

GRADUATE STUDENT SERIES IN PHYSICS

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GAUGE THEORIES IN PARTICLE PHYSICS

A PRACTICAL INTRODUCTION
THIRD EDITION

Volume 1

From Relativistic Quantum Mechanics to QED

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CONTENTS

Preface to the Third Edition

xiii

PART 1

INTRODUCTORY SURVEY, ELECTROMAGNETISM AS A GAUGE THEORY, AND RELATIVISTIC QUANTUM MECHANICS

1	Quarks and Leptons	3
1.1	The Standard Model	3
1.2	Levels of structure: from atoms to quarks	5
1.2.1	Atoms \rightarrow nucleus	5
1.2.2	Nuclei \rightarrow nucleons	8
1.2.3	Nucleons \rightarrow quarks	12
1.3	The generations and flavours of quarks and leptons	18
1.3.1	Lepton flavour	18
1.3.2	Quark flavour	21
2	Particle Interactions in the Standard Model	28
2.1	Introduction	28
2.2	The Yukawa theory of force as virtual quantum exchange	30
2.3	The one-quantum exchange amplitude	34
2.4	Electromagnetic interactions	35
2.5	Weak interactions	37
2.6	Strong interactions	40
2.7	Gravitational interactions	44
2.8	Summary	48
	Problems	49
3	Electromagnetism as a Gauge Theory	53
3.1	Introduction	53
3.2	The Maxwell equations: current conservation	54
3.3	The Maxwell equations: Lorentz covariance and gauge invariance	56
3.4	Gauge invariance (and covariance) in quantum mechanics	60
3.5	The argument reversed: the gauge principle	63
3.6	Comments on the gauge principle in electromagnetism	67
	Problems	73

4	Relativistic Quantum Mechanics	74
4.1	The Klein–Gordon equation	74
4.1.1	Solutions in coordinate space	75
4.1.2	Probability current for the KG equation	76
4.2	The Dirac equation	77
4.2.1	Free-particle solutions	80
4.2.2	Probability current for the Dirac equation	81
4.3	Spin	82
4.4	Lorentz transformation properties of spinors	85
4.5	The negative-energy solutions	91
4.5.1	Positive-energy spinors	92
4.5.2	Negative-energy spinors	92
4.5.3	Dirac’s interpretation of the negative-energy solutions of the Dirac equation	93
4.5.4	Feynman’s interpretation of the negative-energy solutions of the KG and Dirac equations	95
4.6	Inclusion of electromagnetic interactions via the gauge principle: the Dirac prediction of $g = 2$ for the electron	98
	Problems	101

PART 2

INTRODUCTION TO QUANTUM FIELD THEORY

5	Quantum Field Theory I: the Free Scalar Field	109
5.1	The quantum field: (i) descriptive	109
5.2	The quantum field: (ii) Lagrange–Hamilton formulation	119
5.2.1	The action principle: Lagrangian particle mechanics	119
5.2.2	Quantum mechanics à la Heisenberg–Lagrange–Hamilton	122
5.2.3	Interlude: the quantum oscillator	124
5.2.4	Lagrange–Hamilton classical field mechanics	127
5.2.5	Heisenberg–Lagrange–Hamilton quantum field mechanics	129
5.3	Generalizations: four dimensions, relativity and mass	136
	Problems	139
6	Quantum Field Theory II: Interacting Scalar Fields	141
6.1	Interactions in quantum field theory: qualitative introduction	141
6.2	Perturbation theory for interacting fields: the Dyson expansion of the S -matrix	144
6.2.1	The interaction picture	145
6.2.2	The S -matrix and the Dyson expansion	147
6.3	Applications to the ‘ABC’ theory	150
6.3.1	The decay $C \rightarrow A + B$	151
6.3.2	$A + B \rightarrow A + B$ scattering: the amplitudes	155

6.3.3	$A + B \rightarrow A + B$ scattering: the Yukawa exchange mechanism, s and u channel processes	164
6.3.4	$A + B \rightarrow A + B$ scattering: the differential cross section	166
6.3.5	$A + B \rightarrow A + B$ scattering: loose ends	168
	Problems	171
7	Quantum Field Theory III: Complex Scalar Fields, Dirac and Maxwell Fields; Introduction of Electromagnetic Interactions	173
7.1	The complex scalar field: global $U(1)$ phase invariance, particles and anti-particles	174
7.2	The Dirac field and the spin-statistics connection	181
7.3	The Maxwell field $A^\mu(x)$	186
7.3.1	The classical field case	186
7.3.2	Quantizing $A^\mu(x)$	189
7.4	Introduction of electromagnetic interactions	195
	Problems	200
PART 3		
TREE-LEVEL APPLICATIONS IN QED		
8	Elementary Processes in Scalar and Spinor Electrodynamics	205
8.1	Coulomb scattering of charged spin-0 particles	205
8.1.1	Coulomb scattering of s^+ (wavefunction approach)	205
8.1.2	Coulomb scattering of s^+ (field-theoretic approach)	208
8.1.3	Coulomb scattering of s^-	209
8.2	Coulomb scattering of charged spin- $\frac{1}{2}$ particles	210
8.2.1	Coulomb scattering of e^- (wavefunction approach)	210
8.2.2	Coulomb scattering of e^- (field-theoretic approach)	213
8.2.3	Trace techniques for spin summations	214
8.2.4	Coulomb scattering of e^+	217
8.3	e^-s^+ scattering	218
8.3.1	The amplitude for $e^-s^+ \rightarrow e^-s^+$	218
8.3.2	The cross section for $e^-s^+ \rightarrow e^-s^+$	223
8.4	Scattering from a non-point-like object: the pion form factor in $e^-\pi^+ \rightarrow e^-\pi^+$	225
8.4.1	Rutherford scattering from a charge distribution	225
8.4.2	Lorentz invariance	227
8.4.3	Current conservation	228
8.5	The form factor in the time-like region: $e^+e^- \rightarrow \pi^+\pi^-$ and crossing symmetry	230
8.6	Electron Compton scattering	233
8.6.1	The lowest-order amplitudes	233
8.6.2	Gauge invariance	234
8.6.3	The Compton cross section	235

8.7	Electron muon elastic scattering	237
8.8	Electron–proton elastic scattering and nucleon form factors	240
8.8.1	Lorentz invariance	241
8.8.2	Current conservation	241
	Problems	244
9	Deep Inelastic Electron–Nucleon Scattering and the Quark Parton Model	249
9.1	Inelastic electron–proton scattering: kinematics and structure functions	249
9.2	Bjorken scaling and the parton model	252
9.3	The quark parton model	260
9.4	The Drell–Yan process	262
9.5	e^+e^- annihilation into hadrons	267
	Problems	272
PART 4		
LOOPS AND RENORMALIZATION		
10	Loops and Renormalization I: the ABC Theory	279
10.1	The propagator correction in ABC theory	280
10.1.1	The $O(g^2)$ self-energy $\Pi_C^{[2]}(q^2)$	280
10.1.2	Mass shift	287
10.1.3	Field strength renormalization	288
10.2	The vertex correction	291
10.3	Dealing with the bad news: a simple example	293
10.3.1	Exposure of the divergence in $\Pi_C^{[2]}(q^2)$	293
10.3.2	Regularization and renormalization	296
10.4	Bare and renormalized perturbation theory	297
10.4.1	Reorganizing perturbation theory	297
10.4.2	The $O(g_{\text{ph}}^2)$ renormalized self-energy revisited: how counter terms are determined by renormalization conditions	300
10.5	Renormalizability	303
	Problems	304
11	Loops and Renormalization II: QED	306
11.1	Counter terms	306
11.2	The $O(e^2)$ fermion self-energy	308
11.3	The $O(e^2)$ photon self-energy	309
11.4	The $O(e^2)$ renormalized photon self-energy	312
11.5	The physics of $\tilde{\Pi}_\gamma^{[2]}(q^2)$	314
11.5.1	Modified Coulomb’s law	315
11.5.2	Radiatively induced charge form factor	317

11.5.3	The running coupling constant	317
11.5.4	$\bar{\Pi}_\gamma^{[2]}$ in the s -channel	322
11.6	The $O(e^2)$ vertex correction, and $Z_1 = Z_2$	323
11.7	The anomalous magnetic moment and tests of QED	326
11.8	Which theories are renormalizable—and does it matter?	329
	Problems	336
Appendix A	Non-relativistic Quantum Mechanics	338
Appendix B	Natural Units	342
Appendix C	Maxwell's Equations: Choice of Units	345
Appendix D	Special Relativity: Invariance and Covariance	347
Appendix E	Dirac δ-Function	352
Appendix F	Contour Integration	360
Appendix G	Green Functions	366
Appendix H	Elements of Non-relativistic Scattering Theory	372
H.1	Time-independent formulation and differential cross section	372
H.2	Expression for the scattering amplitude: Born approximation	374
H.3	Time-dependent approach	375
Appendix I	The Schrödinger and Heisenberg Pictures	377
Appendix J	Dirac Algebra and Trace Identities	379
J.1	Dirac algebra	379
J.1.1	γ matrices	379
J.1.2	γ_5 identities	379
J.1.3	Hermitian conjugate of spinor matrix elements	380
J.1.4	Spin sums and projection operators	380
J.2	Trace theorems	381
Appendix K	Example of a Cross Section Calculation	383
K.1	The spin-averaged squared matrix element	385
K.2	Evaluation of two-body Lorentz-invariant phase space in 'laboratory' variables	385
Appendix L	Feynman Rules for Tree Graphs in QED	389
L.1	External particles	389
L.2	Propagators	390
L.3	Vertices	390
	References	391
	Index	395