sim R^2/\beta$ , and the other one is the small (unstable) AdS black hole with  $r_0 \sim \beta$ . The small AdS black hole is a local maximum of  $I(r_0)$  and the value of the action there is positive. Thus, for temperatures slightly below the Hagedorn temperature, the transition from the hot AdS space to the large AdS black hole proceeds initially by tunneling through a barrier, just like in a first order phase transition (of course, there is also the Jeans' instability, but this occurs on a larger length scale).

The transition amplitude was actually calculated in [36], in the case of Schwarzschild black holes in flat space. In the AdS spacetime the tunneling rate can be estimated as

$$\Gamma \sim e^{-I(r_0)} \sim \exp \left[ -\frac{C}{g_s^2} \frac{\beta^3 R^5}{\ell_s^8} \right].$$

Small ten-dimensional Schwarzschild black holes have a rate

$$\Gamma \sim \exp \left[ -\frac{C'}{g_s^2} \frac{\beta^8}{\ell_s^8} \right],$$

which actually dominates for  $\beta < R$ . On closer scrutiny, one finds that these rates are still non-perturbatively suppressed,  $\Gamma \sim \exp(-1/g_s^2)$  for  $\beta \sim \ell_s$ . However, we cannot trust the WKB estimation to this extent. To see this, notice that the variation of the Euclidean action around the maximum is of order  $\Delta I \sim g_s^{-2}$  when the horizon varies on the string scale  $\Delta r_0 \sim \ell_s$ . This means that the low-energy approximation breaks down in the calculation of the barrier when  $\beta \sim \ell_s$ . A natural expectation at this point is that the barrier is completely washed out by  $\alpha'$  corrections, so that the tachyon condensation proceeds without any barriers when  $\beta < \ell_s$ , i.e. it becomes a classical process.

One interesting aspect of this scenario is that it potentially reconciles the (historically orthogonal) microcanonical and canonical pictures of the Hagedorn transition. The essential ingredient in the microcanonical picture of the AdS Hagedorn transition is the string/black-hole correspondence principle [6]. In a sense, we have just incorporated this correspondence principle into the Euclidean canonical picture.

## 4.2. Open-String Tachyons and Topology Change

An interesting testing ground for these ideas is the much better understood condensation of open-string tachyons in the  $Dp - \overline{D}p$  system. States of stretched strings with ground-state mass

$$m^2 = \left( \frac{d_\perp}{2\pi\alpha'} \right)^2 - \frac{c}{\alpha'} \quad (4.5)$$

become tachyonic for  $d_\perp < \mathcal{O}(\ell_s)$ , representing the instability towards brane-antibrane annihilation. For  $d_\perp \gg \ell_s$  the D-branes are essentially stable, except for the small gravitational attraction. They are also non-perturbatively unstable: if they pay the energy necessary to reconnect via a “wormhole” (by virtual tunneling), then they can subsequently lower their energy by the wormhole running off to infinity.

The basic tools to study this process can be found in [37]. Using the non-linear Dirac–Born–Infeld action, one can derive explicit expressions for the wormholes. For a pair of parallel branes separated a distance  $d_\perp$ , one finds two types of wormholes. There is a large unstable solution with a neck of size  $r_0 \sim d_\perp$  and energy (normalized to the bare tension of the parallel branes):

$$E_{\text{large}} \sim \frac{1}{g_s \ell_s} \left( \frac{r_0}{\ell_s} \right)^p \sim \frac{1}{g_s \ell_s} \left( \frac{d_\perp}{\ell_s} \right)^p .$$

(In the derivation of this formula it is actually needed to assume  $p \geq 3$ ). The semiclassical nucleation of such bubbles has rate

$$\Gamma \sim \exp \left[ -\frac{C}{g_s} \left( \frac{d_\perp}{\ell_s} \right)^{p+1} \right] .$$

Keeping the string coupling small, it is natural to assume that this process matches the standard open-string tachyon roll when  $d_\perp \sim \ell_s$ . Again, the WKB barrier must be smoothed out by  $\alpha'$  corrections.

The interesting fact about this example is that it makes this correspondence principle basically inescapable. Consider the case of rotated branes, say by a  $\pi/2$  angle, in such a way that the  $c$  appearing in (4.5) is positive and there are tachyons for  $d_\perp < \mathcal{O}(\ell_s)$ . Now the tachyonic open strings are supported *only* in the vicinity of the “intersection” (the region of the branes within  $\mathcal{O}(d_\perp)$  distance). We know that whatever the microscopic description of the decay would be, at very large distances the decay proceeds by reconnection of the branes that subsequently run off to infinity, i.e. by the analogue of the wormhole process (c.f. Figure 13.4 in [38]). Therefore, there must be a matching between the boundary RG flow that describes the open-string tachyon condensation, and the run-away of the reconnected branes.



The small wormhole solution turns out to be the solitonic description of the fundamental open string stretching between the D-brane pair. This is a bion with neck size  $r_0$  given by  $(r_0/\ell_s)^{p-2} \sim g_s \ell_s / d_\perp$ . It is locally stable for large  $d_\perp$  and has energy

$$E_{\text{small}} \sim \frac{d_\perp}{\ell_s^2}$$

as expected for the stretched fundamental string. When  $d_\perp$  is lowered to the critical value

$$(d_\perp)_{\text{critical}} \sim \ell_s g_s^{\frac{1}{p-1}}$$

the two solutions coalesce and all wormholes of any neck size become unstable towards growth! Hence, it is tempting to regard this process of “blow up” of the tachyonic fundamental string as the geometrical interpretation of the tachyon condensation process. Because the system is non-supersymmetric, the matching length scale could receive  $\mathcal{O}(1)$  corrections in string units, so that the true instability should take place at  $d_\perp \sim \ell_s$ .

We see that open-string tachyon condensation, a much better understood phenomenon, is also compatible with our principle of off-shell topology change. The analogue of the “cylinder capping” in this case is the formation of the wormhole between the two parallel D-branes. The tachyonic modes are topologically supported, since they are just the fundamental strings stretched between the two D-branes. Once the wormhole is formed, a stretched string in the far region can be “unwrapped” through the wormhole neck.

## 5. An Off-Shell Correspondence Principle?

We can abstract from the previous examples the following rule: if the condensation of topologically-supported tachyons has a semiclassical regime, the geometrical interpretation of this regime involves a dynamical topology-change process in the background. The background dynamics consists primarily on the formation of “holes” or “bubbles” in such a way that the topologically supported states can be unwrapped.

In the remainder of this section we speculate on a more quantitative version of this principle. Nonperturbative instabilities in string theory that can be studied in the semiclassical approximation have rates of the form

$$\exp [-I(\ell_{\text{eff}})] = \exp \left[ -\frac{C}{g_s^a} \left( \frac{\ell_{\text{eff}}}{\ell_s} \right)^b \right],$$

where  $\ell_{\text{eff}}$  is a characteristic length scale of the decay process. This formula covers the previously considered cases of closed-string instabilities, with  $a = 2$ , and also cases of

open-string instabilities, for which  $a = 1$ . The exponent  $b$  is related to the effective dimensionality on which the process takes place.

A natural conjecture at this point would be that all these processes match tree-level instabilities (tachyon instabilities) as  $\ell_{\text{eff}} \sim \ell_s$ , in a dynamical generalization of the string/black-hole correspondence principle. For this matching to make sense one has to assume two properties. First,  $\alpha'$  corrections should wash out the WKB barrier at the matching point. Second, we should ensure that the dilaton is small throughout the background in order to apply the classical approximation to the estimate of the decay rate.

It has been proposed that the semiclassical regime and the string tachyon condensation should be dually related. The simplest idea is that some semiclassical decay process may become a tachyon condensation process when the theory is continued to strong coupling  $g_s \rightarrow \infty$  so that the nonperturbative exponent becomes small (the barrier collapses). Characteristic examples of such conjectures are bosonic M-theory and the interpretation of various tachyonic Type 0 models via duality to Type II Scherk–Schwarz compactifications [24,35]. These proposals involve strong-coupling extrapolations of processes that are not protected by any supersymmetry and change radically the properties of the asymptotic vacuum.

We propose here a weaker version of the correspondence between semiclassical decay processes and tachyon condensation. In this version one mimics the correspondence principle between black holes and strings. In that case, one matches *finite energy* configurations as a function of some control parameter, but one stays at weak string coupling, so that the asymptotic vacuum is described by the same string theory on both sides of the correspondence. In other words, it is a change in description from microscopic to semiclassical for a class of states of the Hilbert space, but the vacuum does not change in the process.

Analogously, we propose a matching between the semiclassical effective action  $I(\ell_{\text{eff}})$  and the corresponding effective potential of the microscopic RG flow, without any change of the asymptotic boundary conditions, i.e. with no change of asymptotic vacuum, which is supposed weakly coupled. We think that this principle, although somewhat qualitative, is useful because it seems to capture the essential geometry of tachyonic instabilities in string theory, as we have argued over several examples of very different nature.

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## References

- [1] M.B. Green, J.H. Schwarz and E. Witten, “*Superstring Theory*”, Vols I, II. Cambridge Monographs on Mathematical Physics (1987). J. Polchinski, “*String Theory*”, Vols. I, II. Cambridge University Press (1998).
- [2] J. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998) 231 hep-th/9711200. S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Phys. Lett.* **B428** (1998) 105 hep-th/9802109. E. Witten, *Adv. Theor. Math. Phys.* **2** (1998) 253 hep-th/9802150.
- [3] J.D. Bekenstein, *Phys. Rev.* **D7** (1973) 2333. *Phys. Rev.* **D9** (1974) 3292. *Phys. Rev.* **D12** (1975) 3077.
- [4] S.W. Hawking, *Commun. Math. Phys.* **43** (1975) 199. *Phys. Rev.* **D13** (1976) 191.
- [5] G. 't Hooft, gr-qc/9310026. J.D Bekenstein, *Phys. Rev.* **D49** (1994) 1912. L. Susskind, *J. Math. Phys.* **36** (1995) 6377, hep-th/9409089.
- [6] G.T. Horowitz and J. Polchinski, *Phys. Rev.* **D55** (1997) 6189, hep-th/9612146.
- [7] N. Itzhaki, J.M. Maldacena, J. Sonnenschein and S. Yankielowicz, *Phys. Rev.* **D58** (1998) 046004 hep-th/9802042.
- [8] T. Banks, M.R. Douglas, G.T. Horowitz and E. Martinec, hep-th/9808016.
- [9] S.A. Abel, J.L.F. Barbón, I.I. Kogan and E. Rabinovici, *J. High Energy Phys.* **9904** (1999) 015, hep-th/9902058. See also the contribution to “*Many Faces of the Superworld*”: Yuri Golfand Memorial Volume, World Scientific, Singapore (1999) hep-th/9911004.
- [10] I.I. Kogan, *Sov. Phys. JETP Lett.* **45** (1987) 709. B. Sathiapalan, *Phys. Rev.* **D35** (1987) 3277.
- [11] J.J. Atick and E. Witten, *Nucl. Phys.* **B310** (1988) 291.
- [12] E.J. Martinec, hep-th/0210231.
- [13] M. Bowick, L. Smolin and L.C.R. Wijewardhana, *Gen. Rel. Grav.* **19** (1987) 113. G. Veneziano, *Europhys. Lett.* **2** (1986) 199. L. Susskind, hep-th/9309145. G. Veneziano, in *Hot Hadronic Matter: Theory and Experiments*, Divonne, June 1994, eds J. Letessier, H. Gutbrod and J. Rafelsky, NATO-ASI Series B: Physics, **346** (1995), p. 63. A. Sen, *Mod. Phys. Lett.* **A10** (1995) 2081. E. Halyo, A. Rajaraman and L. Susskind, *Phys. Lett.* **B382** (1997) 319, hep-th/9605112. E. Halyo, A. Rajaraman, B. Kol and L. Susskind, *Phys. Lett.* **B401** (1997) 15, hep-th/9609075.
- [14] G.W. Gibbons and S.W. Hawking, *Phys. Rev.* **D15** (1977) 2752.
- [15] S.W. Hawking and G. Horowitz, *Class. Quant. Grav.* **13** (1996) 1487, gr-qc/9501014.

- [16] A. Strominger and C. Vafa, *Phys. Lett.* **B379** (1996) 99 [hep-th/9601029](#).
- [17] S. Elitzur, E. Rabinovici and G. Sarkisian, *Nucl. Phys.* **B541** (1999) 246 [hep-th/9807161](#).
- [18] A. Sen, [hep-th/9904207](#).
- [19] A. Adams, J. Polchinski and E. Silverstein, [hep-th/0108075](#).
- [20] L. Susskind, [hep-th/9309145](#). L. Susskind and J. Uglum, *Phys. Rev.* **D50** (1994) [hep-th/9401070](#). A. Sen, *Mod. Phys. Lett.* **A10** (1995) 2081 [hep-th/9504147](#).
- [21] J.L.F. Barbón and E. Rabinovici, *Nucl. Phys.* **B545** (1999) 371 [hep-th/9805143](#). J.L.F. Barbón, I.I. Kogan and E. Rabinovici, *Nucl. Phys.* **B544** (1999) 104 [hep-th/9809033](#).
- [22] J.L.F. Barbón and E. Rabinovici, *J. High Energy Phys.* **03** (2002) 057 [hep-th/0112173](#).
- [23] E. Witten, *Nucl. Phys.* **B195** (1982) 481.
- [24] G. Horowitz and L. Susskind, *J. Math. Phys.* **42** (2001) 3158 [hep-th/0012037](#). S.P. de Alwis and A.T. Flournoy, [hep-th/0201185](#). R. Emparan and M. Guperle, *J. High Energy Phys.* **0112** (2001) 023 [hep-th/0111177](#). M. Gutperle, [hep-th/0207131](#).
- [25] O. Aharony, M. Fabinger, G.T. Horowitz and E. Silverstein, *J. High Energy Phys.* **0207** (2002) 007 [hep-th/0204158](#).
- [26] A.A. Zamolodchikov, *JETP Lett.* **43** (1986) 730.
- [27] For boundary flows, see I. Affleck and A.W. Ludwig, *Phys. Rev. Lett.* **67** (1991) 161.
- [28] A.M. Polyakov, “*Gauge Fields and Strings*”, Harwood Academic Publishers (1987). Page 265.
- [29] J.A. Harvey, D. Kutasov, E.J. Martinec and G. Moore, [hep-th/0111154](#).
- [30] D.J. Gross and I.R. Klebanov, *Nucl. Phys.* **B344** (1990) 475.
- [31] S. Elitzur, A. Forge and E. Rabinovici, *Nucl. Phys.* **B359** (1991) 581.
- [32] S.W. Hawking and D. Page, *Commun. Math. Phys.* **87** (1983) 577.
- [33] E. Witten, *Adv. Theor. Math. Phys.* **2** (1998) 505 [hep-th/9803131](#).
- [34] M.S. Costa and M. Gutperle, *J. High Energy Phys.* **03** (2001) 027 [hep-th/0012072](#). M. Gutperle and A. Strominger, *J. High Energy Phys.* **06** (2001) 035 [hep-th/0104136](#).
- [35] J.R. David, M. Gutperle, M. Headrick and S. Minwalla, [hep-th/0111212](#).
- [36] D.J. Gross, M.J. Perry and L.G. Yaffe, *Phys. Rev.* **D25** (1982) 330.
- [37] C.G. Callan and J.M. Maldacena, [hep-th/9708147](#).
- [38] J. Polchinski, “*String Theory*”, Volume II, Cambridge University Press (1998).