CEEX Exponentiation in QED*

S. Jadach[a]

[a]Henryk Niewodniczanski Institute of Nuclear Physics,
ul. Radzikowskiego 152, 31-342 Cracow, Poland

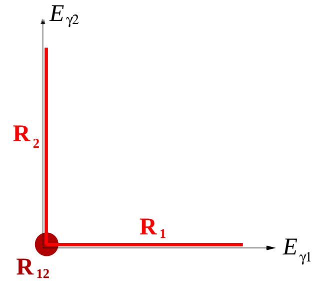
The aim is to summarize briefly on: (a) Main features of YFS Coherent Exclusive EXponentiation in QED, (b) Examples of recent results relevant for LEP/LC physics program.

This presentation is based mainly on three recent papers [1–3]. Standard Model calculations for LEP with YFS exponentiation are implemented in many Monte Carlo event generators: for $e^+e^- \to f\bar{f} + n\gamma$, $f = \tau, \mu, d, u, s, c$ process: (1a) YFS1 (1987-1989) $\mathcal{O}(\alpha^1)_{exp}$ ISR [4], (1b) YFS2∈KORALZ (1989-1990) $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$ ISR [5,6], (1c) YFS3∈KORALZ (1990-1998) $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$ ISR+FSR [7,8], (1d) KKMC (1998-02) $\mathcal{O}(\alpha^2 + h.o.LL)_{exp}$ ISR+FSR+Interf. ($d\sigma/\sigma = 0.2\%$) [9]; for the low angle Bhabha process $e^+e^- \to e^+e^- + n\gamma$ for $\theta < 6°$ the following ones: (2a) BHLUMI 1.x, (1987-1990) $\mathcal{O}(\alpha^1)_{exp}$ [10], (2b) BHLUMI 2.x, (1990-1996) $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$ ($d\sigma/\sigma = 0.07\%$), [11,12]; for the large angle Bhabha $e^+e^- \to e^+e^- + n\gamma$ for $\theta > 6°$ (3) BHWIDE (1994-1998) $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$ [13]; and finally for WW boson production processes $e^+e^- \to W^+W^- + n\gamma$, $W^\pm \to f\bar{f}$ (4a) KORALW (1994-2001) ISR YFS LL ($d\sigma/\sigma = 2\%$) [14,15] and (4b) YFS3WW (1995-2001) YFS expon. + Leading Pole Approx. ($d\sigma/\sigma = 0.4\%$) [16,17].

The detailed description of the Coherent Exclusive Exponentiation (CEEX) can be found in ref. [1], along with the description of the older Exclusive Exponentiation (EEX), closer to the original Yennie-Frautschi-Suura (YFS) scheme [18]. What is in a nutshell YFS exponentiation? The main steps in YFS exponentiation are:

- Reorganization of the perturbative complete $\mathcal{O}(\alpha^\infty)$ series such that IR-finite $\bar{\beta}$ components are isolated (factorization theorem).
- Truncation of the IR-finite $\bar{\beta}$s to finite $\mathcal{O}(\alpha^n)$ and calculation of them from Feynman diagrams recursively.

Apart from disentangling properly ultraviolet divergences from the infrared (IR) ones YFS deals consistently with the overlapping and sub-leading IR singularities. To illustrate this point let is show the "map of IR singularities" for 2γ distribution $D_2(k_1, k_2)$ in the plane of the energies of two real photons:



The point R_{12} is the location of the leading double IR singularity and the lines R_1 and R_2 host non-leading single IR singularities. The above topological picture translates into a decomposition of the differential distributions:
$D_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}) = \bar{\beta}_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4})$,
$D_1(p_f; \mathbf{k_1}) = \bar{\beta}_0(p_f)\tilde{S}(\mathbf{k_1}) + \bar{\beta}_1(p_f; k_1)$,and
$D_2(\mathbf{k_1}, \mathbf{k_2}) = \bar{\beta}_0\tilde{S}(\mathbf{k_1})\tilde{S}(\mathbf{k_2})$
$\quad + \bar{\beta}_1(k_1)\tilde{S}(\mathbf{k_2}) + \bar{\beta}_1(k_2)\tilde{S}(\mathbf{k_1}) + \bar{\beta}_2(k_1, k_2)$
where typicaly $p_{f_1} + p_{f_2} = p_{f_3} + p_{f_4}$ for $\bar{\beta}_0$ in D_0

*Work partly supported by the European Community's Human Potential Programme under contract HPRN-CT-2000-00149 "Physics at Colliders", by Polish Government grants KBN 5P03B09320 and 2P03B00122, and by NATO grant PST.CLG.977751.

and $p_{f_1}+p_{f_2}\neq p_{f_3}+p_{f_4}$ for $\bar\beta_0$ in D_1 and D_2. As we see the IR-finite part $\bar\beta_1$ in the decomposition of the 1-photon D_1 distribution into IR-leading and subleading parts is present as an element in the subleading 2-photon D_2 distribution. This nontrivial fact allows one to find out the IR-finite $\bar\beta_i$ functions order by order from standard perturbative calculations. In this particular case the recursive, order-by-order definition of IR-finite $\bar\beta$s looks as follows:
$\bar\beta_0(p_{f_1},p_{f_2},p_{f_3},p_{f_4}) = D_0(p_{f_1},p_{f_2},p_{f_3},p_{f_4})$,
$\bar\beta_1(p_f;k_1) = D_1(p_f;\mathbf{k_1}) - \bar\beta_0(p_f)\tilde S(\mathbf{k_1})$
$\bar\beta_2(k_1,k_2) = D_2(\mathbf{k_1},\mathbf{k_2}) - \bar\beta_0\tilde S(\mathbf{k_1})\tilde S(\mathbf{k_2}) - \bar\beta_1(k_1)\tilde S(\mathbf{k_2}) + \bar\beta_1(k_2)\tilde S(\mathbf{k_1})$
Having at hand the IR-free $\bar\beta_i$ the YFS scheme provides immediately for the multi-photon differential distributions all over the phase space and the phase space is integraded without any approximations using MC method.

Below we show an example of the ISR $\mathcal{O}(\alpha^1)$ exponentiated multi-photon fully differential distributions for the CEEX variant of YFS exponentiation which is implemented in terms of the IR-free β_i objects constructed at the amplitude level, rather than using the spin-summed squared $\bar\beta_i$s. The CEEX we present here very schematically for the process $e^-(p_1,\lambda_1)+e^+(p_2,\lambda_2)\to f(q_1,\lambda'_1)+\bar f(q_2,\lambda'_2)+\gamma(k_1,\sigma_1)+...\gamma(k_n,\sigma_n)$:

$$\sigma=\sum_{n=0}^{\infty}\int_{m_\gamma} d\Phi_{n+2}\sum_{\lambda,\sigma_1,\dots,\sigma_n}|e^{\alpha B(m_\gamma)}\mathfrak{M}^{\lambda}_{n,\sigma_1,\dots,\sigma_n}(k_1,\dots,k_n)|^2,$$

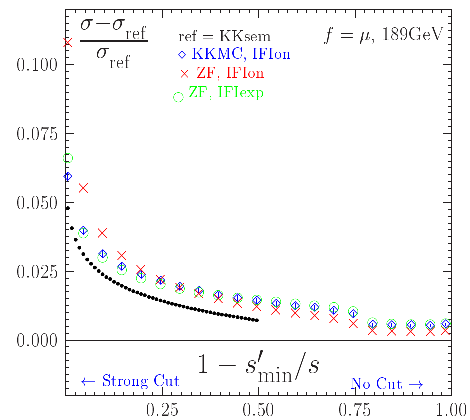
$\mathfrak{M}^{\lambda}_0=\hat\beta^{\lambda}_0,\quad \lambda$=fermion helicities,
$\mathfrak{M}^{\lambda}_{1,\sigma_1}(k_1)=\hat\beta^{\lambda}_0\mathfrak{s}_{\sigma_1}(k_1)+\hat\beta^{\lambda}_{1,\sigma_1}(k_1)$

$$\mathfrak{M}^{\lambda}_{2,\sigma_1,\sigma_2}(k_1,k_2)=\hat\beta^{\lambda}_0\mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2) + \hat\beta^{\lambda}_{1,\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2)+\hat\beta^{\lambda}_{1,\sigma_2}(k_2)\mathfrak{s}_{\sigma_1}(k_1)$$

$$\mathfrak{M}^{\lambda}_{n,\sigma_1,\dots\sigma_n}(k_1,k_2,\dots k_n)=\hat\beta^{\lambda}_0\mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2)\dots\mathfrak{s}_{\sigma_n}(k_n) + \hat\beta^{\lambda}_{1,\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2)\dots\mathfrak{s}_{\sigma_n}(k_n) + \mathfrak{s}_{\sigma_1}(k_1)\hat\beta^{\lambda}_{1,\sigma_2}(k_2)\dots\mathfrak{s}_{\sigma_n}(k_n)\dots \cdots + \mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2)\dots\hat\beta^{\lambda}_{1,\sigma_2}(k_2)$$

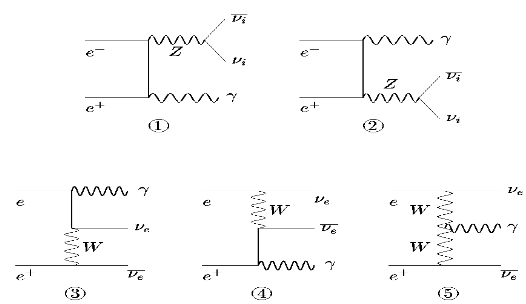
The $\mathcal{O}(\alpha^1)$ IR-finite building blocks are:
$\hat\beta^{\lambda}_0=\left(e^{-\alpha B_4}\mathfrak{M}^{\text{Born+Virt.}}_{\lambda}\right)\big|_{\mathcal{O}(\alpha^1)}$,
$\hat\beta^{\lambda}_{1,\sigma}(k)=\mathfrak{M}^{\lambda}_{1,\sigma}(k)-\hat\beta^{\lambda}_0\mathfrak{s}_{\sigma}(k)$
It is to be stressed that everything here was done in terms of the spin $\mathfrak{M}$-amplitudes! Multiphoton differential distributions are positive by construction (which is not the case for EEX). In KKMC the above CEEX is implemented up to $\mathcal{O}(\alpha^2)$ for ISR and FSR[2].

[2]Multiperipheral contributions and non-IR ISR$\otimes$FSR interferences are still incomplete at the $\mathcal{O}(\alpha^2)$ level. They are unimportant for most of LEP observables.



The example of the CEEX aplication is shown in the figure above from ref. [19] where we present the "state of art" comparison of KK MC vs. Zfitter for ISR+FSR, including IFI=ISR$\otimes$FSR. We conclude that the precision $d\sigma/\sigma = 0.2\%$ was reached, which is enough for LEP2 data analysis. Similar high precision was obtained for the forward-backward asymmetry [19].

Recently, the CEEX matrix element was extended in KKMC to the neutrino channel. The contributions from following diagrams are now included:



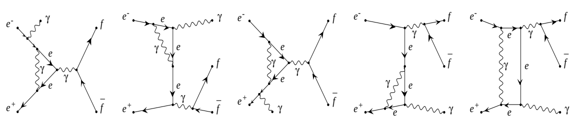
and also all double photon emission contributions feature the exact matrix element. We summarize below on the improvements in KKMC for the neutrino channel.

- The systematic error is estimated to be 1.3% for $\nu_e\bar\nu_e\gamma$ and 0.8% for $\nu_\mu\bar\nu_\mu\gamma$ and $\nu_\tau\bar\nu_\tau\gamma$.
- For observables with two observed photons we estimate the uncertainty to be about 5%.
- These new improved results were obtained thanks to the inclusion of non-photonic electroweak corrections of the `ZFITTER` package

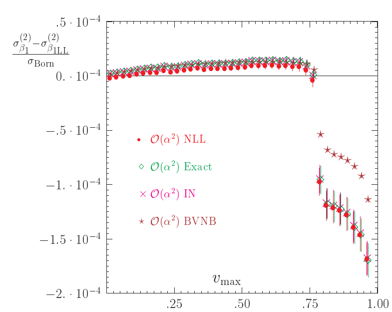
and due to newly constructed, exact, single and double emission photon amplitudes in the KKMC for the contribution with the t-channel W exchange.

- The virtual corrections for the W exchange are at present introduced in the approximated form. The exponentiation scheme CEEX is the same as in the original KKMC program

In the first version of KKMC of ref. [9] the $\mathcal{O}(\alpha^2)$ next-to-leading contributions were incomplete. In ref. [3] the $\mathcal{O}(\alpha^2)$ contribution from the following diagrams were evaluated



and partly included in the development of KKMC – and the remaining contributions were found to be negligible. The contributions in questions [3] are due to one real and one virtual photon emission and the new numerical results are shown in the following figure



where the new calculation is shown to agree with the older calculations of the refs. [20,21]. NB. These contributions were also recently (re)calculated in ref. [22] for the purpose of the low energy electron collider experiments.

Conclusions The YFS inspired EEX and CEEX schemes are successful examples of the Monte Carlos based directly on the factorization theorem (albeit for IR soft case for Abelian QED only). They Work well in practice: KORALZ, BHLUMI, YWSWW3, BHWIDE, KK MC. Extension (as far as possible) to all collinear singularities would be very desirable and practically important! The KKMC program is extended to the neutrino channel. Missing fully differential $2f + 1\gamma_{virt} + 1\gamma_{real}$ distributions for $\mathcal{O}(\alpha^2)$ CEEX are now available.

REFERENCES

1. S. Jadach, B.F.L. Ward and Z. Was, Phys. Rev. D63 (2001) 113009, hep-ph/0006359.
2. D. Bardin et al., Eur. Phys. J. C24 (2002) 373, hep-ph/0110371.
3. S. Jadach et al., Phys. Rev. D65 (2002) 073030, hep-ph/0109279.
4. S. Jadach and B.F.L. Ward, Phys. Rev. D38 (1988) 2897.
5. S. Jadach, B.F.L. Ward and Z. Was, The monte carlo program koralz, for the lepton or quark pair production at lep/slc energies: From version 4.0 to version 4.04, 1999, hep-ph/9905205, Computer. Phys. Commun. in print.
6. S. Jadach and B.F.L. Ward, Comput. Phys. Commun. 56 (1990) 351.
7. S. Jadach, B.F.L. Ward and Z. Wąs, Comput. Phys. Commun. 79 (1994) 503.
8. S. Jadach and B.F.L. Ward, Phys. Lett. B274 (1992) 470.
9. S. Jadach, B.F.L. Ward and Z. Was, Comput. Phys. Commun. 130 (2000) 260, hep-ph/9912214.
10. S. Jadach and B.F.L. Ward, Phys. Rev. D40 (1989) 3582.
11. S. Jadach et al., Comput. Phys. Commun. 70 (1992) 305.
12. S. Jadach et al., Comput. Phys. Commun. 102 (1997) 229.
13. S. Jadach, W. Płaczek and B.F.L. Ward, Phys. Lett. B390 (1997) 298, also hep-ph/9608412; The Monte Carlo program BHWIDE is available from `http://hephp01.phys.utk.edu/pub/BHWIDE`.
14. S. Jadach et al., Comput. Phys. Commun. 119 (1999) 272, hep-ph/9906277.
15. S. Jadach et al., Comput. Phys. Commun. 140 (2001) 475, hep-ph/0104049.
16. S. Jadach et al., Phys. Lett. B417 (1998) 326.
17. S. Jadach et al., Comput. Phys. Commun. 140 (2001) 432, hep-ph/0103163.
18. D.R. Yennie, S. Frautschi and H. Suura, Ann. Phys. (NY) 13 (1961) 379.
19. Two Fermion Working Group, M. Kobel et al., (2000), hep-ph/0007180.
20. M. Igarashi and N. Nakazawa, Nucl. Phys. B288 (1987) 301.
21. F. Berends, G. Burgers and W.V. Neerven, Phys. Lett. 177 (1986) 1191.
22. G. Rodrigo et al., Eur. Phys. J. C24 (2002) 71, hep-ph/0112184.