Quintessence and the Underlying Particle Physics Theory

D.J.H. Chung[(1)*], **L.L. Everett**[(1)†], and **A. Riotto**[(2)‡]

[(1)] *CERN Theory Division, CH-1211 Geneva 23, Switzerland*

[(2)] *INFN, Sezione di Padova, via Marzolo 8, I-35131, Padova, Italy*

Abstract

At present we know nothing about the nature of the dark energy accounting for about 70% of the energy density of the Universe. One possibility is that the dark energy is provided by an extremely light field, the quintessence, rolling down its potential. Even though the underlying particle theory responsible for the present quintessential behaviour of our Universe is unknown, such a theory is likely to have contact with supersymmetry, supergravity or (super)string theory. In these theories, there are plenty of scalar fields (moduli) which are gravitationally coupled to all the other degrees of freedom and have vacuum expectation values of the order of the Planck scale. We point out that, in theories which allow a consistent embedding of quintessence, the generic gravitational interaction of the moduli fields with the quintessence field gives rise to a contribution to the energy density from the moduli fields of the order of the critical energy density of the universe today. Furthermore, the interaction contribution can generically enhance the negativity of the equation of state.

October 2002

*E-mail: daniel.chung@cern.ch
†E-mail: Lisa.Everett@cern.ch
‡Email: antonio.riotto@pd.infn.it

1. There is increasing evidence that the energy density of (baryonic plus dark) matter in the universe is smaller than the critical density [1]. If the universe is flat, as predicted by the most natural inflation models [2] and confirmed by the recent measurements of the cosmic microwave background anisotropies [3], an additional dark energy density component is necessary to account for $\Omega_0 = 1$. The data indicates the dark energy component possesses negative pressure and makes up about 70 % of the energy in the present universe. Without modifying gravity, the most obvious way to explain this observation is through the introduction of a quintessence field [4, 5, 6]: a time dependent scalar field whose current homogeneous background configuration dominates the energy density and has an equation of state which is negative. Indeed, if the cosmic scale factor is accelerating today, the equation of state $w \equiv p/\rho < -1/3$.

In this paper, we present a simple, yet intriguing observation: even without specifying the details of the underlying field theory responsible for the present quintessential behaviour of our universe, if this theory has something to do with supersymmetry, supergravity or (super)string theory, then one can deduce that the total energy density must receive contributions of the order of the critical density from the *interactions* between the quintessence field and other naively "decoupled" fields. Furthermore, this contribution can generically enhance the negativity of the equation of state. In effect, supersymmetrizing the quintessence model can generically enhance the negativity of the equation of state! Of course, as we will explain, this statement is contingent upon the assumption that the cosmological constant is canceled by some as of yet unknown mechanism. Other possible effects that enhance the negativity of equation of state in the context of supergravity can be found in [7].

In addition, but somewhat on the flipside, we remind the reader that supersymmetry (which arguably is the best motivated physics beyond the Standard Model) is naturally at odds with quintessence models because of radiative corrections induced by the nonrenormalizable terms arising from the Kähler potential. Although this observation has been made previously (see for example [8, 9]), we would like to reemphasize this important point which is relevant for our main result of the paper.[1] This implies that if an equation of state for the dark energy is observationally determined to be $0 > w > -1$ or if time variation of the equation of state of the dark energy can be observationally confirmed, there is a strong motivation to alter the standard picture of supersymmetry and supergravity or to specify special symmetries protecting the quintessence mass. However, whenever the traditional supersymmetric embedding of the quintessence is possible, our main result applies.

[1] Ref. [14] also discusses other aspects of general difficulties of embedding quintessence into supergravity.

The usual lore of field theory and string theory dictate that only gauged symmetries are fundamental. Hence, supersymmetry can only play a fundamental role if it is a gauge symmetry. Being a spacetime symmetry, gauging supersymmetry produces supergravity. Furthermore, any fundamental supersymmetric theory such as string theory has supergravity as its low energy effective action. Hence any supersymmetric embedding of quintessence is really in the context of supergravity. Of course, in the limit that all field amplitudes and energies are much smaller than the Planck scale (we will denote $M_p = 2 \times 10^{18}$ GeV as the reduced Planck scale), supergravity reduces to a globally supersymmetric theory. However, in the case of most known quintessence models, because the field amplitudes prefer to attain Planckian values [10], the supergravity structure must be taken into account for a supersymmetrized quintessence theory.

One crucial ingredient in our observation is that in any string, superstring, or supersymmetric theory, scalar fields which are gravitationally coupled to all the other degrees of freedom and have vacuum expectation values (vevs) of the order of the Planck scale are ubiquitous. These fields are usually required to have masses much larger than the expansion rate H_0 today to have acceptable cosmology and in practice usually have mass of order electroweak scale of 100 GeV. These fields are commonly called moduli and we collectively denote them by Φ. The important property that the moduli have vevs of order M_p reflects the fact that supergravity has a natural scale of M_p.

For example, in $N = 1$ phenomenological supergravity models [11] supersymmetry (SUSY) is broken in a hidden sector and the gravitational strength force plays the role of a messenger by transmitting SUSY breaking to the visible sector. In these models there exist scalar fields which are responsible for supersymmetry breaking. Their mass is of the order of $(10^2 - 10^3)$ GeV and their coupling to the other fields is only gravitational. Another common example is in string derived supergravity models, all of which have massless fields that parametrize the continuous ground state degeneracies characteristic of supersymmetric theories. These fields, such as the dilaton and massless gauge singlets of string volume compactification, are massless to all orders in perturbation theory and can obtain their mass of order a TeV from the same nonperturbative mechanism which breaks supersymmetry.[2]

Our observation is that, even though we naively expect that only their particle excitations can have any effect on the late time cosmological evolution because the moduli fields Φ have masses much larger than the Hubble rate, the very simple fact that their field amplitude is of order M_p gives any gravitational interactions of the form

$$\Phi^2 H_0^2, \tag{1}$$

[2] See [12] for a discussion of other generic cosmological properties of string moduli fields.

an interaction energy contribution comparable to the critical density of the universe ($\sim M_p^2 H_0^2$). Furthermore, since H_0 is driven by the quintessence by definition, the Φ vevs are important for quintessence dynamics. Since Eq. (1) naturally arises from Kähler potential couplings of Φ to the quintessence field Q, such effects in supersymmetric quintessence is generic. Unfortunately, as we will explain, because of such couplings, radiative corrections to the quintessence mass tend to destabilize the requisite flatness of the quintessence potential. Since symmetries forbidding these particular radiative corrections are rare, phenomenological viability of quintessence in the usual supersymmetry picture is questionable. This can be viewed as good news in the sense that since we expect to have an observational handle on the quintessence sector, we therefore have a new experimental probe of the Kähler potential.

The role of the large mass of Φ is that it causes the Φ dynamics to decouple from the dynamics of the quintessence, leaving only the constant vev of Φ in the interactions of the form Eq. (1) relevant for the quintessence dynamics. Of course, in practice, the actual magnitude and the resulting equation of state for the interaction energy is model dependent not only on the type of coupling represented by Eq. (1), but the potential of the quintessence field itself. This will be illustrated explicitly in this paper.

These conclusions in fact hold for *any* scalar field Φ gravitationally coupled to quintessence as long as its mass is larger than the present-day Hubble rate and its vacuum expectation value is of the order of M_p. This opens up the possibility that a significant role in the present cosmological evolution of the universe is played by scalar fields arising not only in supersymmetric or (super)string theories, but also, for instance, in brane-world scenarios. It is encouraging that, despite the fact that one of the major problems facing Planck scale physics is the lack of predictivity for low-energy physics, some information on high energy physics may be inferred indirectly through its effects on the present-day cosmological evolution of the universe.

The rest of the paper is organized as follows. In section 2, we give an estimate for the energy contribution from the interaction between the Kähler moduli and the quintessence field. In section 3, we argue why the quintessence picture is typically (but not necessarily) at odds with supersymmetry. In section 4, we give a careful calculation of the interaction energy starting from a generically parametrized Kähler potential. We summarize and conclude in section 5.

2. Let us first discuss the form of the moduli potential. In the usual nonrenormalizable hidden sector models, supersymmetry breaking vanishes in the limit $M_p \to \infty$. Since the potential for a generic moduli field Φ is generated by the same physics associated to supersymmetry breaking in the hidden sector, its potential takes the form

$$V(\Phi) = \widetilde{m}^2\, M_p^2\, \mathcal{V}(\Phi/M_p), \tag{2}$$

where $\widetilde{m} \sim$ TeV is the soft supersymmetry breaking mass. The potential for this moduli direction vanishes in the in the limit $M_p \to \infty$ since $\widetilde{m} \to 0$ in this limit. The vacuum expectation value of moduli fields is naturally of the order of the Planck scale, $\Phi_0 \sim M_p$, and their excitations around the minimum of the potential have a mass $\sim \widetilde{m} \gg H$. The potential (2) can be expanded around the minimum as

$$V(\Phi) = \widetilde{m}^2\, (\Phi - \Phi_0)^2\,, \tag{3}$$

where we have assumed that $V(\Phi_0)$ vanishes.

We wish now to convince the reader that the potential of the modulus field, under the assumption that the quintessence field is dominating the energy density of the universe, generically receives contributions[3] of the form

$$\Delta V(\Phi) = \frac{1}{2}\alpha\, H^2 \Phi^2 \tag{4}$$

for the very same reason that the moduli fields are coupled gravitationally to all the other degrees of freedom and therefore to the quintessence field as well. The coefficient α is of order unity and its sign may be either positive or negative. Let us just give an example of one possible source of such new contributions to $V(\Phi)$.

The Lagrangian of any scalar field in low energy supergravity is determined by the (holomorphic) superpotential W and by the (non-holomorphic) Kähler potential K [11]. The Kähler potential determines the kinetic terms of the scalar fields according to the formula

$$\mathcal{L}_{\rm kin} = \frac{\partial^2 K}{\partial \varphi_i^* \partial \varphi_j} \partial_\mu \varphi_i^* \partial^\mu \varphi_j \tag{5}$$

where φ_i are complex scalar fields (such as the modulus Φ or the quintessence Q) of any SUSY multiplet. In general the Kähler potential is an expansion in inverse powers of M_p and contains all possible terms allowed by the symmetries of the system.[4] For instance, usually there is no symmetry forbidding the Kähler potential to take the form

$$K = \Phi^*\Phi + Q^*Q + \lambda\, (\Phi^*\Phi)^m (\,Q^*Q)^n, \tag{6}$$

where the first two terms induce the canonically normalized kinetic terms for the modulus and for the quintessence field, λ is a numerical coefficient naturally of the order

[3] The same kind of contributions might arise during inflation (and spoil the flatness of the potential) [2], during preheating [16], or be relevant for the Affleck-Dine baryogenesis scenario [17].

[4] On even more general grounds, the Kähler potential is expected to be an expansion in inverse powers of $\Lambda_{\rm UV}$, the ultraviolet energy cut-off of the theory.

of unity whose sign may be positive or negative. Because the last two terms exist for *any* canonically normalized Kähler potential, the term $\delta K = \lambda(\Phi^*\Phi)^m (Q^*Q)^n$ is not forbidden by any gauge symmetries that preserve kinetic terms and hence is expected to be there for the very simple reason that gravitational interactions exist.

To see how Eq. (6) gives rise to terms of the form Eq. (4), consider the equation of motion for Q. The quintessence field Q rolls down a potential according to the equation of motion $\ddot{Q} + 3H\dot{Q} + V'(Q) = 0$, where H is the Hubble constant satisfying the Friedmann equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_p^2}\left(\frac{1}{2}\dot{Q}^2 + V(Q) + \rho_B\right), \tag{7}$$

where a is the scale factor and ρ_B is the remaining background energy density. Since at present the quintessence field Q dominates the energy density of the universe, we can write

$$\frac{1}{2}\dot{Q}^2 = \frac{3}{2}\left(1 + w_Q\right) H^2 M_p^2 \tag{8}$$

and $V(Q) = \frac{3}{2}\left(1 - w_Q\right) H^2 M_p^2$. Note that the mass of the quintessence field Q should naturally be of the order of the current Hubble rate $H_0 \sim 10^{-42}$ GeV.

Eqs. (5), (6), and (8) give rise to a new contribution to the modulus potential of the form (4) with

$$\alpha \sim 3\lambda(1 + w_Q)\left(\frac{\Phi^*\Phi}{M_p^2}\right)^{(m-1)}\left(\frac{Q^*Q}{M_p^2}\right)^{(n-1)} \tag{9}$$

where we have purposely been careless about factors of 2 in the kinetic normalization[6] (we do a careful analysis in section 4). Since both Q and Φ typically have vevs near M_p, the coefficient α is generically not suppressed.

We stress again that this is only one of the possible new contributions to the potential of the modulus. All of them are expected to be parametrized by the expression (4). For example, this type of contribution may arise from nonminimal coupling to gravity of the form

$$\xi R \Phi^2 \tag{10}$$

where R is the Ricci scalar and ξ is a constant. Note that this term is generated at one loop order even when absent at tree level. (Terms of the form in Eq. (4) arising from nonminimal coupling without any reference to supersymmetry have been utilized, for example, by [13].) Since $\langle\Phi\rangle = \Phi_0$ is naturally of the order of M_p, we find that the

[5] As is customary, we set $M_p = 1$ whenever dimensional quantities are not being discussed.

[6] We have not been careful in treating Q and Φ consistently as real or complex fields when they should be consistently complex.

$$\langle V + \Delta V \rangle \sim H^2 M_p^2 \tag{11}$$

while because of the large mass $\widetilde{m}$, the small shift in the Φ vev is negligible.

3. Note that the example of Eq. (6) with $m = n = 1$ is already phenomenologically unsatisfactory because there is an one loop mass contribution from the resulting effective Lagrangian term

$$\lambda |Q|^2 |\partial \Phi|^2 \tag{12}$$

that is too large for the quintessential behavior to be maintained. Even in the most optimistic scenario, we expect this coupling (with the attendant partial cancellation contributions from SUSY partners) to generate quintessence mass corrections of at least of order

$$\delta m_Q \sim \frac{\sqrt{\lambda}}{4\pi} \left(\frac{\widetilde{m}}{M_p} \right) \widetilde{m} \tag{13}$$

which for $\widetilde{m}$ ∼TeV yields $\delta m_Q \sim 10^{-5}$ eV. Although this is not necessarily disastrous for the quintessence energy, it then becomes difficult to explain why the quintessence has not settled to its minimum already thereby making the quintessence energy contribution more like a cosmological constant.

Indeed, that is why typically the quintessence mass is required to be of the order

$$m_Q \sim H_0 \sim 10^{-33} \text{eV} \tag{14}$$

which is indeed a very tiny mass scale compared to any other mass scales that have been measured experimentally. The fact that any tiny effect can destabilize this tiny mass scale makes quintessence a very sensitive probe of the Kähler potential. It is important to keep in mind that because the first two terms in Eq. (6) are gauge invariant and always present, one cannot eliminate the term proportional to λ simply by using gauge symmetries that act only on the kinetic term. Furthermore, experience with the Standard Model has taught us that any terms not forbidden by fundamental symmetries always exist in the Lagrangian. Unless symmetry principles can be found to eliminate the generic nonminimal term in Eq. (6) or symmetry principle cancels the radiative corrections coming from these nonminimal terms exactly even in the presence of SUSY breaking, observational confirmation of the quintessence picture may make the standard picture of supersymmetry quite unfavorable.[7]

[7] One possibility for protecting the quintessence mass is through a global symmetry which can realize quintessence as a pseudo-Nambu-Goldstone boson. Model building along these lines has been considered, for example, in [14, 15].

For the rest of the paper, we will assume that the required symmetry exists to protect the quintessence mass. Hence, in the next section we will solve the general problem with the Kähler potential of Eq. (6) where m and n are natural numbers not necessarily equal to 1.

4. We devote this section to a more complete analysis of the dynamics of the system made of the quintessence and the moduli fields. As we shall see, this detailed analysis confirms the conclusions in the previous sections. The main objective of the analysis is to compute the equation of state with the interaction energies taken into account. The choice of the toy model will be based on the aim of demonstrating the natural existence of the enhancement of the negativity of the equation of state rather than complete generality. As we advertised previously, although the exact numerical value of the energy and pressure contribution due to the interaction is sensitive to the details of the quintessence potential, its order of magnitude is not.

Let us consider the generic action

$$S_{\rm M} = \int d^4x\, \sqrt{-g} \left[G^i_{\ j} D_\mu \phi_i D^\mu \phi^{j*} + e^G (3 - G_i (G^{-1})^i_{\ j} G^j) \right], \tag{15}$$

where $G = K + \ln |W|^2$, $G^i \equiv \partial_{\phi_i} G$, $G_i \equiv \partial_{\phi^{i*}} G$, and $G^i_{\ j} \equiv \partial_{\phi_i} \partial_{\phi^{j*}} G$. We have set the reduced Planck constant $M_p = 1$. Choosing the a Kähler potential of the form Eq. (6) we find the kinetic terms to be

$$\begin{aligned} S_{\rm kin} &= \int d^4x \sqrt{g} g^{ab} \Big[(1 + \lambda n^2 |\Phi|^{2m} |Q|^{2n-2}) \partial_a Q \partial_b Q^* + (1 + \lambda n^2 |Q|^{2n} |\Phi|^{2m-2}) \partial_a \Phi \partial_b \Phi^* \\ &\quad + \lambda nm |\Phi|^{2m-2} |Q|^{2n-2} \left(\Phi^* Q \partial_a \Phi \partial_b Q^* + \Phi Q^* \partial_a \Phi^* \partial_b Q \right) \Big] . \end{aligned} \tag{16}$$

As for the potential, we will assume that there is a contribution to the effective potential of the form

$$V(\Phi) = \widetilde{m}^2 \left| \Phi - \Phi_0 \right|^2 , \tag{17}$$

where Φ_0 is of order M_p. For the quintessence, we also add by hand a potential $V(Q)$ that leads to a negative equation of state and energy density order of the critical density today. Based on the criterion of ease of mathematical manipulation, we choose $V(Q) = V_0\, e^{-\beta R}$ where $R = \frac{1}{2}(Q + Q^*)$ [18].[8] We neglect the background energy density contributions such as those from cold dark matter, baryons, and radiation since we are not interested in the global tracking properties but more on local properties. The qualitative aspects of the present demonstration should not depend upon the details

[8]This type of potential is phenomenologically undesirable for couple of reasons. One is that it does not give any potential to the imaginary part of Q. Another is that big bang nucleosynthesis bounds make this potential undesirable [18]. Nonetheless, since we are not concerned with the global behavior but rather the local behavior, these choices should suffice to illustrate the effect of interest.

of these choices as supported by the general arguments given by the previous sections. Furthermore, we justify not specifying the details of the superpotential by the fact that we do not know how most of the cosmological constant is cancelled. The kinetic term effect that we analyze here is likely to be unaffected by the solution to the cosmological constant problem as long as the solution to the cosmological constant problem does not involve derivatively coupled terms. Finally, we will assume that the quintessence energy density dominates and will neglect the background matter and radiation energy density. This is justified since we are concerned with local properties without worrying about tracking behavior.

The equations of motion for the modulus and for the quintessence field reads

$$\frac{1}{a^3}\partial_t(a^3\partial_t\Phi) + \frac{m}{a^3}(Q^*)^n(\Phi^*)^{m-1}\partial_t(a^3\partial_t(Q^n\Phi^m)) + \widetilde{m}^2(\Phi - \Phi_0) = 0 \tag{18}$$

$$\frac{1}{a^3}\partial_t(a^3\partial_t Q) + \frac{n}{a^3}((Q^*)^{n-1}(\Phi^*)^m\partial_t(a^3\partial_t(Q^n\Phi^m)) - \frac{\beta}{2}V_0 e^{-\beta R} = 0 \tag{19}$$

Note that there is a separation of scales in that $\widetilde{m} \gg H$. Hence, we will define the perturbation order bookkeeping variable s that reflects this hierarchy. In other words, we introduce a homogeneous perturbation $\phi(t)$ about the constant vev (which solves the equation of motion in the limit $\widetilde{m} \to \infty$) as

$$\Phi = \Phi_0 + s\phi(t) \tag{20}$$

and expand everything to first order in s.

Furthermore, since the Kähler expansion is uncontrolled for

$$\lambda(\Phi^*\Phi)^m(Q^*Q)^n > 1 \tag{21}$$

we will impose a hierarchy for the computational sake that

$$\lambda\Phi_0^{2(m-1)}Q_0^{2n} \sim \lambda\Phi_0^{2m}Q_0^{2(n-1)} \sim \lambda\Phi_0^{2m-1}Q^{2n-1} \sim O(1/10) \tag{22}$$

where the Q_0 is the zeroth order solution with $\lambda = 0$. To be conservative, we will treat the perturbation in λ on the same order as perturbation in s and assign a bookkeeping device r for the order of λ:

$$Q = Q_0(t) + rq(t) \tag{23}$$

where q is the homogeneous perturbation. As we will see, even then, the Φ perturbation to first order in s becomes unimportant for the energy and the equation of state. Finally, we also expand the expansion rate $H = \dot{a}/a$ as a perturbation series in r as

$$H = H_1(t) + rh(t) \tag{24}$$

The expansion to zeroth order in r and s gives rise to the usual quintessence equations of motion

$$\ddot{Q}_0 + 3H_1\dot{Q}_0 - \frac{\beta}{2}V_0e^{-\beta R} = 0 \tag{25}$$

$$H_1^2 = \frac{1}{3}\left(|\dot{Q}|^2 + V_0e^{-\beta R}\right) \tag{26}$$

which have the cosmological solutions

$$Q_0 = \frac{2}{\beta}\ln\left(\frac{t}{\tau}\right) \tag{27}$$

$$H_1 = \frac{4}{\beta^2 t} \tag{28}$$

$$V_0 = \frac{4(12-\beta^2)}{\beta^4\tau^2} \tag{29}$$

where we must keep in mind that Q_0 has to be chosen to satisfy Eq. (22). Since $\beta \sim O(1)$, we then should choose $t/\tau \sim O(1)$.

The equations of motion to first order in r and s yield

$$\begin{aligned}\ddot{\phi} + 3H_1\dot{\phi} + \lambda mnQ_0^{2n-1}\Phi_0^{2m-1}(\ddot{Q}_0 + 3H_1\dot{Q}_0) + &\\ \lambda mn(n-1)Q_0^{2n-2}\Phi_0^{2m-1}\dot{Q}_0^2 + \widetilde{m}^2\phi &= 0 \end{aligned}\tag{30}$$

$$\begin{aligned}\ddot{q} + 3h\dot{Q}_0 + 3H_1\dot{q} + \lambda n^2(n-1)Q_0^{2n-3}\Phi_0^{2m}\dot{Q}_0^2 + &\\ \lambda n^2Q_0^{2n-2}\Phi_0^{2m}(\ddot{Q}_0 + 3H_1\dot{Q}_0) + \frac{\beta^2}{4}(q+q^*)V_0e^{-\beta R} &= 0\end{aligned}\tag{31}$$

$$\frac{1}{3}\left[\lambda n^2Q_0^{2n-2}\Phi_0^{2m}\dot{Q}_0^2 + \dot{Q}_0(\dot{q}+\dot{q}^*) - \frac{\beta}{2}V_0e^{-\beta Q_0}(q+q^*)\right] = 2hH_1 \tag{32}$$

where we have assumed $|\frac{\beta}{2}(q+q^*)| < 1$. As for the boundary conditions to these perturbation equations, we set

$$\phi(t_i) = 0 \qquad \dot{\phi}(t_i) = 0 \tag{33}$$

$$q(t_i) = 0 \qquad \dot{q}(t_i) = 0 \tag{34}$$

although any parts of the perturbation solutions that depend on the boundary condition tend to die away faster than the non-boundary condition dependent terms (sourced part of the solution), and therefore is not important unless $V_0 \to 0$.

Using the usual one dimensional Green's function technique, one can solve these equations. We find for the perturbation to Φ_0 the solution

$$\phi(t) = \frac{-\lambda mn\Phi_0^{2m-1}}{t^2}\left(\frac{2}{\beta}\right)^{2n}\left[\tilde{x}\left(\ln\left[\frac{t}{\tau}\right]\right)^{2n-1} + (n-1)\left(\ln\left[\frac{t}{\tau}\right]\right)^{2n-2}\right] + \text{b.c. dep. terms} \tag{35}$$

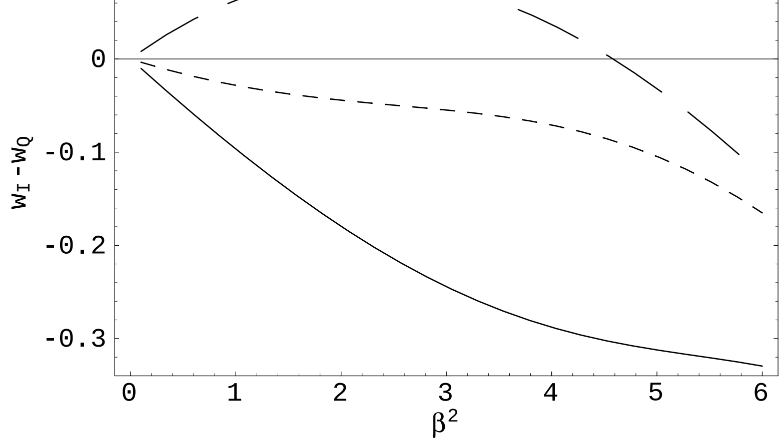

Figure 1: Plot of $w_I - w_Q$ as a function of β^2 for $n = 2$. The long dashed curve corresponds to $t/\tau = 1$, the short dashed curve corresponds to $t/\tau = 5$, and the solid curve corresponds to $t/\tau = 10$.

where $\tilde{x} \equiv 12/\beta^2 - 1$ and the "b.c. dep. terms" represent boundary condition dependent subleading terms that die away as a function of time if $\tilde{x} > 0$. For the perturbation to Q_0, we have

$$q(t) = \frac{-\lambda n^2 \Phi_0^{2m}}{\beta} \left(\frac{2}{\beta}\right)^{2n-2} \left[\frac{2\tilde{x}+1}{\tilde{x}} \left(\ln\left[\frac{t}{\tau}\right]\right)^{2n-2} + (2n-2)!\tilde{x} \sum_{l=0}^{2n-3} (\tilde{x}^{l-2n} + \sum_{y=1}^{2n-l-1} 2\tilde{x}^{-y}) \frac{(-1)^l (\ln[\frac{t}{\tau}])^l}{l!}\right] + \text{b.c. dep. terms} \quad (36)$$

where the "b.c. dep. terms" again indicate boundary condition dependent terms which die away if $\tilde{x} > 0$.[9]

Note that the validity of this solution is only in the regime when $|\beta q| < 1$. The resulting energy density and pressure can be calculated to be

$$\rho = \rho_Q + \rho_I \quad (37)$$

$$p = p_Q + p_I \quad (38)$$

$$\rho_Q \equiv \frac{48}{\beta^4 t^2} \quad (39)$$

$$p_Q \equiv \frac{8(\beta^2 - 6)}{\beta^4 t^2} \quad (40)$$

[9]Note that the singularity at $\beta^2 = 12$ is an artifact of neglecting the boundary condition dependent terms and not a true singularity. In the limit that $\beta^2 \to 12$, Eq. (29) forces $V_0 \to 0$, in which case the only source term is the background solution sensitive to the boundary conditions. In that case, boundary condition dependent terms naturally become important.

$$\rho_I \equiv 2\lambda n^2 \frac{(1+\tilde{x})^{2n-1}}{\tilde{x}^{2n-2}} \left(\frac{2}{\beta}\right)^{2n} \frac{\Phi_0^{2m}}{t^2} \left(\frac{\tau}{t}\right)^{\frac{1}{1+\tilde{x}}} \Gamma\left(2n-1, \frac{-\tilde{x}}{1+\tilde{x}} \ln[\frac{t}{\tau}]\right) \tag{41}$$

$$p_I \equiv -2\lambda n^2 \frac{\tilde{x}\Phi_0^{2m}}{t^2} (\frac{2}{\beta})^{2n} \frac{\tau}{t} \Gamma(2n-1, -\ln\frac{t}{\tau}) \tag{42}$$

where ρ_I and p_I represent interaction energy and pressure, and $\Gamma(a,x)$ is the incomplete gamma function.[10]

First, note that since we are working in the regime $\tilde{x} > 0$ and $\Gamma > 0$, the signs of ρ_I and p_I take the signs of λ and $-\lambda$, respectively. This means that for $\lambda > 0$, the pressure contribution is negative while the energy contribution is positive. Secondly, we can write the equation of state to leading order in λ as

$$w = \frac{p}{\rho} = w_Q + (w_I - w_Q)\Delta \tag{43}$$

$$w_Q \equiv \frac{p_Q}{\rho_Q} = -1 + \frac{\beta^2}{6} < 0 \tag{44}$$

$$w_I \equiv \frac{\rho_I}{p_I} = -\left(\frac{\tilde{x}}{1+\tilde{x}}\right)^{2n-1} \left(\frac{\tau}{t}\right)^{\frac{1}{1+\tilde{x}}} \frac{\Gamma(2n-1, -\ln\frac{t}{\tau})}{\Gamma(2n-1, \frac{-\tilde{x}}{1+\tilde{x}}\ln\frac{t}{\tau})} < 0 \tag{45}$$

$$\Delta \equiv \frac{\rho_I}{\rho_Q} \tag{46}$$

where $\mathrm{sign}(\Delta) = \mathrm{sign}(\lambda)$ and w_I is the interaction equation of state. Hence, if $\lambda > 0$ and $w_I - w_Q < 0$ (equivalently $|w_I| > |w_Q|$), the total equation of state becomes more negative. Since $t/\tau \sim O(1)$, we plot in Fig. 1, $(w_I - w_Q)$ as a function of β^2 for $t/\tau = 1, 5, 10$ with $n = 2$. It clearly shows that a negative contribution to the equation of state is quite generic. Finally, note that the power m of the modulus field Φ enters the total equation of state w only through the relative energy ratio Δ.

5. In this brief paper we have pointed out that any scalar field with gravitational coupling to quintessence and vacuum expectation values of the order of the Planck scale play a significant role in the present-day cosmological evolution of the universe. Our findings suggest that in a realistic particle theory approach to the dark energy problem, the use of a single quintessence field for models is likely to miss significant contributions to the negative equation of state. Furthermore, we have argued that supersymmetric embedding of the quintessence is generically difficult because of sensitivity of the quintessence mass to generic terms in the Kähler potential. This presents the exciting possibility that confirmation of quintessential picture may lead to new probes into the underlying high energy physics. Our observations open up new possibilities, such as

[10] The incomplete gamma function is defined as $\Gamma(a,x) \equiv \int_x^\infty dt t^{a-1} e^{-t}$.

verse or significantly changing the quintessential phenomenology, allowing naively ruled out quintessence parameter space to become viable.

Acknowledgments

We would like to thank R. Rattazzi for helpful discussions.

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