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Problems with Time-Varying Extra Dimensions or "Cardassian Expansion" as Alternatives to Dark Energy

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It has recently been proposed that the Universe might be accelerating as a consequence of extra dimensions with time varying size. We show that although these scenarios can lead to acceleration, they run into serious difficulty when taking into account limits on the time variation of the four dimensional Newton's constant. On the other hand, models of "Cardassian" expansion based on extra dimensions violate the weak energy condition for the bulk stress energy, for parameters that give an accelerating universe.

I.   INTRODUCTION

In recent years, the physics community has witnessed a spectacular revival in interest for extra dimensions. This revival was sparked by the realization that extra dimensions need not be small or compactified to agree with current experimental and observational constraints [1]-[3]. The fact that they might lie within the reach of upcoming experiments while at the same time providing an elegant solution to the hierarchy problem has led to intense study of a number of models containing large and/or warped extra dimensions.

Another subject which lately has attracted much attention is the observation that the Universe is accelerating [4]. A number of hypotheses have been put forward in trying to explain this most unexpected result [5]-[10]. Not surprisingly, given the current interest in branes and extra dimensions, many models have appeared which make use of these very ideas to obtain an accelerating Universe [11]-[14].

Here, we will be concerned with the mechanisms proposed in [12] and [14], where the acceleration is due to the presence of extra dimensions. In [12] the essential new ingredient is some kind of bulk stress energy which changes the form of the Friedmann equations at late times, whereas in [14] the acceleration comes from time-variation of the size of the extra dimension. We will examine whether these ideas can lead to acceleration while complying with well-known physical constraints, namely on the time variation of the four dimensional Newton's constant or on the possible equation of state of the new form of stress energy.

The plan of this note is as follows. In section 2, we will give a brief review of the idea presented in [14]. In section 3, we will confront this idea with experimental and observational constraints on the constancy over time of the gravitational force, showing that it runs into serious difficulty. In section 4, we will analyze the relationship between this model and Brans-Dicke theory. In section 5, we explore whether dropping the assumption that the extra dimensions are isotropic can alleviate some of the problems. In section 6 we show that the model which has been proposed for Cardassian acceleration violates the weak energy condition in the bulk. We discuss our results and conclude in section 7.

II.   ACCELERATION FROM TIME VARIATION OF EXTRA DIMENSIONS

The idea starts with a Universe with our usual four spacetime dimensions, supplemented by n compact dimensions with time varying size. The metric will then have the form [14]

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr_a^2}{1 - k_a r_a^2} + r_a^2 d\Omega_a^2 \right) - b^2(t) \left(\frac{dr_b^2}{1 - k_b r_b^2} + r_b^2 d\Omega_b^2 \right) \tag{1}$$

where $k_b = -1, 0, +1$ characterizes the extra dimensions' spatial curvature. This is to be used in the action

$$S = \int d^{4+n}x \sqrt{-g} \left[-\frac{1}{2\kappa^2} \mathcal{R} + \mathcal{L}_m \right] \tag{2}$$

For a perfect fluid, with stress-energy tensor given by

$$T_N^M = \mathrm{diag}(\bar{\rho}, -\bar{p}_a, -\bar{p}_a, -\bar{p}_a, -\bar{p}_b, -\bar{p}_b, \ldots) \tag{3}$$

the Einstein equations will lead to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\bar{G}}{3}\bar{\rho} - \frac{k_a}{a^2} - n\frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{n(n-1)}{6}\left[\left(\frac{\dot{b}}{b}\right)^2 + \frac{k_b}{b^2}\right] \tag{4}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi\bar{G}}{3}\left(\bar{\rho} + 3\bar{p}_a\right) - \frac{n}{2}\frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{n(n-1)}{6}\left[\left(\frac{\dot{b}}{b}\right)^2 + \frac{k_b}{b^2}\right] - \frac{n}{2}\frac{\ddot{b}}{b} \tag{5}$$

$$\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\frac{8\pi\bar{G}}{3}\bar{p}_b - \frac{k_a}{a^2} - (n-1)\frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{(n-1)(n-2)}{6}\left[\left(\frac{\dot{b}}{b}\right)^2 + \frac{k_b}{b^2}\right] - \frac{(n-1)}{3}\frac{\ddot{b}}{b} \tag{6}$$

Here, $\bar{G}, \bar{\rho}, \bar{p}_a$ and $\bar{p}_b$ refer to the $(4+n)$-dimensional quantities. The corresponding 4-dimensional quantities will be written as G_N, ρ, p_a, p_b where $G_N = \bar{G}/b^n$ and $[\rho, p_a, p_b] = b^n \times [\bar{\rho}, \bar{p}_a, \bar{p}_b]$. Notice in particular the new terms in eq. (5) which depend on the extra dimensions. We see that there is the potential for a new source of acceleration beyond those of standard cosmology. Let us now consider how constraints on the time dependence of Newton's constant constrain this possibility.

III.   CONFRONTATION WITH EXPERIMENTAL CONSTRAINTS

It is well known that in models with extra dimensions, the effective strength of gravity is related to the volume of the extra dimensions. This dependence can be negligible in warped geometries [2, 3] but not in the type of factorizable geometry considered here. Indeed, in such a case, the effective 4-D Newton's constant G_N will be inversely proportional to the total volume of the extra dimensions (see, e.g. [1]). Consequently, any variation in the extra dimensions' volume will show up as a variation of G_N,

$$G_N \sim b^{-n} \Rightarrow \frac{\dot{G}_N}{G_N} = -n\frac{\dot{b}}{b}. \tag{7}$$

There are tight constraints on $\dot{G}_N/G_N$ from a number of experimental and observational considerations (see [15] for a thorough review). The most generous upper bounds currently give, roughly, $\left|\dot{G}_N/G_N\right| < 3 \times 10^{-19}s^{-1}$. Combining this with the accepted value for the current Hubble rate [16] $\frac{\dot{a}_0}{a_0} \approx 2,3 \times 10^{-18}s^{-1}$ leads to

$$\left|\frac{\dot{b}_0}{b_0}\right| \lesssim \frac{1}{10n}\frac{\dot{a}_0}{a_0}. \tag{8}$$

Given this bound, we can see immediately by looking at (5) that in the absence of curvature in the extra dimensions, only the term involving $\ddot{b}$ is capable of providing a significant positive contribution to the acceleration. However, this term must then be of the same order as $\left(\frac{\dot{a}_0}{a_0}\right)^2$, a situation which appears somewhat unnatural given the fact that we demand that the first derivative of b be much smaller. This might not be so unnatural if the extra dimensions were oscillating, since then we could accidentally be living at a time when $(\dot{b}/b)^2 \ll \ddot{b}/b$. However, for oscillations whose period is a significant fraction of the age of the Universe, to account for acceleration that was present at a redshift of $z = 1$, the effective four dimensional theory would have to contain a nearly massless $(m \sim H)$ radion with gravitational strength couplings to Standard Model particles. This option is clearly ruled out by experimental constraints on Brans-Dicke-like theories, as we will discuss in more detail below.

Let us now look at what conditions will lead to acceleration, given (4)-(6). Using (6) to eliminate the second derivative of b from (5), and (4) to eliminate ρ, we find that the conditions for getting a positive value of $\ddot{a}/a$ are

$$\frac{\dot{b}}{b} > \frac{\dot{a}}{a} + \sqrt{\frac{(n+1)(n+2)}{n(n-1)}\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{2n+1}{n(n-1)}\frac{k_a}{a^2} - \frac{k_b}{b^2} - \frac{16\pi G_N}{n(n-1)}\left((n-1)p_a - np_b\right)} \tag{9}$$

$$\frac{\dot{b}}{b} < \frac{\dot{a}}{a} - \sqrt{\frac{(n+1)(n+2)}{n(n-1)}\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{2n+1}{n(n-1)}\frac{k_a}{a^2} - \frac{k_b}{b^2} - \frac{16\pi G_N}{n(n-1)}\left((n-1)p_a - np_b\right)}. \tag{10}$$

Comparing with the constraint (8), we can immediately rule out the first inequality (9).

Since we are concerned with acceleration in our current era of matter domination, we will now set p_a to zero. Furthermore, measurements of the CMB imply that our three large spatial dimensions are flat, or have negligibly small curvature so that it is appropriate to set $k_a = 0$. Demanding that (10) not conflict with (8), we find the following:

$$16\pi G_N p_b - \frac{k_b}{b_0^2}(n-1) < \left(\frac{\dot{a}_0}{a_0}\right)^2 \left[\left(1 + \frac{1}{10n}\right)^2 (n-1) - \frac{(n+2)(n+1)}{n}\right] \tag{11}$$

The right hand side is negative for all $n > 0$, so this condition cannot be satisfied in the case studied in [14], where $k_b = p_b = 0$.

Having shown that the simple flat and pressureless option is excluded, we now examine the more general cases where there can be pressure and/or curvature in the extra dimensions. First, suppose the universe has positive spatial curvature. CMB constraints tell us that the term $\frac{n(n-1)}{6}\frac{k_b}{b^2}$ in (4) can be no more than 5% of H^2 in magnitude. I.e., $n/6$ times the term in brackets ([...]) in (11) must exceed -0.05; however the factor in question has a maximum value of -0.63, so the condition cannot be satisfied.

Since the case $p_b = 0$, $k_b = +1$ is unacceptable, suppose instead that the extra dimensions are flat, i.e. $k_b = 0$. Then (11) requires that $p_b < 0$. The only obvious way to do this is through the introduction of some extra matter component to the energy density which would provide negative pressure along the extra dimensions, and most likely along the ordinary dimensions as well. But this is precisely what the model was trying to avoid in the first place, by proposing the kinematics of the extra dimensions as the sole origin of the acceleration. In light of this fact, it is hard to argue that such a model supplemented with some new form of stress energy represents an improvement over other proposed explanations for dark energy.

IV. RELATION TO BRANS-DICKE THEORY

We now analyze the relationship between the model presented above and Brans-Dicke theory [17]. Brans-Dicke (BD) gravity is a modification of general relativity where a scalar field couples to the Ricci scalar, thus yielding a theory in which the gravitational "constant" will be time dependent. The action is given in the simplest case by

$$S = \int d^4x \left(-\phi\mathcal{R} + \frac{\omega}{\phi}\partial_\mu\phi\partial^\mu\phi + \mathcal{L}_m\right), \tag{12}$$

where we assume that $\mathcal{L}_m$ refers to a perfect fluid. We will generalize this slightly by adding a potential for the BD scalar

$$S = \int d^4x \left(-\phi\mathcal{R} + \frac{\omega}{\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \mathcal{L}_m\right). \tag{13}$$

Models of this type have been the subject of a number of papers (see, e.g., [18]). We will now show explicitly the correspondence between the models obtained from (2) and (13). The equations of motion for BD theory with a potential and a typical FRW metric $ds^2 = dt^2 - a(t)^2 dx_i^2$ are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\bar{G}}{3\phi}\rho + \frac{\omega}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{V(\phi)}{6\phi} \tag{14}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi\bar{G}}{3\phi}(\rho + 3p) - \frac{\omega}{3}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{1}{2}\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} - \frac{1}{2}\frac{\ddot{\phi}}{\phi} + \frac{V(\phi)}{6\phi} \tag{15}$$

$$(2\omega + 3) + \left(\frac{1}{3}\frac{\ddot{\phi}}{\phi} + \frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}\right) = \frac{8\pi\bar{G}}{3\phi}(\rho - 3p) + \frac{1}{3}\left(2\frac{V(\phi)}{\phi} - \frac{\partial V(\phi)}{\partial\phi}\right). \tag{16}$$

Comparing these [27] with (4)-(6), we see that the following choice (which was also obtained in [19]) leads to complete equivalence between both sets of equations, as long as the extra dimensional pressure term $\bar{p}_b$ vanishes:

$$\begin{aligned}
\phi &\equiv b^n \\
\omega &\equiv \frac{1}{n} - 1 \\
V(\phi) &\equiv -n(n-1)k_b b^{n-2} \\
&= -n(n-1)k_b \phi^{1-2/n} \tag{17}
\end{aligned}$$

We can now apply all known constraints on the parameters of BD theory to the model of [14]. We note here two important points. The first is that for a vanishing potential, experimental limits demand that $\omega > 1500$ [19]. Clearly, this is not possible in the present context. Furthermore, the potential given here is non-trivial for $n > 2$, in which case its minimum is located at $\phi = 0$, for which G_N diverges.

What can we say about the case $\bar{p}_b \neq 0$? From the point of view of BD theory, we can look at this in the following way. BD theory is explicitly constructed to limit the effect of the scalar field to inducing time dependence in the strength of gravity. This means that matter will not be directly affected by the presence of the scalar, so that conservation of energy will be given by the standard

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$
$$\Rightarrow \rho \sim a^{-3(1+\omega)}. \tag{18}$$

If we now look at the conservation of energy equation in the extra dimensional theory, it will be given by

$$\dot{\bar{\rho}} = -3\frac{\dot{a}}{a}(\bar{\rho} + \bar{p}_a) - n\frac{\dot{b}}{b}(\bar{\rho} + \bar{p}_b). \tag{19}$$

Writing $\bar{p}_a = \omega_a \bar{\rho}$ and $\bar{p}_b = \omega_b \bar{\rho}$, and assuming ω_a and ω_b to be constant, we have

$$\bar{\rho} \sim a^{-3(1+\omega_a)} b^{-n(1+\omega_b)} \tag{20}$$
$$\Rightarrow \rho \sim a^{-3(1+\omega_a)} b^{-n\omega_b}. \tag{21}$$

We can see that if $\bar{p}_b = 0$ then these will reduce to the standard form (18). However, if $\bar{p}_b \neq 0$, conservation of energy will be modified. Indeed, in order for a scalar-tensor theory to behave as the extra dimensional model does in the case where $\bar{p}_b \neq 0$, the energy density would need to scale as (using (17))

$$\rho \sim a^{-3(1+\omega_a)} \phi^{-\omega_b}$$
$$\Rightarrow \dot{\rho} = -3H(1+\omega_a)\rho - \omega_b\frac{\dot{\phi}}{\phi}\rho \tag{22}$$

leading to a theory qualitatively different from BD gravity.

V. ANISOTROPIC EXTRA DIMENSIONS

In the model we have examined above, it was assumed that all the extra dimensions had the same scale factor. We will now drop this assumption and see whether some of the difficulties we have pointed out for isotropic extra dimensions can be relieved. Such models, dubbed *multidimensional cosmology* have already been extensively studied (see [20, 21] and references therein), but, as far as we know, not with a view toward obtaining cosmic acceleration (however see "Note Added" at end).

A particularly interesting aspect of such models is the fact that in the corresponding multi-scalar-tensor theory, only one scalar (corresponding to the total volume of the extra dimensions) couples to matter. This means that constraints on the time-variation of Newton's constant will only apply to this one field. From the point of view of the extra dimensional theory, one might therefore hope that as long as the total volume remains approximately constant, the variations of the individual dimensions can be large enough to have the desired effect on the dynamics of the Universe.

The metric for this model will be given by [28]

$$ds^2 = dt^2 - a^2(t)dx^2 - \sum_i c_i^2(t)d\theta_i^2 \tag{23}$$

Where i runs from 1 to n, the number of extra dimensions. The stress-energy tensor will be that of a 4+n dimensional perfect fluid

$$T_n^m = \mathrm{diag}(\rho, -p, -p, -p, -p^{\theta_1}, \ldots, -p^{\theta_n}). \tag{24}$$

The Einstein equations then lead to the following:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\dot{a}}{a}\sum_i\frac{\dot{c}_i}{c_i} - \frac{1}{6}\left(\sum_i\frac{\dot{c}_i}{c_i}\right)^2 + \frac{1}{6}\sum_i\left(\frac{\dot{c}_i}{c_i}\right)^2 \tag{25}$$

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}[\rho + 3p] - \frac{1}{2}\frac{\dot{a}}{a}\sum_i\frac{\dot{c_i}}{c_i} - \frac{1}{6}\left(\sum_i\frac{\dot{c_i}}{c_i}\right)^2 - \frac{1}{2}\frac{d}{dt}\sum_i\frac{\dot{c_i}}{c_i} - \frac{1}{3}\sum_i\left(\frac{\dot{c_i}}{c_i}\right)^2 \tag{26}$$

$$\frac{8\pi G}{3}p^{\theta_k} - \frac{1}{3}\frac{d}{dt}\left(\frac{\dot{c_k}}{c_k}\right) - \frac{\dot{c_k}}{c_k}\left(\frac{\dot{a}}{a} + \frac{1}{3}\sum_i\frac{\dot{c_i}}{c_i}\right) = -\frac{4\pi G}{3}(\rho - 3p) + \frac{1}{6}\left(\sum_i\frac{\dot{c_i}}{c_i}\right)^2$$
$$+ \frac{1}{2}\frac{\dot{a}}{a}\sum_i\frac{\dot{c_i}}{c_i} + \frac{1}{6}\frac{d}{dt}\sum_i\frac{\dot{c_i}}{c_i}. \tag{27}$$

Note that there will be one equation of the form (27) for each of the n extra dimensions, whereas for isotropic extra dimensions, these equations were degenerate.

The same reasoning we employed in the previous section allows us to state that

$$\left|\sum_i\frac{\dot{c_i}}{c_i}\right| \lesssim \frac{1}{10}\frac{\dot{a_0}}{a_0}. \tag{28}$$

Notice, however, that due to the independence of the $\frac{\dot{c_i}}{c_i}$'s, this in no way constrains the $\sum_i\left(\frac{\dot{c_i}}{c_i}\right)^2$ terms appearing in the Friedmann equations. This means that it is in principle possible that the sum of the squares term in (25) accounts for the missing energy density, while the $\frac{d}{dt}\sum_i\frac{\dot{c_i}}{c_i}$ term in (26) accounts for the acceleration. As in the isotropic case though, it seems unnatural to ask that the time derivative of a small quantity be large. Furthermore, we have not yet taken (27) into account.

Adding the n equations (27) leads to

$$\frac{8\pi G}{3}\sum_i p^{\theta_i} - \frac{1}{3}\frac{d}{dt}\sum_i\left(\frac{\dot{c_i}}{c_i}\right) - \sum_i\frac{\dot{c_i}}{c_i}\left(\frac{\dot{a}}{a} + \frac{1}{3}\sum_i\frac{\dot{c_i}}{c_i}\right) = -n\frac{4\pi G}{3}(\rho - 3p) + \frac{n}{6}\left(\sum_i\frac{\dot{c_i}}{c_i}\right)^2$$
$$+ \frac{n}{2}\frac{\dot{a}}{a}\sum_i\frac{\dot{c_i}}{c_i} + \frac{n}{6}\frac{d}{dt}\sum_i\frac{\dot{c_i}}{c_i} \tag{29}$$

which allows us to write

$$\frac{\ddot{a}}{a} = -4\pi G\left[\frac{\rho}{3}\left(1 + \frac{3n}{2+n}\right) + p\left(1 - \frac{3n}{2+n}\right) + \frac{2}{2+n}\sum_i p^{\theta_i}\right]$$
$$+ \frac{\dot{a}}{a}\sum_i\frac{\dot{c_i}}{c_i} + \frac{1}{3}\left(\sum_i\frac{\dot{c_i}}{c_i}\right)^2 - \frac{1}{3}\sum_i\left(\frac{\dot{c_i}}{c_i}\right)^2. \tag{30}$$

Using (28) and the fact that $p = 0$ in our current matter-dominated epoch, we see that in order to obtain a positive value for $\ddot{a}/a$, we must have $\sum_i p^{\theta_i} < 0$, as was the case for isotropic extra dimensions.

VI. "CARDASSIAN" EXPANSION FROM EXTRA DIMENSIONS

In the previous sections we considered time-varying extra dimensions as a source of cosmic acceleration. Also in the context of extra dimensional effects, it was recently proposed [12, 13] that the current era of acceleration arises from modifying the Friedmann equation by adding a term proportional to ρ^n:

$$H^2 = \frac{8\pi}{3}G(\rho + C\rho^n) \tag{31}$$

The authors have dubbed this kind of expansion "Cardassian." The new term dominates at late times if $n < 1$, and if $n < 2/3$, it gives rise to a positive acceleration. This kind of behavior is qualitatively very different from the standard braneworld result which has $n = 2$ [22], because it implies a modification of gravity at very low energy scales rather than very high ones. Actually eq. (31) is a bit misleading, in giving the impression that ρ is the sole source of expansion. All of the particle physics models which have been proposed to give the modified Friedmann equation introduce an exotic new source of energy which contributes significantly to the expansion; the effects of the new

matter are merely parametrized in terms of the conventional matter energy density ρ. In the present discussion we will treat only the extra-dimensional model of Cardassian expansion on which [12] is based. (In the alternative model of [13], a new form of dark matter with an exotic confining force is hypothesized, in which the interaction energy redshifts more slowly than cold dark matter, and this gives rise to the ρ^α term. We note that there exists no realistic microscopic model for obtaining the kind of dark matter needed by [13], since they require a new confining force with a confinement scale larger than the present horizon, and this force must be mediated by seemingly implausible objects with spatial dimensionality greater than 1.)

The Cardassian model in question is based on work of ref. [23], which showed that if one parametrizes the Hubble rate in terms of the brane energy density ρ (as though the bulk contribution to the energy density was hidden), then an arbitrary power n can be obtained for the Friedmann equation, $H^2 \sim \rho^n$. This conclusion was reached by writing down a candidate solution for the 5D Einstein equations and Israel junction conditions, and then seeing what form of the bulk stress energy tensor would be required to make this indeed a solution to the bulk equations. Specifically, they consider a 5D metric of the form

$$ds^2 = e^{2\nu(t,r)}dt^2 - e^{2\alpha(t,r)}d\mathbf{x}^2 - e^{2\beta(t,r)}dr^2 \tag{32}$$

where the extra dimension is bounded by branes at $r = 0$ and at $r = l$. The metric functions are given by

$$\beta(t,r) = \nu(t,r) = crt^{-2/n}; \qquad \alpha(t,r) = -\frac{1}{2}\beta(t,r) + \frac{2}{3n}\ln t \tag{33}$$

with $c = \frac{1}{3}\kappa_5^2\mu\left(\frac{2}{3n}\right)^{2/n}$, where μ is a dimensionful constant such that the Friedmann equation is exactly $H^2 = (\rho/\mu)^n$. (No solution has been proposed which gives what one really wants, namely a transition from $H^2 \sim \rho$ to $H^2 \sim \rho^n$.)

The size of the extra dimension is time-dependent in the solution given by (33); this may be incompatible with stabilizing its size, as is usually required for a braneworld model to be compatible with fifth force constraints. However β asymptotically approaches a fixed value at large times, so it may be possible that such a solution is consistent with having a stabilized radion, as well as satisfying constraints on the time-dependence of the gravitational force. In the present model, Newton's constant is proportional to

$$G_N \propto \int_0^l e^\beta dr = G_N|_{t\to\infty} \frac{t^{2/n}}{cl}\left(e^{clt^{-2/n}} - 1\right) \tag{34}$$

which approaches a constant as $t \to \infty$. There seems to be sufficient freedom in the choice of parameters to insure the relative constancy of G_N, even since the era of big bang nucleosynthesis.

However, the model runs into difficulties when we examine the equation of state of the bulk stress energy, in particular the $T_5{}^5$ component: we find that the weak energy condition is violated, with the pressure component $|T_5{}^5|$ being larger in magnitude than $T_0{}^0$. Specifically,

$$\kappa_5^2 T_0{}^0 = \frac{e^{-2ct^{-q}r}}{3t^2}\left(q^2 - \frac{9}{4}t^{-2q}c^2r^2q^2 - 9t^{-2q+2}c^2\right) \tag{35}$$

$$\kappa_5^2 T_5{}^5 = -\frac{e^{-2ct^{-q}r}}{3t^2}\left(\frac{9}{2}t^{-q}crq - 9t^{-2q}c^2r^2q^2 - \frac{9}{4}t^{-2q+2}c^2 - 2q^2 - \frac{9}{2}t^{-q}crq^2 + 3q\right)$$

where $q = 2/n$. In the large-time limit, the ratio of $T_5{}^5$ to $T_0{}^0$ is

$$\frac{T_5{}^5}{T_0{}^0} = 2 - \frac{3}{2}n \tag{36}$$

Interestingly, this quantity starts to violate the weak energy condition just for the values of n where acceleration begins, namely $n < 2/3$! We remind the reader that such a violation implies unphysical behavior; in field theory a negative kinetic energy is implied, corresponding to a wrong-sign kinetic term which spoils unitarity.

VII. CONCLUSION

The idea put forth in [14] that the Universe's current accelerated expansion might have purely geometrical origins initially sounds quite attractive. Indeed, it is easy to show that general multidimensional models can lead to our 3-space accelerating without invoking new matter components with exotic equations of state. However, as we have argued, the most simple of such models are completely equivalent to a class of Brans-Dicke theories which are ruled out

by experiment, either because the BD parameter ω is too small, or because the scalar field potential has a minimum for which Newton's constant diverges. The only way to make these models compatible with experimental constraints while still leading to acceleration is to invoke non-zero pressure along the extra dimensions. This, however, means that we are no longer dealing with "normal" matter, thus spoiling the model's initial motivation. This is true for anisotropic as well as for isotropic extra dimensions. From the point of view of scalar-tensor theory, we have shown that this non-vanishing pressure along the extra dimensions corresponds to modifying conservation of energy such that the theory differs from standard BD gravity.

As far as "Cardassian" expansion is concerned, it appears that it is possible to tune the parameters in such a way that Newton's constant tends toward a constant value fast enough to avoid any conflict with experimental evidence. However, we have shown that upon closer inspection, the bulk stress energy tensor behaves in a way which leads to a serious difficulty. Indeed, for exactly the values of the parameter n which make acceleration possible, it violates the weak energy condition.

There exist alternative proposals [11] which make use of extra dimensions to explain acceleration, relying on the extrinsic curvature of branes in the bulk to achieve the desired effect. Other works (e.g. [19, 24, 25]) have also investigated the cosmological implications of the evolution of the radion. In light of our results, it appears that these represent more promising routes to follow than the ones we have analysed here.

VIII.   ACKNOWLEDGEMENTS

IX.   NOTE ADDED

After completion of this work, we were informed by M. Pietroni of an anisotropic model [26] which gives a more negative equation of state ($w = -1/3$) than does ordinary dark matter ($w = 0$). This result is consistent with our observation in section V that negative values for the extra-dimensional pressure components are needed to get cosmic acceleration. The model of ref. [26] has positive pressures, and it does not have acceleration, since $w = -1/3$ is the borderline value between deceleration and acceleration. We thank M. Pietroni for bringing this work to our attention.

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[27] Note that here as in the extra dimensional theory, $\bar{G}$ refers to the bare gravitational coupling. The effective gravitational constant will thus be $G_N = \bar{G}/\phi$.

[28] Following the discussion in section 3, we will assume from the start that the extra dimensions are flat.