Yukawa structure, flavour changing and CP violation in Supergravity.

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The hierarchical structure of fermion masses and mixings strongly suggests an underlying family symmetry. In supergravity any familon field spontaneously breaking this symmetry necessarily acquires an F-term which contributes to the soft trilinear couplings. We show, as a result, $\mu \to e\gamma$ decay can receive large contributions from this source at the level of current experimental bounds and thus this channel may provide the first indication of supersymmetry and a clue to the structure of the soft breaking sector. Using the mercury EDM bounds we find strong bounds on the right handed down quark quark mixing angles that are inconsistent with models relating them to neutrino mixing angles and favour a near-symmetric form for the magnitude of the down quark mass matrix.

Our knowledge of the Supersymmetry breaking sector in the Minimal Supersymmetric extension of the Standard Model (MSSM) is still very limited. Only after the discovery of SUSY particles and the measurement of the Supersymmetric spectrum we will be able to explore it in detail. Nevertheless, we have already a lot of useful information on this sector from experiments looking for indirect effects of SUSY particles in low-energy experiments [1]. In fact, it was readily realized at the beginning of the SUSY phenomenology era that large contributions to Flavour Changing Neutral Currents (FCNC) and CP violation phenomena were expected in Supersymmetric theories with a generic soft breaking sector. The absence of these effects much below the level most theorists considered reasonable came to be known as the SUSY flavour and CP problems. These problems are closely related to the flavour structure of the soft breaking terms and therefore to the source of flavour itself. The main solutions to these problems are either universality of the soft breaking terms or alignment with the Yukawa matrices.

In this paper we ask whether either of these solutions is sufficent in the case the theory possesses a broken family symmetry which generates the hierarchical structure of Yukawa couplings through a superpotential of the form

$$W = W^{hid}(\eta_k) + \left(\frac{\theta}{M}\right)^{\alpha_{ij}} H_a Q_{Li} q_{Rj}^c + \dots \quad (1)$$

The hierarchy arises through effective operators ordered in terms of powers of (θ/M) where θ is a familon field which spontaneously breaks the family symmetry and M is the mass of the mediator transmitting the symmetry breaking to the quarks and leptons ($M \leq M_{Planck}$). We will show that neither universality of the soft breaking terms nor alignment with the Yukawa matrices solutions can be exact and that the soft breaking terms are necessarily nonuniversal and not aligned with the Yukawa couplings. This is due to the fact that, in a theory with broken local Supersymmetry, the familon field(s) necessarily acquires a non-vanishing F-term [2]. This F-term then induces nonuniversal soft SUSY breaking A-terms which are not diagonalised when the fermion masses are

diagonalised, leading to FCNC and CP violating processes.

We first prove the existence of a non-vanishing F term. Consider the supergravity lagrangian specified in terms of the superpotential W and the Kähler potential $K = \hat{K}(\eta, \eta^*) + \sum_i K_i^i(\eta, \eta^*)|\varphi_i|^2$ where K is a real function of the chiral superfields and η are general fields belonging to the hidden sector. The F-term contribution to the scalar potential is simply given by,

$$V = e^K \left[\sum_i (K^{-1})_i^j \tilde{F}^i \tilde{F}_j - 3|W|^2 \right] \quad (2)$$

where $\tilde{F}^i = \partial W/\partial\phi_i + K_i^i\phi^{i*}W$ and $\tilde{F}_i = \partial W/\partial\phi^{i*} + K_i^i\phi_i W^*$ are related to the normalized supergravity F-terms by $F^i = e^{K/2}(K^{-1})_i^i\tilde{F}^i$ and $(K^{-1})_j^i$ is the inverse of the Kähler metric. In the following, we require cancellation of the cosmological constant in the physical minimum, therefore from Eq. (2), this implies that we do not need to consider the derivatives of e^K in the minimization of the scalar potential. We study the case of broken supergravity in which $m_{3/2} = e^{K/2}\langle W \rangle \neq 0$. We will also have a non-vanishing vev for a certain familon field θ after minimization of the scalar potential. This field should have a small vev in units of the Plank mass to generate the hierarchy in the Yukawa couplings which are proportional to powers of θ. The field θ belongs to a non-trivial representation of the family group and is a singlet under the Standard Model gauge group. We will also allow for further familon fields, χ_i, transforming non-trivially under the family symmetry to acquire vevs. Given this, the form of the θ and χ dependence of the Kähler potential is fixed by the requirement that the potential should be invariant under the family symmetry and the leading term is given by $K = \theta\theta^* + \sum_i \chi_i\chi_i^*$ where we have absorbed the constants of proportionality in a redefinition of the fields. With this form of Kähler potential we have $F^\theta = \langle \partial W/\partial\theta + W \theta^* \rangle$, and the only way to avoid the conclusion that $F^\theta \geq m_{3/2} \theta$ is if a cancellation occurs between the two terms [16].

It is instructive to consider first the case that the θ dependent part of the superpotential contributing to the

potential does not involve the vev any other fields, giving $V(\theta) = \left| F^\theta \right|^2 - 3|W|^2$, and minimisation with respect to θ leads to

$$F^\theta \simeq -3m_{3/2}^2 \; \theta \; \left(\frac{\partial^2 W^*}{\partial \theta^{*\,2}} \right)^{-1}, \qquad (3)$$

where we have set $\langle W \rangle = m_{3/2}$, used the condition

$$\langle \frac{\partial W}{\partial \theta} \rangle \simeq -m_{3/2} \langle \theta^* \rangle, \qquad (4)$$

that is needed if F^θ is to be reduced below its natural value $\geq m_{3/2}\theta$, and neglected $\theta \; \partial W/\partial \theta \simeq W \; \theta^2$ with respect to W, due to $\theta << 1$. From Eq. (3) we see that the condition for an anomalously small F^θ is

$$|\frac{\partial^2 W}{\partial \theta^2}| >> m_{3/2}. \qquad (5)$$

It is now straightforward to check whether this possibility is realised for various forms for $W(\theta)$.

We first consider the case that there is only one scale in the problem, namely the Planck scale and that the fermion mass hierarchy is due to an expansion in θ/M_{Planck}. One possibility, which has been widely explored, is that there is an anomalous $U(1)$ family symmetry and only the field θ acquires a vev close to the Planck scale, driven by the requirement the anomalous $D-$term should be small. In this case the superpotential cannot depend on θ except in combination with fields which, for the moment, we require to have vanishing vevs, leading immediately to the conclusion $F^\theta = \theta m_{3/2}$.

A second possibility is that the family symmetry is discrete and then one can have a dependence on θ of the form

$$W = a + (\theta/M_{Planck})^p \qquad (6)$$

where we have allowed for a term, a, coming from the hidden supersymmetry breaking sector and p is the order of the discrete group. In this case $\partial^2 W/\partial\theta^2 \propto p(p-1)\theta^{p-2} = (p-1)m_{3/2}$, where we have used Eq. (4) to determine the magnitude of θ. Inserted in Eq. (3) this suggests F^θ is reduced by a factor $1/(p-1)$ compared to its natural value. However, for $p > 3$, the true minimum of the potential following from Eq. (6) is at $\langle\theta\rangle = 0$. If θ has a renormalisable coupling θXY to other fields in the theory, there are radiative corrections to the soft SUSY breaking terms which must be included in the effective potential, namely the soft mass squared and the soft trilinear term [3]. They can lead to the global minimum having nonzero θ as desired. How do they affect our conclusions? The soft mass only changes $\partial V/\partial\theta$ at $\mathcal{O}(m_{3/2}^2\theta)$ and so, in the large p limit, does not affect the derivation of Eq. (3). However the trilinear term contributes to $\partial V/\partial\theta$ at $\mathcal{O}((p-1)m_{3/2}\theta)$ leading to a shift in the position of the minimum at $\mathcal{O}(1)$. In turn this leads to

$\mathcal{O}(\theta m_{3/2})$ corrections to F^θ in Eq. (3), i.e. F^θ is still of $\mathcal{O}(\theta m_{3/2})$. This discusion can be repeated in the case there is a continuous symmetry with two charged fields obtaining vevs of the similar size along a D-flat direction with $W = a + \chi^p \bar{\chi}^q$. In this case D-flatness requires that $Q\langle\chi\rangle + \bar{Q}\langle\bar{\chi}\rangle = 0$ with Q and $\bar{Q}$ the charges under the continuous group. Using this relation we arrive again at the same result in this case.

To avoid this conclusion we must consider the case θ gets a mass in the Supersymmetric limit in order to satisfy Eq. (5) [4]. This requires the existence of additional mass scales in the theory different from M_{Plank} or $m_{3/2}$. We assume, as is expected in a string theory, that all mass scales in the theory are generated through spontaneous symmetry breaking and therefore we consider the case there are additional fields in the theory which acquire different vevs. This leads to the possibility that more than one term in the superpotential is important [17]. To demonstrate the possibilities it is sufficient to consider just one extra field, ϕ, and two terms in the superpotential (setting the Planck scale to unity for clarity)

$$W(\theta, \phi) = \theta^p \phi^q + \theta^{p'} \phi^{q'} \qquad (7)$$

Given that the field vevs are typically much greater than the electroweak scale, we take ϕ, like θ, to be neutral under the SM group. The form of the two terms in the superpotential must be determined by a symmetry which may be the original family symmetry or may be a new one. The symmetry can be either discrete, with p and p' multiples of the order parameter of the discrete group, or continuous if the fields acquires vevs along a D-flat direction (in which case, to avoid $\langle\theta\rangle \approx \langle\phi\rangle$ which would cause one of the terms in Eq. (7) to dominate, there must be additional non singlet fields which acquire vevs). We include in the effective potential for ϕ a term V_ϕ which forces it to acquires a vacuum expectation value. Replacing ϕ by its vev Eq. (7) becomes

$$W(\theta) = M_1 \theta^p + M_2 \theta^{p'}. \qquad (8)$$

with $M_1 = \langle\phi\rangle^q / M_{Planck}^{p+q-3}$, $M_2 = \langle\phi\rangle^{q'} / M_{Planck}^{p'+q'-3}$. The potential following from $W(\theta)$ has a non-trivial minima in the globally supersymmetric limit given by

$$\langle\theta_0\rangle^{p'-p} = -pM_1/p'M_2 \qquad (9)$$

For a minimum with $\langle\theta\rangle$ and $\langle\phi\rangle$ less than the Planck mass $(q-q')$ and $(p'-p)$ must be positive. The minimum occurs through a cancellation between the two terms in $\partial W/\partial\theta$. One can readily show that, due to the relatively small effect of the last term in Eq. (2), the global minimum of the full supergravity potential can be close to this minimum. In this case, due to the cancellation between the two terms, the fact that $F^\theta \simeq 0$ given by Eq. (4), does not relate the magnitudes of $\partial^2 W/\partial\theta^2$ and $m_{3/2}$ as was done in the discrete case from Eq. (6). Therefore the

θ mass is allowed to be much larger than $m_{3/2}$ and F^θ is suppressed. Solving for the minimum of the potential following from Eq. (8) one finds (in Planck units)

$$
\begin{aligned}
F^\theta &= \left(\frac{3m_{3/2}}{W''(\theta_0)}\right) m_{3/2}\theta_0 \\
&\simeq \frac{3m_{3/2}}{p'(p'-p)\left(\frac{p}{p'}\right)^{\frac{(p'-2)}{(p'-p)}} \langle\phi\rangle^{q'+\frac{(p'-2)(q-q')}{(p'-p)}}}\; m_{3/2}\theta_0
\end{aligned}
$$

The suppression depends on $\langle\phi\rangle$. To bound $\langle\phi\rangle$ requires minimisation of the full $V(\theta,\phi)$ potential following from Eq. (7) and V_ϕ. In the global limit

$$
V(\theta,\phi) = \left|F^\theta\right|^2 + \left|F^\phi\right|^2 + V_\phi \tag{10}
$$

If $\theta < \phi$ (requiring $(q-q')/(p'-p) > 1$) the individual terms in F^θ and their derivatives are greater than those in F^ϕ. Thus minimisation with respect to θ will be approximately equivalent to minimisation of $\left|F^\theta\right|^2$ keeping ϕ fixed as was done following Eq. (8). Since F^ϕ breaks supersymmetry and contributes to scalar masses, the solution to the hierarchy problem requires $\left|F^\phi\right| \leq m_{3/2}$, bounding the allowed vevs. This, together with $\theta < \phi$, requires, up to a constant factor,

$$
\phi^{\frac{qp'-pq'}{p'-p}-1} < m_{3/2}. \tag{11}
$$

Using Eq. (9), this bound allows F^θ to be smaller than $m_{3/2}\theta_0$ provided $2(q-q')/(p'-p) > 1$, consistent with $\theta < \phi$. To take a simple example, the case $p = 3, q = 2, p' = 4, q' = 0$ gives $\theta = 3\phi^2/4$, with $2\theta^3\phi < m_{3/2}$. Thus $\theta/M_{Planck} \leq 10^{-4}$ and $F^\theta \geq \sqrt{3}\,(\theta/M_{Planck})^{3/2} m_{3/2}\theta_0$ allowing for a strong suppression of F^θ.

So, it is possible in the general case to suppress F^θ below $\langle\theta\rangle\, m_{3/2}$. However this is not the case in many models. If the expansion parameter determining the fermion hierarchy is θ/M_{Planck}, q must be large ($q \geq 8$ for $\theta/M_{Planck} \simeq 0.1$). If the family symmetry is responsible for making q large in Eq. (7) there must be no fields acquiring vevs with family charge allowing lower order terms. Effectively this means that the family structure will be determined by the field θ carrying a single sign of family charge. Such models have been used to generate fermion mass structure but they cannot reproduce the phenomenologically successful Gatto, Sartori, Tonin (GST) relation which requires familon fields of both charges [5]. Moreover the multiplet structure needed to get $q \geq 8$ looks very contrived. In the case there are fields of both signs of family charge, for example if χ and $\overline{\chi}$ acquire vevs along the D-flat direction, we can generate the GST relation. To get a reduction in F^χ again requires further field(s), ϕ. One can reproduce the argument following from Eq. (7) interpreting $\theta^p = \sqrt{\chi\overline{\chi}}$. This time there must be additional symmetries requiring

$q \geq 8$ for $\chi/M_{Planck} \simeq 0.1$. To avoid this conclusion requires the introduction of yet another scale $M < M_{Planck}$ to order the family hierarchy which complicates the theory further, perhaps making it less believable.

Given this, we consider it very likely that the familon field(s) will have an F-term greater than or equal to its natural value and so we turn now to a discussion of the phenomenological implications that follow if $F^\theta = \beta\langle\theta\rangle m_{3/2}$ with $\beta = \mathcal{O}(1)$. In this case terms involving θ contribute to the soft SUSY breaking terms and these terms violate flavour conservation and CP via the couplings in Eq. (1). Using Eqs. (1,2) we can determine the soft breaking terms in the observable sector after SUSY breaking. This leads to the trilinear terms [6],

$$
A_{ij}\hat{Y}^{ij} = F^\eta \hat{K}_\eta Y^{ij} + \alpha_{ij}\,\frac{e^{K/2}}{M}\left(\frac{\theta}{M}\right)^{\alpha_{ij}-1}\beta m_{3/2}\theta \tag{12}
$$

with $\hat{K}_\eta = \partial\hat{K}/\partial\eta$ and $Y^{ij} = e^{K/2}(\theta/M)^{\alpha_{ij}}$. The presence of α_{ij} in the right hand side is due to the dependence of the effective Yukawa couplings on θ. Note that the small parameter θ in F^θ does not affect the trilinear couplings because is reabsorbed in the Yukawa coupling itself,

$$
m_{3/2}\,\langle\theta\rangle\,\frac{\partial Y^{ij}}{\partial\theta} = \alpha_{i,j}\,m_{3/2}\,Y^{ij}. \tag{13}
$$

From Eq. (12) we see that, in any model which explains the hierarchy in the Yukawa textures through nonrenormalizable operators, the trilinear couplings are necessarily nonuniversal. In a similar way, we also expect nonrenormalizable contributions to the Kähler potential of the kind $(\theta\theta^*/M^2)^{\alpha(i,j)}$. However these contributions appear only at order $2\alpha(i,j)$ in θ/M with respect to the dominant term $\mathcal{O}(1)$. So, in the following we concentrate in the nonuniversal trilinear couplings.

Next we must check whether this breaking of universality does not contradict any of the very stringent bounds from low energy phenomenology. Although we do not have a complete theory of flavour [7] that provides the full field dependence of the low energy effective Yukawas, a fit to the fermion masses and mixing angles points to a definite texture for the Yukawa matrices [8],

$$
\frac{M}{m_3} = \begin{pmatrix} 0 & b\epsilon^3 & c\epsilon^3 \\ b'\epsilon^3 & d\epsilon^2 & a\epsilon^2 \\ f\epsilon^m & g\epsilon^n & 1 \end{pmatrix}, \tag{14}
$$

with $\epsilon_d = \sqrt{m_s/m_b} = 0.15$ and $\epsilon_u = \sqrt{m_c/m_t} = 0.05$ at the unification scale, and a, b, b', d, g, f coefficients $\mathcal{O}(1)$ and complex in principle. The coefficient c is very sensitive to the magnitude of the elements below the diagonal and can be of $\mathcal{O}(1)$ or smaller. Our analysis will not depend on its value. In this texture the two undetermined elements, $(3,1)$ and $(3,2)$, determine the unmeasured right-handed quark mixings. However, from the

magnitude of the eigenvalues we can constrain $m \geq 1$. In the context of a broken flavour symmetry the hierarchy in the Yukawa matrices is generated by different powers in the vevs of the familon fields, $\epsilon_a = \langle\theta_a\rangle/M$. For the case of a single familon we can immediately calculate the nonuniversality in the trilinear terms, $(Y^A)_{ij} \equiv Y_{ij}A_{ij}$,

$$(Y^A)_{ij} = A_0 Y_{ij} + \beta m_{3/2}\, Y_{33} \begin{pmatrix} 0 & 3b\epsilon^3 & 3c\epsilon^3 \\ 3b'\epsilon^3 & 2d\epsilon^2 & 2a\epsilon^2 \\ fm\epsilon^m & gn\epsilon^n & 0 \end{pmatrix} \tag{15}$$

with $A_0 = F^\eta \hat{K}_\eta$. For the case of several familons the F-terms of different fields are expected to differ and although the coefficients will change from Eq. (15) the proportionality of Y^A and Y will also be lost. Our results will not depend sensitively on variations of $\mathcal{O}(1)$ in these coefficients so we analyse the particular case of Eq. (15). The Yukawa texture in Eq. (14) is diagonalized by superfield rotations in the so-called SCKM basis, $\tilde{Y} = V_L^\dagger \cdot Y \cdot V_R$. However, in this basis large off-diagonal terms necessarily remain in the trilinear couplings, $\tilde{Y}^A = V_L^\dagger \cdot Y^A \cdot V_R$. The phenomenologically relevant flavour off-diagonal entries in the basis of diagonal Yukawa matrices are,

$$(\tilde{Y}^A)_{32} \simeq Y_{33}\beta\, m_{3/2}\, g\, n\, \epsilon^n + \dots \tag{16}$$

$$(\tilde{Y}^A)_{21} \simeq Y_{33}\,\beta m_{3/2}\, \epsilon^3 (b' + a\,(\frac{b'}{d}\, g\, n\, \epsilon^n - f\, m\, \epsilon^{m-1})) \tag{17}$$

The form of these matrices applies at the messenger scale which, being due to supergravity, is M_{Planck}. Then we must use the MSSM Renormalization Group Equations (RGE) [9] to obtain the corresponding matrices at M_W. The main effect in this RGE evolution is a large flavour universal gaugino contribution to the diagonal elements in the sfermion mass matrices (see for instance Tables I and IV in [10]). In this minimal supergravity scheme we take gaugino masses as $m_{1/2} = \sqrt{3}m_{3/2}$ and sfermion masses $m_0^2 = m_{3/2}^2$. So, the average masses at M_W are,

$$m_{\tilde{q}}^2 \simeq 6 \cdot m_{1/2}^2 + m_0^2 \simeq 19\, m_{3/2}^2$$
$$m_{\tilde{l}}^2 \simeq 1.5 \cdot m_{1/2}^2 + m_0^2 \simeq 5.5\, m_{3/2}^2 \tag{17}$$

The RG evolution of the trilinear terms is also similarly dominated by gluino contributions and 3rd generation Yukawa couplings. However, the offdiagonal elements in the down and lepton trilinear matrices are basically unchanged for $\tan\beta \leq 30$ [10]. From here we can obtain the full trilinear couplings and compare with the experimental observables at low energies. The so-called Mass Insertions (MI) formalism is very useful in this framework [11]. The left–right MI are defined in the SCKM basis as $(\delta_{LR})_{ij} = (m_{LR}^2)_{ij}/m_{\tilde{f}}^2$, with $m_{\tilde{f}}^2$ the average sfermion mass. We estimate the value of the (2, 1) LR MI as,

$$(\delta_{LR}^d)_{21} \simeq \frac{m_b\,\epsilon_d^3}{19\,m_{3/2}}(b' + a\,\frac{b'}{d}\,g\,n\,\epsilon_d^n - a\,f\,m\,\epsilon_d^{m-1}) \simeq$$

x	$\sqrt{\left|\mathrm{Im}\,(\delta_{LR}^d)_{12}^2\right|}$	$|(\delta_{LR}^d)_{13}|$	$|(\delta_{LR}^l)_{12}|$	$|(\delta_{LR}^l)_{23}|$
0.3	1.1×10^{-5}	1.3×10^{-2}	6.9×10^{-7}	8.7×10^{-3}
1.0	2.0×10^{-5}	1.6×10^{-2}	8.4×10^{-7}	1.0×10^{-2}
4.0	6.3×10^{-5}	3.0×10^{-2}	1.9×10^{-6}	2.3×10^{-2}

TABLE I: MI bounds from ε'/ε, $b \to s\gamma$, $\mu \to e\gamma$ and $\tau \to \mu\gamma$ with $m_{\tilde{q}} = 500$ GeV and $m_{\tilde{l}} = 100$ GeV and different values of $x = m_{\tilde{g}(\tilde{\gamma})}^2/m_{\tilde{q}(\tilde{l})}^2$. These bounds scale as $(m_{\tilde{f}}(\mathrm{GeV})/500(100))^2$ for different average sfermion masses.

$$(b' + a\,\frac{b'}{d}\,g\,n\,\epsilon_d^n - a\,f\,m\,\epsilon_d^{m-1})\,7.5 \times 10^{-6} \tag{18}$$

using $m_{\tilde{q}} \simeq 500$ GeV corresponding to $m_{3/2} \simeq 120$ GeV and $\epsilon_d \simeq 0.15$ [8]. We can compare our estimate for the mass insertion with the phenomenological bounds in Table I with $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2 \simeq 1$. Even allowing a phase $\mathcal{O}(1)$, necessary to contribute to ε'/ε, we can see the bound requires only $m \geq 1$ which is already required to fit the fermion masses. Note however that in the presence of a phase, ε'/ε naturally receives a sizeable contribution from the b' term [12].

Similarly, the MI corresponding to the $b \to s\gamma$ decay are,

$$(\delta_{LR}^d)_{32} \simeq \frac{m_{3/2}\, m_b\, g\, n\, \epsilon_d^n}{19\, m_{3/2}^2} \simeq 2.2 \times 10^{-3}\, g\, n\, \epsilon_d^n \tag{19}$$

again with $m_{3/2} \simeq 120$ GeV. This estimate is already of the same order of the phenomenological bound for any n and we do not get any new constraint on n.

The situation is more interesting in the leptonic sector. Here, the photino contribution is dominant for LR mass insertions. In Table I, we show the rescaled bounds from Ref. [11] for the present limits on the branching ratio. In this case, it seems reasonable to expect some kind of lepton-quark Yukawa unification. To generate the correct muon and electron mass we follow Georgi and Jarlskog's suggestion and put a relative factor of 3 in the (2, 2) entry. We also put a factor of 3 in the (2, 3) and (3, 2) entries as is required by non-Abelian models which seek to explain the near equality in the down quark mass matrix of the (2, 2) and (2, 3) elements. With this we obtain,

$$(\delta_{LR}^e)_{12} \simeq \frac{m_\tau\,\epsilon_d^3}{5.5\,m_{3/2}}(b' + 9a\,\frac{b'}{d}\,g\,n\,\epsilon_d^n - 3\,a\,f\,m\,\epsilon_d^{m-1}) \simeq$$
$$(b' + 9\,a\,\frac{b'}{d}\,g\,n\,\epsilon_d^n - 3\,a\,f\,m\,\epsilon_d^{m-1})\,8.7 \times 10^{-6} \tag{20}$$

where we take $m_{3/2} \simeq 120$ GeV corresponding to $m_{\tilde{l}} = 280$ GeV. This result should be compared with the experimental bound from the non-observation of $\mu \to e\gamma$ given by $(\delta_{LR}^e)_{12} \leq 7 \times 10^{-7}\,(280/100)^2 = 5.5 \times 10^{-6}$. Note that in Eq. (20) there is an unavoidable contribution from the b' entry, $(\delta_{LR}^e)_{12} \simeq b'\,8.7 \times 10^{-6}$, exceeding the experimental bound. To avoid this requires a larger value of

the slepton mass. For $m_{\tilde{l}} = 320$ GeV our estimate would be just below the MI bound ($m_{3/2} = 136$ GeV, $m_{\tilde{q}} = 600$ GeV). In this conditions, the contributions of the second term in Eq. (20) does not lead to a new constraints on the value of n. However, the last term would be subdominant for $m \geq 2$ but $m = 1$ would not be allowed up to $m_{\tilde{l}} = 550$ GeV ($m_{\tilde{q}} = 1000$ GeV). To summarise, assuming a quark-lepton unification at M_{GUT}, nonuniversality in the trilinear terms predicts a large $\mu \to e\gamma$ branching ratio even beyond the values expected from other sources such as SUSY seesaw [13]. This illustrates the point that $\mu \to e\gamma$ is a particularly sensitive probe of SUSY and the soft breaking sector.

Another interesting constraint is provided by Electric Dipole Moment (EDM) bounds. Even in the most conservative case, where all soft SUSY breaking parameters and μ are real, we know that the Yukawa matrices contain phases $\mathcal{O}(1)$. If the trilinear terms are nonuniversal, these phases are not completely removed from the diagonal elements of Y^A in the SCKM basis and hence can give rise to large EDMs [4, 14]. However the phase in the trilinear terms will be exactly zero at leading order in θ for any diagonal element [18]. To see this we must take into account the fact that the eigenvalues, $D(\theta)$, and mixing matrices, $V_{L,R}(\theta)$ of the Yukawa matrix depend on θ, $Y(\theta) = V_L(\theta) D(\theta) V_R(\theta)^{\dagger}$. The contribution to the trilinear terms is proportional to $\theta \, \partial Y / \partial\theta$. Evaluating this in the SCKM basis, we have,

$$V_L^{\dagger} \theta \frac{\partial Y}{\partial \theta} V_R = V_L^{\dagger} \frac{\theta \partial V_L}{\partial \theta} D + \frac{\theta \partial D}{\partial \theta} + D \frac{\theta \partial V_R^{\dagger}}{\partial \theta} V_R \quad (21)$$

In this expression the dominant contribution in θ to a diagonal element is controlled by the second term. This follows because $\theta \; \partial V / \partial \theta$ always adds at least a power of θ to the diagonal element and therefore the first and third terms in the above equation can only contribute to subdominant terms in the θ expansion for the diagonal elements. The dominant second term is proportional to the leading θ term in Y_{ii} with a coefficient of proportionality equal to its power in θ i.e. the phase is unchanged and is real in the basis that the Yukawa couplings are real. Therefore any observable phase in the diagonal elements will only appear at higher orders, requiring $n \geq 1$ or $m \geq 2$ or through higher order contributions to entries of the Yukawa matrix. Using this result, and assuming real μ and soft breaking terms, the EDMs have the form

$$\text{Im} \left(\delta_{LR}^{q,l} \right)_{11} \simeq \frac{m_1}{R_{q,l} \, m_{3/2}} \left(\epsilon^n \, n + \epsilon^{m-1} \, (m-1) \right) \quad (22)$$

where we use $m_1 = m_3 \epsilon^4 bb'/d$, we take all unknown coefficients to be unity and assume an $\mathcal{O}(1)$ phase which is observable in the basis of real masses. The coefficients $R_q = 19$ and $R_l = 5.5$ take care of the RGE effects in the eigenvalues as before. So with $\epsilon_d = 0.15$, $\epsilon_u = 0.05$ and $m_d \simeq 10$ MeV, $m_u \simeq 5$ MeV and $m_e = 0.5$ MeV we get,

$$\text{Im} \left(\delta_{LR}^d \right)_{11} \simeq \left(\epsilon_d^n \, n + \epsilon_d^{m-1} \, (m-1) \right) 3.9 \times 10^{-6}$$

x	$\vert\text{Im}(\delta_{11}^d)_{LR}\vert$	$\vert\text{Im}(\delta_{11}^u)_{LR}\vert$	$\vert\text{Im}(\delta_{22}^d)_{LR}\vert$	$\vert\text{Im}(\delta_{11}^l)_{LR}\vert$
0.3	4.3×10^{-8}	4.3×10^{-8}	3.6×10^{-6}	4.2×10^{-7}
1	8.0×10^{-8}	8.0×10^{-8}	6.7×10^{-6}	5.1×10^{-7}
3	1.8×10^{-7}	1.8×10^{-7}	1.6×10^{-5}	8.3×10^{-7}

TABLE II: MI bounds from the mercury EDM for an average squark mass of 600 GeV and for the electron EDM with an average slepton mass of 320 GeV and different values of $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$. The bounds scale as $(m_{\tilde{q}(\tilde{l})}(\text{GeV})/600(320))$.

$$\text{Im} \left(\delta_{LR}^u \right)_{11} \simeq \left(\epsilon_u^n \, n + \epsilon_u^{m-1} \, (m-1) \right) 1.9 \times 10^{-6}$$
$$\text{Im} \left(\delta_{LR}^e \right)_{11} \simeq \left(\epsilon_d^n \, n + \epsilon_d^{m-1} \, (m-1) \right) 6.7 \times 10^{-7} \quad (23)$$

We compare these estimates with the phenomenological bounds in Table II [11, 15]. The bounds from the neutron and electron EDM do not provide any new information on the structure of the Yukawa textures. However, the mercury EDM bounds are much more restrictive and taking $x \simeq 1$ we find that, in the down sector, the case $n = 1$, $m = 2$ is not allowed by EDM experiments and we require $n \geq 2$, $m \geq 3$. As we said above, the same applies to subdominant corrections to Y_{22} and Y_{12}, Y_{21} where the first correction to the dominant terms can only be ϵ^4 or ϵ^5 respectively to satisfy EDM bounds.

In summary, we have shown here that in a broken supergravity theory any field that acquires a vev also has an F-term of order $\langle \theta \rangle m_{3/2}$. These F-terms contribute unsuppressed to the trilinear couplings and have observable effects in low energy phenomenology. The most significant contribution, assuming a GUT type relation between the quark and lepton masses, is to $\mu \to e\gamma$. To keep this at the level of current experimental bounds requires a slepton mass greater than $320 \, GeV$. At this level $\mu \to e\gamma$ should be seen by the proposed experiments in the near future. There are also significant bounds coming from the mercury EDM bounds which impose significant limits on the down quark matrix elements below the diagonal responsible for right handed mixing. These bounds disfavour large right handed mixing and thus disfavour $SU(5)$ based models in which the large (left handed) neutrino mixing angles are related to large down quark right handed mixing angles. Of course these strong bounds apply only to supergravity models with gravity as the supersymmetry breaking messenger. Models with light messenger states, such as gauge mediation models, have a much lower value for $m_{3/2}$ and this determines the size of these flavour and CP violating effects.

[1] A. Masiero and O. Vives, Ann. Rev. Nucl. Part. Sci. **51**, 161 (2001).

[2] For an example of this in type I string models see S. A. Abel and G. Servant, Nucl. Phys. B **611**, 43 (2001).

[3] L. E. Ibanez and G. G. Ross, Phys. Lett. B **110**, 215 (1982).

[4] Y. Nir and R. Rattazzi, Phys. Lett. B **382**, 363 (1996).

[5] R. Gatto, G. Sartori and M. Tonin, Phys. Lett. B **28**, 128 (1968); For a review see G. G. Ross, in *TASI 2000* ed. J. L. Rosner (World Scientific, New Jersey, 2001)

[6] S. K. Soni and H. A. Weldon, Phys. Lett. B **126**, 215 (1983); V. S. Kaplunovsky and J. Louis, Phys. Lett. B **306**, 269 (1993); A. Brignole, L. E. Ibanez and C. Munoz, Nucl. Phys. B **422**, 125 (1994) [Erratum-ibid. B **436**, 747 (1995)].

[7] S. F. King and G. G. Ross, Phys. Lett. B **520**, 243 (2001); Z. Berezhiani and A. Rossi, Nucl. Phys. B **594**, 113 (2001); R. Barbieri and L. J. Hall, Nuovo Cim. A **110**, 1 (1997); G. G. Ross and L. Velasco-Sevilla, hep-ph/0208218.

[8] R. G. Roberts *et. al*, Nucl. Phys. B **615**, 358 (2001).

[9] N.K. Falck, Z Phys. **C30**, 247 (1986); S. Bertolini *et. al*, Nucl. Phys. **B353**, 591 (1991).

[10] A. Bartl *et. al*, Phys. Rev. D **64**, 076009 (2001).

[11] F. Gabbiani *et. al*, Nucl. Phys. **B477**, 321 (1996); J. Hagelin, S. Kelley and T. Tanaka, Nucl. Phys. **B415**, 293 (1994).

[12] A. Masiero and H. Murayama, Phys. Rev. Lett. **83**, 907 (1999); S. Khalil, T. Kobayashi and O. Vives, Nucl. Phys. **B580**, 275 (2000); A. Masiero and O. Vives, Phys. Rev. Lett. **86**, 26 (2001).

[13] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986); J. Hisano *et. al*, Phys. Rev. D **53**, 2442 (1996); J. A. Casas and A. Ibarra, Nucl. Phys. B **618**, 171 (2001); S. Lavignac, I. Masina and C. A. Savoy, Phys. Lett. B **520**, 269 (2001): J. R. Ellis and M. Raidal, Nucl. Phys. B **643**, 229 (2002); A. Masiero, S. K. Vempati and O. Vives, Nucl. Phys. B **649**, 189 (2003).

[14] S. Abel, S. Khalil and O. Lebedev, Phys. Rev. Lett. **89**, 121601 (2002);

[15] T. Falk *et. al*, Nucl. Phys. B **560**, 3 (1999); S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B **606**, 151 (2001).

[16] For general Kähler potentials depending on the hidden sector, minimization of the potential with respect to the unnormalized θ is equal to minimization with respect to the rescaled θ up to higher order corrections in θ

[17] In the previous case one term dominates since θ/M_{Planck} is the only dimensionless parameter and is small.

[18] This leads to a weaker bound than that found in [14].