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Exploration of the MSSM with Non-Universal Higgs Masses

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Abstract

We explore the parameter space of the minimal supersymmetric extension of the Standard Model (MSSM), allowing the soft supersymmetry-breaking masses of the Higgs multiplets, $m_{1,2}$, to be non-universal (NUHM). Compared with the constrained MSSM (CMSSM) in which $m_{1,2}$ are required to be equal to the soft supersymmetry-breaking masses m_0 of the squark and slepton masses, the Higgs mixing parameter μ and the pseudoscalar Higgs mass m_A, which are calculated in the CMSSM, are free in the NUHM model. We incorporate accelerator and dark matter constraints in determining allowed regions of the (μ, m_A), (μ, M_2) and $(m_{1/2}, m_0)$ planes for selected choices of the other NUHM parameters. In the examples studied, we find that the LSP mass cannot be reduced far below its limit in the CMSSM, whereas m_A may be as small as allowed by LEP for large $\tan\beta$. We present in Appendices details of the calculations of neutralino-slepton, chargino-slepton and neutralino-sneutrino coannihilation needed in our exploration of the NUHM.

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1 Introduction

The hierarchy of mass scales in physics is preserved in a natural way if supersymmetric particles weigh less than about a TeV. Many supersymmetric models conserve the quantity $R = (-1)^{3B+L+2S}$, where B is the baryon number, L the lepton number and S the spin. If this R parity is conserved, the lightest supersymmetric particle (LSP) is expected to be absolutely stable. The most plausible candidate for the LSP is the lightest neutralino χ, which is a good candidate [1] for the cold dark matter (CDM) that is thought to dominate over baryonic and hot dark matter.

In this paper, we refine and extend the many previous calculations of the relic LSP density in the framework of the minimal supersymmetric extension of the Standard Model (MSSM). In particular, we expand the recent analysis of the MSSM parameter space in [2], where we allowed non-universal input soft supersymmetry-breaking scalar masses for the Higgs multiplets. Here we explore in more detail the constraints imposed by accelerator experiments - including searches at LEP, $b \to s\gamma$ and $g_\mu - 2$ - and the cosmological bound on the LSP relic density.

Before discussing our calculations in more detail, we first review the range of the relic LSP density that we prefer in our calculations. An important new constraint on this is provided by data on the cosmic microwave background (CMB), which have recently been used to obtained the following preferred 95% confidence range: $\Omega_{\rm CDM} h^2 = 0.12 \pm 0.04$ [3]. Values much smaller than $\Omega_{\rm CDM} h^2 = 0.10$ seem to be disfavoured by earlier analyses of structure formation in the CDM framework, so we restrict our attention to $\Omega_{\rm CDM} h^2 > 0.1$. However, one should note that the LSP may not constitute all the CDM, in which case $\Omega_{\rm LSP}$ could be reduced below this value. On the upper side, we prefer to remain very conservative, in particular because the upper limit on $\Omega_{\rm LSP}$ sets the upper limit for the sparticle mass scale. In this paper, we use $\Omega_{\rm CDM} h^2 < 0.3$, while being aware that the lower part of this range currently appears the most plausible.

The parameter space of the MSSM with non-universal soft supersymmetry-breaking masses for the two Higgs multiplets has two additional dimensions, beyond those in the constrained MSSM (CMSSM), in which all the soft supersymmetry-breaking scalar masses m_0 are assumed to be universal. In the CMSSM, the underlying parameters may be taken as m_0, the soft supersymmetry-breaking gaugino mass $m_{1/2}$ that is also assumed to be universal, the trilinear supersymmetry-breaking parameters A_0 that we set to zero at the GUT scale in this paper, the ratio $\tan\beta$ of Higgs vacuum expectation values, the Higgs superpotential coupling μ and the pseudoscalar Higgs boson mass m_A. Two relations between these parameters follow from the electroweak symmetry-breaking vacuum conditions, which are normally used in the CMSSM to fix the values of μ (up to a sign ambiguity) and m_A in terms of the other parameters $(m_0, m_{1/2}, A_0, \tan\beta)$.

In the more general MSSM with non-universal Higgs masses (NUHM), the parameters μ and m_A become independent again [4, 5, 6]. Thus one may use the parameters $(m_0, m_{1/2}, \mu, m_A, A_0, \tan\beta)$ to parametrize this more general NUHM. The underlying theory is likely to specify the non-universalities of the Higgs masses: $\hat{m}_i \equiv \text{sign}(m_i^2)|m_i/m_0|$: $i = 1, 2$, so it is important to know how the different values of the $\hat{m}_i$ map into the $(m_0, m_{1/2}, \mu, m_A, A_0, \tan\beta)$ parameter space, a point we discuss in Section 2. Furthermore, this non-universality leads to new coannihilation processes becoming important, which are discussed in Section 3.

We review and update in Section 4 the experimental and phenomenological constraints on the MSSM parameter space that we use, applying them to the CMSSM. Then, in Section 5, we explore the NUHM parameter space. Previously, we gave priority to a first scan of the extra dimensions of the parameter space, and postponed a complete discussion of the NUHM at large $\tan\beta$. Here we also show how our results in the (μ, m_A), (μ, M_2) and $(m_{1/2}, m_0)$ planes for $\tan\beta = 10$ change at larger $\tan\beta$, concentrating on the behaviour of the relic LSP density, but also incorporating constraints on the NUHM from accelerators. In our discussions of these planes, we emphasize the novel features not present in the CMSSM, such as the forms of the regions in which the LSP is charged, e.g., the lighter $\widetilde{\tau}$, regions where the LSP is a sneutrino $\widetilde{\nu}$ (thus necessitating the inclusion of additional $\chi - \widetilde{\nu}$ coannihilation processes), and the potential importance of Higgsino coannihilation processes. These are not usually relevant in the CMSSM, where the relic LSP is usually mainly a $\widetilde{B}$. Section 6 summarizes some conclusions from our analysis, including comments on the range of LSP and pseudoscalar Higgs masses allowed in the NUHM.

The Appendices provide the information needed to reproduce our calculations of coannihilation processes relevant to this NUHM analysis. In particular, Appendix A lists the MSSM couplings we use, Appendix B extends previous results on neutralino-slepton coannihilation to include left-right (L-R) mixing, Appendix C discusses chargino-slepton coannihilation processes, and Appendix D concerns neutralino-sneutrino coannihilation.

2 Vacuum Conditions for Non-Universal Higgs Masses

We assume that the soft supersymmetry-breaking parameters are specified at some large input scale M_X, that may be identified with the supergravity or grand unification scale.

The low levels of flavour-changing neutral interactions provide good reasons to think that sparticles with the same Standard Model quantum numbers have universal soft scalar masses, e.g., for the $\widetilde{e}_L$, $\widetilde{\mu}_L$ and $\widetilde{\tau}_L$. Specific grand unification models may equate the soft scalar masses of matter sparticles with different Standard Model quantum numbers, e.g., $(\widetilde{d}, \widetilde{s}, \widetilde{b})_L$ and $(\widetilde{e}, \widetilde{\mu}, \widetilde{\tau})_L$ in SU(5), and all the Standard Model matter sparticles in SO(10). However, there are no particularly good reasons to expect that the soft supersymmetry-breaking scalar masses of the Higgs multiplets should be equal to those of the matter sparticles. This is, however, the assumption made in the CMSSM, which we relax in the NUHM studied here [1].

One of the attractive features of the CMSSM is that it provides a mechanism for generating electroweak symmetry breaking via the running of the effective Higgs masses-squared m_1^2 and m_2^2 from M_X down to low energies. We use this mechanism also in the NUHM, which enables us to relate $m_1^2(M_X)$ and $m_2^2(M_X)$ to the Higgs supermultiplet mixing parameter μ and the pseudoscalar Higgs mass m_A. Therefore, we can and do choose as our independent parameters $\mu(m_Z) \equiv \mu$ and $m_A(Q) \equiv m_A$, where $Q \equiv (m_{\widetilde{t}_R} m_{\widetilde{t}_L})^{1/2}$; as well as the CMSSM parameters $(m_0(M_X), m_{1/2}(M_X), A_0, \tan\beta)$. In fact, in this paper we set $A_0 = 0$ for definiteness.

The electroweak symmetry breaking conditions may be written in the form:

$$m_A^2(Q) = m_1^2(Q) + m_2^2(Q) + 2\mu^2(Q) + \Delta_A(Q) \tag{1}$$

and

$$\mu^2 = \frac{m_1^2 - m_2^2 \tan^2\beta + \frac{1}{2} m_Z^2 (1 - \tan^2\beta) + \Delta_\mu^{(1)}}{\tan^2\beta - 1 + \Delta_\mu^{(2)}}, \tag{2}$$

where Δ_A and $\Delta_\mu^{(1,2)}$ are loop corrections [8, 9, 10] and $m_{1,2} \equiv m_{1,2}(m_Z)$. We incorporate the known radiative corrections [8, 11, 12] c_1, c_2 and c_μ relating the values of the NUHM parameters at Q to their values at m_Z:

$$\begin{aligned} m_1^2(Q) &= m_1^2 + c_1 \\ m_2^2(Q) &= m_2^2 + c_2 \\ \mu^2(Q) &= \mu^2 + c_\mu \,. \end{aligned} \tag{3}$$

Solving for m_1^2 and m_2^2, one has

$$\begin{aligned} m_1^2(1+\tan^2\beta) &= m_A^2(Q)\tan^2\beta - \mu^2(\tan^2\beta + 1 - \Delta_\mu^{(2)}) - (c_1 + c_2 + 2c_\mu)\tan^2\beta \\ &\quad -\Delta_A(Q)\tan^2\beta - \frac{1}{2}m_Z^2(1-\tan^2\beta) - \Delta_\mu^{(1)} \end{aligned} \tag{4}$$

[1] For models with non-universality also in the sfermion masses, see [4, 7].

and

$$m_2^2(1+\tan^2\beta) = m_A^2(Q) - \mu^2(\tan^2\beta + 1 + \Delta_\mu^{(2)}) - (c_1 + c_2 + 2c_\mu) \\ -\Delta_A(Q) + \frac{1}{2}m_Z^2(1-\tan^2\beta) + \Delta_\mu^{(1)}, \quad (5)$$

which we use to perform our numerical calculations.

It can be seen from (4) and (5) that, if m_A is too small or μ is too large, then m_1^2 and/or m_2^2 can become negative and large. This could lead to $m_1^2(M_X) + \mu^2(M_X) < 0$ and/or $m_2^2(M_X) + \mu^2(M_X) < 0$, thus triggering electroweak symmetry breaking at the GUT scale. The requirement that electroweak symmetry breaking occurs far below the GUT scale forces us to impose the conditions $m_1^2(M_X) + \mu(M_X), m_2^2(M_X) + \mu(M_X) > 0$ as extra constraints, which we call the GUT stability constraint [2].

Specific models for the origin of supersymmetry breaking should be able to predict the amounts by which universality is violated in $m_{1,2}^2$, which can be read off immediately from (4, 5). Alternatively, for a given amount of universality breaking, these equations may easily be inverted to yield the corresponding values of μ and m_A. In this paper, we plot quantities in terms of μ and m_A.

In the CMSSM, to obtain a consistent low energy model given GUT scale inputs, we must run down the full set of renormalization group equations (RGE's) from the GUT scale and use the electroweak symmetry breaking constraints which fix μ and m_A. Consistency requires the RGE's to be run back up to the GUT scale, where the input parameters are reset and the RGE's are run back down. Many models require running this cycle about 3 times, though in some cases convergence may be much slower, particularly at large $\tan\beta$. Indeed, there are no solutions for $\mu < 0$ when $\tan\beta$ is large ($\gtrsim 40$) because of diverging Yukawas. In the NUHM case considered here, we have boundary conditions at both the GUT and low-energy scales. Once again, the numerical calculations of the RGE's must be iterated until they converge. However, in this case, it is not always possible to arrive at a solution, especially for large $\tan\beta$. In our subsequent calculations, we start by making guesses for the values of $m_{1,2}(M_X)$ for use in the first run from M_X down to m_Z, and it can happen that the iteration pushes the solution away from the convergence point instead of towards it. Therefore, the first few iterations must be monitored for any potential blow-ups.

[2] For a different point of view, however, see [13].

3 Renormalization and Coannihilations in the NUHM Model

The RGEs for the NUHM have additional terms beyond those appearing in the CMSSM, and the resulting sparticle spectrum may exhibit some novel features, as we now discuss.

The new terms in the RGEs which vanish in the CMSSM involve the following combination of soft supersymmetry-breaking parameters [12]:

$$\begin{aligned} S \equiv & \frac{g_1^2}{4}(m_2^2 - m_1^2 + 2(m^2_{\widetilde{Q}_L} - m^2_{\widetilde{L}_L} - 2m^2_{\widetilde{u}_R} + m^2_{\widetilde{d}_R} + m^2_{\widetilde{e}_R}) \\ & + (m^2_{\widetilde{Q}_{3L}} - m^2_{\widetilde{L}_{3L}} - 2m^2_{\widetilde{t}_R} + m^2_{\widetilde{b}_R} + m^2_{\widetilde{\tau}_R})) \end{aligned} \quad (6)$$

Here $\widetilde{Q}_L$, $\widetilde{L}_L$ are the first two generations left-handed sfermions, and $\widetilde{Q}_{3L}$, $\widetilde{L}_{3L}$ are the third-generation sfermions. These new terms appear as follows in the RGEs for the NUHM:

$$\begin{aligned}
\frac{dm_1^2}{dt} &= \frac{1}{8\pi^2}(-3g_2^2M_2^2 - g_1^2M_1^2 + h_\tau^2(m^2_{\widetilde{\tau}_L} + m^2_{\widetilde{\tau}_R} + m_1^2 + A_\tau^2) \\
&\quad +3h_b^2(m^2_{\widetilde{b}_L} + m^2_{\widetilde{b}_R} + m_1^2 + A_b^2) - 2S) \\
\frac{dm_2^2}{dt} &= \frac{1}{8\pi^2}(-3g_2^2M_2^2 - g_1^2M_1^2 + 3h_t^2(m^2_{\widetilde{t}_L} + m^2_{\widetilde{t}_R} + m_2^2 + A_t^2) + 2S) \\
\frac{dm^2_{\widetilde{L}_L}}{dt} &= \frac{1}{8\pi^2}(-3g_2^2M_2^2 - g_1^2M_1^2 - 2S) \\
\frac{dm^2_{\widetilde{e}_R}}{dt} &= \frac{1}{8\pi^2}(-4g_1^2M_1^2 + 4S) \\
\frac{dm^2_{\widetilde{L}_{3L}}}{dt} &= \frac{1}{8\pi^2}(-3g_2^2M_2^2 - g_1^2M_1^2 + h_\tau^2(m^2_{\widetilde{L}_{3L}} + m^2_{\widetilde{\tau}_R} + m_1^2 + A_\tau^2) - 2S) \\
\frac{dm^2_{\widetilde{\tau}_R}}{dt} &= \frac{1}{8\pi^2}(-4g_1^2M_1^2 + 2h_\tau^2(m^2_{\widetilde{L}_{3L}} + m^2_{\widetilde{\tau}_R} + m_1^2 + A_\tau^2) + 4S) \\
\frac{dm^2_{\widetilde{Q}_L}}{dt} &= \frac{1}{8\pi^2}(-\frac{16}{3}g_3^2M_3^2 - 3g_2^2M_2^2 - \frac{1}{9}g_1^2M_1^2 + \frac{2}{3}S) \\
\frac{dm^2_{\widetilde{u}_R}}{dt} &= \frac{1}{8\pi^2}(-\frac{16}{3}g_3^2M_3^2 - \frac{16}{9}g_1^2M_1^2 - \frac{8}{3}S) \\
\frac{dm^2_{\widetilde{d}_R}}{dt} &= \frac{1}{8\pi^2}(-\frac{16}{3}g_3^2M_3^2 - \frac{4}{9}g_1^2M_1^2 + \frac{4}{3}S) \\
\frac{dm^2_{\widetilde{Q}_{3L}}}{dt} &= \frac{1}{8\pi^2}(-\frac{16}{3}g_3^2M_3^2 - 3g_2^2M_2^2 - \frac{1}{9}g_1^2M_1^2 + h_b^2(m^2_{\widetilde{Q}_{3L}} + m^2_{\widetilde{b}_R} + m_1^2 + A_b^2) \\
&\quad +h_t^2(m^2_{\widetilde{Q}_{3L}} + m^2_{\widetilde{t}_R} + m_2^2 + A_t^2) + \frac{2}{3}S) \\
\frac{dm^2_{\widetilde{t}_R}}{dt} &= \frac{1}{8\pi^2}(-\frac{16}{3}g_3^2M_3^2 - \frac{16}{9}g_1^2M_1^2 + 2h_t^2(m^2_{\widetilde{Q}_{3L}} + m^2_{\widetilde{t}_R} + m_2^2 + A_t^2) - \frac{8}{3}S)
\end{aligned}$$

$$\frac{dm^2_{\widetilde{b}_R}}{dt} = \frac{1}{8\pi^2}(-\frac{16}{3}g_3^2M_3^2 - \frac{4}{9}g_1^2M_1^2 + 2h_b^2(m^2_{\widetilde{Q}_{3L}} + m^2_{\widetilde{b}_R} + m_1^2 + A_b^2) + \frac{4}{3}S) \qquad (7)$$

where the $M_{1,2,3}$ are gaugino masses that we assume to be universal at the GUT scale.

In the CMSSM, with all scalar masses set equal to m_0 at the GUT scale, $S = 0$ initially and remains zero at any scale [14], since $S = 0$ is a fixed point of the RGEs at the one-loop level. However, in the NUHM, with $m_1 \neq m_2$, as seen in Eq. (6) $S \neq 0$ and can cause the low-energy NUHM spectrum to differ significantly from that in the CMSSM. For example, if $S < 0$ the left-handed slepton can be lighter than the right-handed one. Also, m_1^2 and m_2^2 appear in the Yukawa parts of the RGEs for the third generation (Eq. (7)), so NUHM initial conditions may cause their spectrum to differ from that in the CMSSM. In the NUHM case, depending on the parameters, we may find the LSP to be either (i) the lightest neutralino χ, (ii) the lighter stau $\widetilde{\tau}_1$, (iii) the right-handed selectron $\widetilde{e}_R$ and smuon $\widetilde{\mu}_R$ [3], (iv) the left-handed selectron $\widetilde{e}_L$ and smuon $\widetilde{\mu}_L$, (v) the electron and muon sneutrinos $\widetilde{\nu}_{e,\mu}$, (vi) the tau sneutrino $\widetilde{\nu}_\tau$, or (vii) one of the squarks, especially the stop and the sbottom [4]. Note that in the cases that we consider here, the $\widetilde{\nu}_{e,\mu}$ are generally lighter than the $\widetilde{\nu}_\tau$ in the regions in which they are the LSP (that is, when $m_1^2 < 0$, cf. Eq. (7) and Fig. 2 of ref. [2]), unless m_A is very large ($\gtrsim 1000$ GeV).

We assume that R parity is conserved, so that the LSP is stable and is present in the Universe today as a relic from the Big Bang. Searches for anomalous heavy isotopes tell us that the dark matter should be weakly-interacting and neutral, and therefore eliminate all but the neutralino and the sneutrinos as possible LSPs. LEP and direct dark-matter searches together exclude a sneutrino LSP [15], at least if the majority of the CDM is the LSP. Thus we require in our analysis that the lightest neutralino be the LSP.

Nevertheless there are new coannihilation processes to be considered when one or more of these 'wannabe' LSPs is almost degenerate with the lightest neutralino. These include $\chi - \widetilde{\tau}_1$, $\chi - \widetilde{e}_L - \widetilde{\mu}_L$, $\chi - \widetilde{e}_R - \widetilde{\mu}_R$, $\chi - \widetilde{\nu}_{e,\mu}$, $\chi - \chi' - \chi^\pm$ coannihilations and all possible combinations [5]. However, not all of these combinations are important as they are significant only in very small regions for a particular set of parameters. For this reason we do not include, for example, the sneutrino-slepton coannihilation in our calculations.

We include in our subsequent calculations neutralino-slepton $\chi - \widetilde{\ell}$ [17, 7, 18], $\chi - \chi' -$

[3] We neglect left-right (L-R) mixing for the first two generations of sfermions, so the right-handed selectron and smuon are degenerate. Here and elsewhere, by 'right-handed' sfermion we mean the superpartner of right-handed fermion.

[4] A squark LSP is possible only if $|A_0|$ is large, a possibility we do not study in this paper.

[5] Again, because we set $A_0 = 0$ here, squark coannihilations are not important, but see [16] for a calculation of neutralino-stop coannihilation.

$\chi^\pm$ [19], $\chi - \tilde{\nu}_{e,\mu}$, $\chi' - \tilde{\ell}$ and $\chi^\pm - \tilde{\ell}$ coannihilations[6]. The χ' (co)annihilation rates can be derived from the corresponding χ (co)annihilations by appropriate mass and coupling replacements. Details of our calculations are given in the Appendices. Following a summary of the relevant couplings in Appendix A, in Appendix B we update the neutralino-slepton coannihilation calculation of [17] to include L-R mixing. These are not very important at relatively low $\tan\beta$ but are potentially important for large values of $\tan\beta$. These improved coannihilation calculations were in fact already used in [21], but no details were given there. Appendix C provides chargino-slepton coannihilation processes, whilst Appendix D deals with neutralino-sneutrino coannihilation processes.

4 Summary of Constraints and Review of the CMSSM Parameter Space

We impose in our analysis the constraints on the MSSM parameter space that are provided by direct sparticle searches at LEP, including that on the lightest chargino $\chi^\pm$: $m_{\chi^\pm} \gtrsim 103.5$ GeV [22], and that on the selectron $\tilde{e}$: $m_{\tilde{e}} \gtrsim 99$ GeV [23]. Another important constraint is provided by the LEP lower limit on the Higgs mass: $m_H > 114.4$ GeV [24] in the Standard Model[7]. The lightest Higgs boson h in the general MSSM must obey a similar limit, which may in principle be relaxed for larger $\tan\beta$. However, as we discussed in our previous analysis of the NUHM [2], the relaxation in the LEP limit is not relevant in the regions of MSSM parameter space of interest to us. We recall that m_h is sensitive to sparticle masses, particularly $m_{\tilde{t}}$, via loop corrections [25, 26], implying that the LEP Higgs limit constrains the MSSM parameters.

We also impose the constraint imposed by measurements of $b \to s\gamma$ [27], $\mathrm{BR}(B \to X_s\gamma) = (3.11 \pm 0.42 \pm 0.21) \times 10^{-4}$, which agree with the Standard Model calculation $\mathrm{BR}(B \to X_s\gamma)_{\mathrm{SM}} = (3.29 \pm 0.33) \times 10^{-4}$ [28]. We recall that the $b \to s\gamma$ constraint is more important for $\mu < 0$, but it is also relevant for $\mu > 0$, particularly when $\tan\beta$ is large, as we see again in this paper.

We also take into account the latest value of the anomalous magnetic moment of the muon reported [29] by the BNL E821 experiment. The world average of $a_\mu \equiv \frac{1}{2}(g_\mu - 2)$ now deviates by $(33.9 \pm 11.2) \times 10^{-10}$ from the Standard Model calculation of [30] using e^+e^- data, and by $(17 \pm 11) \times 10^{-10}$ from the Standard Model calculation of [30] based on

[6]See [20] for recent work which includes all coannihilation channels.

[7]In view of the theoretical uncertainty in calculating m_h, we apply this bound with three significant digits, i.e., our figures use the constraint $m_h > 114$ GeV.

τ decay data. Other recent analyses of the e^+e^- data yield similar results. On some of the subsequent plots, we display the formal 2-σ range $11.5 \times 10^{-10} < \delta a_\mu < 56.3 \times 10^{-10}$. However, in view of the chequered theoretical history of the Standard Model calculations of a_μ, we do not impose this as an absolute constraint on the supersymmetric parameter space.

As a standard of comparison for our NUHM analysis, we first consider the impacts of the above constraints on the parameter space of the CMSSM, in which the soft supersymmetry-breaking Higgs scalar masses are assumed to be universal at the input scale. In this case, as mentioned in the Introduction, one may use the parameters $(m_{1/2}, m_0, A_0, \tan\beta)$ and the sign of μ. We assume for simplicity that $A_0 = 0$, and plot in Fig. 1 the $(m_{1/2}, m_0)$ planes for certain choices of $\tan\beta$ and the sign of μ. These plots are similar to those published previously [31], but differ in using the latest version of `FeynHiggs` [26] and the latest information on a_μ discussed above.

The shadings and lines in Fig. 1 are as follows. The dark (brick red) shaded regions have a charged LSP, i.e. $\tilde{\tau}_1$, so these regions are excluded. The $b \to s\gamma$ exclusion is presented by the medium (dark green) shaded regions. The light (turquoise) shaded areas are the cosmologically preferred regions with $0.1 \le \Omega_\chi h^2 \le 0.3$. The regions allowed by the E821 measurement of a_μ at the 2-σ level, $11.5 \times 10^{-10} < \delta a_\mu < 56.3 \times 10^{-10}$, are shaded (pink) and bounded by solid black lines. Only panel (a) and (d) have regions allowed by a_μ. The near-vertical (red) dot-dashed lines are the contours $m_h = 114$ GeV, and the near-vertical (black) dashed line in panel (a) is the contour $m_{\chi^\pm} = 103.5$ GeV (though we do not plot this constraint in panels (b,c,d), the position of the chargino contour would be very similar). Regions on the left of these lines are excluded.

We see in panel (a) of Fig. 1 for $\tan\beta = 10$ and $\mu > 0$ that all the experimental constraints are compatible with the CMSSM for $m_{1/2} \sim 300$ to 400 GeV and $m_0 \sim 100$ GeV, with larger values of $m_{1/2}$ also being allowed if one relaxes the a_μ condition. In the case of $\tan\beta = 10$ and $\mu < 0$ shown in panel (b) of Fig. 1, valid only if one discards the a_μ condition, the m_h and $b \to s\gamma$ constraints both require $m_{1/2} \gtrsim 400$ GeV and $m_0 \gtrsim 100$ GeV. In the case $\tan\beta = 35$ and $\mu < 0$ shown in panel (c) of Fig. 1, the $b \to s\gamma$ constraint is much stronger than the m_h constraint, and imposes $m_{1/2} \gtrsim 700$ GeV, with the $\Omega_\chi h^2$ constraint then allowing bands of parameter space emanating from $m_0 \sim 600, 300$ GeV. Finally, in panel (d) for $\tan\beta = 50$ and $\mu > 0$, we see again that the m_h and $b \to s\gamma$ constraints are almost equally important, imposing $m_{1/2} \gtrsim 300$ GeV for $m_0 \sim 400$ GeV. As in the case of panel (a), there is again a region compatible with the a_μ constraint, extending in this case as far as $m_{1/2} \sim 800$ GeV and $m_0 \sim 500$ GeV.

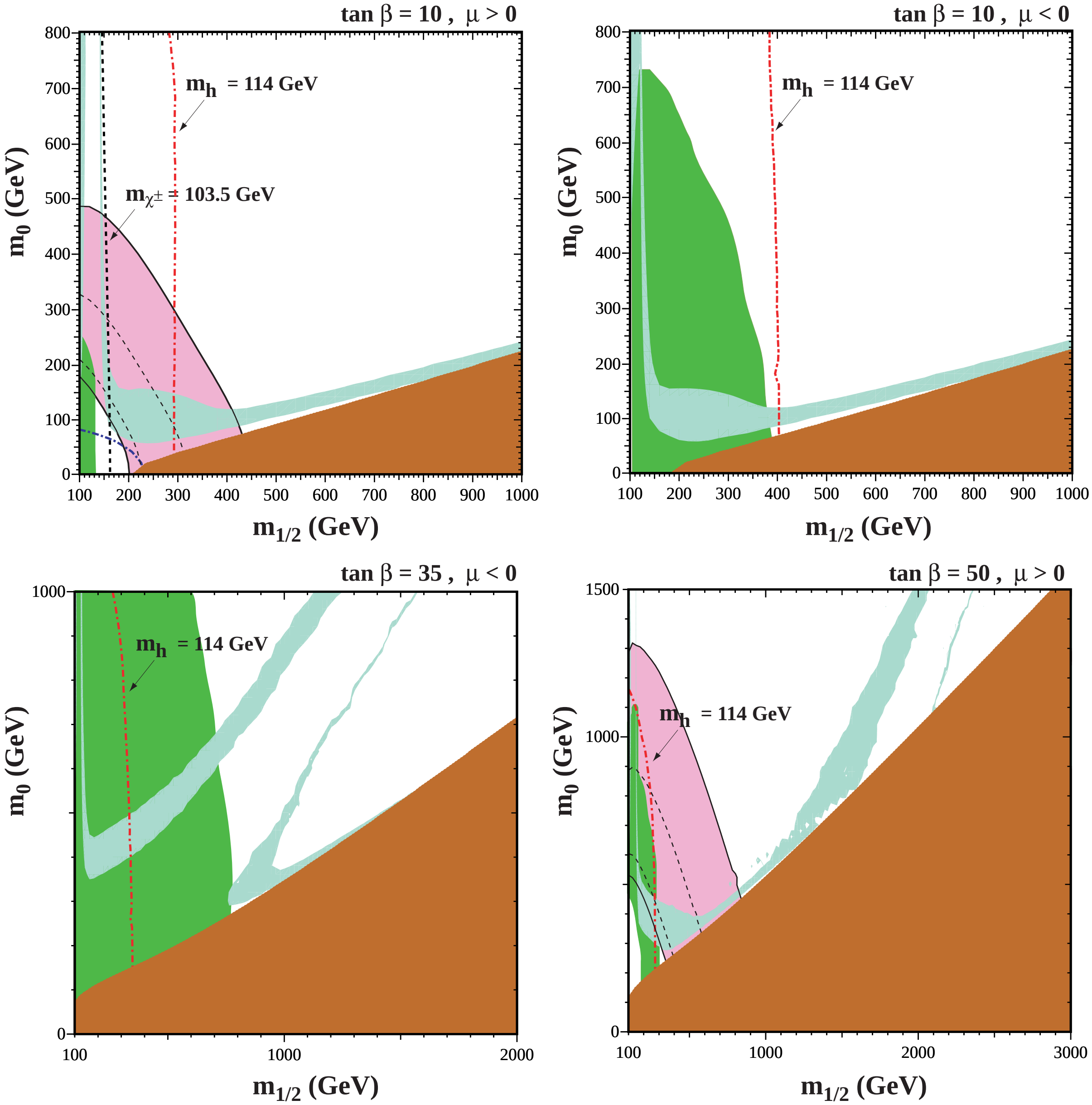

Figure 1: *The CMSSM* $(m_{1/2}, m_0)$ *planes for (a)* $\tan\beta = 10$ *and* $\mu > 0$, *(b)* $\tan\beta = 10$ *and* $\mu < 0$, *(c)* $\tan\beta = 35$ *and* $\mu < 0$ *and (d)* $\tan\beta = 50$ *and* $\mu > 0$, *assuming* $A_0 = 0, m_t = 175$ *GeV and* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ *GeV. The near-vertical (red) dot-dashed lines are the contours* $m_h = 114$ *GeV as calculated using* `FeynHiggs` *[26], and the near-vertical (black) dashed line in panel (a) is the contour* $m_{\chi^\pm} = 103.5$ *GeV. The medium (dark green) shaded regions are excluded by* $b \to s\gamma$, *and the light (turquoise) shaded areas are the cosmologically preferred regions with* $0.1 \leq \Omega_\chi h^2 \leq 0.3$. *In the dark (brick red) shaded regions, the LSP is the charged* $\tilde{\tau}_1$, *so these regions are excluded. In panels (a) and (d), the regions allowed by the E821 measurement of* a_μ *at the 2-*σ *level, as discussed in the text, are shaded (pink) and bounded by solid black lines, with dashed lines indicating the 1-*σ *ranges.*

5 Exploration of the NUHM Parameter Space

Following our discussion of the CMSSM parameter space in the previous Section, we now discuss how that analysis changes in the NUHM. We extend our previous analysis [2] in two ways: (i) fixing $\tan\beta = 10$ and $\mu > 0$, but choosing different values of μ and m_A, rather than assuming the CMSSM values, and (ii) varying $\tan\beta$ for representative fixed values of μ and m_A. We make such selections for three projections of the NUHM, onto the $(m_{1/2}, m_0)$ plane, the (μ, m_A) plane and the (μ, M_2) plane.

5.1 The $(m_{1/2}, m_0)$ Plane

Panel (a) of Fig. 2 shows the $(m_{1/2}, m_0)$ plane for $\tan\beta = 10$ and the particular choice $\mu =$ 400 GeV and $m_A = 400$ GeV, assuming $A_0 = 0, m_t = 175$ GeV and $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ GeV as usual. Again as usual, the light (turquoise) shaded area is the cosmologically preferred region with $0.1 \leq \Omega_\chi h^2 \leq 0.3$. There is a bulk region satisfying this preference at $m_{1/2} \sim 50$ GeV to 350 GeV, $m_0 \sim 50$ GeV to 150 GeV. The dark (red) shaded regions are excluded because a charged sparticle is lighter than the neutralino. As in the CMSSM shown in Fig. 1, the $\tilde{\tau}_1$ is the LSP in the bigger area at larger $m_{1/2}$, and there are light (turquoise) shaded strips close to these forbidden regions where coannihilation suppresses the relic density sufficiently to be cosmologically acceptable. Further away from these regions, the relic density is generally too high. However, for larger $m_{1/2}$ there is another suppression, discussed below, which makes the relic density too low. At small $m_{1/2}$ and m_0 the left handed sleptons, and also the sneutrinos, become lighter than the neutralino. The darker (dark blue) shaded area is where a sneutrino is the LSP. Within these excluded regions there are also areas with tachyonic sparticles.

The near-vertical dark (black) dashed and light (red) dot-dashed lines in Fig. 2 are the LEP exclusion contours $m_{\chi^\pm} > 104$ GeV and $m_h > 114$ GeV respectively. As in the CMSSM case, they exclude low values of $m_{1/2}$, and hence rule out rapid relic annihilation via direct-channel h and Z^0 poles. The solid lines curved around small values of $m_{1/2}$ and m_0 bound the light (pink) shaded region favoured by a_μ and recent analyses of the e^+e^- data.

A striking feature in Fig. 2(a) when $m_{1/2} \sim 500$ GeV is a strip with low $\Omega_\chi h^2$, which has bands with acceptable relic density on either side. The low-$\Omega_\chi h^2$ strip is due to rapid annihilation via the direct-channel A, H poles which occur when $m_\chi = m_A/2 = 200$ GeV, indicated by the near-vertical solid (blue) line. Analogous rapid-annihilation strips have been noticed previously in the CMSSM [32, 21], but at larger $\tan\beta$ as seen in Fig. 1. There, they are diagonal in the $(m_{1/2}, m_0)$ plane, reflecting a CMSSM link between m_0 and m_A

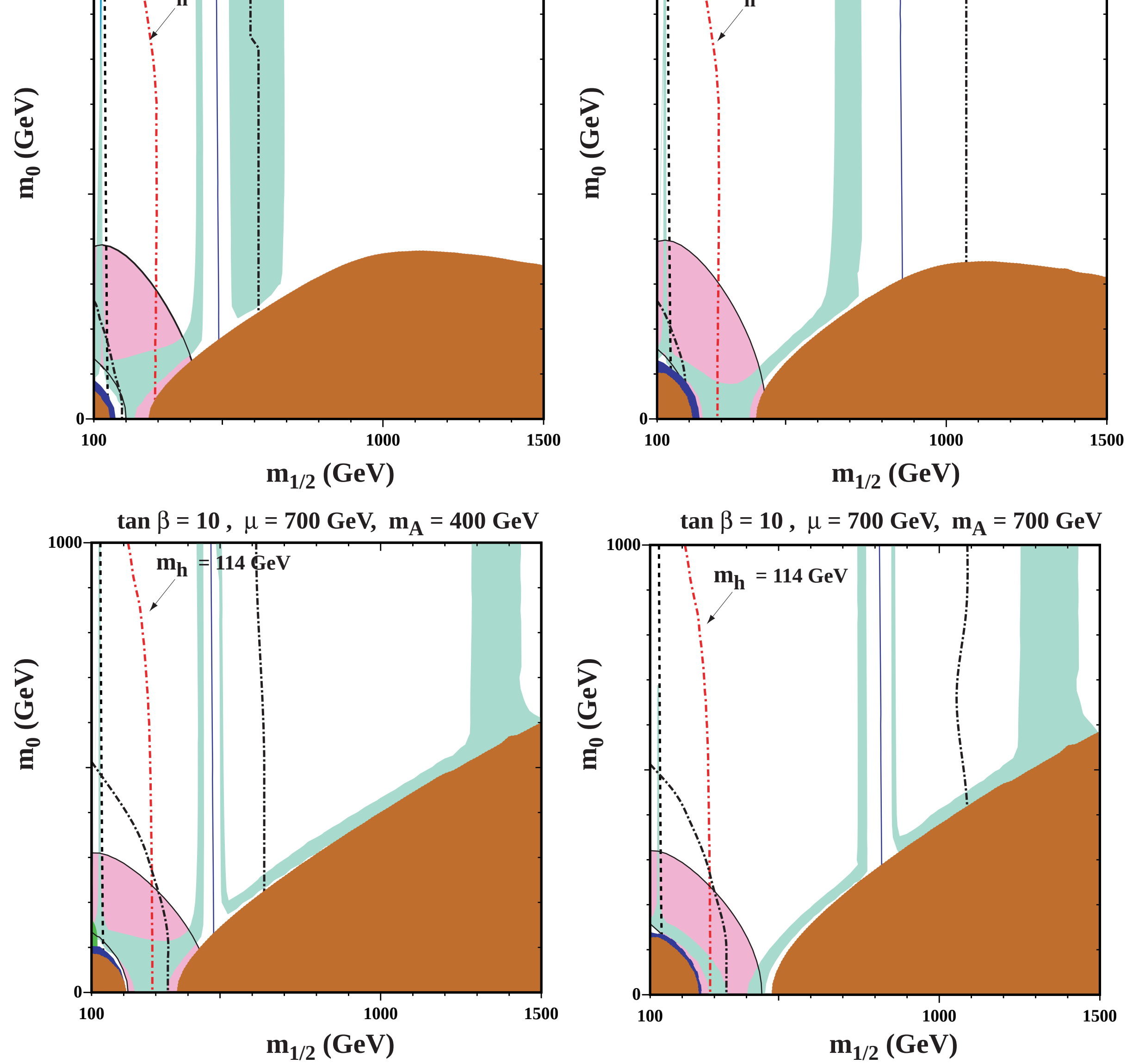

Figure 2: *Projections of the NUHM model on the* $(m_{1/2}, m_0)$ *planes for* $\tan\beta = 10$ *and (a)* $\mu = 400$ *GeV and* $m_A = 400$ *GeV, (b)* $\mu = 400$ *GeV and* $m_A = 700$ *GeV, (c)* $\mu = 700$ *GeV and* $m_A = 400$ *GeV and (d)* $\mu = 700$ *GeV and* $m_A = 700$ *GeV, assuming* $A_0 = 0, m_t = 175$ *GeV and* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ *GeV. The near-vertical (red) dot-dashed lines are the contours* $m_h = 114$ *GeV as calculated using* `FeynHiggs` *[26], and the near-vertical (black) dashed lines are the contours* $m_{\chi^\pm} = 103.5$ *GeV. The dark (black) dot-dashed lines indicate the GUT stability constraint. There are two such lines for each panel and only the areas in between are allowed by this constraint. The light (turquoise) shaded areas are the cosmologically preferred regions with* $0.1 \leq \Omega_\chi h^2 \leq 0.3$*. The dark (brick red) shaded regions is excluded because a charged particle is lighter than the neutralino, and the darker (dark blue) shaded regions is excluded because the LSP is a sneutrino. In panel (c) there is a very small medium (green) shaded region excluded by* $b \to s\gamma$*, at small* $m_{1/2}$*. The regions allowed by the E821 measurement of* a_μ *at the 2-*σ *level, as discussed in the text, are shaded (pink) and bounded by solid black lines.*

that is absent in our implementation of the NUHM. The right-hand band in Fig. 2(a) with acceptable $\Omega_\chi h^2$ is broadened because the neutralino acquires significant Higgsino content, and the relic density is suppressed by the increased W^+W^- production. Hereafter, we will call this the 'transition' band, which in this case is incidentally coincident with the right-hand rapid annihilation band [8]. As $m_{1/2}$ increases, the neutralino becomes almost degenerate with the second lightest neutralino and the lighter chargino, and the $\chi - \chi' - \chi^\pm$ coannihilation processes eventually push $\Omega_\chi h^2 < 0.1$ when $m_{1/2} \gtrsim 700$ GeV. We note that chargino-slepton coannihilation processes become important at the junction between the vertical bands in Fig. 2(a) and the neutralino-slepton coannihilation strip that parallels the $m_\chi = m_{\tilde{\tau}_1}$ boundary of the forbidden (red) charged-LSP region.

There are two dark (black) dash-dotted lines in Fig. 2(a) that indicate where scalar squared masses become negative at the input GUT scale for one of the Higgs multiplets, specifically when either $(m_1(M_X)^2 + \mu(M_X)^2) < 0$ or $(m_2(M_X)^2 + \mu(M_X)^2) < 0$. One of these GUT stability lines is near-vertical at $m_{1/2} \sim 600$ GeV, and the other is a curved line at $m_{1/2} \sim 150$ GeV, $m_0 \sim 200$ GeV. We take the point of view that regions outside either of these lines are excluded, because the preferred electroweak vacuum should be energetically favoured and not bypassed early in the evolution of the Universe, but a different point of view is argued in [13].

Thus, combining all the constraints, the allowed regions are those between the m_h line at $m_{1/2} \sim 300$ GeV and the stability line at $m_{1/2} \sim 600$ GeV, which include two rapid-annihilation bands, some of the transition band and the junction between the bulk and coannihilation regions around $m_{1/2} \sim 350$ GeV, $m_0 \sim 150$ GeV. If one incorporates also the putative a_μ constraint, only the latter region survives. We note however, that if the τ data were used in the $g-2$ analysis, the constraint from a_μ only excludes the lower left corner of the plane and large values of $m_{1/2}$ and m_0 survive at the 2σ level.

Panel (b) of Fig. 2 is for $\mu = 400$ GeV and $m_A = 700$ GeV. We notice immediately that the heavy Higgs pole and the right-hand boundary of the GUT stability region move out to larger $m_{1/2} \sim 850, 1050$ GeV, respectively, as one would expect for larger m_A. At this value of m_A, the transition strip and and the rapid annihilation ('funnel') strip are separate. However the latter would be to the right of the transition strip and hence the $\Omega_\chi h^2$ bands on both sides of the rapid-annihilation strip that was prominent in panel (a) have disappeared,

[8] As the neutralino acquires more Higgsino content, annihilation to W^+W^- production increases, whilst fermion-pair production decreases (except for $t\bar{t}$). Around $m_{1/2} \sim 625$ GeV, there is a threshold for hA, hH production, which decreases $\Omega_\chi h^2$ to ~ 0.08, not far below the preferred range. However, the decrease in fermion production quickly raises $\Omega_\chi h^2$ again for larger $m_{1/2}$. There is a very narrow stripe with $\Omega_\chi h^2 < 0.1$ of width $\delta m_{1/2} \sim 2$ GeV, which is not shown in the figure due to problems of resolution.

due to enhanced chargino-neutralino coannihilation effects. Panel (b) has the interesting feature that there is a region of $m_{1/2} \sim 300$ to 400 GeV where $m_0 = 0$ is allowed. As discussed in [33] this possibility, which would be favoured in some specific no-scale models of supersymmetry breaking, is *disallowed* in the CMSSM. The small-m_0 region is even favoured in this variant of the NUHM by the putative a_μ constraint.

Panel (c) of Fig. 2 is for $\mu = 700$ GeV and $m_A = 400$ GeV. In this case, we see that the rapid-annihilation strip is back to $m_{1/2} \sim 500$ GeV, reflecting the smaller value of m_A, whereas the transition band has separated off to large $m_{1/2}$, reflecting the larger value of μ. However, this band is excluded in this case by the GUT stability requirement. GUT stability also excludes the possibility that $m_0 = 0$. In this case, the putative a_μ constraint would restrict one to around $m_{1/2} \sim 400$ GeV and $m_0 \sim 100$ GeV.

Finally, panel (d) of Fig. 2 is for $\mu = 700$ GeV and $m_A = 700$ GeV. In this case, the rapid-annihilation strip has again moved to larger $m_{1/2}$, related to the larger value of m_A, and the transition band at large $m_{1/2}$ is again excluded by the GUT stability requirement. The bulk region has disappeared in this panel, reflecting the fact that the values of μ and m_A here has strayed away from their values in the bulk region for the CMSSM. GUT stability no longer excludes $m_0 = 0$, and this possibility would be selected by the putative a_μ constraint.

We now turn, in Fig. 3, to the impact of varying $\tan\beta$, keeping $\mu = 400$ GeV and $m_A = 700$ GeV. For convenience, panel (a) reproduces Fig. 2(b) with $\tan\beta = 10$. In general as $\tan\beta$ is increased, two major trends are visible. One is for the region excluded by the requirement that the LSP be neutral to spread up to larger values of m_0, and the other is for the a_μ constraint to move out to larger values of $m_{1/2}$ and m_0.

Specifically, we see in panel (b) of Fig. 3 for $\tan\beta = 20$ that, whereas the heavy Higgs pole at $m_{1/2} \sim 850$ GeV essentially does not move, the $\tilde{\tau}_1$ LSP and coannihilation strip lying above the excluded charged-LSP region rise to larger m_0. This has the effect of excluding the $m_0 = 0$ option that was present in panel (a). At low $m_{1/2}$, the m_h constraint is stronger than the GUT stability and other constraints. The a_μ constraint would allow a larger range of $m_{1/2}$ than in panel (a), extending up to ~ 550 GeV.

Continuing in panel (c) of Fig. 3 to $\tan\beta = 35$, we see that the minimum value of m_0 has now risen to ~ 200 GeV. We also see that the $b \to s\gamma$ constraint is now important, enforcing $m_{1/2} > 300$ GeV in the region preferred by the relic density. Because of this and the m_h constraint, the GUT stability constraint is now irrelevant at low $m_{1/2}$, whereas at high $m_{1/2}$ it has vanished off the screen, and is in any case also irrelevant because of chargino-neutralino coannihilation. The a_μ constraint would now allow part of the cosmological band on the left side of the rapid-annihilation strip. These trends are strengthened in panel (d) of Fig. 3 for

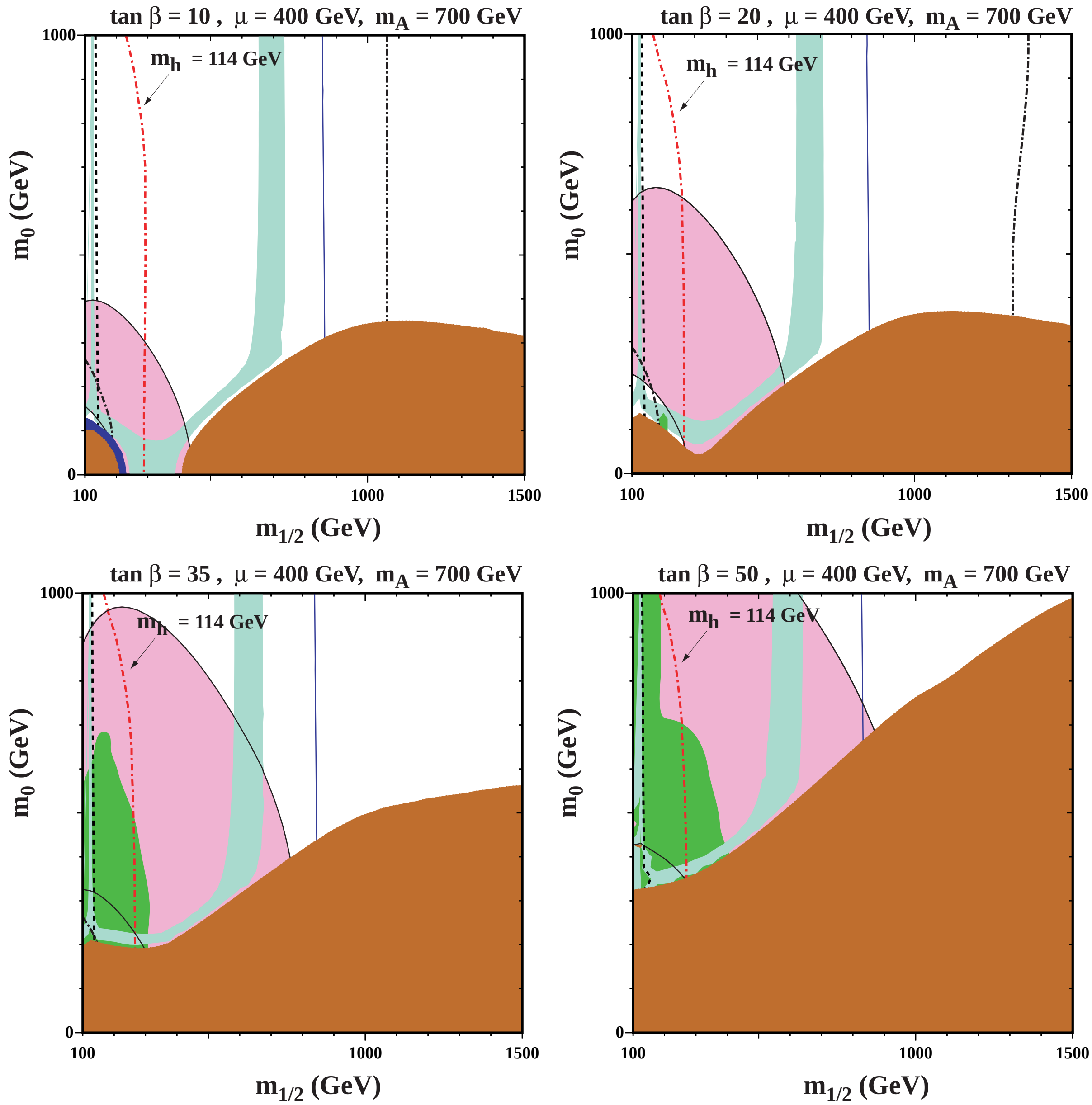

Figure 3: *The NUHM* $(m_{1/2}, m_0)$ *planes for (a)* $\tan\beta = 10$, *(b)* $\tan\beta = 20$, *(c)* $\tan\beta = 35$ *and (d)* $\tan\beta = 50$, *for* $\mu = 400$ *GeV*, $m_A = 700$ *GeV, assuming* $A_0 = 0, m_t = 175$ *GeV and* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ *GeV. The shadings and line styles are the same as in Fig. 2.*

$\tan\beta = 50$, where we see that $m_0, m_{1/2} \gtrsim 400$ GeV because of the $b \to s\gamma$ constraint, and a_μ would allow $m_0 \lesssim 1100$ GeV along the transition band.

5.2 The (μ, m_A) Plane

We now analyze the range of possibilities in the (μ, m_A) plane for various fixed choices of $m_{1/2}$ and m_0, first choosing $\tan\beta = 10$. Panel (a) of Fig. 4 displays the (μ, m_A) plane for $m_{1/2} = 300$ GeV, $m_0 = 100$ GeV. As we saw in Fig. 1(a), there is a CMSSM point with $\mu > 0$ that is compatible with all the constraints for these values of $m_{1/2}$ and m_0. The corresponding CMSSM point for $\mu < 0$ in Fig. 1(b) is, however, incompatible with the m_h and $b \to s\gamma$ constraints, as well as the putative a_μ constraint. The CMSSM equivalent points are shown as crosses in panel (a) of Fig. 4, and have $(\mu, m_A) \simeq (\pm 390, 450)$ GeV.

As usual, there are dark (red) regions where there is one or more charged sparticle lighter than the neutralino χ so that χ is no longer the LSP. First, there are 'shark's teeth' at $|\mu| \sim 300$ GeV, $m_A \lesssim 300$ GeV in panel (a) of Fig. 4 where the $\tilde{\tau}_1$ is the LSP. At small $|\mu|$, particularly at small m_A when the mass difference $m_2^2 - m_1^2$ is small, the $\widetilde{\tau}_R$ mass is driven small, making the $\widetilde{\tau}_1$ the LSP [9]. At even smaller $|\mu|$, however, the lightest neutralino gets lighter again, since $m_\chi \simeq \mu$ when $\mu < M_1 \simeq 0.4\, m_{1/2}$ [10]. In addition, there are extended dark (red) shaded regions at large $|\mu|$ where left-handed sleptons become lighter than the neutralino. However, the electron sneutrino $\widetilde{\nu}_e$ and the muon sneutrino $\widetilde{\nu}_\mu$ (which are degenerate within the approximations used here) have become joint LSPs at a slightly smaller $|\mu|$. Since the possibility of sneutrino dark matter has been essentially excluded by a combination of 'ν counting' at LEP, which excludes $m_{\tilde{\nu}} < 43$ GeV [34], and searches for cold dark matter, which exclude heavier $\tilde{\nu}$ weighing $\lesssim 1$ TeV [15], we still demand that the LSP be a neutralino χ. The darker (dark blue) shaded regions are where the sneutrinos are the LSPs, and therefore excluded.

To explain the possible spectra better, we plot in Fig. 5 some sparticle masses as functions of μ, with other parameters fixed. In particular, we plot the neutralino mass m_χ (dark solid curve), the chargino mass $m_{\chi^\pm}$ (dark dashed curve), the lighter stau mass $m_{\widetilde{\tau}_1}$ (light sold curve), the right-handed selectron mass $m_{\widetilde{e}_R}$ (light dashed curve), and the sneutrino masses $m_{\widetilde{\nu}_\tau}$ and $m_{\widetilde{\nu}_e}$ (thin solid and dashed curves respectively). We have omitted the curve for $m_{\widetilde{e}_L}$, which is very similar to the sneutrino curves, for reasons of clarity. In both panels of

[9] Note that the $\widetilde{e}_R$ and $\widetilde{\mu}_R$ masses are also driven to small values, along with the $\widetilde{\tau}_R$, and in fact there are small regions where the degenerate $\widetilde{e}_R$ and $\widetilde{\mu}_R$ become the LSP.

[10] We note that regions in Figs. 4(a,b) and 7(b) at small $|\mu|$ and m_A are excluded by the LEP slectron search [23]. However, these regions are all excluded by other limits, such as GUT stability, the LEP chargino limit [22] and $b \to s\gamma$, and so are not shown separately.

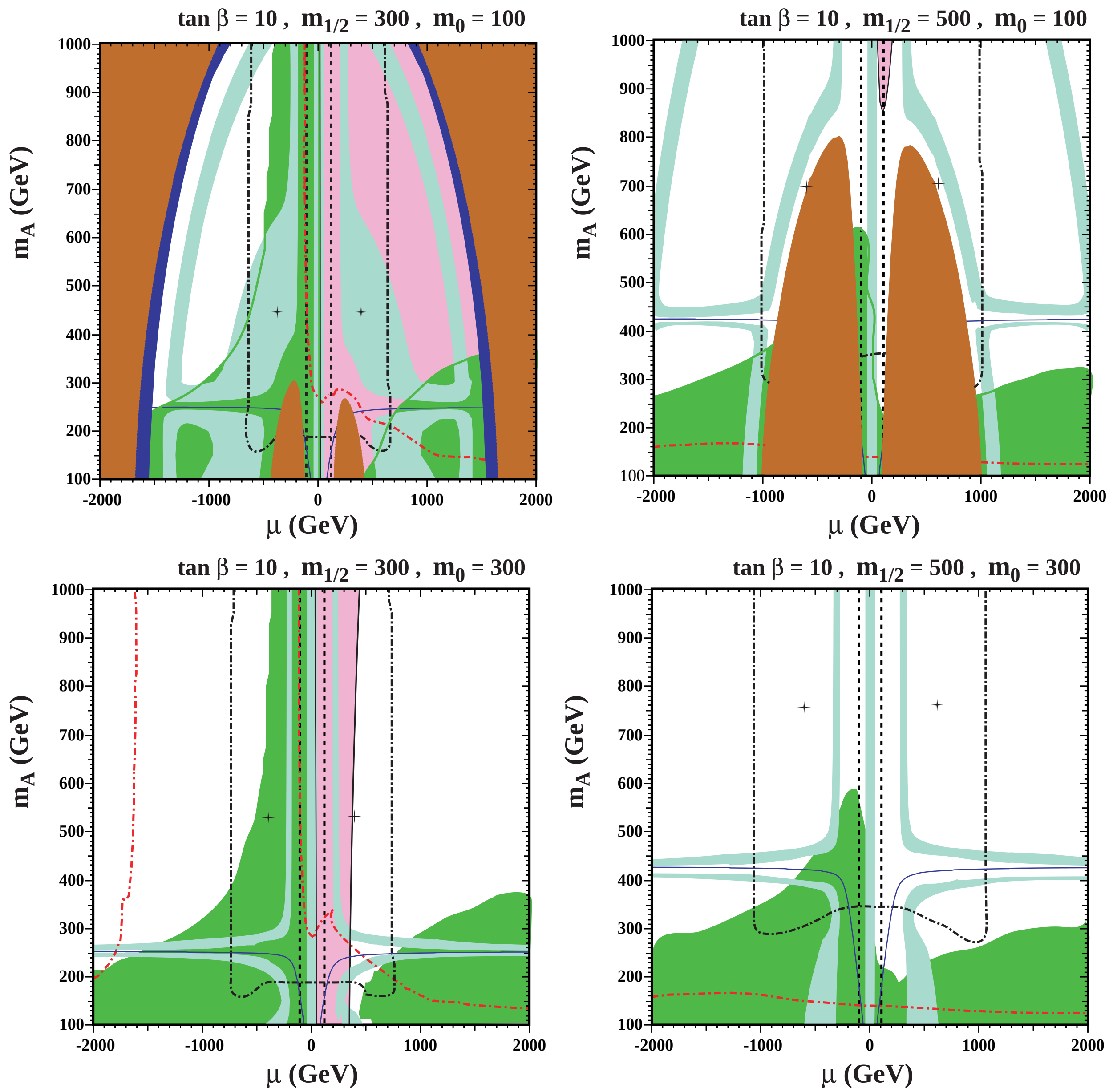

Figure 4: *The NUHM* (μ, m_A) *planes for* $\tan\beta = 10$, *(a)* $m_0 = 100$ *GeV and* $m_{1/2} = 300$ *GeV, (b)* $m_0 = 100$ *GeV and* $m_{1/2} = 500$ *GeV, (c)* $m_0 = 300$ *GeV and* $m_{1/2} = 300$ *GeV and (d)* $m_0 = 300$ *GeV and* $m_{1/2} = 500$ *GeV, assuming* $A_0 = 0, m_t = 175$ *GeV and* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ *GeV. The shadings and line styles are the same as in Fig. 2.*

Fig. 5, we have fixed most parameters as in Fig. 4(a). Fig. 5(a) is for $m_A = 200$ GeV, and we see that the lighter stau and the sneutrinos are lighter than the neutralino for very large values of $|\mu|$. As $|\mu|$ is decreased, there is a small window in which $\widetilde{\nu}_e$ is the LSP. The curious behaviour of the stau mass curve at large $|\mu|$ is due to a large and negative value for S which drives $m_{\widetilde{\tau}_L}$ down while driving $m_{\widetilde{\tau}_R}$ up (cf. Eq. (7)). For large values of $|\mu|$, the lighter stau is mostly left-handed, whilst it is mostly right-handed for smaller $|\mu| \lesssim 1000$ GeV. When $|\mu| \lesssim 400$ GeV, both the stau and the right-handed selectron are lighter than the neutralino until $|\mu|$ is very small and the neutralino becomes Higgsino-like, with its mass scaling as μ. The 'shark's teeth' in Fig. 4(a) correspond to the range around $|\mu| \sim 200$ GeV where the lighter stau is the LSP, with the right-handed selectron slightly heavier.

In Fig. 5(b), we have fixed $m_A = 600$ GeV. Here, one sees a similar pattern of masses at large $|\mu|$. However, at small $|\mu|$ the lighter stau and right-handed selectron are much heavier, so the neutralino remains the LSP. This reflects the fact that $m_A = 600$ GeV is above the tips of the 'shark's teeth' in Fig. 4(a). It is clear that the sizes of these 'shark's teeth' must depend sensitively on the NUHM parameters, as seen in the other panels of Fig. 4 and subsequent figures.

Returning to panel (a) of Fig. 4, we see strips adjacent to the $\widetilde{\nu}_{e,\mu}$ LSP regions, where neutralino-sneutrino coannihilation is important in suppressing the relic density to an acceptable level. Appendix D presents details of our calculations of $\tilde{\nu} - \chi$ coannihilation channels. We consider here only $\widetilde{\nu}_e$ and $\widetilde{\nu}_\mu$, for which the effects of m_e, m_μ and L-R mixing in the slepton mass matrix are negligible, leading to some simplifications compared to the $\widetilde{\nu}_\tau$ (whose inclusion would have little effect on our figures). The inclusion of neutralino-sneutrino coannihilation in panel (a) squeezes inwards slightly the coannihilation strips. As we see later, the effect of neutralino-sneutrino coannihilation is more noticeable at larger $\tan\beta$. Again as usual, the light (turquoise) shaded regions are those for which $0.1 < \Omega_\chi h^2 < 0.3$.

The thick cosmological region at smaller μ in panel (a) corresponds to the 'bulk' region familiar from CMSSM studies. Note that this region is squeezed outward near the 'shark's teeth' by the neutralino-slepton coannihilation. Extending upward in m_A from this bulk region, there is another light (turquoise) shaded band at smaller $|\mu|$. This is the transition band, where the neutralino gets more Higgsino-content and the annihilation to W^+W^- becomes important, yielding a relic density in the allowed range, as happens in the focus-point region [35] in the CMSSM. For smaller $|\mu|$, the relic density becomes too small due to $\chi - \chi' - \chi^+$ coannihilations, and the chargino-slepton annihilations described in Appendix C must be taken into account where this strip meets the neutralino-slepton coannihilation strip discussed earlier. The LEP limit on the chargino mass excludes a strip at even smaller

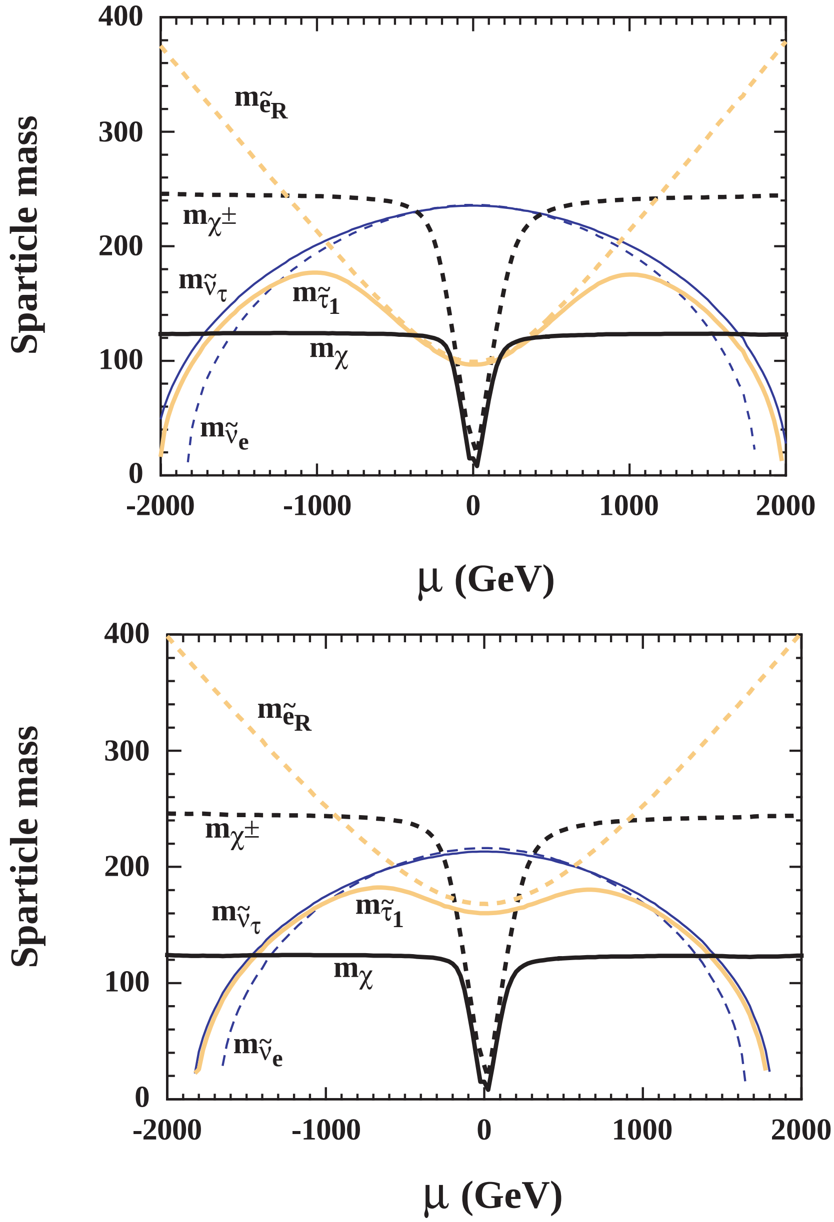

Figure 5: *The masses* m_χ *(dark solid),* $m_{\chi^\pm}$ *(dark dashed),* $m_{\tilde{\tau}_1}$ *(light solid),* $m_{\tilde{e}_R}$ *(light dashed),* $m_{\tilde{\nu}_\tau}$ *(thin solid) and* $m_{\tilde{\nu}_e}$ *(thin dashed) as functions of* μ *for* $\tan\beta = 10$, $m_{1/2} = 300$ *GeV,* $m_0 = 100$ *GeV for (a)* $m_A = 200$ *GeV and (b)* $m_A = 600$ *GeV, assuming* $A_0 = 0, m_t = 175$ *GeV and* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ *GeV.*

$|\mu|$ where the relic density would again have come into the cosmologically preferred region. There are also horizontal bands of acceptable relic density when $m_A \sim 250$ GeV, that are separated by strips of low relic density, due to rapid annihilation through the A (indicated by solid (blue) lines) and H poles.

Underlying the cosmological regions are dark (green) shaded regions excluded by $b \to s\gamma$, which are more important for $\mu < 0$, as expected from previous analyses. Also important is the m_h constraint, calculated using the `FeynHiggs` programme [26], which excludes the option $\mu < 0$ in panel (a) of Fig. 4. The putative a_μ constraint would also exclude the $\mu < 0$ half-plane, while allowing all of the $\mu > 0$ parameter space for this particular choice of $\tan\beta$, $m_{1/2}$ and m_0.

The darker (black) dot-dashed lines in Fig. 4(a) indicates where one or the other of the Higgs mass-squared becomes negative at the input GUT scale. We see that these constraints exclude much of the cosmological region still permitted, apart from the 'bulk' region and part of the 'transition' region for small $\mu > 0$.

We conclude from Fig. 4(a) that moderate positive values of $\mu < 700$ GeV are favoured and m_A is unlikely to be very small, though there is a very small allowed region below the rapid A, H annihilation strip where $m_A \sim 230$ GeV. The cosmological relic density lies within the range favoured by astrophysics and cosmology in a large fraction of the remaining NUHM parameter space for $\mu > 0$, generalizing the CMSSM point that is indicated by the cross.

The notations used for the constraints illustrated in the other panels of Fig. 4 are the same, but the constraints interplay in different ways. In panel (b), we have chosen a larger value of $m_{1/2}$. In this case, the dark (red) shaded 'shark's teeth' at moderate $|\mu|$ and small m_A where the LSP is charged have expanded greatly, and are flanked by pale (turquoise) shaded regions where neutralino-slepton coannihilation produces an acceptable relic density. On the other hand, the dark (red) shaded and the darker (dark blue) shaded regions at large $|\mu|$ have moved out of the panel displayed, and one sees only parts of the adjacent coannihilation strips. The relatively large value of $m_{1/2}$ keeps the rate of $b \to s\gamma$ under control unless m_A is small and $\mu < 0$. The chargino constraint is similar to that in panel (a), whereas the m_h constraint is irrelevant due to the large value of $m_{1/2}$. The putative a_μ constraint would allow only a very small region in this panel, but without any cosmological preferred region. Finally, we observe that the GUT stability constraint now allows larger values of $|\mu| \lesssim 1000$ GeV and $m_A \gtrsim 300$ GeV.

We now turn to panel (c) of Fig. 4, which is for $\tan\beta = 10, m_{1/2} = 300$ GeV and $m_0 = 300$ GeV. In this case, the 'shark's teeth' have disappeared, as have the neutralino-slepton strips at large $|\mu|$ (due to the large value of m_0). Negative values of μ are excluded by m_h and partially by $b \to s\gamma$, as well as by the putative a_μ constraint, which would permit a strip with $\mu > 0$. GUT stability enforces $\mu \lesssim 800$ GeV, and also provides a lower limit on m_A that is irrelevant because of the other constraints. In contrast, in panel (d) of Fig. 4 for $m_{1/2} = 500$ GeV and $m_0 = 300$ GeV, we see a similar 'cruciform' pattern for the regions allowed by cosmology, but $b \to s\gamma$ has only rather limited impact for $\mu < 0$ and m_h is irrelevant: there is no region consistent with the putative a_μ constraint. In this case, m_A could be as small as the ~ 200 GeV allowed by the GUT stability constraint, and $|\mu|$ could be as large as ~ 1100 GeV.

We now discuss the variation with $\tan\beta$ shown in Fig. 6, keeping fixed $m_{1/2} = 300$ GeV and $m_0 = 100$ GeV, and starting (for convenience) in panel (a) with the case $\tan\beta = 10$ shown previously in panel (a) of Fig. 4. Increasing $\tan\beta$ to 20 in panel (b), we note that the 'shark's teeth' are somewhat expanded, whereas the regions at large $|\mu|$ and/or m_A where the lightest neutralino is no longer the LSP change in shape. The allowed cosmological region is bounded at larger m_A by one where the LSP is the $\tilde{\tau}_1$, and at larger $|\mu|$ by one where the LSP is a sneutrino: these different regions had shapes similar to each other in panel (a). As usual, neutralino-slepton coannihilations - as calculated in Appendix B - and neutralino-sneutrino coannihilations - as calculated in Appendix D - suppress the relic density close to these boundaries, and the relic density is within the preferred range over most of the region outside the 'shark's teeth' and the chargino exclusion for $m_A > 2m_\chi$. As in previous cases, $b \to s\gamma$ and m_h together exclude the option $\mu < 0$, which would also be disfavoured by a_μ. The $\mu > 0$ region surviving all the constraints when $\tan\beta = 20$ lies between the GUT stability constraint at $\mu \sim 600$ GeV and $b \to s\gamma$ constraint around $\mu \sim 350$ GeV, and has 130 GeV$\lesssim m_A \lesssim$900 GeV.

When $\tan\beta = 35$, in panel (c) of Fig. 6, we find no consistent electroweak vacuum for a large (polka-dotted) region with $\mu < 0$, and the condition that the LSP not be the $\tilde{\tau}_1$ excludes a broad swathe with small $\mu > 0$. The condition that the LSP not be a sneutrino provides an upper bound $\mu \lesssim 1500$ GeV. The m_h constraint imposes $m_A \gtrsim 120$ GeV and the $b \to s\gamma$ constraint excludes a region at large μ and small m_A. The GUT stability constraint allows only a narrow sliver of $\mu \sim 700$ GeV for 120 GeV $\lesssim m_A \lesssim 200$ GeV, where the lower bound comes from the LEP Higgs search. This reach is not compatible with a_μ. Here, the predicted discrepancy with the Standard Model is in excess of that allowed at the 2σ level by the $g-2$ experiment.

In the case of $\tan\beta = 50$, shown in panel (d) of Fig. 6, the problem with the non-existence of a consistent electroweak vacuum extends to most of the $\mu < 0$ half-plane, as shown by

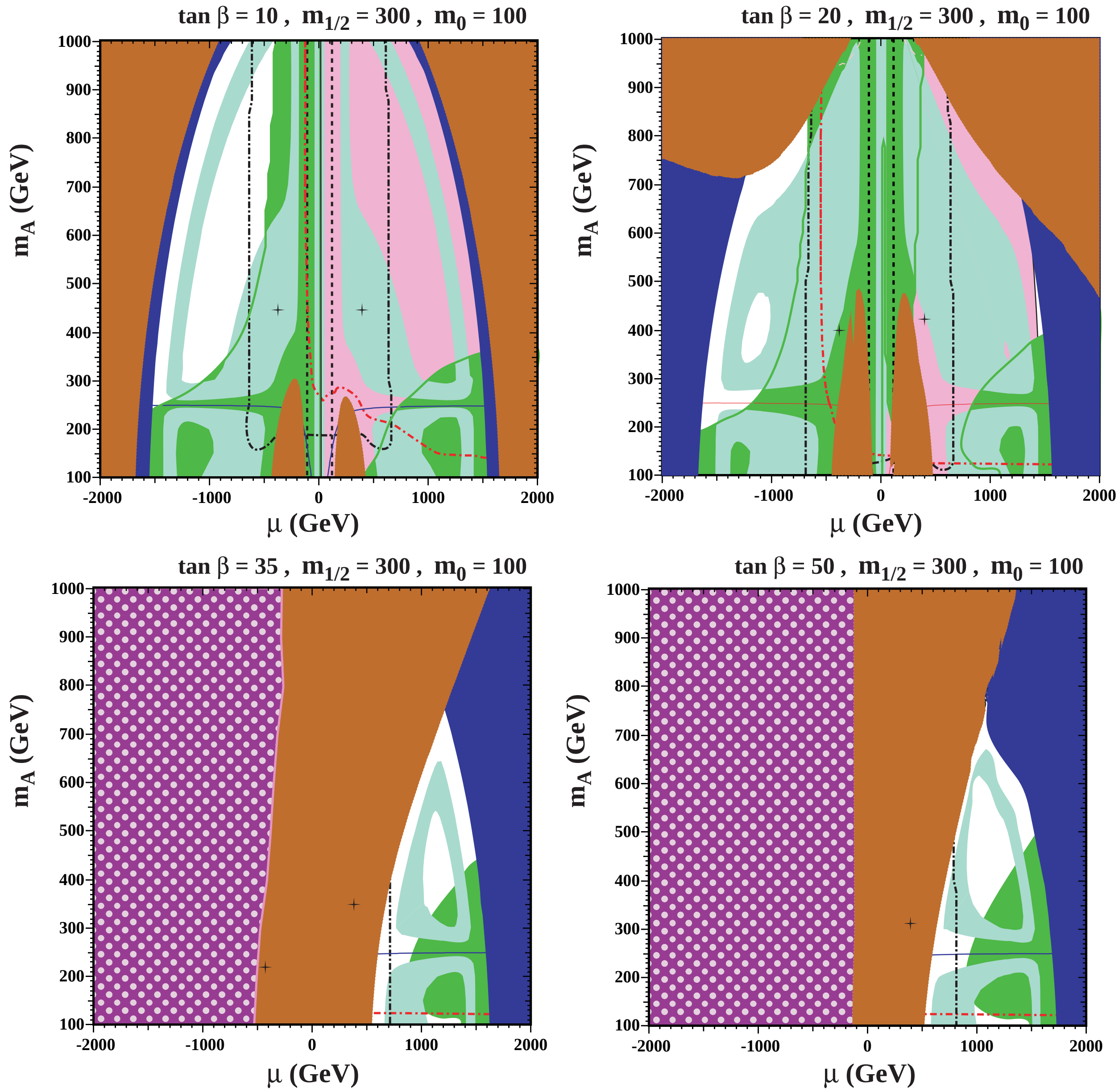

Figure 6: *The NUHM* (μ, m_A) *planes for* $m_0 = 100$ *GeV and* $m_{1/2} = 300$ *GeV, for (a)* $\tan\beta = 10$*, (b)* $\tan\beta = 20$*, (c)* $\tan\beta = 35$ *and (d)* $\tan\beta = 50$*, assuming* $A_0 = 0, m_t = 175$ *GeV and* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ *GeV. The shadings and line styles are the same as in Fig. 2, and there is no consistent electroweak vacuum in the polka-dotted region.*

the polka dots. The regions excluded because the LSP is either the $\tilde{\tau}_1$ or a sneutrino hem in a small region with $\mu > 0$. The region that survives all the experimental and cosmological constraints is similar to that for $\tan\beta = 35$, but extends to somewhat larger m_A and broader μ. Once again, these regions are not compatible with the $g-2$ result.

Another series of plots for different values of $\tan\beta$ is shown in Fig. 7, this time for the fixed values $m_{1/2} = 500$ GeV and $m_0 = 300$ GeV. All of these have the 'cruciform shape' of cosmological region familiar from panels (c, d) of Fig. 4. The m_h constraint is irrelevant for this larger value of $m_{1/2}$, but much of the parameter space for $\mu < 0$ is excluded by $b \to s\gamma$, particularly for larger $\tan\beta$. Likewise, a_μ consistently favours $\mu > 0$. One of the GUT stability constraints requires $|\mu| \lesssim 1200$ GeV, the exact value increasing slightly with $\tan\beta$. The other GUT stability constraint requires $m_A \gtrsim 300$ GeV for $\tan\beta = 10$, but weakens for larger $\tan\beta$, becoming irrelevant when $\tan\beta \geq 35$. As in Fig. 6, the absence of a consistent electroweak vacuum also becomes a problem for $\mu < 0$ when $\tan\beta \geq 35$. Finally, we see in panel (d) of Fig. 7 that small values of positive μ are disallowed because the LSP becomes the $\tilde{\tau}_1$. As in Fig. 6, values of m_A as low as allowed by the Higgs search are consistent with all the constraints for $\tan\beta \geq 35$, as long as μ has a suitable positive value. Unlike the case shown, in Fig. 6, here the entire $\mu > 0$ half-plane is within the 2σ range for a_μ when $\tan\beta = 35$ and 50.

5.3 The (μ, M_2) Plane

Panel (a) of Fig. 8 displays the (μ, M_2) plane for the choices $\tan\beta = 10$, $m_0 = 100$ GeV and $m_A = 500$ GeV. We restrict our attention to the region allowed by the GUT stability constraints, which is roughly triangular, extending from the origin to vertices at $(\mu, M_2) = (\pm 1500, 600)$ GeV. Within this region, there are two large triangular regions with $M_2 \gtrsim 300$ GeV and either sign of μ that are excluded because the LSP is not the lightest neutralino. Also, a band around the $\mu = 0$ axis is excluded by the LEP chargino constraint, regions with $M_2 \lesssim 250(300)$ GeV for $\mu > (<) 0$ are excluded by the LEP Higgs constraint, and most of the surviving $\mu < 0$ region is eroded by the $b \to s\gamma$ constraint. In the lower corners (for both signs of μ) the LSP is a sneutrino and we see the effects of $\chi - \widetilde{\nu}$ coannihilation running alongside these regions. However, these regions are in conflict with the GUT stability constraint.

Much of the remaining area of the plane is consistent with the cosmological relic density constraint, mainly along strips where neutralino-slepton is important. These terminate around $M_2 \sim 450$ GeV, where rapid annihilations through the direct-channel A, H resonances cut down the relic density. The putative a_μ constraint would favour $\mu > 0$ and

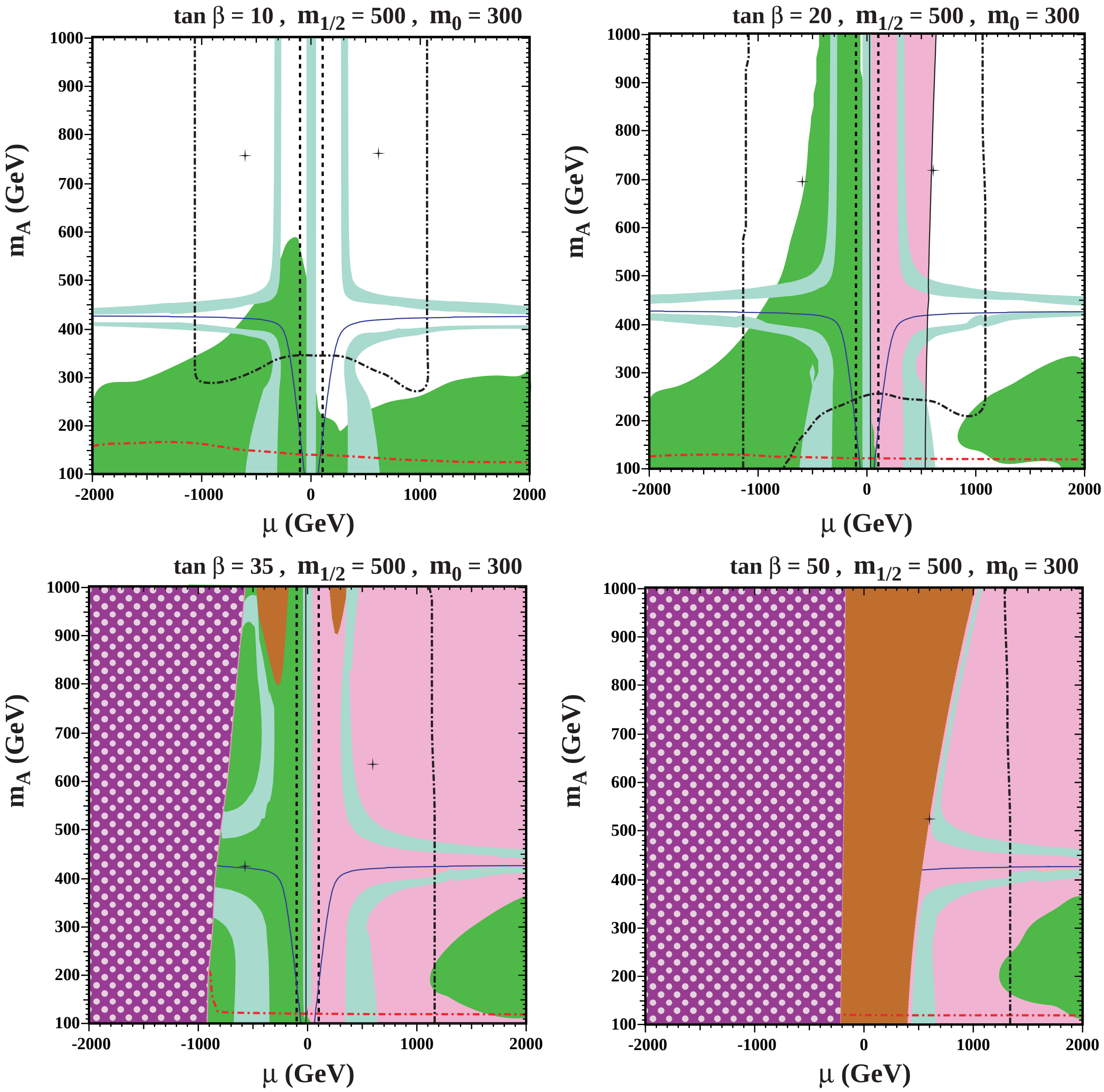

Figure 7: *The NUHM* (μ, m_A) *planes for* $m_0 = 300$ *GeV and* $m_{1/2} = 500$ *GeV, for (a)* $\tan\beta = 10$*, (b)* $\tan\beta = 20$*, (c)* $\tan\beta = 35$ *and (d)* $\tan\beta = 50$*, assuming* $A_0 = 0, m_t = 175$ *GeV and* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ *GeV. The remaining shadings and line styles of Fig. 2 are used here, and there is no consistent electroweak vacuum in the polka-dotted region.*

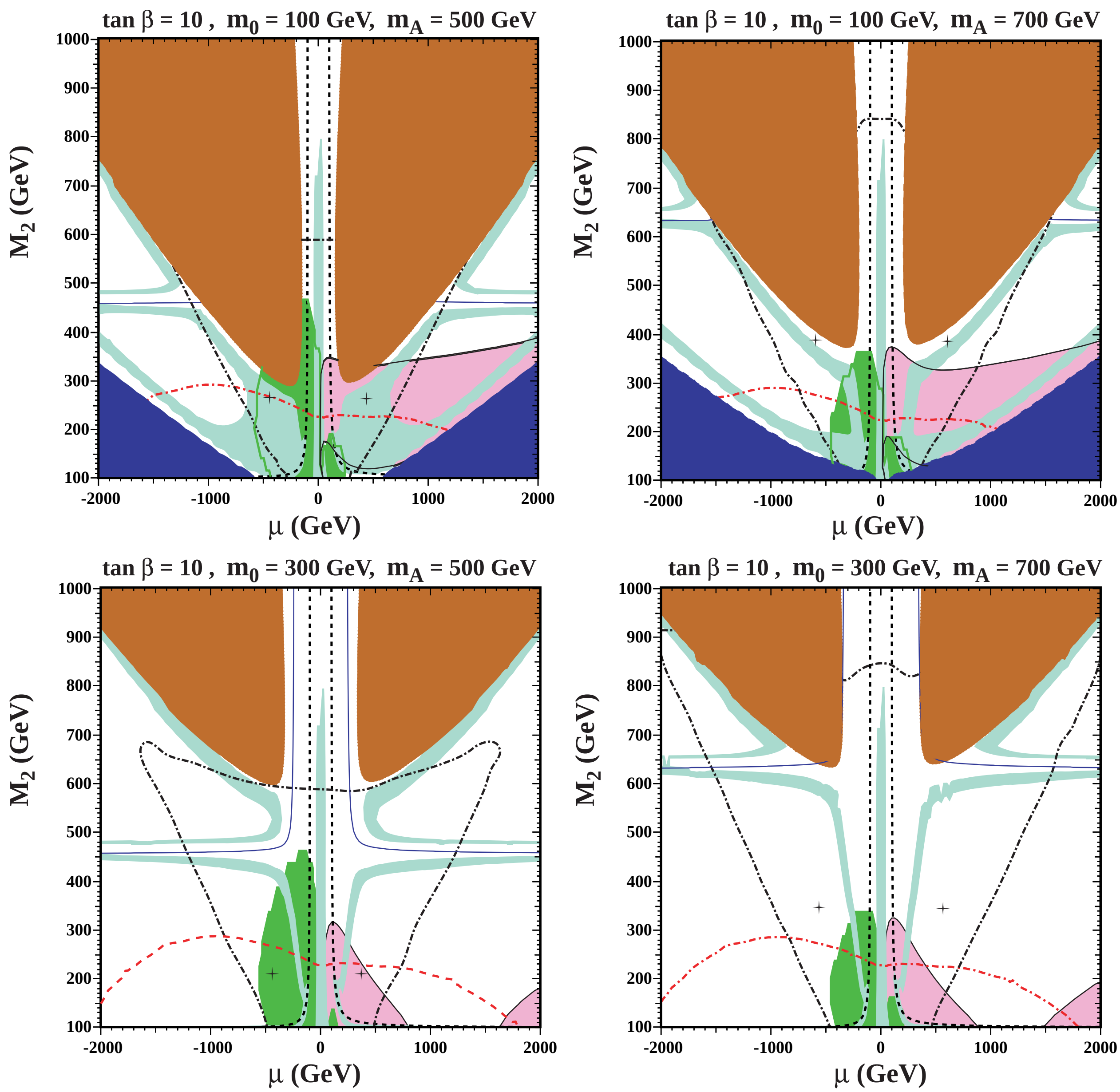

Figure 8: *The NUHM* (μ, M_2) *planes for* $\tan\beta = 10$, *(a)* $m_0 = 100$ *GeV and* $m_A = 500$ *GeV, (b)* $m_0 = 100$ *GeV and* $m_A = 700$ *GeV, (c)* $m_0 = 300$ *GeV and* $m_A = 500$ *GeV and (d)* $m_0 = 300$ *GeV and* $m_A = 700$ *GeV, assuming* $A_0 = 0, m_t = 175$ *GeV and* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ *GeV. The shadings and line styles are the same as in Fig. 7.*

$M_2 \lesssim 350$ GeV.

The above constraints interplay analogously in panel (b) of Fig. 8, for $m_0 = 100$ GeV and $m_A = 700$ GeV. We note, however, that the GUT stability limit has risen with m_A to above 800 GeV, and that the tips of the non-neutralino LSP triangles have also moved up slightly. The neutralino-slepton coannihilation strip now extends up to $M_2 \sim 640$ GeV, where it is cut off by rapid A, H annihilations. We also see the 'transition' band at lower M_2. In fact, this region with $\mu > 0$ is the one favoured by all constraints including a_μ.

In panel (c) of Fig. 8, for $m_0 = 300$ GeV and $m_A = 500$ GeV, the triangular GUT stability region is very similar to that in panel (a), whereas the non-neutralino LSP triangles have moved to significantly higher M_2, reflecting the larger value of m_0. The regions where the cosmological relic density falls within the preferred range are now narrow strips in the neutralino-slepton coannihilation regions, on either side of the rapid A, H annihilation strips, and the 'transition' bands. This tendency towards 'skinnier' cosmological regions is also apparent in panel (d), where $m_0 = 300$ GeV and $m_A = 700$ GeV are assumed. In both these panels, the $b \to s\gamma$ constraints disfavours $\mu < 0$ and small m_A, whilst the putative a_μ constraint would favour $\mu > 0$ and small m_A.

We now explore the variation of these results with $\tan\beta$, as displayed in Fig. 9 for the choices $m_0 = 100$ Gev and $m_A = 500$ GeV. Panel (a) reproduces for convenience the case $\tan\beta = 10$ that was shown also in panel (a) of Fig. 8. When $\tan\beta = 20$, as seen in panel (b) of Fig. 9, we first notice that the GUT stability region now extends to larger M_2. Next, we observe that the triangular non-neutralino LSP regions have extended down to lower M_2. As one would expect on general grounds and from earlier plots, the m_h constraint at low M_2 is weaker than in panel (a), whereas the $b \to s\gamma$ constraint is stronger, ruling out much of the otherwise allowed region with $\mu < 0$. The allowed region with $\mu > 0$ is generally compatible with the putative a_μ constraint.

Panel (c) for $\tan\beta = 35$ again shows the feature that no consistent electroweak vacuum is found over much of the half-plane with $\mu < 0$. The GUT stability constraint now allows $M_2 \lesssim 1000$ GeV. The non-neutralino LSP region is no longer triangular in shape, but now requires $\mu \gtrsim 800$ GeV. This happens as the regions with light left-handed slepton at low M_2 meet with the ones with light right-handed slepton at higher M_2. There is a minuscule allowed region for $\mu < 0$. The residual regions with relic density in the preferred range for $\mu > 0$ are limited to strips in the neutralino-slepton coannihilation regions, and on either side of the rapid A, H annihilation strip. All this preferred region is compatible with the putative a_μ constraint.

A rather similar pattern is visible in panel (d) of Fig. 9 for $\tan\beta = 50$. The most

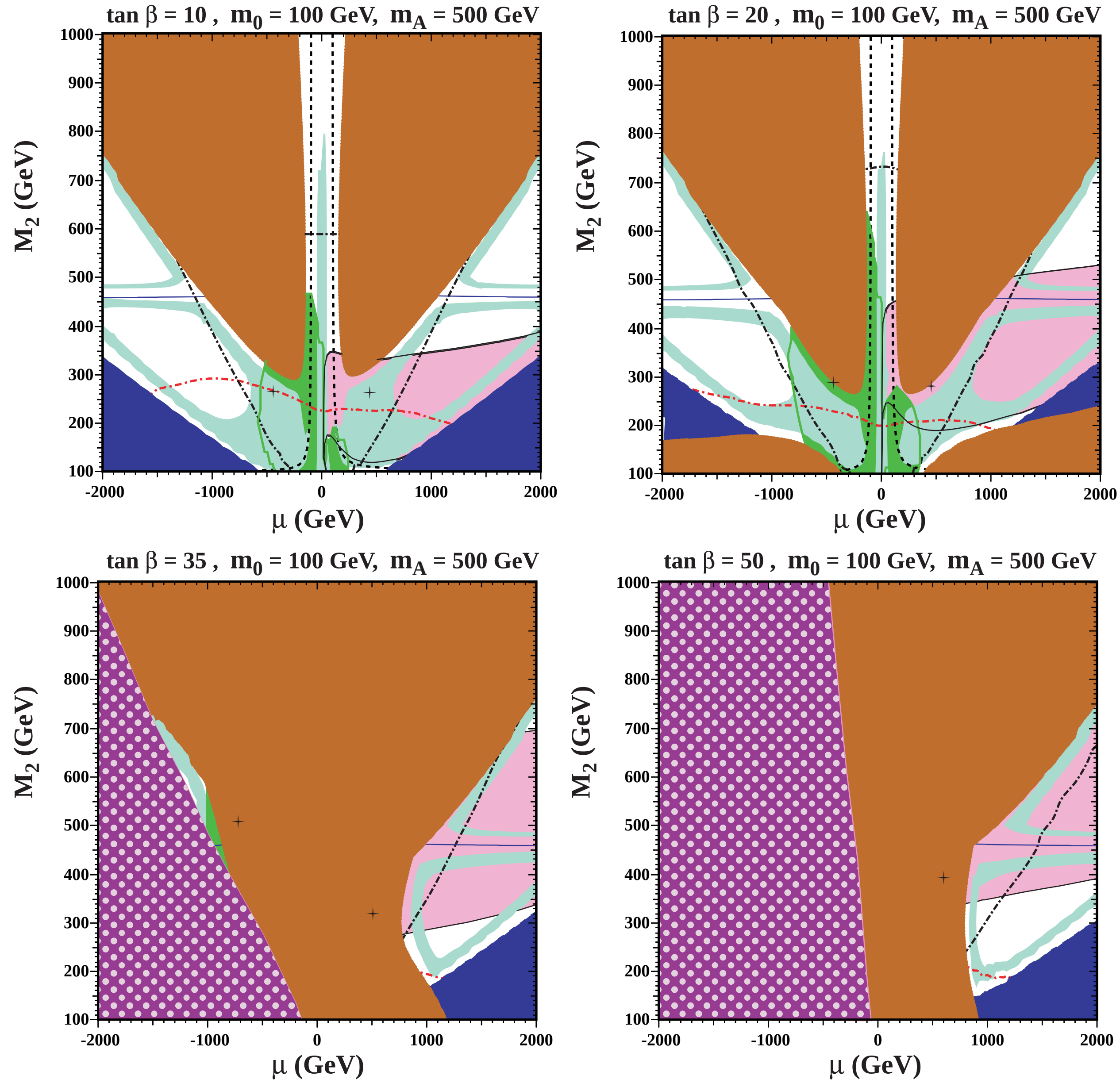

Figure 9: *The NUHM* (μ, M_2) *planes for* $m_0 = 100$ *GeV and* $m_A = 500$ *GeV, for (a)* $\tan\beta = 10$*, (b)* $\tan\beta = 20$*, (c)* $\tan\beta = 35$ *and (d)* $\tan\beta = 50$*, assuming* $A_0 = 0, m_t = 175$ *GeV and* $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ *GeV. The shadings and line styles are the same as in Fig. 7.*

noticeable differences are that the absence of a consistent electroweak vacuum is even more marked for $\mu > 0$, and that the putative a_μ constraint would suggest a lower limit $M_2 \gtrsim 350$ GeV, whereas lower values would have been permitted in panel (c) for $\tan\beta = 35$.

6 Conclusions and Open Issues

We have provided in this paper the tools needed for a detailed study of the NUHM, in the form of complete calculations of the most important coannihilation processes. However, in this paper we have only been able to scratch the surface of the phenomenology of the NUHM. Even this exploratory study has shown that many new features appear compared with the CMSSM, as results of the two additional parameters in the NUHM, but much more remains to be studied. For example, we have not studied the NUHM with a nonzero trilinear coupling A_0. Nevertheless, some interesting preliminary conclusions about the NUHM can be drawn, though many questions remain open.

The lower limit on the LSP mass m_χ in the CMSSM has been much discussed, and it is interesting to consider whether this could be greatly reduced in the NUHM. The top panel of Fig. 10 compiles the bounds on m_χ for the various sample parameter choices explored in this paper. We focus on the case $\mu > 0$, which is favoured by m_h and $b \to s\gamma$ as well as the dubious $g_\mu - 2$ constraint. The solid (black) line connects the lower limits on m_χ for the particular choice $\mu = 400$ GeV and $m_A = 700$ GeV shown for the four choices of $\tan\beta$ in Fig. 3. This lower limit is provided by m_h for $\tan\beta = 10, 20$, but by $b \to s\gamma$ for $\tan\beta = 35, 50$. The CMSSM lower bound on m_χ is indicated by a thick (blue) line. We see that this CMSSM lower bound on m_χ is similar to that in the NUHM when $\tan\beta = 10$, but weaker when $\tan\beta = 50$ because of the different behaviour in the CMSSM of the $b \to s\gamma$ constraint in this case.

For $\tan\beta = 10$, we also indicate by different black symbols the lower bounds on m_χ for the other choices of (μ, m_A) studied in Fig. 2, namely $(\mu, m_A) = (400, 400)$ GeV (square), $(\mu, m_A) = (700, 400)$ GeV (diamond) and $(\mu, m_A) = (700, 700)$ GeV (star). The two latter lower bounds are significantly higher than in the default case $(\mu, m_A) = (400, 700)$ GeV, due to the impacts of the GUT stability condition and the $\Omega_\chi h^2$ constraint, respectively. In none of the NUHM examples studied was the lower limit on m_χ relaxed compared with the CMSSM, though this might be found possible in a more complete survey of the NUHM parameter space.

We also show in the top panel of Fig. 10 the upper bounds on m_χ found in the NUHM for the various parameter choices explored in this paper. The (red) dot-dashed line is the upper

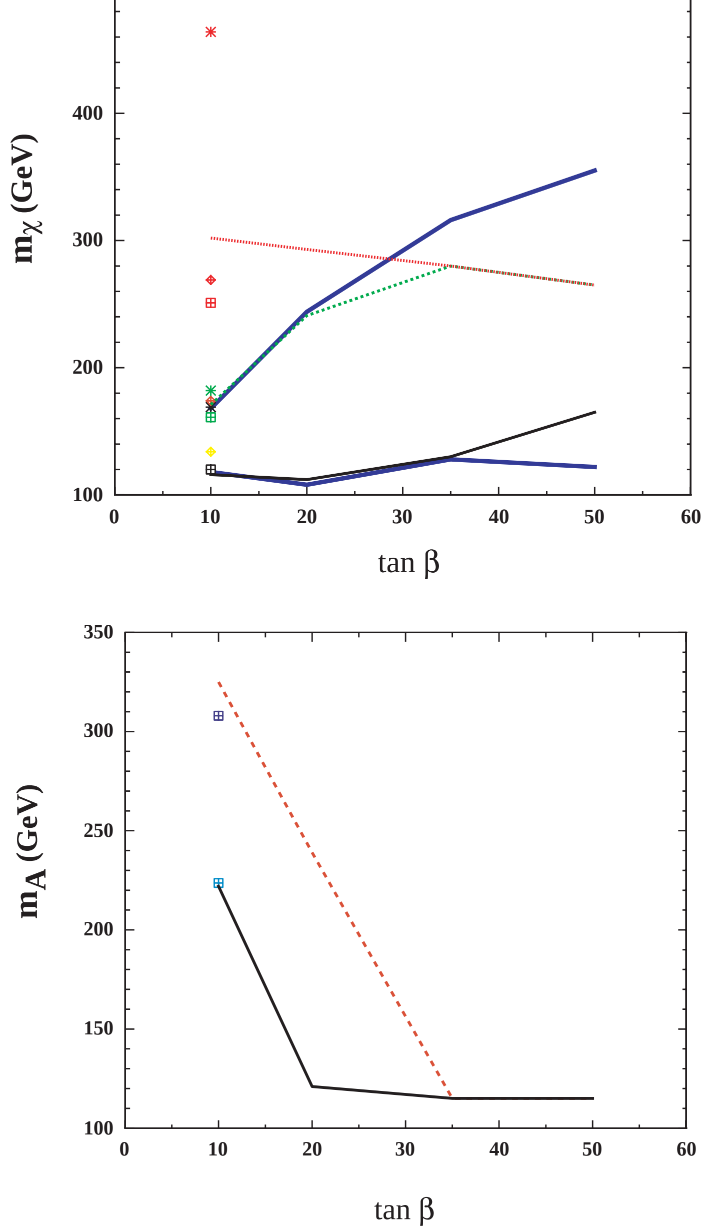

Figure 10: *Top panel: Bounds on the LSP mass m_χ found in the NUHM for the particular parameter choices displayed in Figs. 2 and 3. The solid (black) line is the lower limit on m_χ for $(\mu, m_A) = (400, 700)$ GeV, The (red) dotted line is the upper bound for the same values of (μ, m_A), but without imposing the putative $g_\mu - 2$ constraint, and the (green) dashed line shows how this upper bound would be strengthened if the $g_\mu - 2$ constraint were imposed. The symbols shown for $\tan\beta = 10$ mark the lower ($g_\mu - 2$ upper, overall upper) limits on m_χ for $(\mu, m_A) = (400, 400)$ GeV (squares), $(\mu, m_A) = (700, 400)$ GeV (diamonds) and $(\mu, m_A) = (700, 700)$ GeV (stars). The thick (blue) lines correspond to CMSSM upper and lower limits. Bottom panel: Lower bounds on m_A for $(m_{1/2}, m_0) = (300, 100)$ GeV (solid black line) and $(m_{1/2}, m_0) = (500, 300)$ GeV (dashed red line). The extra symbols for $\tan\beta = 10$ are lower bounds for $(m_{1/2}, m_0) = (500, 100)$ GeV (upper point) and $(300, 300)$ GeV (lower point).*

bound for $\mu = 400$ GeV and $m_A = 400$ GeV, as obtained without imposing the putative $g_\mu - 2$ constraint. The upper bound on m_χ is attained in the rapid-annihilation strip where $m_\chi < m_A/2$. Also shown in Fig. 10, as a (green) dashed line, is the strengthening of this upper bound that would be found if the $g_\mu - 2$ constraint were imposed. This constraint would not strengthen the upper limit for $\tan\beta \geq 35$. We also show as a thick (blue) line the CMSSM upper limit on m_χ, implementing the $g_\mu - 2$ constraint. We see that it is similar to that in the NUHM for $\tan\beta = 10, 20$, but is weaker for higher $\tan\beta$. The CMSSM upper bound would be much weaker still if the $g_\mu - 2$ constraint were relaxed, because of the different behaviours of the rapid-annihilation strips in the NUHM and CMSSM, as seen by comparing Figs. 1 and 3.

For $\tan\beta = 10$, we also show in the top panel of Fig. 10 the upper bounds that hold with and without $g_\mu - 2$ for the other three choices of (μ, m_A) made in Fig. 2, using the same symbols as for the lower bounds (squares, diamonds and stars, respectively). The largest upper bound is for the choice $(\mu, m_A) = (700, 700)$ GeV, for which there is an extension of the allowed region along the coannihilation tail, extending up to the GUT stability line.

In the second panel of Fig. 10, we summarize the lower limits on m_A found in the NUHM models that we have studied. The solid (black) line is for the choice $(m_{1/2}, m_0) = (300, 100)$ GeV displayed previously in Fig. 7. The lower bounds shown for $\tan\beta = 10, 20$ are compatible with $g_\mu - 2$, but we find that no region would be allowed by $g_\mu - 2$ in the cases $\tan\beta = 35, 50$, as seen in panels (c) and (d) of Fig. 7. The (red) dashed line in the second panel of Fig. 10 is for the choice $(m_{1/2}, m_0) = (500, 300)$ GeV shown previously in Fig. 6. In this case, we find that no region would be allowed by $g_\mu - 2$ for $\tan\beta = 10$, whereas the lower bounds for $\tan\beta = 20, 35, 50$ are compatible with $g_\mu - 2$. The extra symbols for $\tan\beta = 10$ correspond to the other choices $(m_{1/2}, m_0) = (500, 100)$ GeV and $(300, 300)$ GeV shown in panels (b) and (c) of Fig. 4, at $m_A = 308$ GeV and $m_A = 224$ GeV, respectively. We note that neither of these choices satisfies the $g - 2$ constraint.

An interesting feature of the second panel of Fig. 10 is the fact that the lower bounds coincide for $\tan\beta \geq 35$, and correspond to the lower limit established by direct searches at LEP. For comparison, we note that in the CMSSM for $(m_{1/2}, m_0) = (300, 100)$ GeV one would have $m_A = 449, 424, 377, 315$ GeV for $\tan\beta = 10, 20, 35, 50$, whilst for $(m_{1/2}, m_0) = (500, 300)$ GeV one would have $m_A = 762, 720, 639, 526$ GeV for $\tan\beta = 10, 20, 35, 50$. We conclude that the NUHM allows m_A to be considerably smaller than in the CMSSM, particularly at large $\tan\beta$.

This brief survey is no substitute for a detailed study of the NUHM. However, we have provided in the Appendices the technical tools required for such a study, and in this Section we have presented some preliminary observations based on a cursory exploration of the NUHM. This has already provided some interesting indications, for example that it may prove difficult to relax significantly the CMSSM lower bound on the LSP, but that m_A may be greatly reduced. In our view, it would be interesting to pursue these questions more deeply in a detailed study of the NUHM, a project that lies beyond the scope of this work.

Acknowledgments

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Appendix A: Couplings used in the Calculations

Here we list the couplings used in the calculations. For clarity, we have not written factors such as i, γ's and momenta. These are taken into account in the calculations of the amplitudes squared in the other Appendices.

Couplings for chargino-slepton coannihilation:

$$
\begin{aligned}
C^L_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_i^-} &= 0 \\
C^R_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_i^-} &= (g_2\, m_\ell/(\sqrt{2}\, m_W \cos\beta))U_{i2}\sin\theta_\ell - g_2\, U_{i1}\cos\theta_\ell \\
C^L_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-Z} &= g_2/\cos\theta_W(-V_{i1}V_{11} - V_{i2}V_{12}/2 + \delta_{i1}\sin^2\theta_W) \\
C^R_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-Z} &= g_2/\cos\theta_W(-U_{i1}U_{11} - U_{i2}U_{12}/2 + \delta_{i1}\sin^2\theta_W) \\
C_{\widetilde{\ell}_1-Z-\widetilde{\ell}_1^-} &= g_2/(2\cos\theta_W)(\cos^2\theta_\ell - 2\sin^2\theta_W) \\
C_{\widetilde{\ell}_1-Z-\widetilde{\ell}_2^-} &= g_2/(2\cos\theta_W)\sin\theta_\ell\cos\theta_\ell \\
C^L_{\widetilde{\chi}_1^--\nu_\ell-\widetilde{\ell}_i} &= 0 \\
C^R_{\widetilde{\chi}_1^--\nu_\ell-\widetilde{\ell}_1} &= (g_2 m_\ell/(\sqrt{2} m_W \cos\beta))U_{12}\sin\theta_\ell - g_2 U_{11}\cos\theta_\ell \\
C^R_{\widetilde{\chi}_1^--\nu_\ell-\widetilde{\ell}_2} &= (g_2 m_\ell/(\sqrt{2} m_W \cos\beta))U_{12}\cos\theta_\ell + g_2 U_{11}\sin\theta_\ell \\
C_{\widetilde{\chi}_i^+-\widetilde{\chi}_i^+-\gamma} &= -e \\
C_{\widetilde{\ell}_i-\widetilde{\ell}_i-\gamma} &= e \\
C^L_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-H} &= -g_2(V_{11}U_{i2}/\sqrt{2}\cos\alpha + V_{12}U_{i1}/\sqrt{2}\sin\alpha) \\
C^R_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-H} &= -g_2(V_{i1}U_{12}/\sqrt{2}\cos\alpha + V_{i2}U_{11}/\sqrt{2}\sin\alpha) \\
C^L_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-h} &= g_2(V_{11}U_{i2}/\sqrt{2}\sin\alpha - V_{12}U_{i1}/\sqrt{2}\cos\alpha) \\
C^R_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-h} &= g_2(V_{i1}U_{12}/\sqrt{2}\sin\alpha - V_{i2}U_{11}/\sqrt{2}\cos\alpha) \\
C_{\widetilde{\ell}_1-H-\widetilde{\ell}_1} &= g_2 m_Z/\cos\theta_W(1/2 - \sin^2\theta_W)\cos(\alpha+\beta)\cos^2\theta_\ell \\
&\quad -g_2 m_Z/\cos\theta_W(-1)\sin^2\theta_W\cos(\alpha+\beta)\sin^2\theta_\ell \\
&\quad -g_2 m_\ell^2/(m_W\cos\beta)\cos\alpha \\
&\quad +g_2 m_\ell/(m_W\cos\beta)(\mu\sin\alpha + A_\ell\cos\alpha)\sin\theta_\ell\cos\theta_\ell \\
C_{\widetilde{\ell}_1-H-\widetilde{\ell}_2} &= g_2 m_Z/\cos\theta_W(1/2 - \sin^2\theta_W)\cos(\alpha+\beta)\cos\theta_\ell(-\sin\theta_\ell) \\
&\quad -g_2 m_Z/\cos\theta_W(-1)\sin^2\theta_W\cos(\alpha+\beta)\sin\theta_\ell\cos\theta_\ell \\
&\quad +g_2 m_\ell/(2m_W\cos\beta)(\mu\sin\alpha + A_\ell\cos\alpha)(\cos^2\theta_\ell - \sin^2\theta_\ell) \\
C_{\widetilde{\ell}_1-h-\widetilde{\ell}_1} &= -g_2 m_Z/\cos\theta_W(1/2 - \sin^2\theta_W)\sin(\alpha+\beta)\cos^2\theta_\ell
\end{aligned}
$$

$$
\begin{aligned}
& & +g_2 m_Z/\cos\theta_W(-1)\sin^2\theta_W\sin(\alpha+\beta)\sin^2\theta_\ell \\
& & +g_2 m_\ell^2/(m_W\cos\beta)\sin\alpha \\
& & +g_2 m_\ell/(m_W\cos\beta)(\mu\cos\alpha - A_\ell\sin\alpha)\sin\theta_\ell\cos\theta_\ell \\
C_{\widetilde{\ell}_1-h-\widetilde{\ell}_2} &=& -g_2 m_Z/\cos\theta_W(1/2-\sin^2\theta_W)\sin(\alpha+\beta)\cos\theta_\ell(-\sin\theta_\ell) \\
& & +g_2 m_Z/\cos\theta_W(-1)\sin^2\theta_W\sin(\alpha+\beta)\sin\theta_\ell\cos\theta_\ell \\
& & +g_2 m_\ell/(2m_W\cos\beta)(\mu\cos\alpha - A_\ell\sin\alpha)(\cos^2\theta_\ell-\sin^2\theta_\ell) \\
C^L_{\widetilde{\chi}^+_1-\widetilde{\chi}^+_i-A} &=& g_2(V_{11}U_{i2}/\sqrt{2}\sin\beta + V_{12}U_{i1}/\sqrt{2}\cos\beta) \\
C^R_{\widetilde{\chi}^+_1-\widetilde{\chi}^+_i-A} &=& -g_2(V_{i1}U_{12}/\sqrt{2}\sin\beta + V_{i2}U_{11}/\sqrt{2}\cos\beta) \\
C_{\widetilde{\ell}_1-A-\widetilde{\ell}^-_2} &=& -g_2 m_\ell/(2m_W)(\mu - A_\ell\tan\beta) \\
C^L_{\widetilde{\ell}_1-\ell-\widetilde{\chi}^0_i} &=& \cos\theta_\ell(g_2/\sqrt{2}(N_{i2}+\tan\theta_W N_{i1})) \\
& & +\sin\theta_\ell(-g_2/\sqrt{2}m_\ell N_{i3}/(m_W\cos\beta)) \\
C^R_{\widetilde{\ell}_1-\ell-\widetilde{\chi}^0_i} &=& \cos\theta_\ell(-g_2/\sqrt{2}m_\ell N_{i3}/(m_W\cos\beta)) \\
& & +\sin\theta_\ell(-g_2/\sqrt{2}(2)\tan\theta_W N_{i1}) \\
C_{\nu_\ell-\ell-W^+} &=& -g_2/\sqrt{2} \\
C^L_{W-\widetilde{\chi}^-_1-\widetilde{\chi}^0_i} &=& g_2(N_{i2}V_{11} - N_{i4}V_{12}/\sqrt{2}) \\
C^R_{W-\widetilde{\chi}^-_1-\widetilde{\chi}^0_i} &=& g_2(N_{i2}U_{11} + N_{i3}U_{12}/\sqrt{2}) \\
C_{\nu_\ell-\ell-H^+} &=& g_2/(\sqrt{2}m_W)m_\ell\tan\beta \\
C^L_{H^+-\widetilde{\chi}^-_1-\widetilde{\chi}^0_i} &=& -g_2(N_{i4}V_{11} + (N_{i2}+N_{i1}\tan\theta_W)V_{12}/\sqrt{2})\cos\beta \\
C^R_{H^+-\widetilde{\chi}^-_1-\widetilde{\chi}^0_i} &=& -g_2(N_{i3}U_{11} - (N_{i2}+N_{i1}\tan\beta)U_{12}/\sqrt{2})\sin\beta \\
C^L_{\ell-\widetilde{\nu}_\ell-\widetilde{\chi}^-_1} &=& -g_2 V_{11} \\
C^R_{\ell-\widetilde{\nu}_\ell-\widetilde{\chi}^-_1} &=& g_2 m_\ell/(\sqrt{2}m_W\cos\beta)U_{12} \\
C_{\widetilde{\nu}_\ell-\widetilde{\ell}_1-W} &=& -g_2/\sqrt{2}\cos\theta_\ell \\
C_{\widetilde{\nu}_\ell-\widetilde{\ell}_1-H^+} &=& -g_2 m_W/\sqrt{2}(\sin(2\beta) - m_\ell^2\tan\beta/m_W^2)\cos\theta_\ell \\
& & +g_2 m_\ell/(\sqrt{2}m_W)(\mu - A_\ell\tan\beta)\sin\theta_\ell
\end{aligned}
$$

Couplings for neutralino-sneutrino coannihilation:

$$
\begin{aligned}
C_{H-W-W} &=& g_2 m_W\cos(\beta-\alpha) \\
C_{h-W-W} &=& g_2 m_W\sin(\beta-\alpha)
\end{aligned}
$$

$$
\begin{aligned}
C_{\widetilde{\nu}-\widetilde{\nu}-H} &= -g_2 m_Z/\cos\theta_W(1/2)\cos(\beta+\alpha) \\
C_{\widetilde{\nu}-\widetilde{\nu}-h} &= g_2 m_Z/\cos\theta_W(1/2)\sin(\beta+\alpha) \\
C_{\widetilde{\nu}-\widetilde{\nu}-Z} &= -g_2/\cos\theta_W(1/2) \\
C_{\widetilde{\nu}-\widetilde{\nu}-W-W} &= g_2^2/2 \\
C_{Z-W-W} &= g_2\cos\theta_W \\
C_{\widetilde{\nu}-\widetilde{e}-W} &= -g_2/\sqrt{2} \\
C_{H-Z-Z} &= g_2 m_Z/\cos\theta_W\cos(\beta-\alpha) \\
C_{h-Z-Z} &= g_2 m_Z/\cos\theta_W\sin(\beta-\alpha) \\
C_{\widetilde{\nu}-\widetilde{\nu}-Z-Z} &= 2g_2^2/(\cos^2\theta_W)(1/2)^2 \\
C^L_{Z-f-f} &= -g_2/\cos\theta_W(T_{3f}-Q_f\sin^2\theta_W) \\
C^R_{Z-f-f} &= g_2/\cos\theta_W Q_f\sin^2\theta_W \\
C_{Z-\nu-\nu} &= -g_2/(2\cos\theta_W) \\
C_{H-f-f} &= -g_2 m_f/(2m_W)\sin\alpha/\sin\beta \\
C_{h-f-f} &= -g_2 m_f/(2m_W)\cos\alpha/\sin\beta \\
C_{\widetilde{\nu}-\widetilde{\chi}_i^+-e} &= -g_2 V_{i1} \\
C_{\widetilde{\nu}-\widetilde{\chi}_i^0-\nu} &= -g_2/\sqrt{2}(N_{i2}-\tan\theta_W N_{i1}) \\
C_{H-W^+-H^-} &= -g_2 m_Z/\cos\theta_W(1/2)\cos(\alpha+\beta) \\
C_{h-W^+-H^-} &= g_2 m_Z/\cos\theta_W(1/2)\sin(\alpha+\beta) \\
C_{\widetilde{\nu}-\widetilde{e}_L-H^+} &= -g_2 m_W/\sqrt{2} \\
C_{H-H^+-H^-} &= -g_2(m_W\cos(\beta-\alpha)-m_Z/(2\cos\theta_W)\cos(2\beta)\cos(\beta+\alpha)) \\
C_{h-H^+-H^-} &= -g_2(m_W\sin(\beta-\alpha)+m_Z/(2\cos\theta_W)\cos(2\beta)\sin(\beta+\alpha)) \\
C_{\widetilde{\nu}-\widetilde{\nu}-H^+-H^-} &= g_2^2/2\cos(2\beta)(-2(1/2)+(1/2)/\cos^2\theta_W) \\
C_{H-H-H} &= -3g_2 m_Z/(2\cos\theta_W)\cos(2\alpha)\cos(\beta+\alpha) \\
C_{H-H-h} &= g_2 m_Z/(2\cos\theta_W)(2\sin(2\alpha)\cos(\beta+\alpha)+\sin(\beta+\alpha)\cos(2\alpha)) \\
C_{H-h-h} &= -g_2 m_Z/(2\cos\theta_W)(2\sin(2\alpha)\sin(\beta+\alpha)-\cos(\beta+\alpha)\cos(2\alpha)) \\
C_{h-h-h} &= -3g_2 m_Z/(2\cos\theta_W)\cos(2\alpha)\sin(\beta+\alpha) \\
C_{\widetilde{\nu}-\widetilde{\nu}-H-H} &= g_2^2/2((1/2)/\cos^2\theta_W(-\cos(2\alpha)))
\end{aligned}
$$

$$
\begin{aligned}
C_{\widetilde{\nu}-\widetilde{\nu}-h-h} &= g_2^2/2((1/2)/\cos^2\theta_W\cos(2\alpha)) \\
C_{\widetilde{\nu}-\widetilde{\nu}-H-h} &= g_2^2/2\sin(2\alpha)((1/2)/\cos^2\theta_W) \\
C_{H-A-A} &= g_2 m_Z/(2\cos\theta_W)\cos(2\beta)\cos(\beta+\alpha) \\
C_{h-A-A} &= -g_2 m_Z/(2\cos\theta_W)\cos(2\beta)\sin(\beta+\alpha) \\
C_{\widetilde{\nu}-\widetilde{\nu}-A-A} &= g_2^2/2((1/2)/\cos^2\theta_W\cos(2\beta)) \\
C_{H-Z-A} &= g_2\sin(\beta-\alpha)/(2\cos\theta_W) \\
C_{h-Z-A} &= -g_2\cos(\beta-\alpha)/(2\cos\theta_W) \\
C^L_{\widetilde{\chi}^0_1-\widetilde{\chi}^0_i-Z} &= g_2/(2\cos\theta_W)(N_{i4}N_{14}-N_{i3}N_{13}) \\
C^R_{\widetilde{\chi}^0_1-\widetilde{\chi}^0_i-Z} &= -g_2/(2\cos\theta_W)(N_{i4}N_{14}-N_{i3}N_{13}) \\
C_{\nu-e-W} &= -g_2/\sqrt{2} \\
C_{\widetilde{\nu}-\widetilde{e}-W} &= -g_2/\sqrt{2} \\
C^L_{\widetilde{e}_1-\widetilde{\chi}^0_1-e} &= -g_2/\sqrt{2}(\sin\theta_e 2\tan\theta_W N_{11}) \\
C^L_{\widetilde{e}_2-\widetilde{\chi}^0_1-e} &= -g_2/\sqrt{2}(\cos\theta_e 2\tan\theta_W N_{11}) \\
C^R_{\widetilde{e}_1-\widetilde{\chi}^0_1-e} &= -g_2/\sqrt{(2}(\cos\theta_e(-N_{12}-\tan\theta_W N_{11})) \\
C^R_{\widetilde{e}_2-\widetilde{\chi}^0_1-e} &= -g_2/\sqrt{2}(-\sin\theta_e(-N_{12}-\tan\theta_W N_{11})) \\
C^L_{\widetilde{\chi}^0_1-\widetilde{\chi}^+_i-W} &= g_2(-1/\sqrt{2}N_{14}V_{i2}+N_{12}V_{i1}) \\
C^R_{\widetilde{\chi}^0_1-\widetilde{\chi}^+_i-W} &= g_2(1/\sqrt{2}N_{13}U_{i2}+N_{12}U_{i1}) \\
C^L_{\widetilde{\chi}^0_1-\widetilde{\chi}^0_i-h} &= g_2/2((N_{13}(N_{i2}-N_{i1}\tan\theta_W)+N_{i3}(N_{12}-N_{11}\tan\theta_W))\sin\alpha \\
&\quad +(N_{14}(N_{i2}-N_{i1}\tan\theta_W)N_{i4}(N_{12}-N_{11}\tan\theta_W))\cos\alpha) \\
C^R_{\widetilde{\chi}^0_1-\widetilde{\chi}^0_i-h} &= g_2/2((N_{i3}(N_{12}-N_{11}\tan\theta_W)+N_{13}(N_{i2}-N_{i1}\tan\theta_W))\sin\alpha \\
&\quad +(N_{i4}(N_{12}-N_{11}\tan\theta_W)+N_{14}(N_{i2}-N_{i1}\tan\theta_W))\cos\alpha) \\
C^L_{\widetilde{\chi}^0_1-\widetilde{\chi}^0_i-H} &= g_2/2((N_{13}(N_{i2}-N_{i1}\tan\theta_W)+N_{i3}(N_{12}-N_{11}\tan\theta_W))(-\cos\alpha) \\
&\quad +(N_{14}(N_{i2}-N_{i1}\tan\theta_W)N_{i4}(N_{12}-N_{11}\tan\theta_W))\sin\alpha) \\
C^R_{\widetilde{\chi}^0_1-\widetilde{\chi}^0_i-H} &= g_2/2((N_{i3}(N_{12}-N_{11}\tan\theta_W)+N_{13}(N_{i2}-N_{i1}\tan\theta_W))(-\cos\alpha) \\
&\quad +(N_{i4}(N_{12}-N_{11}\tan\theta_W)+N_{14}(N_{i2}-N_{i1}\tan\theta_W))\sin\alpha) \\
C^L_{\widetilde{\chi}^0_1-\widetilde{\chi}^0_i-A} &= g_2/2((N_{13}(N_{i2}-N_{i1}\tan\theta_W)+N_{i3}(N_{12}-N_{11}\tan\theta_W))\sin\beta \\
&\quad +(N_{14}(N_{i2}-N_{i1}\tan\theta_W)N_{i4}(N_{12}-N_{11}\tan\theta_W))(-\cos\beta))
\end{aligned}
$$

$$
\begin{aligned}
C^R_{\widetilde{\chi}^0_1-\widetilde{\chi}^0_i-A} &= g_2/2((N_{i3}(N_{12}-N_{11}\tan\theta_W)+N_{13}(N_{i2}-N_{i1}\tan\theta_W))(-\sin\beta) \\
&\quad +(N_{i4}(N_{12}-N_{11}\tan\theta_W)+N_{14}(N_{i2}-N_{i1}\tan\theta_W))\cos\beta) \\
C_{\widetilde{\nu}-\widetilde{e}-H^+} &= -g_2/\sqrt{2}\sin(2\beta) \\
C^L_{\widetilde{\chi}^0_1-\widetilde{\chi}^-_i-H^+} &= -g_2(N_{14}V_{i1}\sqrt{1/2}(N_{12}+N_{11}\tan\theta_W)V_{i2})\cos\beta \\
C^R_{\widetilde{\chi}^0_1-\widetilde{\chi}^-_i-H^+} &= -g_2(N_{13}U_{i1}-\sqrt{1/2}(N_{12}+N_{11}\tan\theta_W)U_{i2})\sin\beta
\end{aligned}
$$

Appendix B: Neutralino-Slepton Coannihilation with L-R Mixing

We list below some modifications and additions to the slepton-slepton and neutralino-slepton coannihilation channels previously given in the Appendix of [17]. There the lighter sleptons were assumed to be pure partners of the right-handed leptons, which is a very good approximation for CMSSM when $\tan\beta$ is small. Here, we include L-R mixing and denote the lighter sleptons by $\widetilde{\ell}_1$, denoting $\ell \equiv e, \mu, \tau$. The obvious change needed is then replacing $\widetilde{\tau}_{\rm R}$ in [17] with $\widetilde{\ell}_1$. Furthermore, the presence of the left component entails some modifications in the couplings and opens up some new channels. We note that L-R mixing had already been included in some of our previous work [21, 2]. Our convention for the slepton mixing angle is

$$
\begin{pmatrix} \widetilde{\ell}_1 \\ \widetilde{\ell}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_\ell & \sin\theta_\ell \\ -\sin\theta_\ell & \cos\theta_\ell \end{pmatrix} \begin{pmatrix} \widetilde{\ell}_L \\ \widetilde{\ell}_R \end{pmatrix} \tag{B1}
$$

The sign of $m_{\widetilde{\chi}^0_i}$ must be kept in all of the formulae below.

$\widetilde{\ell}_1\widetilde{\ell}^*_1 \longrightarrow W^+W^-$

There are two additional channels:

III. t-channel $\widetilde{\nu}_\ell$ exchange

IV. point interaction

The couplings f_1, f_2 and f_5 are modified, while f_6 remains the same.

$$
\begin{aligned}
f_1 &= (-g_2 m_W \cos(\beta-\alpha))(g_2 m_Z/\cos\theta_W((1/2-\sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell \\
&\quad +\sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)+g_2 m_\ell^2/(m_W\cos\beta)(-\cos\alpha) \\
&\quad -g_2 m_\ell/(m_W\cos\beta)(-A_\ell\cos\alpha-\mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell) \\
f_2 &= (-g_2 m_W \sin(\beta-\alpha))(g_2 m_Z/\cos\theta_W((-1/2+\sin^2\theta_W)\sin(\beta+\alpha)\cos^2\theta_\ell
\end{aligned}
$$

$$
\begin{aligned}
& -\sin^2\theta_W \sin(\beta+\alpha)\sin^2\theta_\ell) + g_2 m_\ell^2/(m_W\cos\beta)\sin\alpha \\
& -g_2 m_\ell/(m_W\cos\beta)(A_\ell\sin\alpha - \mu\cos\alpha)\sin\theta_\ell\cos\theta_\ell) \\
f_3 &= (-g_2\cos\theta_\ell/\sqrt{2})^2 \\
f_4 &= -g_2^2\cos^2\theta_\ell/2 \\
f_5 &= (g_2\cos\theta_W)(-g_2/\cos\theta_W(\sin^2\theta_W - \cos^2\theta_\ell/2)) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= (m_{\tilde\ell_1}^8 - 4m_{\tilde\ell_1}^6 m_W^2 + 6m_{\tilde\ell_1}^4 m_W^4 - 4m_{\tilde\ell_1}^2 m_W^6 + m_W^8 - 4m_{\tilde\ell_1}^6 u + 4m_{\tilde\ell_1}^4 m_W^2 u \\
& +4m_{\tilde\ell_1}^2 m_W^4 u - 4m_W^6 u + 6m_{\tilde\ell_1}^4 u^2 + 4m_{\tilde\ell_1}^2 m_W^2 u^2 + 6m_W^4 u^2 - 4m_{\tilde\ell_1}^2 u^3 \\
& -4m_W^2 u^3 + u^4)/(m_W^4(m_{\tilde\nu_\ell}^2 - u)^2) \\
\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} &= (12m_W^4 - 4m_W^2 s + s^2)/(4m_W^4) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} &= (-6m_{\tilde\ell_1}^4 m_W^2 - 20m_{\tilde\ell_1}^2 m_W^4 - 6m_W^6 + m_{\tilde\ell_1}^4 s + 2m_{\tilde\ell_1}^2 m_W^2 s + 5m_W^4 s + 4m_{\tilde\ell_1}^2 m_W^2 t \\
& +4m_W^4 t + 8m_{\tilde\ell_1}^2 m_W^2 u + 8m_W^4 u - 2m_{\tilde\ell_1}^2 su - 2m_W^2 su - 4m_W^2 tu - 2m_W^2 u^2 \\
& +su^2)/(2m_W^4(m_H^2 - s)(m_{\tilde\nu_\ell}^2 - u)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} &= (-6m_{\tilde\ell_1}^4 m_W^2 - 20m_{\tilde\ell_1}^2 m_W^4 - 6m_W^6 + m_{\tilde\ell_1}^4 s + 2m_{\tilde\ell_1}^2 m_W^2 s + 5m_W^4 s + 4m_{\tilde\ell_1}^2 m_W^2 t \\
& +4m_W^4 t + 8m_{\tilde\ell_1}^2 m_W^2 u + 8m_W^4 u - 2m_{\tilde\ell_1}^2 su - 2m_W^2 su - 4m_W^2 tu - 2m_W^2 u^2 \\
& +su^2)/(2m_W^4(m_h^2 - s)(m_{\tilde\nu_\ell}^2 - u)) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} &= (-6m_{\tilde\ell_1}^4 m_W^2 - 20m_{\tilde\ell_1}^2 m_W^4 - 6m_W^6 + m_{\tilde\ell_1}^4 s + 2m_{\tilde\ell_1}^2 m_W^2 s + 5m_W^4 s + 4m_{\tilde\ell_1}^2 m_W^2 t \\
& +4m_W^4 t + 8m_{\tilde\ell_1}^2 m_W^2 u + 8m_W^4 u - 2m_{\tilde\ell_1}^2 su - 2m_W^2 su - 4m_W^2 tu - 2m_W^2 u^2 \\
& +su^2)/(2m_W^4(m_{\tilde\nu_\ell}^2 - u)) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V} &= (64m_{\tilde\ell_1}^4 m_W^4 m_Z^2 + 64m_{\tilde\ell_1}^2 m_W^6 m_Z^2 - 16m_{\tilde\ell_1}^4 m_W^2 m_Z^2 s - 32m_{\tilde\ell_1}^2 m_W^4 m_Z^2 s \\
& -16m_W^6 m_Z^2 s + 4m_{\tilde\ell_1}^2 m_W^2 m_Z^2 s^2 + 4m_W^4 m_Z^2 s^2 - 2m_{\tilde\ell_1}^4 m_W^2 m_Z^2 t - 28m_{\tilde\ell_1}^2 m_W^4 m_Z^2 t \\
& -2m_W^6 m_Z^2 t + m_{\tilde\ell_1}^4 m_Z^2 st + 2m_{\tilde\ell_1}^2 m_W^2 m_Z^2 st + 5m_W^4 m_Z^2 st + 2m_{\tilde\ell_1}^4 m_W^2 m_Z^2 u \\
& -36m_{\tilde\ell_1}^2 m_W^4 m_Z^2 u + 2m_W^6 m_Z^2 u - m_{\tilde\ell_1}^4 m_Z^2 su + 14m_{\tilde\ell_1}^2 m_W^2 m_Z^2 su + 11m_W^4 m_Z^2 su \\
& -4m_W^2 m_Z^2 s^2 u + 4m_{\tilde\ell_1}^2 m_W^2 m_Z^2 tu + 4m_W^4 m_Z^2 tu - 2m_{\tilde\ell_1}^2 m_Z^2 stu - 2m_W^2 m_Z^2 stu \\
& -4m_{\tilde\ell_1}^2 m_W^2 m_Z^2 u^2 - 4m_W^4 m_Z^2 u^2 + 2m_{\tilde\ell_1}^2 m_Z^2 su^2 + 2m_W^2 m_Z^2 su^2 - 2m_W^2 m_Z^2 tu^2 \\
& +m_Z^2 stu^2 + 2m_W^2 m_Z^2 u^3 - m_Z^2 su^3)/(2m_W^4 m_Z^2(m_Z^2 - s)(m_{\tilde\nu_\ell}^2 - u)) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm VI} &= (-64m_{\tilde\ell_1}^4 m_W^4 - 64m_{\tilde\ell_1}^2 m_W^6 + 16m_{\tilde\ell_1}^4 m_W^2 s + 32m_{\tilde\ell_1}^2 m_W^4 s + 16m_W^6 s - 4m_{\tilde\ell_1}^2 m_W^2 s^2 \\
& -4m_W^4 s^2 + 2m_{\tilde\ell_1}^4 m_W^2 t + 28m_{\tilde\ell_1}^2 m_W^4 t + 2m_W^6 t - m_{\tilde\ell_1}^4 st - 2m_{\tilde\ell_1}^2 m_W^2 st - 5m_W^4 st
\end{aligned}
$$

$$-2m_{\tilde{\ell}_1}^4 m_W^2 u + 36m_{\tilde{\ell}_1}^2 m_W^4 u - 2m_W^6 u + m_{\tilde{\ell}_1}^4 su - 14m_{\tilde{\ell}_1}^2 m_W^2 su - 11m_W^4 su$$
$$+4m_W^2 s^2 u - 4m_{\tilde{\ell}_1}^2 m_W^2 tu - 4m_W^4 tu + 2m_{\tilde{\ell}_1}^2 stu + 2m_W^2 stu + 4m_{\tilde{\ell}_1}^2 m_W^2 u^2$$
$$+4m_W^4 u^2 - 2m_{\tilde{\ell}_1}^2 su^2 - 2m_W^2 su^2 + 2m_W^2 tu^2 - stu^2 - 2m_W^2 u^3 + su^3)$$
$$/(2m_W^4 s(m_{\tilde{\nu}_\ell}^2 - u))$$
$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} = (12m_W^4 - 4m_W^2 s + s^2)/(4m_W^4(m_H^2 - s))$$
$$\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} = (12m_W^4 - 4m_W^2 s + s^2)/(4m_W^4(m_h^2 - s))$$
$$\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} = (-12m_W^4 m_Z^2 t + m_Z^2 s^2 t + 12m_W^4 m_Z^2 u - m_Z^2 s^2 u)/(4m_W^4 m_Z^2(m_Z^2 - s))$$
$$\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm VI} = (12m_W^4 t - s^2 t - 12m_W^4 u + s^2 u)/(4m_W^4 s)$$
$$|\mathcal{T}|^2 = f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + f_4^2\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} + f_5^2\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V} + f_6^2\mathcal{T}_{\rm VI}\times\mathcal{T}_{\rm VI}$$
$$+2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} + 2f_1f_4\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} + 2f_1f_5\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V} + 2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III}$$
$$+2f_2f_4\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} + 2f_2f_5\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V} + 2f_3f_4\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} + 2f_3f_5\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V}$$
$$+2f_4f_5\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} + 2f_1f_6\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm VI} + 2f_2f_6\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm VI} + 2f_3f_6\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm VI}$$
$$+2f_4f_6\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm VI} + 2f_5f_6\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm VI} \qquad \text{(B2)}$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow ZZ$

Besides the $\widetilde{\ell}_1$ exchange (III and IV in [17]), we also have $\widetilde{\ell}_2$ exchange, written here as VI and VII.

VI. t-channel $\widetilde{\ell}_1$ exchange

VII. u-channel $\widetilde{\ell}_1$ exchange

The couplings $f_1, \ldots f_5$ are modified, and we have new f_6 and f_7 couplings.

$$f_1 = (-g_2 m_Z \cos(\beta-\alpha)/\cos\theta_W)(g_2 m_Z/\cos\theta_W((1/2-\sin^2\theta_W)\cos(\beta+\alpha)$$
$$\cos^2\theta_\ell + \sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell) + g_2 m_\ell^2/(m_W\cos\beta)(-\cos\alpha)$$
$$-g_2 m_\ell/(m_W\cos\beta)(-A_\ell\cos\alpha - \mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell)$$
$$f_2 = (-g_2 m_Z \sin(\beta-\alpha)/\cos\theta_W)(g_2 m_Z/\cos\theta_W((-1/2+\sin^2\theta_W)\sin(\beta+\alpha)$$
$$\cos^2\theta_\ell - \sin^2\theta_W\sin(\beta+\alpha)\sin^2\theta_\ell) + g_2 m_\ell^2/(m_W\cos\beta)\sin\alpha$$
$$-g_2 m_\ell/(m_W\cos\beta)(A_\ell\sin\alpha - \mu\cos\alpha)\sin\theta_\ell\cos\theta_\ell)$$
$$f_3 = (-g_2/\cos\theta_W(\sin^2\theta_W - \cos^2\theta_\ell/2))^2$$
$$f_4 = (-g_2/\cos\theta_W(\sin^2\theta_W - \cos^2\theta_\ell/2))^2$$

$$
\begin{aligned}
f_5 &= -2g_2^2/\cos^2\theta_W(\sin^4\theta_W+\cos^2\theta_\ell(1/4-\sin^2\theta_W))\\
f_6 &= (-(g_2/\cos\theta_W)\cos\theta_\ell\sin\theta_\ell/2)^2\\
f_7 &= (-(g_2/\cos\theta_W)\cos\theta_\ell\sin\theta_\ell/2)^2\\
\mathcal{T}_{\rm VI}\times\mathcal{T}_{\rm VI} &= (m_{\tilde\ell_1}^8-4m_{\tilde\ell_1}^6m_Z^2+6m_{\tilde\ell_1}^4m_Z^4-4m_{\tilde\ell_1}^2m_Z^6+m_Z^8-4m_{\tilde\ell_1}^6t+4m_{\tilde\ell_1}^4m_Z^2t\\
&\quad+4m_{\tilde\ell_1}^2m_Z^4t-4m_Z^6t+6m_{\tilde\ell_1}^4t^2+4m_{\tilde\ell_1}^2m_Z^2t^2+6m_Z^4t^2-4m_{\tilde\ell_1}^2t^3-4m_Z^2t^3\\
&\quad+t^4)/(m_Z^4(m_{\tilde\ell_2}^2-t)^2)\\
\mathcal{T}_{\rm VII}\times\mathcal{T}_{\rm VII} &= (m_{\tilde\ell_1}^8-4m_{\tilde\ell_1}^6m_Z^2+6m_{\tilde\ell_1}^4m_Z^4-4m_{\tilde\ell_1}^2m_Z^6+m_Z^8-4m_{\tilde\ell_1}^6u+4m_{\tilde\ell_1}^4m_Z^2u\\
&\quad+4m_{\tilde\ell_1}^2m_Z^4u-4m_Z^6u+6m_{\tilde\ell_1}^4u^2+4m_{\tilde\ell_1}^2m_Z^2u^2+6m_Z^4u^2-4m_{\tilde\ell_1}^2u^3-4m_Z^2u^3\\
&\quad+u^4)/(m_Z^4(m_{\tilde\ell_2}^2-u)^2)\\
\mathcal{T}_{\rm VI}\times\mathcal{T}_{\rm VII} &= (m_{\tilde\ell_1}^8+12m_{\tilde\ell_1}^6m_Z^2+38m_{\tilde\ell_1}^4m_Z^4+12m_{\tilde\ell_1}^2m_Z^6+m_Z^8-4m_{\tilde\ell_1}^4m_Z^2s-24m_{\tilde\ell_1}^2m_Z^4s\\
&\quad-4m_Z^6s+4m_Z^4s^2-2m_{\tilde\ell_1}^6t-14m_{\tilde\ell_1}^4m_Z^2t-14m_{\tilde\ell_1}^2m_Z^4t-2m_Z^6t+4m_{\tilde\ell_1}^2m_Z^2st\\
&\quad+4m_Z^4st+m_{\tilde\ell_1}^4t^2+2m_{\tilde\ell_1}^2m_Z^2t^2+m_Z^4t^2-2m_{\tilde\ell_1}^6u-14m_{\tilde\ell_1}^4m_Z^2u-14m_{\tilde\ell_1}^2m_Z^4u\\
&\quad-2m_Z^6u+4m_{\tilde\ell_1}^2m_Z^2su+4m_Z^4su+4m_{\tilde\ell_1}^4tu+16m_{\tilde\ell_1}^2m_Z^2tu+4m_Z^4tu\\
&\quad-4m_Z^2stu-2m_{\tilde\ell_1}^2t^2u-2m_Z^2t^2u+m_{\tilde\ell_1}^4u^2+2m_{\tilde\ell_1}^2m_Z^2u^2+m_Z^4u^2-2m_{\tilde\ell_1}^2tu^2\\
&\quad-2m_Z^2tu^2+t^2u^2)/(m_Z^4(m_{\tilde\ell_2}^2-t)(m_{\tilde\ell_2}^2-u))\\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm VI} &= (-6m_{\tilde\ell_1}^4m_Z^2-20m_{\tilde\ell_1}^2m_Z^4-6m_Z^6+m_{\tilde\ell_1}^4s+2m_{\tilde\ell_1}^2m_Z^2s+5m_Z^4s+8m_{\tilde\ell_1}^2m_Z^2t\\
&\quad+8m_Z^4t-2m_{\tilde\ell_1}^2st-2m_Z^2st-2m_Z^2t^2+st^2+4m_{\tilde\ell_1}^2m_Z^2u+4m_Z^4u-4m_Z^2tu)\\
&\quad/(2m_Z^4(m_H^2-s)(m_{\tilde\ell_2}^2-t))\\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm VI} &= (-6m_{\tilde\ell_1}^4m_Z^2-20m_{\tilde\ell_1}^2m_Z^4-6m_Z^6+m_{\tilde\ell_1}^4s+2m_{\tilde\ell_1}^2m_Z^2s+5m_Z^4s+8m_{\tilde\ell_1}^2m_Z^2t\\
&\quad+8m_Z^4t-2m_{\tilde\ell_1}^2st-2m_Z^2st-2m_Z^2t^2+st^2+4m_{\tilde\ell_1}^2m_Z^2u+4m_Z^4u-4m_Z^2tu)\\
&\quad/(2m_Z^4(m_h^2-s)(m_{\tilde\ell_2}^2-t))\\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm VI} &= -((m_{\tilde\ell_1}^4+(m_Z^2-t)^2-2m_{\tilde\ell_1}^2(m_Z^2+t))^2/(m_Z^4(m_{\tilde\ell_1}^2-t)(-m_{\tilde\ell_2}^2+t)))\\
\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm VI} &= (m_{\tilde\ell_1}^4+m_Z^4+m_{\tilde\ell_1}^2(6m_Z^2-t-u)+tu-m_Z^2(2s+t+u))^2\\
&\quad/(m_Z^4(m_{\tilde\ell_2}^2-t)(m_{\tilde\ell_1}^2-u))\\
\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm VI} &= (-6m_{\tilde\ell_1}^4m_Z^2-20m_{\tilde\ell_1}^2m_Z^4-6m_Z^6+m_{\tilde\ell_1}^4s+2m_{\tilde\ell_1}^2m_Z^2s+5m_Z^4s+8m_{\tilde\ell_1}^2m_Z^2t\\
&\quad+8m_Z^4t-2m_{\tilde\ell_1}^2st-2m_Z^2st-2m_Z^2t^2+st^2+4m_{\tilde\ell_1}^2m_Z^2u+4m_Z^4u-4m_Z^2tu)\\
&\quad/(2m_Z^4(m_{\tilde\ell_2}^2-t))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm VII} &= (-6m_{\tilde{\ell}_1}^4 m_Z^2 - 20m_{\tilde{\ell}_1}^2 m_Z^4 - 6m_Z^6 + m_{\tilde{\ell}_1}^4 s + 2m_{\tilde{\ell}_1}^2 m_Z^2 s + 5m_Z^4 s + 4m_{\tilde{\ell}_1}^2 m_Z^2 t \\
&\quad +4m_Z^4 t + 8m_{\tilde{\ell}_1}^2 m_Z^2 u + 8m_Z^4 u - 2m_{\tilde{\ell}_1}^2 su - 2m_Z^2 su - 4m_Z^2 tu - 2m_Z^2 u^2 + su^2) \\
&\quad /(2m_Z^4(m_H^2 - s)(m_{\tilde{\ell}_2}^2 - u)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm VII} &= (-6m_{\tilde{\ell}_1}^4 m_Z^2 - 20m_{\tilde{\ell}_1}^2 m_Z^4 - 6m_Z^6 + m_{\tilde{\ell}_1}^4 s + 2m_{\tilde{\ell}_1}^2 m_Z^2 s + 5m_Z^4 s + 4m_{\tilde{\ell}_1}^2 m_Z^2 t \\
&\quad +4m_Z^4 t + 8m_{\tilde{\ell}_1}^2 m_Z^2 u + 8m_Z^4 u - 2m_{\tilde{\ell}_1}^2 su - 2m_Z^2 su - 4m_Z^2 tu - 2m_Z^2 u^2 + su^2) \\
&\quad /(2m_Z^4(m_h^2 - s)(m_{\tilde{\ell}_2}^2 - u)) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm VII} &= (m_{\tilde{\ell}_1}^4 + m_Z^4 + m_{\tilde{\ell}_1}^2(6m_Z^2 - t - u) + tu - m_Z^2(2s + t + u))^2 \\
&\quad /(m_Z^4(m_{\tilde{\ell}_1}^2 - t)(m_{\tilde{\ell}_2}^2 - u)) \\
\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm VII} &= -((m_{\tilde{\ell}_1}^4 + (m_Z^2 - u)^2 - 2m_{\tilde{\ell}_1}^2(m_Z^2 + u))^2/(m_Z^4(m_{\tilde{\ell}_1}^2 - u)(-m_{\tilde{\ell}_2}^2 + u))) \\
\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm VII} &= (-6m_{\tilde{\ell}_1}^4 m_Z^2 - 20m_{\tilde{\ell}_1}^2 m_Z^4 - 6m_Z^6 + m_{\tilde{\ell}_1}^4 s + 2m_{\tilde{\ell}_1}^2 m_Z^2 s + 5m_Z^4 s + 4m_{\tilde{\ell}_1}^2 m_Z^2 t \\
&\quad +4m_Z^4 t + 8m_{\tilde{\ell}_1}^2 m_Z^2 u + 8m_Z^4 u - 2m_{\tilde{\ell}_1}^2 su - 2m_Z^2 su - 4m_Z^2 tu - 2m_Z^2 u^2 + su^2) \\
&\quad /(2m_Z^4(m_{\tilde{\ell}_2}^2 - u)) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + f_4^2\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} + f_5^2\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V} + f_6^2\mathcal{T}_{\rm VI}\times\mathcal{T}_{\rm VI} \\
&\quad +f_7^2\mathcal{T}_{\rm VII}\times\mathcal{T}_{\rm VII} + 2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} + 2f_1f_4\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} + 2f_1f_5\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V} \\
&\quad +2f_1f_6\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm VI} + 2f_1f_7\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm VII} + 2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} + 2f_2f_4\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} + 2f_2f_5\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V} \\
&\quad +2f_2f_6\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm VI} + 2f_2f_7\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm VII} + 2f_3f_4\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} + 2f_3f_5\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V} \\
&\quad +2f_3f_6\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm VI} + 2f_3f_7\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm VII} + 2f_4f_5\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} + 2f_4f_6\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm VI} \\
&\quad +2f_4f_7\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm VII} + 2f_5f_6\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm VI} + 2f_5f_7\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm VII} \qquad \text{(B3)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow \gamma\gamma$

There is no change for this channel, except that $\widetilde{\tau}_{\rm R} \to \widetilde{\ell}_1$.

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow Z\gamma$

There is no change for this channel, except that $\widetilde{\tau}_{\rm R} \to \widetilde{\ell}_1$.

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow Zh[H]$

Besides the $\widetilde{\ell}_1$ exchange (I and II in [17]), we also have $\widetilde{\ell}_2$ exchanged, written here as IV and V.

IV. t-channel $\widetilde{\ell}_2$ exchange

V. u-channel $\widetilde{\ell}_2$ exchange

The couplings are modified.

$$
\begin{aligned}
f_1 &= (-g_2/\cos\theta_W(\sin^2\theta_W-\cos^2\theta_\ell/2))(g_2 m_Z/\cos\theta_W((-1/2+\sin^2\theta_W) \\
&\quad \sin[-\cos](\beta+\alpha)\cos^2\theta_\ell-\sin^2\theta_W\sin[-\cos](\beta+\alpha)\sin^2\theta_\ell) \\
&\quad +g_2 m_\ell^2/(m_W\cos\beta)\sin[-\cos]\alpha-g_2 m_\ell/(m_W\cos\beta)(A_\ell\sin[-\cos]\alpha \\
&\quad -\mu\cos[\sin]\alpha)\sin\theta_\ell\cos\theta_\ell) \\
f_2 &= -(-g_2/\cos\theta_W(\sin^2\theta_W-\cos^2\theta_\ell/2))(g_2 m_Z/\cos\theta_W((-1/2+\sin^2\theta_W) \\
&\quad \sin[-\cos](\beta+\alpha)\cos^2\theta_\ell-\sin^2\theta_W\sin[-\cos](\beta+\alpha)\sin^2\theta_\ell) \\
&\quad +g_2 m_\ell^2/(m_W\cos\beta)\sin[-\cos]\alpha-g_2 m_\ell/(m_W\cos\beta)(A_\ell\sin[-\cos]\alpha \\
&\quad -\mu\cos[\sin]\alpha)\sin\theta_\ell\cos\theta_\ell) \\
f_3 &= (-g_2/\cos\theta_W(\sin^2\theta_W-\cos^2\theta_\ell/2))(-g_2 m_Z\sin[\cos](\beta-\alpha)/\cos\theta_W) \\
f_4 &= (-g_2\cos\theta_\ell\sin\theta_\ell/(2\cos\theta_W))(g_2 m_Z/\cos\theta_W((-1/2+\sin^2\theta_W) \\
&\quad \sin[-\cos](\beta+\alpha)(-\cos\theta_\ell\sin\theta_\ell)-\sin^2\theta_W\sin[-\cos](\beta+\alpha)\cos\theta_\ell\sin\theta_\ell) \\
&\quad -g_2 m_\ell/(2m_W\cos\beta)(A_\ell\sin[-\cos]\alpha-\mu\cos[\sin]\alpha)\cos(2\theta_\ell)) \\
f_5 &= (-)(-g_2\cos\theta_\ell\sin\theta_\ell/(2\cos\theta_W))(g_2 m_Z/\cos\theta_W((-1/2+\sin^2\theta_W) \\
&\quad \sin[-\cos](\beta+\alpha)(-\cos\theta_\ell\sin\theta_\ell)-\sin^2\theta_W\sin[-\cos](\beta+\alpha)\cos\theta_\ell\sin\theta_\ell) \\
&\quad -g_2 m_\ell/(2m_W\cos\beta)(A_\ell\sin[-\cos]\alpha-\mu\cos[\sin]\alpha)\cos(2\theta_\ell)) \\
\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} &= (m_{\tilde\ell_1}^4+(m_Z^2-t)^2-2m_{\tilde\ell_1}^2(m_Z^2+t))/(m_Z^2(m_{\tilde\ell_2}^2-t)^2) \\
\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V} &= (m_{\tilde\ell_1}^4+(m_Z^2-u)^2-2m_{\tilde\ell_1}^2(m_Z^2+u))/(m_Z^2(m_{\tilde\ell_2}^2-u)^2) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} &= -((m_{\tilde\ell_1}^4+(m_Z^2-t)^2-2m_{\tilde\ell_1}^2(m_Z^2+t))/(m_Z^2(m_{\tilde\ell_1}^2-t)(-m_{\tilde\ell_2}^2+t))) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V} &= (m_{\tilde\ell_1}^4+m_Z^4+m_{\tilde\ell_1}^2(6m_Z^2-t-u)+tu-m_Z^2(2s+t+u)) \\
&\quad /(m_Z^2(m_{\tilde\ell_1}^2-t)(m_{\tilde\ell_2}^2-u)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} &= (m_{\tilde\ell_1}^4+m_Z^4+m_{\tilde\ell_1}^2(6m_Z^2-t-u)+tu-m_Z^2(2s+t+u)) \\
&\quad /(m_Z^2(m_{\tilde\ell_2}^2-t)(m_{\tilde\ell_1}^2-u)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V} &= -((m_{\tilde\ell_1}^4+(m_Z^2-u)^2-2m_{\tilde\ell_1}^2(m_Z^2+u))/(m_Z^2(m_{\tilde\ell_1}^2-u)(-m_{\tilde\ell_2}^2+u))) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} &= (t(t-u)+m_{\tilde\ell_1}^2(-8m_Z^2-t+u)+m_Z^2(2s-t+u)) \\
&\quad /(2m_Z^2(m_Z^2-s)(m_{\tilde\ell_2}^2-t))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V} &= ((t-u)u + m_{\tilde{\ell}_1}^2(8m_Z^2 - t + u) + m_Z^2(-2s - t + u)) \\
&\quad /(2m_Z^2(m_Z^2 - s)(m_{\tilde{\ell}_2}^2 - u)) \\
\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} &= (m_{\tilde{\ell}_1}^4 + m_Z^4 + m_{\tilde{\ell}_1}^2(6m_Z^2 - t - u) + tu - m_Z^2(2s + t + u)) \\
&\quad /(m_Z^2(m_{\tilde{\ell}_2}^2 - t)(m_{\tilde{\ell}_2}^2 - u)) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + f_4^2\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} + f_5^2\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V} \\
&\quad +2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} + 2f_1f_4\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} + 2f_1f_5\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V} \\
&\quad +2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} + 2f_2f_4\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} + 2f_2f_5\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V} + 2f_3f_4\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} \\
&\quad +2f_3f_5\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V} + 2f_4f_5\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} \qquad \text{(B4)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow \gamma h[H]$

There is no new channels, however the couplings are modified.

$$
\begin{aligned}
f_1 &= (e)(g_2m_Z/\cos\theta_W((-1/2 + \sin^2\theta_W)\sin[-\cos](\beta+\alpha)\cos^2\theta_\ell \\
&\quad - \sin^2\theta_W\sin[-\cos](\beta+\alpha)\sin^2\theta_\ell) + g_2m_\ell^2/(m_W\cos\beta)\sin[-\cos]\alpha \\
&\quad -g_2m_\ell/(m_W\cos\beta)(A_\ell\sin[-\cos]\alpha - \mu\cos[\sin]\alpha)\sin\theta_\ell\cos\theta_\ell) \\
f_2 &= -(e)(g_2m_Z/\cos\theta_W((-1/2 + \sin^2\theta_W)\sin[-\cos](\beta+\alpha)\cos^2\theta_\ell \\
&\quad - \sin^2\theta_W\sin[-\cos](\beta+\alpha)\sin^2\theta_\ell) + g_2m_\ell^2/(m_W\cos\beta)\sin[-\cos]\alpha \\
&\quad -g_2m_\ell/(m_W\cos\beta)(A_\ell\sin[-\cos]\alpha - \mu\cos[\sin]\alpha)\sin\theta_\ell\cos\theta_\ell) \qquad \text{(B5)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow ZA$

We now have t- and u-channel $\widetilde{\ell}_2$ exchanges, written here as III and IV.
III. t-channel $\widetilde{\ell}_2$ exchange
IV. u-channel $\widetilde{\ell}_2$ exchange
The couplings f_1 and f_2 are also modified.

$$
\begin{aligned}
f_1 &= (g_2m_Z/\cos\theta_W((-1/2 + \sin^2\theta_W)\sin(\beta+\alpha)\cos^2\theta_\ell - \sin^2\theta_W\sin(\beta+\alpha)\sin^2\theta_\ell) \\
&\quad +g_2m_\ell^2/(m_W\cos\beta)\sin\alpha - g_2m_\ell/(m_W\cos\beta)(A_\ell\sin\alpha - \mu\cos\alpha)\sin\theta_\ell\cos\theta_\ell) \\
&\quad (-g_2\cos(\alpha-\beta)/(2\cos\theta_W)) \\
f_2 &= (g_2m_Z/\cos\theta_W((1/2 - \sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell + \sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell) \\
&\quad -g_2m_\ell^2/(m_W\cos\beta)\cos\alpha - g_2m_\ell/(m_W\cos\beta)(-A_\ell\cos\alpha - \mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell)
\end{aligned}
$$

$$
\begin{aligned}
& (-g_2 \sin(\alpha-\beta)/(2\cos\theta_W)) \\
f_3 &= (-g_2 \cos\theta_\ell \sin\theta_\ell/(2\cos\theta_W))(-g_2 m_\ell/(2m_W)(A_\ell \tan\beta - \mu)) \\
f_4 &= (-g_2 \cos\theta_\ell \sin\theta_\ell/(2\cos\theta_W))(-g_2 m_\ell/(2m_W)(A_\ell \tan\beta - \mu)) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= (m^4_{\tilde\ell_1} + (m_Z^2 - t)^2 - 2m^2_{\tilde\ell_1}(m_Z^2 + t))/(m_Z^2(m^2_{\tilde\ell_2} - t)^2) \\
\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} &= (m^4_{\tilde\ell_1} + (m_Z^2 - u)^2 - 2m^2_{\tilde\ell_1}(m_Z^2 + u))/(m_Z^2(m^2_{\tilde\ell_2} - u)^2) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} &= (m^4_{\tilde\ell_1} + m_Z^4 + m^2_{\tilde\ell_1}(6m_Z^2 - t - u) + tu - m_Z^2(2s + t + u)) \\
& /(m_Z^2(m^2_{\tilde\ell_2} - t)(m^2_{\tilde\ell_2} - u)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} &= (-m_Z^4 + m_Z^2 s + m^2_{\tilde\ell_1}(-3m_Z^2 + s) + m_Z^2 t - st + m_A^2(-m^2_{\tilde\ell_1} - 3m_Z^2 + t) \\
& +2m_Z^2 u)/(m_Z^2(m_h^2 - s)(m^2_{\tilde\ell_2} - t)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} &= (-m_Z^4 + m_Z^2 s + m^2_{\tilde\ell_1}(-3m_Z^2 + s) + m_Z^2 t - st + m_A^2(-m^2_{\tilde\ell_1} - 3m_Z^2 + t) \\
& +2m_Z^2 u)/(m_Z^2(m_H^2 - s)(m^2_{\tilde\ell_2} - t)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} &= (-m_Z^4 + m_Z^2 s + m^2_{\tilde\ell_1}(-3m_Z^2 + s) + 2m_Z^2 t + m_Z^2 u - su + m_A^2(-m^2_{\tilde\ell_1} - 3m_Z^2 \\
& +u))/(m_Z^2(m_h^2 - s)(m^2_{\tilde\ell_2} - u)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} &= (-m_Z^4 + m_Z^2 s + m^2_{\tilde\ell_1}(-3m_Z^2 + s) + 2m_Z^2 t + m_Z^2 u - su + m_A^2(-m^2_{\tilde\ell_1} - 3m_Z^2 \\
& +u))/(m_Z^2(m_H^2 - s)(m^2_{\tilde\ell_2} - u)) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + f_4^2\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} + 2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} \\
& +2f_1f_4\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} + 2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} + 2f_2f_4\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} + 2f_3f_4\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} \qquad \text{(B6)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow \ell\overline{\ell}$

The s-channel h and H annihilations are neglected, due to the small Yukawa couplings for leptons. The t-channel χ exchange is modified, with more couplings introduced $(K, K') \rightarrow (K_a, K_b, K'_a, K'_b)$. The terms with m_ℓ were neglected in [17], but not here. The couplings f_{3c} and f_{3d} are modified while f_{4c} remains the same.

$$
\begin{aligned}
f_{3c} &= (-g_2(\sin^2\theta_W - \cos^2\theta_\ell/2)/\cos\theta_W)(g_2(1 - 4\sin^2\theta_W)/(4\cos\theta_W)) \\
f_{3d} &= (-g_2(\sin^2\theta_W - \cos^2\theta_\ell/2)/\cos\theta_W)(-g_2/(4\cos\theta_W)) \\
K_a &= -\sin\theta_\ell(g_2 m_\ell/(2m_W\cos\beta)N_{i3} + g_1 N_{i1})/\sqrt{2} \\
& -\cos\theta_\ell(g_2 m_\ell/(2m_W\cos\beta)N_{i3} - (g_1 N_{i1} + g_2/2(N_{i2} - N_{i1}\tan\theta_W)))/\sqrt{2} \\
K_b &= -\sin\theta_\ell(g_2 m_\ell/(2m_W\cos\beta)N_{i3} - g_1 N_{i1})/\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
&\quad -\cos\theta_\ell(-g_2 m_\ell/(2m_W\cos\beta)N_{i3}-(g_1N_{i1}+g_2/2(N_{i2}-N_{i1}\tan\theta_W)))/\sqrt{2}\\
K'_a &= -\sin\theta_\ell(g_2 m_\ell/(2m_W\cos\beta)N_{j3}+g_1N_{j1})/\sqrt{2}\\
&\quad -\cos\theta_\ell(g_2 m_\ell/(2m_W\cos\beta)N_{j3}-(g_1N_{j1}+g_2/2(N_{j2}-N_{j1}\tan\theta_W)))/\sqrt{2}\\
K'_b &= -\sin\theta_\ell(g_2 m_\ell/(2m_W\cos\beta)N_{j3}-g_1N_{j1})/\sqrt{2}\\
&\quad -\cos\theta_\ell(-g_2 m_\ell/(2m_W\cos\beta)N_{j3}-(g_1N_{j1}+g_2/2(N_{j2}-N_{j1}\tan\theta_W)))/\sqrt{2}\\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V} &= (2f_{3d}K_bK_a(-16m_{\tilde\ell_1}^2m_\ell^2+4m_{\tilde\ell_1}^2s+4m_\ell^2s-s^2+t^2-2tu+u^2)\\
&\quad +f_{3c}(K_a^2(-4m_{\tilde\ell_1}^2s+s^2-4m_\ell^2t-4m_\ell m_{\tilde\chi_i^0}t-t^2+4m_\ell^2u+4m_\ell m_{\tilde\chi_i^0}u+2tu-u^2)\\
&\quad +K_b^2(-4m_{\tilde\ell_1}^2s+s^2-4m_\ell^2t+4m_\ell m_{\tilde\chi_i^0}t-t^2+4m_\ell^2u-4m_\ell m_{\tilde\chi_i^0}u+2tu-u^2)))\\
&\quad /((m_Z^2-s)(m_{\tilde\chi_i^0}^2-t))\\
\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} &= -(f_{4c}(K_a^2(-4m_{\tilde\ell_1}^2s+s^2-4m_\ell^2t-4m_\ell m_{\tilde\chi_i^0}t-t^2+4m_\ell^2u+4m_\ell m_{\tilde\chi_i^0}u+2tu\\
&\quad -u^2)+K_b^2(-4m_{\tilde\ell_1}^2s+s^2-4m_\ell^2t+4m_\ell m_{\tilde\chi_i^0}t-t^2+4m_\ell^2u-4m_\ell m_{\tilde\chi_i^0}u+2tu\\
&\quad -u^2))/(s(m_{\tilde\chi_i^0}^2-t)))\\
\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V} &= (2(K_a'^2K_b^2(-m_{\tilde\ell_1}^4-2m_{\tilde\ell_1}^2m_\ell^2+3m_\ell^4-m_\ell^3m_{\tilde\chi_i^0}-3m_\ell^3m_{\tilde\chi_i^0}+m_\ell^3m_{\tilde\chi_j^0}+3m_\ell^3m_{\tilde\chi_j^0}\\
&\quad -2m_\ell^2m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}-2m_\ell^2m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}+m_{\tilde\ell_1}^2s-m_\ell^2s+m_\ell m_{\tilde\chi_i^0}s-m_\ell m_{\tilde\chi_j^0}s\\
&\quad +m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}s+m_{\tilde\ell_1}^2t+2m_\ell^2t+m_\ell^2t-m_\ell m_{\tilde\chi_i^0}t+m_\ell m_{\tilde\chi_j^0}t+m_{\tilde\ell_1}^2u-m_\ell^2u\\
&\quad +m_\ell m_{\tilde\chi_i^0}u-m_\ell m_{\tilde\chi_j^0}u-tu)+K'_aK_aK'_bK_b(-2m_{\tilde\ell_1}^4-4m_{\tilde\ell_1}^2m_\ell^2+6m_\ell^4+2m_{\tilde\ell_1}^2s\\
&\quad -2m_\ell^2s+2m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}s+2m_{\tilde\ell_1}^2t-4m_\ell^2t+2m_\ell^2t+2m_{\tilde\ell_1}^2u-2m_\ell^2u-2tu)\\
&\quad +K_a^2K_b'^2(-m_{\tilde\ell_1}^4-2m_{\tilde\ell_1}^2m_\ell^2+3m_\ell^4+4m_\ell^3m_{\tilde\chi_i^0}-4m_\ell^3m_{\tilde\chi_j^0}-4m_\ell^2m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}\\
&\quad +m_{\tilde\ell_1}^2s-m_\ell^2s-m_\ell m_{\tilde\chi_i^0}s+m_\ell m_{\tilde\chi_j^0}s+m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}s+m_{\tilde\ell_1}^2t+3m_\ell^2t+m_\ell m_{\tilde\chi_i^0}t\\
&\quad -m_\ell m_{\tilde\chi_j^0}t+m_{\tilde\ell_1}^2u-m_\ell^2u-m_\ell m_{\tilde\chi_i^0}u+m_\ell m_{\tilde\chi_j^0}u-tu)\\
&\quad +(K_aK'_a+K_bK'_b)(K_aK'_a+K_bK'_b)(-m_\ell^2s-m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}s+m_{\tilde\ell_1}^2t+m_\ell^2t\\
&\quad +m_{\tilde\ell_1}^2u-m_\ell^2u-tu)+(K_aK'_a+K_bK'_b)(K_aK'_a-K_bK'_b)(4m_\ell^3m_{\tilde\chi_i^0}\\
&\quad +4m_\ell^3m_{\tilde\chi_j^0}-m_\ell m_{\tilde\chi_i^0}s-m_\ell m_{\tilde\chi_j^0}s+m_\ell m_{\tilde\chi_i^0}t+m_\ell m_{\tilde\chi_j^0}t-m_\ell m_{\tilde\chi_i^0}u-m_\ell m_{\tilde\chi_j^0}u)\\
&\quad +(K_aK'_a-K_bK'_b)(K_aK'_a-K_bK'_b)(2m_\ell^2m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}+2m_\ell^2t)))\\
&\quad /((m_{\tilde\chi_i^0}^2-t)(-m_{\tilde\chi_j^0}^2+t))\\
|\mathcal{T}|^2 &= \mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III}+\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV}+\sum_{i,j}\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V}+2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV}+2\sum_i(\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V}+\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V}) \qquad \text{(B7)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow f\bar{f}$

When f is a fermion other than a lepton or t quark, again the s-channel h and H annihilations can be neglected. We separate the channel $\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow \nu_\ell\bar{\nu}_\ell$ because it has t-channel chargino exchange. The couplings f_{3c} and f_{3d} are modified while f_{4c} remains the same.

$$\begin{aligned} f_{3c} &= (-g_2/\cos\theta_W(\sin^2\theta_W - \cos^2\theta_\ell/2))(g_2(-2T_3^f + 4Q_f\sin^2\theta_W)/(4\cos\theta_W)) \\ f_{3d} &= (-g_2/\cos\theta_W(\sin^2\theta_W - \cos^2\theta_\ell/2))(g_2(2T_3^f)/(4\cos\theta_W)) \end{aligned} \tag{B8}$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow \nu_\ell\bar{\nu}_\ell$

With $\widetilde{\ell}_R$ we have t-channel chargino exchange.
III. s-channel Z annihilation
IV. t-channel $\widetilde{\chi}^-_{(1,2)}$ exchange

$$\begin{aligned} f_3 &= (-g_2/\cos\theta_W(\sin^2\theta_W - \cos^2\theta_\ell/2))(-g_2/(2\cos\theta_W)) \\ f_4(i) &= (g_2 m_\ell/(\sqrt{2}m_W\cos\beta)U_{i2}\sin\theta_\ell - g_2U_{i1}\cos\theta_\ell)^2 \\ \mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= (-4m^2_{\tilde{\ell}_1}s + s^2 + 2tu - t^2 - u^2)/(s-m_Z^2)^2 \\ \mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} &= (-m^2_{\tilde{\ell}_1}s - m^2_{\tilde{\ell}_1}t - m^2_{\tilde{\ell}_1}u + m^4_{\tilde{\ell}_1} + tu)/((t-m^2_{\tilde{\chi}_i^+})(t-m^2_{\tilde{\chi}_j^+})) \\ \mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} &= -(-2m^2_{\tilde{\ell}_1}s + 1/2s^2 + tu - 1/2t^2 - 1/2u^2)/((s-m_Z^2)(t-m^2_{\tilde{\chi}_i^+})) \\ |\mathcal{T}|^2 &= f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + \sum_{i,j} f_4(i)f_4(j)\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} + 2f_3\sum_i f_4(i)\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} \end{aligned} \tag{B9}$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow t\bar{t}$

The couplings f_{1a}, f_{2a}, f_{3c} and f_{3d} are modified while f_{4c} remains the same.

$$\begin{aligned} f_{1a} &= (-g_2m_t\sin\alpha/(2m_W\sin\beta))(g_2m_Z((1/2-\sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell \\ &\quad +\sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W - g_2m_\ell^2\cos\alpha/(m_W\cos\beta) \\ &\quad -g_2m_\ell(-A_\ell\cos\alpha - \mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\ f_{2a} &= (-g_2m_t\cos\alpha/(2m_W\sin\beta))(g_2m_Z((-1/2+\sin^2\theta_W)\sin(\beta+\alpha)\cos^2\theta_\ell \\ &\quad -\sin^2\theta_W\sin(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W + g_2m_\ell^2\sin\alpha/(m_W\cos\beta) \\ &\quad -g_2m_\ell(A_\ell\sin\alpha - \mu\cos\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\ f_{3c} &= (-g_2(\sin^2\theta_W - \cos^2\theta_\ell/2)/\cos\theta_W)(g_2(-1+8/3\sin^2\theta_W)/(4\cos\theta_W)) \\ f_{3d} &= (-g_2(\sin^2\theta_W - \cos^2\theta_\ell/2)/\cos\theta_W)(g_2/(4\cos\theta_W)) \end{aligned} \tag{B10}$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow hh$

The couplings are modified. Channels IV (t-channel $\widetilde{\ell}$ exchange) and V (u-channel $\widetilde{\ell}$ exchange) are now summed over $\widetilde{\ell}_1$ and $\widetilde{\ell}_2$, with appropriate propagators.

$$
\begin{aligned}
f_1 &= (-3g_2 m_z \cos(2\alpha)\sin(\alpha+\beta)/(2\cos\theta_W))(g_2 m_z((1/2-\sin^2\theta_W)\cos(\beta+\alpha) \\
&\quad \cos^2\theta_\ell + \sin^2\theta_W \cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W - g_2 m_\ell^2 \cos\alpha/(m_W\cos\beta) \\
&\quad -g_2 m_\ell(-A_\ell\cos\alpha - \mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
f_2 &= (g_2 m_z(\cos(2\alpha)\cos(\alpha+\beta) - 2\sin(2\alpha)\sin(\alpha+\beta))/(2\cos\theta_W)) \\
&\quad (g_2 m_z((1/2-\sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell + \sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
&\quad -g_2 m_\ell^2\cos\alpha/(m_W\cos\beta) - g_2 m_\ell(-A_\ell\cos\alpha - \mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
f_3 &= (g_2^2/2)((-1/2+\sin^2\theta_W)\cos(2\alpha)/\cos^2\theta_W - m_\ell^2\sin^2\alpha/(m_W^2\cos^2\beta))\cos^2\theta_\ell \\
&\quad +(g_2^2/2)(-\sin^2\theta_W\cos(2\alpha)/\cos^2\theta_W - m_\ell^2\sin^2\alpha/(m_W^2\cos^2\beta))\sin^2\theta_\ell \\
f_4(1) &= (g_2 m_z((1/2-\sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell + \sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
&\quad -g_2 m_\ell^2\cos\alpha/(m_W\cos\beta) - g_2 m_\ell(-A_\ell\cos\alpha - \mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta))^2 \\
f_4(2) &= (g_2 m_z\cos\theta_\ell\sin\theta_\ell((1/2-\sin^2\theta_W)\sin(\beta+\alpha) - \sin^2\theta_W\sin(\beta+\alpha))/\cos\theta_W \\
&\quad -g_2 m_\ell(A_\ell\sin\alpha - \mu\cos\alpha)\cos(2\theta_\ell)/(2m_W\cos\beta))^2 \\
f_5(1) &= (g_2 m_z((1/2-\sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell + \sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
&\quad -g_2 m_\ell^2\cos\alpha/(m_W\cos\beta) - g_2 m_\ell(-A_\ell\cos\alpha - \mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta))^2 \\
f_5(2) &= (g_2 m_z\cos\theta_\ell\sin\theta_\ell((1/2-\sin^2\theta_W)\sin(\beta+\alpha) - \sin^2\theta_W\sin(\beta+\alpha))/\cos\theta_W \\
&\quad -g_2 m_\ell(A_\ell\sin\alpha - \mu\cos\alpha)\cos(2\theta_\ell)/(2m_W\cos\beta))^2 \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + \sum_{i,j}(f_4(i)f_4(j)\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} + f_5(i)f_5(j)\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V}) \\
&\quad +2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} + 2\sum_i(f_1f_4(i)\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} + f_1f_5(i)\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V}) \\
&\quad +2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} + 2\sum_i(f_2f_4(i)\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} + f_2f_5(i)\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V}) \\
&\quad +2\sum_i(f_3f_4(i)\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} + f_3f_5(i)\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V}) + 2\sum_{i,j}f_4(i)f_5(j)\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} \qquad \text{(B11)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow hA[HA]$

Besides the s-channel Z annihilation, we have t- and u-channel $\widetilde{\ell}_2$ exchanges.

II. t-channel $\widetilde{\ell}_2$ exchange

III. u-channel $\widetilde{\ell}_2$ exchange

The coupling f_1 is also modified.

$$\begin{aligned}
f_1 &= (-g_2\cos[\sin](\alpha-\beta)/(2\cos\theta_W))(-g_2(\sin^2\theta_W - \cos^2\theta_\ell/2)/\cos\theta_W) \\
f_2 &= (-g_2 m_\ell(A_\ell\tan\beta-\mu)/(2m_W))(g_2 m_Z((1/2-\sin^2\theta_W)\sin[-\cos](\beta+\alpha) \\
&\quad -\sin^2\theta_W\sin[-\cos](\beta+\alpha))\cos\theta_\ell\sin\theta_\ell/\cos\theta_W \\
&\quad -g_2 m_\ell(A_\ell\sin[-\cos]\alpha-\mu\cos[\sin]\alpha)\cos(2\theta_\ell)/(2m_W\cos\beta)) \\
f_3 &= (-g_2 m_\ell(A_\ell\tan\beta-\mu)/(2m_W))(g_2 m_Z((1/2-\sin^2\theta_W)\sin[-\cos](\beta+\alpha) \\
&\quad -\sin^2\theta_W\sin[-\cos](\beta+\alpha))\cos\theta_\ell\sin\theta_\ell/\cos\theta_W \\
&\quad -g_2 m_\ell(A_\ell\sin[-\cos]\alpha-\mu\cos[\sin]\alpha)\cos(2\theta_\ell)/(2m_W\cos\beta)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= 1/(m_{\tilde{\ell}_2}^2-t)^2 \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= 1/(m_{\tilde{\ell}_2}^2-u)^2 \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= (t-u)/((m_Z^2-s)(m_{\tilde{\ell}_2}^2-t)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} &= (t-u)/((m_Z^2-s)(m_{\tilde{\ell}_2}^2-u)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} &= 1/((m_{\tilde{\ell}_2}^2-t)(m_{\tilde{\ell}_2}^2-u)) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + 2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} \\
&\quad +2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} \qquad \text{(B12)}
\end{aligned}$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow W^+H^-$

Besides the s-channel H and h annihilation, we have t-channel sneutrino exchange.

III. t-channel $\widetilde{\nu}_\ell$ exchange.

The couplings f_1 and f_2 are modified.

$$\begin{aligned}
f_1 &= (g_2 m_Z((1/2-\sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell + \sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
&\quad -g_2 m_\ell^2\cos\alpha/(m_W\cos\beta) - g_2 m_\ell(-A_\ell\cos\alpha-\mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
&\quad (g_2\sin(\alpha-\beta)/2) \\
f_2 &= (g_2 m_Z((-1/2+\sin^2\theta_W)\sin(\beta+\alpha)\cos^2\theta_\ell - \sin^2\theta_W\sin(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W
\end{aligned}$$

$$
\begin{aligned}
& +g_2 m_\ell^2 \sin\alpha/(m_W \cos\beta) - g_2 m_\ell (A_\ell \sin\alpha - \mu\cos\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
& (g_2\cos(\alpha-\beta)/2) \\
f_3 &= (-g_2/\sqrt{2}\cos\theta_\ell)(-g_2 m_W \cos\theta_\ell(-m_\ell^2\tan\beta/m_W^2 + \sin(2\beta))/\sqrt{2} \\
& +g_2 m_\ell/(\sqrt{2}m_W)\sin\theta_\ell(\mu - A_\ell\tan\beta)) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= (m_{\tilde\ell_1}^4 + (m_W^2 - t)^2 - 2m_{\tilde\ell_1}^2(m_W^2+t))/(m_W^2(m_{\tilde\nu_\ell}^2 - t)^2) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} &= (-m_W^4 + m_W^2 s + m_{\tilde\ell_1}^2(-3m_W^2+s) + m_W^2 t - st + m_{H^+}^2(-m_{\tilde\ell_1}^2 - 3m_W^2 + t) \\
& +2m_W^2 u)/(m_W^2(m_H^2 - s)(m_{\tilde\nu_\ell}^2 - t)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} &= (-m_W^4 + m_W^2 s + m_{\tilde\ell_1}^2(-3m_W^2+s) + m_W^2 t - st + m_{H^+}^2(-m_{\tilde\ell_1}^2 - 3m_W^2 + t) \\
& +2m_W^2 u)/(m_W^2(m_h^2 - s)(m_{\tilde\nu_\ell}^2 - t)) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + 2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} \\
& +2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III}
\end{aligned}
\tag{B13}
$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow AA$

The new channels are

IV. t-channel $\widetilde{\ell}_2$ exchange

V. u-channel $\widetilde{\ell}_2$ exchange

The couplings are modified.

$$
\begin{aligned}
f_1 &= (g_2 m_Z((1/2 - \sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell + \sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
& -g_2 m_\ell^2\cos\alpha/(m_W\cos\beta) - g_2 m_\ell(-A_\ell\cos\alpha - \mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
& (g_2 m_Z\cos(2\beta)\cos(\alpha+\beta)/(2\cos\theta_W)) \\
f_2 &= (g_2 m_Z((-1/2 + \sin^2\theta_W)\sin(\beta+\alpha)\cos^2\theta_\ell - \sin^2\theta_W\sin(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
& +g_2 m_\ell^2\sin\alpha/(m_W\cos\beta) - g_2 m_\ell(A_\ell\sin\alpha - \mu\cos\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
& (-g_2 m_Z\cos(2\beta)\sin(\alpha+\beta)/(2\cos\theta_W)) \\
f_3 &= g_2^2/2((-1/2+\sin^2\theta_W)\cos(2\beta)/\cos^2\theta_W - m_\ell^2\tan^2\beta/m_W^2)\cos^2\theta_\ell \\
& +g_2^2/2(-\sin^2\theta_W\cos(2\beta)/\cos^2\theta_W - m_\ell^2\tan^2\beta/m_W^2)\sin^2\theta_\ell \\
f_4 &= (-g_2 m_\ell(A_\ell\tan\beta - \mu)/(2m_W))^2 \\
f_5 &= (-g_2 m_\ell(A_\ell\tan\beta - \mu)/(2m_W))^2 \\
\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} &= 1/(m_{\tilde\ell_2}^2 - t)^2
\end{aligned}
$$

$$\begin{aligned}
\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V} &= 1/(m^2_{\tilde{\ell}_2}-u)^2 \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} &= 1/((m^2_H-s)(m^2_{\tilde{\ell}_2}-t)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V} &= 1/((m^2_H-s)(m^2_{\tilde{\ell}_2}-u)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} &= 1/((m^2_h-s)(m^2_{\tilde{\ell}_2}-t)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V} &= 1/((m^2_h-s)(m^2_{\tilde{\ell}_2}-u)) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} &= 1/(m^2_{\tilde{\ell}_2}-t) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V} &= 1/(m^2_{\tilde{\ell}_2}-u) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III}+f_4^2\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV}+f_5^2\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V} \\
&\quad +2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II}+2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III}+2f_1f_4\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV}+2f_1f_5\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V} \\
&\quad +2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III}+2f_2f_4\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV}+2f_2f_5\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V}+2f_3f_4\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} \\
&\quad +2f_3f_5\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V}+2f_4f_5\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} \qquad \text{(B14)}
\end{aligned}$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow hH$

The t- and u-channels $\widetilde{\ell}$ exchanges are now summed over $\widetilde{\ell}_1$ and $\widetilde{\ell}_2$ with appropriate propagators. The couplings are

$$\begin{aligned}
f_1 &= (g_2m_Z((1/2-\sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell+\sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
&\quad -g_2m_\ell^2\cos\alpha/(m_W\cos\beta)-g_2m_\ell(-A_\ell\cos\alpha-\mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
&\quad (g_2m_Z(\cos(2\alpha)\sin(\alpha+\beta)+2\sin(2\alpha)\cos(\alpha+\beta))/(2\cos\theta_W)) \\
f_2 &= (g_2m_Z((-1/2+\sin^2\theta_W)\sin(\beta+\alpha)\cos^2\theta_\ell-\sin^2\theta_W\sin(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
&\quad +g_2m_\ell^2\sin\alpha/(m_W\cos\beta)-g_2m_\ell(A_\ell\sin\alpha-\mu\cos\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
&\quad (g_2m_Z(\cos(2\alpha)\cos(\alpha+\beta)-2\sin(2\alpha)\sin(\alpha+\beta))/(2\cos\theta_W)) \\
f_3 &= g_2^2\sin(2\alpha)((-1/2+\sin^2\theta_W)/\cos^2\theta_W+m_\ell^2/(2m_W^2\cos^2\beta))\cos^2\theta_\ell/2 \\
&\quad +g_2^2\sin(2\alpha)(-\sin^2\theta_W/\cos^2\theta_W+m_\ell^2/(2m_W^2\cos^2\beta))\sin^2\theta_\ell/2 \\
f_4(1) &= (g_2m_Z((1/2-\sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell+\sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
&\quad -g_2m_\ell^2\cos\alpha/(m_W\cos\beta)-g_2m_\ell(-A_\ell\cos\alpha-\mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
&\quad (g_2m_Z((-1/2+\sin^2\theta_W)\sin(\beta+\alpha)\cos^2\theta_\ell-\sin^2\theta_W\sin(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
&\quad +g_2m_\ell^2\sin\alpha/(m_W\cos\beta)-g_2m_\ell(A_\ell\sin\alpha-\mu\cos\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
f_4(2) &= (g_2m_Z((-1/2+\sin^2\theta_W)\cos(\beta+\alpha)+\sin^2\theta_W\cos(\beta+\alpha))\cos\theta_\ell\sin\theta_\ell/\cos\theta_W
\end{aligned}$$

$$
\begin{aligned}
& -g_2 m_\ell(-A_\ell \cos\alpha - \mu \sin\alpha)\cos(2\theta_\ell)/(2m_W \cos\beta)) \\
& (g_2 m_Z((1/2 - \sin^2\theta_W)\sin(\beta+\alpha) - \sin^2\theta_W \sin(\beta+\alpha))\cos\theta_\ell \sin\theta_\ell/\cos\theta_W \\
& -g_2 m_\ell(A_\ell \sin\alpha - \mu\cos\alpha)\cos(2\theta_\ell)/(2m_W\cos\beta)) \\
f_5(1) \;=\; & (g_2 m_Z((1/2 - \sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell + \sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
& -g_2 m_\ell^2 \cos\alpha/(m_W\cos\beta) - g_2 m_\ell(-A_\ell\cos\alpha - \mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
& (g_2 m_Z((-1/2 + \sin^2\theta_W)\sin(\beta+\alpha)\cos^2\theta_\ell - \sin^2\theta_W\sin(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
& +g_2 m_\ell^2\sin\alpha/(m_W\cos\beta) - g_2 m_\ell(A_\ell\sin\alpha - \mu\cos\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
f_5(2) \;=\; & (g_2 m_Z((-1/2 + \sin^2\theta_W)\cos(\beta+\alpha) + \sin^2\theta_W\cos(\beta+\alpha))\cos\theta_\ell\sin\theta_\ell/\cos\theta_W \\
& -g_2 m_\ell(-A_\ell\cos\alpha - \mu\sin\alpha)\cos(2\theta_\ell)/(2m_W\cos\beta)) \\
& (g_2 m_Z((1/2 - \sin^2\theta_W)\sin(\beta+\alpha) - \sin^2\theta_W\sin(\beta+\alpha))\cos\theta_\ell\sin\theta_\ell/\cos\theta_W \\
& -g_2 m_\ell(A_\ell\sin\alpha - \mu\cos\alpha)\cos(2\theta_\ell)/(2m_W\cos\beta)) \\
|\mathcal{T}|^2 \;=\; & f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + \sum_{i,j}(f_4(i)f_4(j)\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} + f_5(i)f_5(j)\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V}) \\
& +2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} + 2\sum_i f_1(f_4(i)\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} + f_5(i)\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V}) \\
& +2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} + 2\sum_i f_2(f_4(i)\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} + f_5(i)\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V}) \\
& +2f_3\sum_i(f_4(i)\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} + f_5(i)\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V}) + 2\sum_{i,j} f_4(i)f_5(j)\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} \qquad \text{(B15)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow HH$

The t- and u-channels $\widetilde{\ell}$ exchanges are now summed over $\widetilde{\ell}_1$ and $\widetilde{\ell}_2$ with appropriate propagators. The couplings are

$$
\begin{aligned}
f_1 \;=\; & (-3g_2 m_Z\cos(2\alpha)\cos(\alpha+\beta)/(2\cos\theta_W))(g_2 m_Z((1/2 - \sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell \\
& +\sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W - g_2 m_\ell^2\cos\alpha/(m_W\cos\beta) \\
& -g_2 m_\ell(-A_\ell\cos\alpha - \mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
f_2 \;=\; & (g_2 m_Z(\cos(2\alpha)\sin(\alpha+\beta) + 2\sin(2\alpha)\cos(\alpha+\beta))/(2\cos\theta_W)) \\
& (g_2 m_Z((-1/2 + \sin^2\theta_W)\sin(\beta+\alpha)\cos^2\theta_\ell - \sin^2\theta_W\sin(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
& +g_2 m_\ell^2\sin\alpha/(m_W\cos\beta) - g_2 m_\ell(A_\ell\sin\alpha - \mu\cos\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
f_3 \;=\; & g_2^2/2(-(-1/2 + \sin^2\theta_W)\cos(2\alpha)/\cos^2\theta_W - m_\ell^2\cos^2\alpha/(m_W^2\cos^2\beta))\cos^2\theta_\ell \\
& +g_2^2/2(\sin^2\theta_W\cos(2\alpha)/(\cos^2\theta_W) - m_\ell^2(\cos^2\alpha)/(m_W^2\cos^2\beta))\sin^2\theta_\ell
\end{aligned}
$$

$$
\begin{aligned}
f_4(1) &= (g_2 m_Z((1/2-\sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell+\sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
&\quad -g_2 m_\ell^2\cos\alpha/(m_W\cos\beta)-g_2 m_\ell(-A_\ell\cos\alpha-\mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta))^2 \\
f_4(2) &= (g_2 m_Z((1/2-\sin^2\theta_W)\cos(\beta+\alpha)(-\cos\theta_\ell\sin\theta_\ell)+\sin^2\theta_W\cos(\beta+\alpha)\cos\theta_\ell\sin\theta_\ell) \\
&\quad /\cos\theta_W-g_2 m_\ell(-A_\ell\cos\alpha-\mu\sin\alpha)\cos(2\theta_\ell)/(2m_W\cos\beta))^2 \\
f_5(1) &= (g_2 m_Z((1/2-\sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell+\sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
&\quad -g_2 m_\ell^2\cos\alpha/(m_W\cos\beta)-g_2 m_\ell(-A_\ell\cos\alpha-\mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta))^2 \\
f_5(2) &= (g_2 m_Z((1/2-\sin^2\theta_W)\cos(\beta+\alpha)(-\cos\theta_\ell\sin\theta_\ell)+\sin^2\theta_W\cos(\beta+\alpha)\cos\theta_\ell\sin\theta_\ell) \\
&\quad /\cos\theta_W-g_2 m_\ell(-A_\ell\cos\alpha-\mu\sin\alpha)\cos(2\theta_\ell)/(2m_W\cos\beta))^2 \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III}+\sum_{i,j}(f_4(i)f_4(j)\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV}+f_5(i)f_5(j)\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V}) \\
&\quad +2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II}+2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III}+2\sum_i(f_1f_4(i)\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV}+f_1f_5(i)\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V}) \\
&\quad +2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III}+2\sum_i(f_2f_4(i)\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV}+f_2f_5(i)\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V}) \\
&\quad +2\sum_i(f_3f_4(i)\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV}+f_3f_5(i)\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V})+2\sum_{i,j}f_4(i)f_5(j)\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} \qquad \text{(B16)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\ell}_1^* \longrightarrow H^+H^-$

We have one new channel,

VI. t-channel $\widetilde{\nu}_\ell$ exchange

The couplings, except f_4, are modified

$$
\begin{aligned}
f_1 &= (g_2 m_Z((1/2-\sin^2\theta_W)\cos(\beta+\alpha)\cos^2\theta_\ell+\sin^2\theta_W\cos(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
&\quad -g_2 m_\ell^2\cos\alpha/(m_W\cos\beta)-g_2 m_\ell(-A_\ell\cos\alpha-\mu\sin\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
&\quad (-g_2(m_W\cos(\beta-\alpha)-m_Z\cos(2\beta)\cos(\beta+\alpha)/(2\cos\theta_W))) \\
f_2 &= (g_2 m_Z((-1/2+\sin^2\theta_W)\sin(\beta+\alpha)\cos^2\theta_\ell-\sin^2\theta_W\sin(\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W \\
&\quad +g_2 m_\ell^2\sin\alpha/(m_W\cos\beta)-g_2 m_\ell(A_\ell\sin\alpha-\mu\cos\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)) \\
&\quad (-g_2(m_W\sin(\beta-\alpha)+m_Z/(2\cos\theta_W)\cos(2\beta)\sin(\beta+\alpha))) \\
f_3 &= (-g_2(\sin^2\theta_W-\cos^2\theta_\ell/2)/\cos\theta_W)(g_2\cos(2\theta_W)/(2\cos\theta_W)) \\
f_5 &= g_2^2\cos(2\beta)(1+(-1+2\sin^2\theta_W)/(2\cos^2\theta_W))\cos^2\theta_\ell/2 \\
&\quad -(g_2^2\cos(2\beta)\sin^2\theta_W/2+g_2^2m_\ell^2\tan^2\beta/(2m_W^2))\sin^2\theta_\ell \\
f_6 &= (-g_2 m_W\cos\theta_\ell(-m_\ell^2\tan\beta/m_W^2+\sin(2\beta))/\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
& +g_2 m_\ell \sin\theta_\ell(\mu - A_\ell \tan\beta)/(\sqrt{2}m_W))^2 \\
\mathcal{T}_{\rm VI}\times\mathcal{T}_{\rm VI} &= 1/(m_{\tilde{\nu}_\ell}^2 - t)^2 \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm VI} &= 1/((m_H^2 - s)(m_{\tilde{\nu}_\ell}^2 - t)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm VI} &= 1/((m_h^2 - t)(m_{\tilde{\nu}_\ell}^2 - t)) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm VI} &= (t-u)/((m_Z^2 - s)(m_{\tilde{\nu}_\ell}^2 - t)) \\
\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm VI} &= (-t+u)/(s(m_{\tilde{\nu}_\ell}^2 - t)) \\
\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm VI} &= 1/(m_{\tilde{\nu}_\ell}^2 - t) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + f_4^2\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} + f_5^2\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V} + f_6^2\mathcal{T}_{\rm VI}\times\mathcal{T}_{\rm VI} \\
& +2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} + 2f_1f_4\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} + 2f_1f_5\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V} + 2f_1f_6\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm VI} \\
& +2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} + 2f_2f_4\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} + 2f_2f_5\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V} + 2f_2f_6\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm VI} \\
& +2f_3f_4\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} + 2f_3f_5\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V} + 2f_3f_6\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm VI} + 2f_4f_5\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} \\
& +2f_4f_6\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm VI} + 2f_5f_6\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm VI} \qquad \text{(B17)}
\end{aligned}
$$

$\tilde{\ell}_1\tilde{\ell}_1 \longrightarrow \ell\ell$

The channels I. t-channel χ exchange II. u-channel χ exchange are modified as follows.

$$
\begin{aligned}
K_a &= -\sin\theta_\ell(g_2m_\ell/(2m_W\cos\beta)N_{i3} + g_1N_{i1})/\sqrt{2} \\
& -\cos\theta_\ell(g_2m_\ell/(2m_W\cos\beta)N_{i3} + (-g_1N_{i1} - g_2/2(N_{i2} - N_{i1}\tan\theta_W)))/\sqrt{2} \\
K_b &= -\sin\theta_\ell(g_2m_\ell/(2m_W\cos\beta)N_{i3} - g_1N_{i1})/\sqrt{2} \\
& -\cos\theta_\ell(-g_2m_\ell/(2m_W\cos\beta)N_{i3} + (-g_1N_{i1} - g_2/2(N_{i2} - N_{i1}\tan\theta_W)))/\sqrt{2} \\
K'_a &= -\sin\theta_\ell(g_2m_\ell/(2m_W\cos\beta)N_{j3} + g_1N_{j1})/\sqrt{2} \\
& -\cos\theta_\ell(g_2m_\ell/(2m_W\cos\beta)N_{j3} + (-g_1N_{j1} - g_2/2(N_{j2} - N_{j1}\tan\theta_W)))/\sqrt{2} \\
K'_b &= -\sin\theta_\ell(g_2m_\ell/(2m_W\cos\beta)N_{j3} - g_1N_{j1})/\sqrt{2} \\
& -\cos\theta_\ell(-g_2m_\ell/(2m_W\cos\beta)N_{j3} + (-g_1N_{j1} - g_2/2(N_{j2} - N_{j1}\tan\theta_W)))/\sqrt{2} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (-2(4K_aK'_aK_bK'_bm_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}s \\
& +K_b^2K_a'^2(-m_{\tilde{\ell}_1}^4 - 2m_{\tilde{\ell}_1}^2m_\ell^2 + 3m_\ell^4 - 4m_\ell^3m_{\tilde{\chi}_i^0} + 4m_\ell^3m_{\tilde{\chi}_j^0} - 4m_\ell^2m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0} \\
& +m_{\tilde{\ell}_1}^2s - m_\ell^2s + m_\ell m_{\tilde{\chi}_i^0}s - m_\ell m_{\tilde{\chi}_j^0}s + m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}s + m_{\tilde{\ell}_1}^2t + 3m_\ell^2t - m_\ell m_{\tilde{\chi}_i^0}t \\
& +m_\ell m_{\tilde{\chi}_j^0}t + m_{\tilde{\ell}_1}^2u - m_\ell^2u + m_\ell m_{\tilde{\chi}_i^0}u - m_\ell m_{\tilde{\chi}_j^0}u - tu)
\end{aligned}
$$

$$
\begin{aligned}
&+K_b^2K_b'^2(m_{\tilde{\ell}_1}^4+2m_{\tilde{\ell}_1}^2m_\ell^2-3m_\ell^4+4m_\ell^3m_{\tilde{\chi}_i^0}+4m_\ell^3m_{\tilde{\chi}_j^0}-4m_\ell^2m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}\\
&-m_{\tilde{\ell}_1}^2s+m_\ell^2s-m_\ell m_{\tilde{\chi}_i^0}s-m_\ell m_{\tilde{\chi}_j^0}s+m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}s-m_{\tilde{\ell}_1}^2t-3m_\ell^2t+m_\ell m_{\tilde{\chi}_i^0}t\\
&+m_\ell m_{\tilde{\chi}_j^0}t-m_{\tilde{\ell}_1}^2u+m_\ell^2u-m_\ell m_{\tilde{\chi}_i^0}u-m_\ell m_{\tilde{\chi}_j^0}u+tu)\\
&+K_a^2K_b'^2(-m_{\tilde{\ell}_1}^4-2m_{\tilde{\ell}_1}^2m_\ell^2+3m_\ell^4+4m_\ell^3m_{\tilde{\chi}_i^0}-4m_\ell^3m_{\tilde{\chi}_j^0}-4m_\ell^2m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}\\
&+m_{\tilde{\ell}_1}^2s-m_\ell^2s-m_\ell m_{\tilde{\chi}_i^0}s+m_\ell m_{\tilde{\chi}_j^0}s+m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}s+m_{\tilde{\ell}_1}^2t+3m_\ell^2t+m_\ell m_{\tilde{\chi}_i^0}t\\
&-m_\ell m_{\tilde{\chi}_j^0}t+m_{\tilde{\ell}_1}^2u-m_\ell^2u-m_\ell m_{\tilde{\chi}_i^0}u+m_\ell m_{\tilde{\chi}_j^0}u-tu)\\
&+K_a^2K_a'^2(m_{\tilde{\ell}_1}^4+2m_{\tilde{\ell}_1}^2m_\ell^2-3m_\ell^4-4m_\ell^3m_{\tilde{\chi}_i^0}-4m_\ell^3m_{\tilde{\chi}_j^0}-4m_\ell^2m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}\\
&-m_{\tilde{\ell}_1}^2s+m_\ell^2s+m_\ell m_{\tilde{\chi}_i^0}s+m_\ell m_{\tilde{\chi}_j^0}s+m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}s-m_{\tilde{\ell}_1}^2t-3m_\ell^2t-m_\ell m_{\tilde{\chi}_i^0}t\\
&-m_\ell m_{\tilde{\chi}_j^0}t-m_{\tilde{\ell}_1}^2u+m_\ell^2u+m_\ell m_{\tilde{\chi}_i^0}u+m_\ell m_{\tilde{\chi}_j^0}u+tu)))\\
&/((m_{\tilde{\chi}_i^0}^2-t)(-m_{\tilde{\chi}_j^0}^2+t))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} \;=\;& (-2(4K_aK_a'K_bK_b'm_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}s\\
&+K_a^2K_b'^2(-m_{\tilde{\ell}_1}^4-2m_{\tilde{\ell}_1}^2m_\ell^2+3m_\ell^4+4m_\ell^3m_{\tilde{\chi}_i^0}-4m_\ell^3m_{\tilde{\chi}_j^0}-4m_\ell^2m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}\\
&+m_{\tilde{\ell}_1}^2s-m_\ell^2s-m_\ell m_{\tilde{\chi}_i^0}s+m_\ell m_{\tilde{\chi}_j^0}s+m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}s+m_{\tilde{\ell}_1}^2t-m_\ell^2t-m_\ell m_{\tilde{\chi}_i^0}t\\
&+m_\ell m_{\tilde{\chi}_j^0}t+m_{\tilde{\ell}_1}^2u+3m_\ell^2u+m_\ell m_{\tilde{\chi}_i^0}u-m_\ell m_{\tilde{\chi}_j^0}u-tu)\\
&+K_a^2K_a'^2(m_{\tilde{\ell}_1}^4+2m_{\tilde{\ell}_1}^2m_\ell^2-3m_\ell^4-4m_\ell^3m_{\tilde{\chi}_i^0}-4m_\ell^3m_{\tilde{\chi}_j^0}-4m_\ell^2m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}\\
&-m_{\tilde{\ell}_1}^2s+m_\ell^2s+m_\ell m_{\tilde{\chi}_i^0}s+m_\ell m_{\tilde{\chi}_j^0}s+m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}s-m_{\tilde{\ell}_1}^2t+m_\ell^2t+m_\ell m_{\tilde{\chi}_i^0}t\\
&+m_\ell m_{\tilde{\chi}_j^0}t-m_{\tilde{\ell}_1}^2u-3m_\ell^2u-m_\ell m_{\tilde{\chi}_i^0}u-m_\ell m_{\tilde{\chi}_j^0}u+tu)\\
&+K_b^2K_a'^2(-m_{\tilde{\ell}_1}^4-2m_{\tilde{\ell}_1}^2m_\ell^2+3m_\ell^4-4m_\ell^3m_{\tilde{\chi}_i^0}+4m_\ell^3m_{\tilde{\chi}_j^0}-4m_\ell^2m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}\\
&+m_{\tilde{\ell}_1}^2s-m_\ell^2s+m_\ell m_{\tilde{\chi}_i^0}s-m_\ell m_{\tilde{\chi}_j^0}s+m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}s+m_{\tilde{\ell}_1}^2t-m_\ell^2t+m_\ell m_{\tilde{\chi}_i^0}t\\
&-m_\ell m_{\tilde{\chi}_j^0}t+m_{\tilde{\ell}_1}^2u+3m_\ell^2u-m_\ell m_{\tilde{\chi}_i^0}u+m_\ell m_{\tilde{\chi}_j^0}u-tu)\\
&+K_b^2K_b'^2(m_{\tilde{\ell}_1}^4+2m_{\tilde{\ell}_1}^2m_\ell^2-3m_\ell^4+4m_\ell^3m_{\tilde{\chi}_i^0}+4m_\ell^3m_{\tilde{\chi}_j^0}-4m_\ell^2m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}\\
&-m_{\tilde{\ell}_1}^2s+m_\ell^2s-m_\ell m_{\tilde{\chi}_i^0}s-m_\ell m_{\tilde{\chi}_j^0}s+m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}s-m_{\tilde{\ell}_1}^2t+m_\ell^2t-m_\ell m_{\tilde{\chi}_i^0}t\\
&-m_\ell m_{\tilde{\chi}_j^0}t-m_{\tilde{\ell}_1}^2u-3m_\ell^2u+m_\ell m_{\tilde{\chi}_i^0}u+m_\ell m_{\tilde{\chi}_j^0}u+tu)))\\
&/((m_{\tilde{\chi}_i^0}^2-u)(-m_{\tilde{\chi}_j^0}^2+u))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \;=\;& (8K_aK_a'K_bK_b'm_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}s+\\
&K_a^2(-K_b'^2(2m_{\tilde{\ell}_1}^4+4m_{\tilde{\ell}_1}^2m_\ell^2-6m_\ell^4-8m_\ell^3m_{\tilde{\chi}_i^0}+8m_\ell^3m_{\tilde{\chi}_j^0}+8m_\ell^2m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}\\
&+2m_{\tilde{\ell}_1}^2s+2m_\ell^2s+2m_\ell m_{\tilde{\chi}_i^0}s-2m_\ell m_{\tilde{\chi}_j^0}s-2m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0}s-s^2-2m_{\tilde{\ell}_1}^2t-2m_\ell^2t
\end{aligned}
$$

$$
\begin{aligned}
& +2m_\ell m_{\tilde\chi_i^0}t + 2m_\ell m_{\tilde\chi_j^0}t + t^2 - 2m_{\tilde\ell_1}^2 u - 2m_\ell^2 u - 2m_\ell m_{\tilde\chi_i^0}u - 2m_\ell m_{\tilde\chi_j^0}u + u^2)) + \\
& K_a^2 K_a'^2(2m_{\tilde\ell_1}^4 + 4m_{\tilde\ell_1}^2 m_\ell^2 - 6m_\ell^4 - 8m_\ell^3 m_{\tilde\chi_i^0} - 8m_\ell^3 m_{\tilde\chi_j^0} - 8m_\ell^2 m_{\tilde\chi_i^0} m_{\tilde\chi_j^0} \\
& +2m_{\tilde\ell_1}^2 s + 2m_\ell^2 s + 2m_\ell m_{\tilde\chi_i^0}s + 2m_\ell m_{\tilde\chi_j^0}s + 2m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}s - s^2 - 2m_{\tilde\ell_1}^2 t - 2m_\ell^2 t \\
& +2m_\ell m_{\tilde\chi_i^0}t - 2m_\ell m_{\tilde\chi_j^0}t + t^2 - 2m_{\tilde\ell_1}^2 u - 2m_\ell^2 u - 2m_\ell m_{\tilde\chi_i^0}u + 2m_\ell m_{\tilde\chi_j^0}u + u^2) + \\
& K_b^2 K_b'^2(2m_{\tilde\ell_1}^4 + 4m_{\tilde\ell_1}^2 m_\ell^2 - 6m_\ell^4 + 8m_\ell^3 m_{\tilde\chi_i^0} + 8m_\ell^3 m_{\tilde\chi_j^0} - 8m_\ell^2 m_{\tilde\chi_i^0} m_{\tilde\chi_j^0} \\
& +2m_{\tilde\ell_1}^2 s + 2m_\ell^2 s - 2m_\ell m_{\tilde\chi_i^0}s - 2m_\ell m_{\tilde\chi_j^0}s + 2m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}s - s^2 - 2m_{\tilde\ell_1}^2 t - 2m_\ell^2 t \\
& -2m_\ell m_{\tilde\chi_i^0}t + 2m_\ell m_{\tilde\chi_j^0}t + t^2 - 2m_{\tilde\ell_1}^2 u - 2m_\ell^2 u + 2m_\ell m_{\tilde\chi_i^0}u - 2m_\ell m_{\tilde\chi_j^0}u + u^2) - \\
& K_b^2 K_a'^2(2m_{\tilde\ell_1}^4 + 4m_{\tilde\ell_1}^2 m_\ell^2 - 6m_\ell^4 + 8m_\ell^3 m_{\tilde\chi_i^0} - 8m_\ell^3 m_{\tilde\chi_j^0} + 8m_\ell^2 m_{\tilde\chi_i^0} m_{\tilde\chi_j^0} \\
& +2m_{\tilde\ell_1}^2 s + 2m_\ell^2 s - 2m_\ell m_{\tilde\chi_i^0}s + 2m_\ell m_{\tilde\chi_j^0}s - 2m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}s - s^2 - 2m_{\tilde\ell_1}^2 t - 2m_\ell^2 t \\
& -2m_\ell m_{\tilde\chi_i^0}t - 2m_\ell m_{\tilde\chi_j^0}t + t^2 - 2m_{\tilde\ell_1}^2 u - 2m_\ell^2 u + 2m_\ell m_{\tilde\chi_i^0}u + 2m_\ell m_{\tilde\chi_j^0}u + u^2)) \\
& /((m_{\tilde\chi_i^0}^2 - t)(m_{\tilde\chi_j^0}^2 - u))
\end{aligned}
$$

$$
|\mathcal{T}|^2 = \sum_{i,j}(\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + \mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + 2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II}) \qquad \text{(B18)}
$$

$\widetilde\ell_1^A \widetilde\ell_1^{B*} \longrightarrow \ell^A \bar\ell^B$

I. t-channel χ exchange

Here $m_{A,B} \equiv m_{\ell^{A,B}}$ and $m_{\widetilde A,\widetilde B} \equiv m_{\tilde\ell_1^{A,B}}$.

$$
\begin{aligned}
A(i) &= -g_2/\sqrt{2}(\cos\theta_A(-N_{i2} - \tan\theta_W N_{i1}) + \sin\theta_A m_A/(m_W\cos\beta)N_{i3}) \\
B(i) &= -g_2/\sqrt{2}(\cos\theta_A m_A/(m_W\cos\beta)N_{i3} + 2\sin\theta_A\tan\theta_W N_{i1}) \\
C(i) &= -g_2/\sqrt{2}(\cos\theta_B m_B/(m_W\cos\beta)N_{i3} + 2\sin\theta_B\tan\theta_W N_{i1}) \\
D(i) &= -g_2/\sqrt{2}(\cos\theta_B(-N_{i2} - \tan\theta_W N_{i1}) + \sin\theta_B m_B/(m_W\cos\beta)N_{i3}) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (A(i)C(i)A(j)C(j)(-m_A^2 m_{\tilde\chi_i^0}m_{\tilde\chi_j^0} - m_B^2 m_{\tilde\chi_i^0}m_{\tilde\chi_j^0} + m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}s) \\
& +A(i)C(i)A(j)D(j)(-m_{\widetilde A}^2 m_B m_{\tilde\chi_i^0} + m_A^2 m_B m_{\tilde\chi_i^0} + m_B m_{\tilde\chi_i^0}t) \\
& +A(i)C(i)B(j)C(j)(m_{\widetilde A}^2 m_A m_{\tilde\chi_i^0} + 2m_A m_B^2 m_{\tilde\chi_i^0} - m_A m_{\tilde\chi_i^0}s - m_A m_{\tilde\chi_i^0}u + m_A^3 m_{\tilde\chi_i^0}) \\
& +A(i)C(i)B(j)D(j)(-2m_A m_B m_{\tilde\chi_i^0}m_{\tilde\chi_j^0}) \\
& +A(i)D(i)A(j)C(j)(-m_{\widetilde A}^2 m_B m_{\tilde\chi_j^0} + m_A^2 m_B m_{\tilde\chi_j^0} + m_B m_{\tilde\chi_j^0}t) \\
& +A(i)D(i)A(j)D(j)(2m_{\widetilde A}^2 m_B^2 - m_{\widetilde A}^2 s - m_{\widetilde A}^2 t - m_{\widetilde A}^2 u + m_{\widetilde A}^4 - 2m_A^2 m_B^2 + m_A^2 s \\
& +m_A^2 u - m_A^4 - m_B^2 t + tu)
\end{aligned}
$$

$$
\begin{aligned}
&+A(i)D(i)B(j)C(j)(-2m_Am_Bt)\\
&+A(i)D(i)B(j)D(j)(m^2_{\widetilde{A}}m_Am_{\tilde{\chi}^0_j}+2m_Am^2_Bm_{\tilde{\chi}^0_j}-m_Am_{\tilde{\chi}^0_j}s-m_Am_{\tilde{\chi}^0_j}u+m^3_Am_{\tilde{\chi}^0_j})\\
&+B(i)C(i)A(j)C(j)(m^2_{\widetilde{A}}m_Am_{\tilde{\chi}^0_j}+2m_Am^2_Bm_{\tilde{\chi}^0_j}-m_Am_{\tilde{\chi}^0_j}s-m_Am_{\tilde{\chi}^0_j}u+m^3_Am_{\tilde{\chi}^0_j})\\
&+B(i)C(i)A(j)D(j)(-2m_Am_Bt)\\
&+B(i)C(i)B(j)C(j)(2m^2_{\widetilde{A}}m^2_B-m^2_{\widetilde{A}}s-m^2_{\widetilde{A}}t-m^2_{\widetilde{A}}u+m^4_{\widetilde{A}}-2m^2_Am^2_B+m^2_As\\
&+m^2_Au-m^4_A-m^2_Bt+tu)\\
&+B(i)C(i)B(j)D(j)(-m^2_{\widetilde{A}}m_Bm_{\tilde{\chi}^0_j}+m^2_Am_Bm_{\tilde{\chi}^0_j}+m_Bm_{\tilde{\chi}^0_j}t)\\
&+B(i)D(i)A(j)C(j)(-2m_Am_Bm_{\tilde{\chi}^0_i}m_{\tilde{\chi}^0_j})\\
&+B(i)D(i)A(j)D(j)(m^2_{\widetilde{A}}m_Am_{\tilde{\chi}^0_i}+2m_Am^2_Bm_{\tilde{\chi}^0_i}-m_Am_{\tilde{\chi}^0_i}s-m_Am_{\tilde{\chi}^0_i}u+m^3_Am_{\tilde{\chi}^0_i})\\
&+B(i)D(i)B(j)C(j)(-m^2_{\widetilde{A}}m_Bm_{\tilde{\chi}^0_i}+m^2_Am_Bm_{\tilde{\chi}^0_i}+m_Bm_{\tilde{\chi}^0_i}t)\\
&+B(i)D(i)B(j)D(j)(-m^2_Am_{\tilde{\chi}^0_i}m_{\tilde{\chi}^0_j}-m^2_Bm_{\tilde{\chi}^0_i}m_{\tilde{\chi}^0_j}+m_{\tilde{\chi}^0_i}m_{\tilde{\chi}^0_j}s))\\
&/((t-m^2_{\tilde{\chi}^0_i})(t-m^2_{\tilde{\chi}^0_j}))
\end{aligned}
$$

$$|\mathcal{T}|^2 = \sum_{i,j}\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} \tag{B19}$$

$\widetilde{\ell}^A_1\widetilde{\ell}^B_1 \longrightarrow \ell^A\ell^B$

I. t-channel χ exchange

$$
\begin{aligned}
A(i) &= -g_2/\sqrt{2}(\cos\theta_A(-N_{i2}-\tan\theta_W N_{i1})+\sin\theta_A m_A/(m_W\cos\beta)N_{i3})\\
B(i) &= -g_2/\sqrt{2}(\cos\theta_A m_A/(m_W\cos\beta)N_{i3}+2\sin\theta_A\tan\theta_W N_{i1})\\
C(i) &= -g_2/\sqrt{2}(\cos\theta_B m_B/(m_W\cos\beta)N_{i3}+2\sin\theta_B\tan\theta_W N_{i1})\\
D(i) &= -g_2/\sqrt{2}(\cos\theta_B(-N_{i2}-\tan\theta_W N_{i1})+\sin\theta_B m_B/(m_W\cos\beta)N_{i3})\\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (-A(i)(-B(j)m_A(D(i)m_{\tilde{\chi}^0_i}(2C(j)m_Bm_{\tilde{\chi}^0_j}+D(j)(m^2_A+2m^2_B+m^2_{\widetilde{A}}-s-u))\\
&+C(i)(2D(j)m_Bt+C(j)m_{\tilde{\chi}^0_j}(m^2_A+2m^2_B+m^2_{\widetilde{A}}-s-u)))\\
&+A(j)(D(i)m_{\tilde{\chi}^0_i}(D(j)m_{\tilde{\chi}^0_j}(m^2_A+m^2_B-s)+C(j)m_B(m^2_A-m^2_{\widetilde{A}}+t))\\
&+C(i)(D(j)m_Bm_{\tilde{\chi}^0_j}(m^2_A-m^2_{\widetilde{A}}+t)+C(j)(m^4_A-m^4_{\widetilde{A}}+m^2_{\widetilde{A}}s+m^2_{\widetilde{A}}t\\
&+m^2_B(-2m^2_{\widetilde{A}}+t)+m^2_A(2m^2_B-s-u)+m^2_{\widetilde{A}}u-tu))))\\
&-B(i)(C(i)m_{\tilde{\chi}^0_i}(B(j)(C(j)m_{\tilde{\chi}^0_j}(m^2_A+m^2_B-s)+D(j)m_B(m^2_A-m^2_{\widetilde{A}}+t))\\
&-A(j)m_A(2D(j)m_Bm_{\tilde{\chi}^0_j}+C(j)(m^2_A+2m^2_B+m^2_{\widetilde{A}}-s-u)))
\end{aligned}
$$

$$
\begin{aligned}
&+D(i)(C(j)m_B(-2A(j)m_A t+B(j)m_{\tilde{\chi}^0_j}(m_A^2-m_{\widetilde{A}}^2+t))\\
&+D(j)(A(j)m_A m_{\tilde{\chi}^0_j}(-m_A^2-2m_B^2-m_{\widetilde{A}}^2+s+u)+B(j)(m_A^4-m_{\widetilde{A}}^4+m_{\widetilde{A}}^2 s+m_{\widetilde{A}}^2 t\\
&+m_B^2(-2m_{\widetilde{A}}^2+t)+m_A^2(2m_B^2-s-u)+m_{\widetilde{A}}^2 u-tu)))))\\
&/((t-m_{\tilde{\chi}^0_i}^2)(t-m_{\tilde{\chi}^0_j}^2))
\end{aligned}
$$

$$
|\mathcal{T}|^2 = \sum_{i,j}\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} \quad \text{(B20)}
$$

$\widetilde{\ell}_1\chi \longrightarrow Z\ell$

Channel II (the t-channel $\widetilde{\ell}$ exchange) is now summed over $\widetilde{\ell}_1$ and $\widetilde{\ell}_2$, with appropriate propagators. Also added is the neutralino u-channel exchange.

$$
\begin{aligned}
f_A(1,i) &= -g_2/\sqrt{2}(\cos\theta_\ell(-N_{i2}-\tan\theta_W N_{i1})+\sin\theta_\ell m_\ell/(m_W\cos\beta)N_{i3})\\
f_A(2,i) &= -g_2/\sqrt{2}(-\sin\theta_\ell(-N_{i2}-\tan\theta_W N_{i1})+\cos\theta_\ell m_\ell/(m_W\cos\beta)N_{i3})\\
f_B(1,i) &= -g_2/\sqrt{2}(\cos\theta_\ell m_\ell/(m_W\cos\beta)N_{i3}+\sin\theta_\ell(2\tan\theta_W N_{i1}))\\
f_B(2,i) &= -g_2/\sqrt{2}(-\sin\theta_\ell m_\ell/(m_W\cos\beta)N_{i3}+\cos\theta_\ell(2\tan\theta_W N_{i1}))\\
f_C(1) &= -g_2/\cos\theta_W((-1/2)\cos^2\theta_\ell-(-1)\sin^2\theta_W)\\
f_C(2) &= g_2/\cos\theta_W((-1/2)\cos\theta_\ell\sin\theta_\ell)\\
f_L &= -g_2/\cos\theta_W((-1/2)-(-1)\sin^2\theta_W)\\
f_R &= g_2/\cos\theta_W((-1)\sin^2\theta_W)\\
f_{OL}(i) &= g_2/(2\cos\theta_W)(N_{i4}N_{14}-N_{i3}N_{13})\\
f_{OR}(i) &= -g_2/(2\cos\theta_W)(N_{i4}N_{14}-N_{i3}N_{13})\\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (1/2)(2f_A(1,1)f_B(1,1)m_{\tilde{\chi}}m_\ell(2m_Z^4f_L^2+6m_\ell^2m_Z^2f_Lf_R+2m_Z^4f_R^2\\
&+m_{\tilde{\chi}}^2(f_L^2+f_R^2)(m_\ell^2-s)+m_{\tilde{\ell}_1}^2(f_L^2+f_R^2)(m_\ell^2-s)-2m_Z^2f_L^2s+6m_Z^2f_Lf_Rs\\
&-2m_Z^2f_R^2s-m_\ell^2f_L^2t-m_\ell^2f_R^2t+f_L^2st+f_R^2st-m_\ell^2f_L^2u-m_\ell^2f_R^2u+f_L^2su+f_R^2su)\\
&+f_A(1,1)^2(-m_\ell^2m_Z^4f_R^2+m_{\tilde{\ell}_1}^4f_L^2(m_\ell^2-s)-m_Z^4f_L^2s-6m_\ell^2m_Z^2f_Lf_Rs\\
&+m_\ell^2m_Z^2f_R^2s+m_Z^2f_L^2s^2+m_{\tilde{\chi}}^4f_L^2(-m_\ell^2+s)-m_\ell^2m_Z^2f_R^2t+m_\ell^2f_L^2st\\
&+m_Z^2f_L^2st-f_L^2s^2t+m_\ell^4f_R^2u+m_\ell^2m_Z^2f_R^2u-m_Z^2f_L^2su-m_\ell^2f_R^2su\\
&+m_{\tilde{\ell}_1}^2f_L(2m_Z^4f_L+m_Z^2(6m_\ell^2f_R-2f_Ls)-f_L(m_\ell^2-s)(s+t+u))\\
&+m_{\tilde{\chi}}^2(-2m_Z^4f_L^2+2m_Z^2f_L(-3m_\ell^2f_R+f_Ls)-(m_\ell^2-s)(m_\ell^2f_R^2-f_L^2(t+u))))
\end{aligned}
$$

$$
\begin{aligned}
& -f_B(1,1)^2(m_{\tilde\chi}^4 f_R^2(m_\ell^2 - s) - m_\ell^4 f_L^2 u + m_\ell^2(m_Z^4 f_L^2 - m_{\tilde\ell_1}^4 f_R^2 - f_R^2 st + f_L^2 su \\
& + m_{\tilde\ell_1}^2 f_R^2(s+t+u) - m_Z^2 f_L(6m_{\tilde\ell_1}^2 f_R - 6f_R s + f_L(s-t+u))) \\
& + m_{\tilde\chi}^2(m_\ell^4 f_L^2 + m_\ell^2(6m_Z^2 f_L f_R - f_L^2 s - f_R^2(t+u)) + f_R^2(2m_Z^4 \\
& - 2m_Z^2 s + s(t+u))) - f_R^2(-m_{\tilde\ell_1}^4 s - s(m_Z^4 + st - m_Z^2(s+t-u)) \\
& + m_{\tilde\ell_1}^2(2m_Z^4 - 2m_Z^2 s + s(s+t+u))))) / (m_Z^2(s-m_\ell^2)^2)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} \;=\; & (1/2)((f_A(i,1)(-2f_B(j,1)m_{\tilde\chi}m_\ell + f_A(j,1)(m_{\tilde\chi}^2 + m_\ell^2 - t)) \\
& + f_B(i,1)(-2f_A(j,1)m_{\tilde\chi}m_\ell \\
& + f_B(j,1)(m_{\tilde\chi}^2 + m_\ell^2 - t)))(m_{\tilde\ell_1}^4 + m_Z^4 - 2m_Z^2 t + t^2 - 2m_{\tilde\ell_1}^2(m_Z^2 + t))) \\
& /(m_Z^2(t - m_{\tilde\ell_i}^2)(t - m_{\tilde\ell_j}^2))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} \;=\; & 1/2((1/m_Z^2)(f_B(1,i)(f_A(1,j)m_\ell(m_{\tilde\chi}^4(m_{\tilde\chi_j^0} f_{OL}(i)f_{OL}(j) + m_{\tilde\chi_i^0} f_{OR}(i)f_{OR}(j)) \\
& + m_{\tilde\chi}^2(m_{\tilde\chi_j^0} f_{OL}(i)f_{OL}(j) + m_{\tilde\chi_i^0} f_{OR}(i)f_{OR}(j))(m_Z^2 - 2u) \\
& - 6m_{\tilde\chi}m_Z^2(m_{\tilde\chi_i^0}m_{\tilde\chi_j^0} f_{OR}(j)f_{OR}(i) + f_{OL}(i)f_{OR}(j)u) - (m_{\tilde\chi_j^0} f_{OL}(i)f_{OL}(j) \\
& + m_{\tilde\chi_i^0} f_{OR}(i)f_{OR}(j))(2m_Z^4 - m_Z^2 u - u^2)) + f_B(1,j)(m_{\tilde\chi}^6 f_{OL}(i)f_{OL}(j) \\
& - 4m_\ell^2 m_Z^4 f_{OL}(i)f_{OL}(j) - 2m_Z^6 f_{OL}(i)f_{OL}(j) - m_{\tilde\chi_i^0}m_{\tilde\chi_j^0} m_Z^4 f_{OR}(i)f_{OR}(j) \\
& - 3m_{\tilde\chi}^3 m_Z^2(m_{\tilde\chi_i^0} f_{OL}(j)f_{OR}(i) + m_{\tilde\chi_j^0} f_{OL}(i)f_{OR}(j)) + 2m_Z^4 f_{OL}(i)f_{OL}(j)s \\
& + m_{\tilde\chi_i^0}m_{\tilde\chi_j^0} m_Z^2 f_{OR}(i)f_{OR}(j)s - 3m_{\tilde\chi}m_Z^2(m_{\tilde\chi_i^0} f_{OL}(j)f_{OR}(i) \\
& + m_{\tilde\chi_j^0} f_{OL}(i)f_{OR}(j))(2m_\ell^2 + m_Z^2 - s - t) + 2m_Z^4 f_{OL}(i)f_{OL}(j)t \\
& - m_{\tilde\chi_i^0}m_{\tilde\chi_j^0} m_Z^2 f_{OR}(i)f_{OR}(j)t + m_{\tilde\chi}^4 f_{OL}(i)f_{OL}(j)(2m_\ell^2 + 2m_Z^2 - s - t - 2u) \\
& + 2m_\ell^2 m_Z^2 f_{OL}(i)f_{OL}(j)u + 2m_Z^4 f_{OL}(i)f_{OL}(j)u + m_\ell^2 m_{\tilde\chi_i^0}m_{\tilde\chi_j^0} f_{OR}(i)f_{OR}(j)u \\
& + m_{\tilde\chi_i^0}m_{\tilde\chi_j^0} m_Z^2 f_{OR}(i)f_{OR}(j)u - 2m_Z^2 f_{OL}(i)f_{OL}(j)su - m_{\tilde\chi_i^0}m_{\tilde\chi_j^0} f_{OR}(i)f_{OR}(j)su \\
& + m_\ell^2 f_{OL}(i)f_{OL}(j)u^2 - f_{OL}(i)f_{OL}(j)tu^2 + m_{\tilde\chi}^2(-m_Z^4 f_{OL}(i)f_{OL}(j) \\
& + m_{\tilde\chi_i^0}m_{\tilde\chi_j^0} f_{OR}(i)f_{OR}(j)s + f_{OL}(i)f_{OL}(j)su + 2f_{OL}(i)f_{OL}(j)tu \\
& + f_{OL}(i)f_{OL}(j)u^2 - m_Z^2 f_{OL}(i)f_{OL}(j)(s+t+u) + m_\ell^2(2m_Z^2 f_{OL}(i)f_{OL}(j) \\
& - m_{\tilde\chi_i^0}m_{\tilde\chi_j^0} f_{OR}(i)f_{OR}(j) - 3f_{OL}(i)f_{OL}(j)u)))) \\
& + f_A(1,i)(f_B(1,j)m_\ell(m_{\tilde\chi}^4(m_{\tilde\chi_i^0} f_{OL}(i)f_{OL}(j) + m_{\tilde\chi_j^0} f_{OR}(i)f_{OR}(j)) \\
& + m_{\tilde\chi}^2(m_{\tilde\chi_i^0} f_{OL}(i)f_{OL}(j) + m_{\tilde\chi_j^0} f_{OR}(i)f_{OR}(j))(m_Z^2 - 2u) \\
& - 6m_{\tilde\chi}m_Z^2(m_{\tilde\chi_i^0}m_{\tilde\chi_j^0} f_{OL}(i)f_{OR}(j) + f_{OL}(j)f_{OR}(i)u) - (m_{\tilde\chi_i^0} f_{OL}(i)f_{OL}(j)
\end{aligned}
$$

$$
\begin{aligned}
& +m_{\tilde{\chi}_j^0} f_{OR}(i) f_{OR}(j))(2m_Z^4 - m_Z^2 u - u^2)) + f_A(1,j)(m_{\tilde{\chi}}^6 f_{OR}(i) f_{OR}(j) \\
& -3m_{\tilde{\chi}}^3 m_Z^2 (m_{\tilde{\chi}_j^0} f_{OL}(j) f_{OR}(i) + m_{\tilde{\chi}_i^0} f_{OL}(i) f_{OR}(j)) - 3m_{\tilde{\chi}} m_Z^2 (m_{\tilde{\chi}_j^0} f_{OL}(j) f_{OR}(i) \\
& +m_{\tilde{\chi}_i^0} f_{OL}(i) f_{OR}(j))(2m_\ell^2 + m_Z^2 - s - t) \\
& +m_{\tilde{\chi}}^4 f_{OR}(i) f_{OR}(j)(2m_\ell^2 + 2m_Z^2 - s - t - 2u) \\
& +m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} f_{OL}(i) f_{OL}(j)(-m_Z^4 + (m_\ell^2 - s)u + m_Z^2(s - t + u)) \\
& +f_{OR}(i) f_{OR}(j)(-2m_Z^6 - 2m_Z^2 su - tu^2 + 2m_Z^4(s + t + u) \\
& +m_\ell^2(-4m_Z^4 + 2m_Z^2 u + u^2)) - m_{\tilde{\chi}}^2 (m_Z^4 f_{OR}(i) f_{OR}(j) \\
& -m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} f_{OL}(i) f_{OL}(j) s - f_{OR}(i) f_{OR}(j) su - 2 f_{OR}(i) f_{OR}(j) tu \\
& -f_{OR}(i) f_{OR}(j) u^2 + m_Z^2 f_{OR}(i) f_{OR}(j)(s + t + u) + m_\ell^2 (m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} f_{OL}(i) f_{OL}(j) \\
& +f_{OR}(i) f_{OR}(j)(-2m_Z^2 + 3u)))))))/((u - m_{\tilde{\chi}_i^0}^2)(u - m_{\tilde{\chi}_j^0}^2))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm I} \times \mathcal{T}_{\rm II} \;=\; & (1/2)(-f_A(1,1)(2 f_B(j,1) m_{\tilde{\chi}} m_\ell (m_{\tilde{\ell}_1}^4 f_L + 2m_Z^4 f_L - 3m_\ell^2 m_Z^2 f_R - m_Z^4 f_R \\
& -2m_Z^2 f_L s + m_Z^2 f_R s + m_{\tilde{\chi}}^2 f_L (m_{\tilde{\ell}_1}^2 + 3m_Z^2 - t) - 3m_Z^2 f_L t + m_\ell^2 f_R t + m_Z^2 f_R t \\
& -f_R st + f_L t^2 - m_Z^2 f_L u + 2m_Z^2 f_R u + f_L tu + m_{\tilde{\ell}_1}^2 (m_Z^2 (f_L - 3f_R) - m_\ell^2 f_R + f_R s \\
& -2 f_L t - f_L u)) + f_A(j,1)(-2m_\ell^2 m_Z^4 f_R + m_\ell^2 m_Z^2 f_L s + m_Z^4 f_L s + 4m_\ell^2 m_Z^2 f_R s \\
& -m_Z^2 f_L s^2 + m_\ell^2 m_Z^2 f_L t + m_Z^4 f_L t + 2m_\ell^2 m_Z^2 f_R t - m_\ell^2 f_L st - m_Z^2 f_L st + f_L s^2 t \\
& -m_\ell^2 f_L t^2 - 2m_Z^2 f_L t^2 + f_L t^3 + m_{\tilde{\ell}_1}^4 f_L (-2m_\ell^2 + s + t - u) - m_\ell^2 m_Z^2 f_L u \\
& -m_Z^4 f_L u + 2m_\ell^2 m_Z^2 f_R u + m_\ell^2 f_L tu + m_Z^2 f_L tu - 2m_\ell^2 f_R tu + m_Z^2 f_L u^2 - f_L tu^2 \\
& +m_{\tilde{\ell}_1}^2 (m_\ell^2 (2m_Z^2 (f_L - 3f_R) + f_L (s + 3t - u) + 2 f_R u) + f_L (-s^2 - st - 2t^2 \\
& +2m_Z^2 (s - t - u) + tu + u^2)) + m_{\tilde{\chi}}^2 (2m_\ell^2 (f_L - f_R)(3m_Z^2 - t) \\
& +m_{\tilde{\ell}_1}^2 (2m_\ell^2 (f_L - f_R) + f_L (8m_Z^2 - s + t - u)) - f_L (t(-s + t - u) \\
& +m_Z^2 (s - t + 5u))))) + f_B(1,1)(-2 f_A(j,1) m_{\tilde{\chi}} m_\ell (-m_Z^4 f_L + m_{\tilde{\ell}_1}^4 f_R + 3m_{\tilde{\chi}}^2 m_Z^2 f_R \\
& +2m_Z^4 f_R + m_Z^2 f_L s - 2m_Z^2 f_R s + m_Z^2 f_L t - m_{\tilde{\chi}}^2 f_R t - 3m_Z^2 f_R t - f_L st + f_R t^2 \\
& +m_\ell^2 f_L (-3m_Z^2 + t) + 2m_Z^2 f_L u - m_Z^2 f_R u + f_R tu + m_{\tilde{\ell}_1}^2 (-m_\ell^2 f_L + m_{\tilde{\chi}}^2 f_R \\
& +m_Z^2 (-3f_L + f_R) + f_L s - 2 f_R t - f_R u)) + f_B(j,1)(2m_\ell^2 m_Z^4 f_L - 4m_\ell^2 m_Z^2 f_L s \\
& -m_\ell^2 m_Z^2 f_R s - m_Z^4 f_R s + m_Z^2 f_R s^2 - 2m_\ell^2 m_Z^2 f_L t - m_\ell^2 m_Z^2 f_R t - m_Z^4 f_R t \\
& +m_\ell^2 f_R st + m_Z^2 f_R st - f_R s^2 t + m_\ell^2 f_R t^2 + 2m_Z^2 f_R t^2 - f_R t^3 - 2m_\ell^2 m_Z^2 f_L u \\
& +m_\ell^2 m_Z^2 f_R u + m_Z^4 f_R u + 2m_\ell^2 f_L tu - m_\ell^2 f_R tu - m_Z^2 f_R tu - m_Z^2 f_R u^2 + f_R tu^2
\end{aligned}
$$

$$
\begin{aligned}
& +m_{\tilde{\ell}_1}^4 f_R(2m_\ell^2 - s - t + u) + m_{\tilde{\ell}_1}^2(-f_R(-s^2 - st - 2t^2 + 2m_Z^2(s - t - u) + tu \\
& +u^2) + m_\ell^2(m_Z^2(6f_L - 2f_R) - 2f_L u + f_R(-s - 3t + u))) + m_{\tilde{\chi}}^2(2m_\ell^2(f_L \\
& -f_R)(3m_Z^2 - t) + m_{\tilde{\ell}_1}^2(2m_\ell^2(f_L - f_R) + f_R(-8m_Z^2 + s - t + u)) + f_R(t(-s \\
& +t - u) + m_Z^2(s - t + 5u))))))/(2m_Z^2(s - m_\ell^2)(t - m_{\tilde{\ell}_j}^2))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} \;=\; & 1/2((1/(2m_Z^2))(f_A(1,i)(2f_B(1,1)m_\ell(m_{\tilde{\chi}}^5 f_R f_{OR}(i) + m_{\tilde{\chi}}^2 m_{\tilde{\chi}_i^0} f_{OL}(i)(-m_\ell^2 f_L \\
& -3m_Z^2 f_R + f_L s) + m_{\tilde{\chi}}^3 f_{OR}(i)(m_Z^2(-3f_L + f_R) + f_R(m_{\tilde{\ell}_1}^2 - t - 2u)) \\
& +m_{\tilde{\chi}} f_{OR}(i)(-6m_\ell^2 m_Z^2 f_L - m_Z^4(3f_L + 4f_R) + m_Z^2(-3m_{\tilde{\ell}_1}^2 f_R + 3f_L(s + t) \\
& +f_R(s + 2t)) + f_R u(-m_{\tilde{\ell}_1}^2 + t + u)) + m_{\tilde{\chi}_i^0} f_{OL}(i)(-m_Z^4 f_L + f_L(m_\ell^2 - s)u \\
& +m_Z^2(3m_{\tilde{\ell}_1}^2 f_R - 3f_R s + f_L(s - t + u)))) + f_A(1,1)(2m_{\tilde{\chi}}^3 m_{\tilde{\chi}_i^0} f_L f_{OL}(i)(-m_\ell^2 + s) \\
& +m_{\tilde{\chi}}^4 f_{OR}(i)(2m_\ell^2 f_R - f_L(s + t - u)) - 2m_{\tilde{\chi}} m_{\tilde{\chi}_i^0} f_{OL}(i)(2m_Z^4 f_L + m_{\tilde{\ell}_1}^2 f_L(m_\ell^2 - s) \\
& +m_Z^2(6m_\ell^2 f_R - 2f_L s) - f_L(m_\ell^2 - s)(t + u)) + m_{\tilde{\chi}}^2 f_{OR}(i)(m_{\tilde{\ell}_1}^2 f_L(4m_\ell^2 + 8m_Z^2 \\
& -3s - t + u) + m_\ell^2(2m_Z^2 f_R - 4f_R u + f_L(-s - 3t + u)) + f_L(s^2 + t^2 + tu \\
& -2u^2 + s(2t + u) - m_Z^2(5s + t + 3u))) + f_{OR}(i)(m_{\tilde{\ell}_1}^2 f_L(4m_Z^4 - 4m_Z^2(s + t) \\
& +m_\ell^2(8m_Z^2 - 2u) + (s + t - u)u) + m_\ell^2(-4m_Z^4 f_R + m_Z^2(-8f_L s + 2f_R u) \\
& +u(f_L(s + t - u) + 2f_R u)) + f_L(-4m_Z^4 s - u(s^2 + t^2 - u^2) \\
& +m_Z^2(4s^2 + (t - u)u + s(4t + u)))))) + f_B(1,i)(2f_A(1,1)m_\ell(m_{\tilde{\chi}}^5 f_L f_{OL}(i) \\
& +m_{\tilde{\chi}}^2 m_{\tilde{\chi}_i^0} f_{OR}(i)(-3m_Z^2 f_L + f_R(-m_\ell^2 + s)) + m_{\tilde{\chi}} f_{OL}(i)(-m_Z^4(4f_L + 3f_R) \\
& +m_Z^2(-6m_\ell^2 f_R + 3f_R(s + t) + f_L(s + 2t)) - m_{\tilde{\ell}_1}^2 f_L(3m_Z^2 + u) + f_L u(t + u)) \\
& +m_{\tilde{\chi}}^3 f_{OL}(i)(m_{\tilde{\ell}_1}^2 f_L + m_Z^2(f_L - 3f_R) - f_L(t + 2u)) + m_{\tilde{\chi}_i^0} f_{OR}(i)(3m_{\tilde{\ell}_1}^2 m_Z^2 f_L \\
& -m_Z^4 f_R + f_R(m_\ell^2 - s)u + m_Z^2(-3f_L s + f_R(s - t + u)))) \\
& +f_B(1,1)(2m_{\tilde{\chi}}^3 m_{\tilde{\chi}_i^0} f_R f_{OR}(i)(-m_\ell^2 + s) + m_{\tilde{\chi}}^4 f_{OL}(i)(2m_\ell^2 f_L - f_R(s + t - u)) \\
& -2m_{\tilde{\chi}} m_{\tilde{\chi}_i^0} f_{OR}(i)(m_\ell^2(6m_Z^2 f_L + f_R(m_{\tilde{\ell}_1}^2 - t - u)) + f_R(2m_Z^4 - 2m_Z^2 s + s(-m_{\tilde{\ell}_1}^2 \\
& +t + u))) + m_{\tilde{\chi}}^2 f_{OL}(i)(m_\ell^2(2m_Z^2 f_L + 4m_{\tilde{\ell}_1}^2 f_R - f_R s - 3f_R t - 4f_L u + f_R u) \\
& +f_R(s^2 + 2st + t^2 + su + tu - 2u^2 + m_{\tilde{\ell}_1}^2(8m_Z^2 - 3s - t + u) - m_Z^2(5s + t \\
& +3u))) + f_{OL}(i)(m_\ell^2(-4m_Z^4 f_L + 2m_Z^2(4m_{\tilde{\ell}_1}^2 f_R - 4f_R s + f_L u) + u(-2m_{\tilde{\ell}_1}^2 f_R \\
& +f_R(s + t - u) + 2f_L u)) + f_R(-4m_Z^4 s + m_{\tilde{\ell}_1}^2(4m_Z^4 - 4m_Z^2(s + t) + (s + t \\
& -u)u) - u(s^2 + t^2 - u^2) + m_Z^2(4s^2 + (t - u)u + s(4t + u))))))))
\end{aligned}
$$

$$/((u-m^2_{\tilde\chi^0_i})(s-m^2_\ell))$$

$$\begin{aligned}
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} &= 1/2((1/m_Z^2)(f_B(j,1)(f_A(1,i)m_\ell(m_{\tilde\chi^0_i}f_{OL}(i)+m_{\tilde\chi}f_{OR}(i))(m_Z^4-2m_Z^2 s\\
&+m_{\tilde\chi}^2(m^2_{\tilde\ell_1}+3m_Z^2-t)-m_Z^2 t+m^2_{\tilde\ell_1}(3m_Z^2-u)-m_Z^2 u+tu)\\
&+f_B(1,i)(m^4_{\tilde\chi}f_{OL}(i)(m^2_{\tilde\ell_1}+3m_Z^2-t)+m_{\tilde\chi}m_{\tilde\chi^0_i}f_{OR}(i)(-m_Z^4+m_Z^2 s\\
&+m^2_{\tilde\ell_1}(-m_\ell^2-3m_Z^2+s)+m_Z^2 t-st+m_\ell^2(-3m_Z^2+t)+2m_Z^2 u)\\
&+m^2_{\tilde\chi}f_{OL}(i)(3m_Z^4-4m_Z^2 s+m_\ell^2(6m_Z^2-2t)-5m_Z^2 t+st+t^2+m^2_{\tilde\ell_1}(2m_\ell^2\\
&+5m_Z^2-s-t-u)-2m_Z^2 u+tu)+f_{OL}(i)(-m_Z^4 s+m_Z^2 s^2-m_Z^4 t+2m_Z^2 st\\
&+m_Z^2 t^2+m_Z^4 u+m_Z^2 tu-t^2 u-m_Z^2 u^2+m_\ell^2(-m_Z^2(3s+t)+tu)\\
&+m^2_{\tilde\ell_1}(m_\ell^2(4m_Z^2-u)+tu+m_Z^2(-s-3t+u)))))\\
&+f_A(j,1)(f_B(1,i)m_\ell(m_{\tilde\chi}f_{OL}(i)+m_{\tilde\chi^0_i}f_{OR}(i))(m_Z^4-2m_Z^2 s+m^2_{\tilde\chi}(m^2_{\tilde\ell_1}\\
&+3m_Z^2-t)-m_Z^2 t+m^2_{\tilde\ell_1}(3m_Z^2-u)-m_Z^2 u+tu)+f_A(1,i)(m^4_{\tilde\chi}f_{OR}(i)(m^2_{\tilde\ell_1}\\
&+3m_Z^2-t)+m_{\tilde\chi}m_{\tilde\chi^0_i}f_{OL}(i)(-m_Z^4+m_Z^2 s+m^2_{\tilde\ell_1}(-m_\ell^2-3m_Z^2+s)+m_Z^2 t\\
&-st+m_\ell^2(-3m_Z^2+t)+2m_Z^2 u)+m^2_{\tilde\chi}f_{OR}(i)(3m_Z^4-4m_Z^2 s+m_\ell^2(6m_Z^2-2t)\\
&-5m_Z^2 t+st+t^2+m^2_{\tilde\ell_1}(2m_\ell^2+5m_Z^2-s-t-u)-2m_Z^2 u+tu)\\
&+f_{OR}(i)(-m_Z^4 s+m_Z^2 s^2-m_Z^4 t+2m_Z^2 st+m_Z^2 t^2+m_Z^4 u+m_Z^2 tu-t^2 u-m_Z^2 u^2\\
&+m_\ell^2(-m_Z^2(3s+t)+tu)+m^2_{\tilde\ell_1}(m_\ell^2(4m_Z^2-u)+tu+m_Z^2(-s-3t+u)))))))\\
&/((u-m^2_{\tilde\chi^0_i})(t-m^2_{\tilde\ell_j}))\\
|\mathcal{T}|^2 &= \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+\sum_{i,j}f_C(i)f_C(j)\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+\sum_{i,j}\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III}+2\sum_j f_C(j)\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II}\\
&+2\sum_i \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III}+2\sum_{i,j}f_C(j)\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} \qquad \text{(B21)}
\end{aligned}$$

$\tilde\ell_1\chi\longrightarrow\gamma\ell$

The couplings are modified.

$$\begin{aligned}
f_A &= -g_2/\sqrt{2}(\cos\theta_\ell(-N_{12}-\tan\theta_W N_{11})+\sin\theta_\ell m_\ell/(m_W\cos\beta)N_{13})(e)\\
f_B &= -g_2/\sqrt{2}(\cos\theta_\ell m_\ell/(m_W\cos\beta)N_{13}+\sin\theta_\ell(2\tan\theta_W N_{11}))(e)\\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (1/2)(2(4f_Af_Bm_{\tilde\chi}m_\ell(-m^2_{\tilde\chi}-m^2_{\tilde\ell_1}+2s+t+u)+f_A^2(m^4_{\tilde\chi}-m^4_{\tilde\ell_1}+m_\ell^4-3m_\ell^2 s\\
&-m_\ell^2 t-su-m^2_{\tilde\chi}(m_\ell^2+t+u)+m^2_{\tilde\ell_1}(2m_\ell^2+s+t+u))+f_B^2(m^4_{\tilde\chi}-m^4_{\tilde\ell_1}+m_\ell^4\\
&-3m_\ell^2 s-m_\ell^2 t-su-m^2_{\tilde\chi}(m_\ell^2+t+u)+m^2_{\tilde\ell_1}(2m_\ell^2+s+t+u))))/(s-m_\ell^2)^2
\end{aligned}$$

$$\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} = (1/2)(-2(-4f_Af_Bm_{\tilde\chi}m_\ell + f_A^2(m_{\tilde\chi}^2+m_\ell^2-t)+f_B^2(m_{\tilde\chi}^2+m_\ell^2-t))(m_{\tilde\ell_1}^2+t))$$
$$/(t-m_{\tilde\ell_1}^2)^2$$
$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} = (1/2)(1/2(-4f_Af_Bm_{\tilde\chi}m_\ell(3m_{\tilde\chi}^2-3m_{\tilde\ell_1}^2-3m_\ell^2-s-t+u)+f_A^2(-5m_\ell^2s+s^2$$
$$-m_\ell^2t+t^2-m_\ell^2u-u^2+m_{\tilde\ell_1}^2(2m_\ell^2-s+3t+u)+m_{\tilde\chi}^2(-8m_{\tilde\ell_1}^2+s-t+5u))$$
$$+f_B^2(-5m_\ell^2s+s^2-m_\ell^2t+t^2-m_\ell^2u-u^2+m_{\tilde\ell_1}^2(2m_\ell^2-s+3t+u)$$
$$+m_{\tilde\chi}^2(-8m_{\tilde\ell_1}^2+s-t+5u))))/((s-m_\ell^2)(t-m_{\tilde\ell_1}^2))$$
$$|\mathcal{T}|^2 = \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \quad \text{(B22)}$$

$\widetilde{\ell}_1\chi \longrightarrow \ell h[H]$

The s-channel τ annihilation was neglected in [17] due to the small Yukawa coupling. However, at large $\tan\beta$, these couplings are enhanced particularly for the h final state. The t-channel $\widetilde{\ell}$ exchange is now summed over $\widetilde{\ell}_1$ and $\widetilde{\ell}_2$. Also added is the neutralino u-channel exchange.

I. s-channel ℓ annihilation

II. t-channel $\widetilde{\ell}_{1,2}$ exchange

III. u-channel $\chi_{1,2,3,4}$ exchange

$$f_1 = -g_2m_\ell\cos\alpha/(2m_W\cos\beta)$$
$$f_2(1) = g_2m_Z((-1/2+\sin^2\theta_W)\sin[-\cos](\beta+\alpha)\cos^2\theta_\ell$$
$$-\sin^2\theta_W\sin[-\cos](\beta+\alpha)\sin^2\theta_\ell)/\cos\theta_W$$
$$+g_2m_\ell^2\sin[-\cos]\alpha/(m_W\cos\beta)$$
$$-g_2m_\ell(A_\ell\sin[-\cos]\alpha-\mu\cos[\sin]\alpha)\sin\theta_\ell\cos\theta_\ell/(m_W\cos\beta)$$
$$f_2(2) = g_2m_Z((1/2-\sin^2\theta_W)\sin[-\cos](\beta+\alpha)$$
$$-\sin^2\theta_W\sin[-\cos](\beta+\alpha))\cos\theta_\ell\sin\theta_\ell/\cos\theta_W$$
$$-g_2m_\ell(A_\ell\sin[-\cos]\alpha-\mu\cos[\sin]\alpha)\cos(2\theta_\ell)/(2m_W\cos\beta)$$
$$f_A(1,i) = -g_2/\sqrt{2}(\cos\theta_\ell(-N_{i2}-\tan\theta_WN_{i1})+\sin\theta_\ell m_\ell/(m_W\cos\beta)N_{i3})$$
$$f_A(2,i) = -g_2/\sqrt{2}(-\sin\theta_\ell(-N_{i2}-\tan\theta_WN_{i1})+\cos\theta_\ell m_\ell/(m_W\cos\beta)N_{i3})$$
$$f_B(1,i) = -g_2/\sqrt{2}(\cos\theta_\ell m_\ell/(m_W\cos\beta)N_{i3}+\sin\theta_\ell(2\tan\theta_WN_{i1}))$$
$$f_B(2,i) = -g_2/\sqrt{2}(-\sin\theta_\ell m_\ell/(m_W\cos\beta)N_{i3}+\cos\theta_\ell(2\tan\theta_WN_{i1}))$$

$$
\begin{aligned}
f_{CL}(i) &= g_2/2((N_{13}(N_{i2}-N_{i1}\tan\theta_W)+N_{i3}(N_{12}-N_{11}\tan\theta_W))\sin\alpha \\
&+(N_{14}(N_{i2}-N_{i1}\tan\theta_W)+N_{i4}(N_{12}-N_{11}\tan\theta_W))\cos\alpha) \\
f_{CR}(i) &= g_2/2((N_{i3}(N_{12}-N_{11}\tan\theta_W)+N_{13}(N_{i2}-N_{i1}\tan\theta_W))\sin\alpha \\
&+(N_{i4}(N_{12}-N_{11}\tan\theta_W)+N_{14}(N_{i2}-N_{i1}\tan\theta_W))\cos\alpha) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= 1/2(f_A(1,1)(2f_B(1,1)m_{\tilde\chi}m_\ell(m_{\tilde\chi}^2+m_{\tilde\ell_1}^2+3m_\ell^2+s-t-u) \\
&+f_A(1,1)(m_{\tilde\chi}^4-m_{\tilde\ell_1}^4+m_\ell^4+3m_\ell^2 s-m_\ell^2 t+m_{\tilde\chi}^2(5m_\ell^2-t-u)-su \\
&+m_{\tilde\ell_1}^2(-4m_\ell^2+s+t+u)))+f_B(1,1)(2f_A(1,1)m_{\tilde\chi}m_\ell(m_{\tilde\chi}^2+m_{\tilde\ell_1}^2+3m_\ell^2+s \\
&-t-u)+f_B(1,1)(m_{\tilde\chi}^4-m_{\tilde\ell_1}^4+m_\ell^4+3m_\ell^2 s-m_\ell^2 t+m_{\tilde\chi}^2(5m_\ell^2-t-u)-su \\
&+m_{\tilde\ell_1}^2(-4m_\ell^2+s+t+u))))/(s-m_\ell^2)^2 \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= 1/2(f_A(i,1)(2f_B(j,1)m_{\tilde\chi}m_\ell+f_A(j,1)(m_{\tilde\chi}^2+m_\ell^2-t)) \\
&+f_B(i,1)(2f_A(j,1)m_{\tilde\chi}m_\ell+f_B(j,1)(m_{\tilde\chi}^2+m_\ell^2-t)))/((t-m_{\tilde\ell_i}^2)(t-m_{\tilde\ell_j}^2)) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= 1/2(f_A(1,i)(f_B(1,j)m_\ell(f_{CR}(i)m_{\tilde\chi_i^0}(2f_{CL}(j)m_{\tilde\chi}m_{\tilde\chi_j^0}+f_{CR}(j)(m_{\tilde\chi}^2-m_{h[H]}^2+u)) \\
&+f_{CL}(i)(2f_{CR}(j)m_{\tilde\chi}u+f_{CL}(j)m_{\tilde\chi_j^0}(m_{\tilde\chi}^2-m_{h[H]}^2+u))) \\
&+f_A(1,j)(f_{CR}(i)m_{\tilde\chi_i^0}(f_{CR}(j)m_{\tilde\chi_j^0}(m_{\tilde\chi}^2+m_\ell^2-t)+f_{CL}(j)m_{\tilde\chi}(m_{\tilde\chi}^2+m_{h[H]}^2 \\
&+2m_\ell^2-s-t))+f_{CL}(i)(f_{CR}(j)m_{\tilde\chi}m_{\tilde\chi_j^0}(m_{\tilde\chi}^2+m_{h[H]}^2+2m_\ell^2-s-t) \\
&+f_{CL}(j)(m_{\tilde\chi}^4-m_{h[H]}^4+m_{\tilde\chi}^2(2m_\ell^2-s-t)+(m_\ell^2-s)u+m_{h[H]}^2(-2m_\ell^2+s+t \\
&+u)))))+f_B(1,i)(f_A(1,j)m_\ell(f_{CL}(i)m_{\tilde\chi_i^0}(2f_{CR}(j)m_{\tilde\chi}m_{\tilde\chi_j^0}+f_{CL}(j)(m_{\tilde\chi}^2-m_{h[H]}^2 \\
&+u))+f_{CR}(i)(2f_{CL}(j)m_{\tilde\chi}u+f_{CR}(j)m_{\tilde\chi_j^0}(m_{\tilde\chi}^2-m_{h[H]}^2+u))) \\
&+f_B(1,j)(f_{CL}(i)m_{\tilde\chi_i^0}(f_{CL}(j)m_{\tilde\chi_j^0}(m_{\tilde\chi}^2+m_\ell^2-t)+f_{CR}(j)m_{\tilde\chi}(m_{\tilde\chi}^2+m_{h[H]}^2+2m_\ell^2 \\
&-s-t))+f_{CR}(i)(f_{CL}(j)m_{\tilde\chi}m_{\tilde\chi_j^0}(m_{\tilde\chi}^2+m_{h[H]}^2+2m_\ell^2-s-t)+f_{CR}(j)(m_{\tilde\chi}^4 \\
&-m_{h[H]}^4+m_{\tilde\chi}^2(2m_\ell^2-s-t)+(m_\ell^2-s)u+m_{h[H]}^2(-2m_\ell^2+s+t+u)))))) \\
&/((u-m_{\tilde\chi_i^0}^2)(u-m_{\tilde\chi_j^0}^2)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= 1/2(f_B(1,1)(f_B(i,1)m_\ell(2m_{\tilde\chi}^2-m_{\tilde\ell_1}^2+m_\ell^2+s-t)+f_A(i,1)m_{\tilde\chi}(m_{\tilde\chi}^2+m_{\tilde\ell_1}^2 \\
&+4m_\ell^2-t-u))+f_A(1,1)(f_A(i,1)m_\ell(2m_{\tilde\chi}^2-m_{\tilde\ell_1}^2+m_\ell^2+s-t) \\
&+f_B(i,1)m_{\tilde\chi}(m_{\tilde\chi}^2+m_{\tilde\ell_1}^2+4m_\ell^2-t-u)))/((s-m_\ell^2)(t-m_{\tilde\ell_i}^2)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} &= 1/2((1/2)(f_A(1,i)(2f_A(1,1)m_\ell(f_{CR}(i)m_{\tilde\chi_i^0}(2m_{\tilde\chi}^2-m_{\tilde\ell_1}^2+m_\ell^2+s-t) \\
&+f_{CL}(i)m_{\tilde\chi}(m_{\tilde\chi}^2-m_{h[H]}^2-2m_{\tilde\ell_1}^2+2m_\ell^2+u))+f_B(1,1)(2f_{CR}(i)m_{\tilde\chi}m_{\tilde\chi_i^0}(m_{\tilde\chi}^2
\end{aligned}
$$

$$
\begin{aligned}
& +m_{\tilde{\ell}_1}^2+4m_\ell^2-t-u)+f_{CL}(i)(2m_{\tilde{\chi}}^4+m_{\tilde{\ell}_1}^2 s+m_\ell^2 s-s^2-m_{\tilde{\ell}_1}^2 t-m_\ell^2 t+t^2 \\
& +m_{\tilde{\chi}}^2(2m_{h[H]}^2+2m_{\tilde{\ell}_1}^2+6m_\ell^2-s-3t-u)+m_{\tilde{\ell}_1}^2 u+3m_\ell^2 u-u^2+m_{h[H]}^2(-2m_{\tilde{\ell}_1}^2 \\
& -2m_\ell^2+s-t+u))))+f_B(1,i)(2f_B(1,1)m_\ell(f_{CL}(i)m_{\tilde{\chi}_i^0}(2m_{\tilde{\chi}}^2-m_{\tilde{\ell}_1}^2+m_\ell^2+s \\
& -t)+f_{CR}(i)m_{\tilde{\chi}}(m_{\tilde{\chi}}^2-m_{h[H]}^2-2m_{\tilde{\ell}_1}^2+2m_\ell^2+u)) \\
& +f_A(1,1)(2f_{CL}(i)m_{\tilde{\chi}}m_{\tilde{\chi}_i^0}(m_{\tilde{\chi}}^2+m_{\tilde{\ell}_1}^2+4m_\ell^2-t-u)+f_{CR}(i)(2m_{\tilde{\chi}}^4+m_{\tilde{\ell}_1}^2 s \\
& +m_\ell^2 s-s^2-m_{\tilde{\ell}_1}^2 t-m_\ell^2 t+t^2+m_{\tilde{\chi}}^2(2m_{h[H]}^2+2m_{\tilde{\ell}_1}^2+6m_\ell^2-s-3t-u) \\
& +m_{\tilde{\ell}_1}^2 u+3m_\ell^2 u-u^2+m_{h[H]}^2(-2m_{\tilde{\ell}_1}^2-2m_\ell^2+s-t+u)))))) \\
& /((s-m_\ell^2)(u-m_{\tilde{\chi}_i^0}^2))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} &= 1/2(f_B(j,1)(f_B(1,i)(f_{CL}(i)m_{\tilde{\chi}_i^0}(m_{\tilde{\chi}}^2+m_\ell^2-t)+f_{CR}(i)m_{\tilde{\chi}}(m_{\tilde{\chi}}^2+m_{h[H]}^2 \\
& +2m_\ell^2-s-t))+f_A(1,i)m_\ell(2f_{CR}(i)m_{\tilde{\chi}}m_{\tilde{\chi}_i^0}+f_{CL}(i)(m_{\tilde{\chi}}^2-m_{h[H]}^2+u))) \\
& +f_A(j,1)(f_A(1,i)(f_{CR}(i)m_{\tilde{\chi}_i^0}(m_{\tilde{\chi}}^2+m_\ell^2-t)+f_{CL}(i)m_{\tilde{\chi}}(m_{\tilde{\chi}}^2+m_{h[H]}^2+2m_\ell^2 \\
& -s-t))+f_B(1,i)m_\ell(2f_{CL}(i)m_{\tilde{\chi}}m_{\tilde{\chi}_i^0}+f_{CR}(i)(m_{\tilde{\chi}}^2-m_{h[H]}^2+u)))) \\
& /((t-m_{\tilde{\ell}_j}^2)(u-m_{\tilde{\chi}_i^0}^2)) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+\sum_{i,j}f_2(i)f_2(j)\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+\sum_{i,j}\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III}+2\sum_i f_1f_2(i)\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \\
& +2\sum_i f_1\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III}+2\sum_{i,j}f_2(j)\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} \qquad \text{(B23)}
\end{aligned}
$$

$\widetilde{\ell}_1\chi \longrightarrow \ell A$

I. s-channel ℓ annihilation

II. t-channel $\widetilde{\ell}_2$ exchange

III. u-channel $\chi_{1,2,3,4}$ exchange

$$
\begin{aligned}
f_A(1,i) &= -g_2/\sqrt{2}(\cos\theta_\ell(-N_{i2}-\tan\theta_W N_{i1})+\sin\theta_\ell m_\ell/(m_W\cos\beta)N_{i3}) \\
f_A(2,i) &= -g_2/\sqrt{2}(-\sin\theta_\ell(-N_{i2}-\tan\theta_W N_{i1})+\cos\theta_\ell m_\ell/(m_W\cos\beta)N_{i3}) \\
f_B(1,i) &= -g_2/\sqrt{2}(\cos\theta_\ell m_\ell/(m_W\cos\beta)N_{i3}+\sin\theta_\ell(2\tan\theta_W N_{i1})) \\
f_B(2,i) &= -g_2/\sqrt{2}(-\sin\theta_\ell m_\ell/(m_W\cos\beta)N_{i3}+\cos\theta_\ell(2\tan\theta_W N_{i1})) \\
f_1 &= -(g_2 m_\ell\tan\beta)/(2m_W) \\
f_2 &= -g_2 m_\ell/(2m_W)(A_\ell\tan\beta-\mu) \\
f_{CL}(i) &= g_2/2((N_{i3}(N_{12}-N_{11}\tan\theta_W)+N_{13}(N_{i2}-N_{i1}\tan\theta_W))\sin\beta
\end{aligned}
$$

$$
\begin{aligned}
& -(N_{i4}(N_{12}-N_{11}\tan\theta_W)+N_{14}(N_{i2}-N_{i1}\tan\theta_W))\cos\beta) \\
f_{CR}(i) &= -g_2/2((N_{i3}(N_{12}-N_{11}\tan\theta_W)+N_{13}(N_{i2}-N_{i1}\tan\theta_W))\sin\beta \\
& -(N_{i4}(N_{12}-N_{11}\tan\theta_W)+N_{14}(N_{i2}-N_{i1}\tan\theta_W))\cos\beta) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= 1/2(f_A(1,)(2f_B(1,1)m_{\tilde\chi}m_\ell(-m_{\tilde\chi}^2-m_{\tilde\ell_1}^2-m_\ell^2+s+t+u) \\
& +f_A(1,1)(-m_{\tilde\chi}^4+m_{\tilde\ell_1}^4-m_\ell^4+m_\ell^2 s+m_\ell^2 t+su+m_{\tilde\chi}^2(-m_\ell^2+t+u) \\
& -m_{\tilde\ell_1}^2(s+t+u)))+f_B(1,1)(2f_A(1,1)m_{\tilde\chi}m_\ell(-m_{\tilde\chi}^2-m_{\tilde\ell_1}^2-m_\ell^2+s+t+u) \\
& +f_B(1,1)(-m_{\tilde\chi}^4+m_{\tilde\ell_1}^4-m_\ell^4+m_\ell^2 s+m_\ell^2 t+su+m_{\tilde\chi}^2(-m_\ell^2+t+u) \\
& -m_{\tilde\ell_1}^2(s+t+u))))/(s-m_\ell^2)^2 \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= 1/2(f_A(2,1)(2f_B(2,1)m_{\tilde\chi}m_\ell+f_A(2,1)(m_{\tilde\chi}^2+m_\ell^2-t)) \\
& +f_B(2,1)(2f_A(2,1)m_{\tilde\chi}m_\ell+f_B(2,1)(m_{\tilde\chi}^2+m_\ell^2-t)))/((t-m_{\tilde\ell_2}^2)^2) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= 1/2(f_A(1,i)(f_B(1,j)m_\ell(f_{CR}(i)m_{\tilde\chi_i^0}(2f_{CL}(j)m_{\tilde\chi}m_{\tilde\chi_j^0}+f_{CR}(j)(-m_A^2+m_{\tilde\chi}^2+u)) \\
& +f_{CL}(i)(2f_{CR}(j)m_{\tilde\chi}u+f_{CL}(j)m_{\tilde\chi_j^0}(-m_A^2+m_{\tilde\chi}^2+u))) \\
& +f_A(1,j)(f_{CR}(i)m_{\tilde\chi_i^0}(f_{CR}(j)m_{\tilde\chi_j^0}(m_{\tilde\chi}^2+m_\ell^2-t)+f_{CL}(j)m_{\tilde\chi}(m_A^2+m_{\tilde\chi}^2+2m_\ell^2 \\
& -s-t))+f_{CL}(i)(f_{CR}(j)m_{\tilde\chi}m_{\tilde\chi_j^0}(m_A^2+m_{\tilde\chi}^2+2m_\ell^2-s-t)+f_{CL}(j)(-m_A^4 \\
& +m_{\tilde\chi}^4+m_{\tilde\chi}^2(2m_\ell^2-s-t)+(m_\ell^2-s)u+m_A^2(-2m_\ell^2+s+t+u))))) \\
& +f_B(1,i)(f_A(1,j)m_\ell(f_{CL}(i)m_{\tilde\chi_i^0}(2f_{CR}(j)m_{\tilde\chi}m_{\tilde\chi_j^0}+f_{CL}(j)(-m_A^2+m_{\tilde\chi}^2+u)) \\
& +f_{CR}(i)(2f_{CL}(j)m_{\tilde\chi}u+f_{CR}(j)m_{\tilde\chi_j^0}(-m_A^2+m_{\tilde\chi}^2+u))) \\
& +f_B(1,j)(f_{CL}(i)m_{\tilde\chi_i^0}(f_{CL}(j)m_{\tilde\chi_j^0}(m_{\tilde\chi}^2+m_\ell^2-t)+f_{CR}(j)m_{\tilde\chi}(m_A^2+m_{\tilde\chi}^2+2m_\ell^2 \\
& -s-t))+f_{CR}(i)(f_{CL}(j)m_{\tilde\chi}m_{\tilde\chi_j^0}(m_A^2+m_{\tilde\chi}^2+2m_\ell^2-s-t)+f_{CR}(j)(-m_A^4 \\
& +m_{\tilde\chi}^4+m_{\tilde\chi}^2(2m_\ell^2-s-t)+(m_\ell^2-s)u+m_A^2(-2m_\ell^2+s+t+u)))))) \\
& /((u-m_{\tilde\chi_i^0}^2)(u-m_{\tilde\chi_j^0}^2)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= 1/2(f_A(1,1)(f_A(2,1)m_\ell(-m_{\tilde\ell_1}^2-m_\ell^2+s+t)+f_B(2,1)m_{\tilde\chi}(m_{\tilde\chi}^2+m_{\tilde\ell_1}^2-t-u)) \\
& +f_B(1,1)(f_B(2,1)m_\ell(m_{\tilde\ell_1}^2+m_\ell^2-s-t)+f_A(2,1)m_{\tilde\chi}(-m_{\tilde\chi}^2-m_{\tilde\ell_1}^2+t+u))) \\
& /((s-m_\ell^2)(t-m_{\tilde\ell_2}^2)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} &= 1/2((1/2)(f_B(1,i)(2f_B(1,1)m_\ell(f_{CL}(i)m_{\tilde\chi_i^0}(m_{\tilde\ell_1}^2+m_\ell^2-s-t) \\
& +f_{CR}(i)m_{\tilde\chi}(3m_A^2+m_{\tilde\chi}^2+2m_{\tilde\ell_1}^2+2m_\ell^2-2s-2t-u)) \\
& +f_A(1,1)(2f_{CL}(i)m_{\tilde\chi}m_{\tilde\chi_i^0}(m_{\tilde\chi}^2+m_{\tilde\ell_1}^2-t-u)+f_{CR}(i)(2m_{\tilde\chi}^4+m_{\tilde\ell_1}^2 s+m_\ell^2 s
\end{aligned}
$$

$$
\begin{aligned}
&-s^2 - m_{\tilde{\ell}_1}^2 t - m_\ell^2 t + t^2 + m_{\tilde{\chi}}^2(2m_{\tilde{\ell}_1}^2 + 2m_\ell^2 - s - 3t - u) + m_{\tilde{\ell}_1}^2 u - m_\ell^2 u - u^2 \\
&+m_A^2(2m_{\tilde{\chi}}^2 - 2m_{\tilde{\ell}_1}^2 + 2m_\ell^2 + s - t + u)))) \\
&+f_A(1,i)(-2f_A(1,1)m_\ell(f_{CR}(i)m_{\tilde{\chi}_i^0}(m_{\tilde{\ell}_1}^2 + m_\ell^2 - s - t) + f_{CL}(i)m_{\tilde{\chi}}(3m_A^2 \\
&+m_{\tilde{\chi}}^2 + 2m_{\tilde{\ell}_1}^2 + 2m_\ell^2 - 2s - 2t - u)) + f_B(1,1)(2f_{CR}(i)m_{\tilde{\chi}}m_{\tilde{\chi}_i^0}(-m_{\tilde{\chi}}^2 - m_{\tilde{\ell}_1}^2 \\
&+t + u) + f_{CL}(i)(-2m_{\tilde{\chi}}^4 - m_{\tilde{\ell}_1}^2 s - m_\ell^2 s + s^2 + m_{\tilde{\ell}_1}^2 t + m_\ell^2 t - t^2 - m_{\tilde{\ell}_1}^2 u + m_\ell^2 u \\
&+u^2 - m_A^2(2m_{\tilde{\chi}}^2 - 2m_{\tilde{\ell}_1}^2 + 2m_\ell^2 + s - t + u) + m_{\tilde{\chi}}^2(-2m_{\tilde{\ell}_1}^2 - 2m_\ell^2 + s + 3t \\
&+u))))))/((s - m_\ell^2)(u - m_{\tilde{\chi}_i^0}^2))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} &= 1/2(f_B(2,1)(f_B(1,i)(f_{CL}(i)m_{\tilde{\chi}_i^0}(m_{\tilde{\chi}}^2 + m_\ell^2 - t) + f_{CR}(i)m_{\tilde{\chi}}(m_A^2 + m_{\tilde{\chi}}^2 + 2m_\ell^2 \\
&\quad -s - t)) + f_A(1,i)m_\ell(2f_{CR}(i)m_{\tilde{\chi}}m_{\tilde{\chi}_i^0} + f_{CL}(i)(-m_A^2 + m_{\tilde{\chi}}^2 + u))) \\
&\quad +f_A(2,1)(f_A(1,i)(f_{CR}(i)m_{\tilde{\chi}_i^0}(m_{\tilde{\chi}}^2 + m_\ell^2 - t) + f_{CL}(i)m_{\tilde{\chi}}(m_A^2 + m_{\tilde{\chi}}^2 + 2m_\ell^2 \\
&\quad -s - t)) + f_B(1,i)m_\ell(2f_{CL}(i)m_{\tilde{\chi}}m_{\tilde{\chi}_i^0} + f_{CR}(i)(-m_A^2 + m_{\tilde{\chi}}^2 + u)))) \\
&\quad /((t - m_{\tilde{\ell}_2}^2)(u - m_{\tilde{\chi}_i^0}^2)) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + \sum_{i,j}\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + 2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2\sum_i f_1\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} \\
&\quad +2\sum_i f_2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} \qquad \text{(B24)}
\end{aligned}
$$

Appendix C: Chargino-Slepton Coannihilation

Below is the list of the amplitudes squared for chargino-slepton coannihilation. Note that, for identical-particle final states, one needs to divide them by two when performing the momentum integrations.

$\widetilde{\ell}_1\widetilde{\chi}_1^+ \longrightarrow \nu_\ell Z$

I. t-channel $\widetilde{\chi}_{(1,2)}^-$ exchange
II. u-channel $\widetilde{\ell}_{(1,2)}$ exchange

$$
\begin{aligned}
f_{1LL}(i) &= C^L_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_i^-}\ C^L_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-Z} \\
f_{1LR}(i) &= C^L_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_i^-}\ C^R_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-Z} \\
f_{1RL}(i) &= C^R_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_i^-}\ C^L_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-Z} \\
f_{1RR}(i) &= C^R_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_i^-}\ C^R_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-Z}
\end{aligned}
$$

$$
\begin{aligned}
f_{2L}(i) &= C_{\widetilde{\ell}_1-Z-\widetilde{\ell}_i^-}\; C^L_{\widetilde{\chi}_1^- -\nu_\ell-\widetilde{\ell}_i} \\
f_{2R}(i) &= C_{\widetilde{\ell}_1-Z-\widetilde{\ell}_i^-}\; C^R_{\widetilde{\chi}_1^- -\nu_\ell-\widetilde{\ell}_i} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (m_{\tilde{\chi}_i^+}(3f_{1LL}(j)(f_{1LR}(i)+f_{1RL}(i))m_{\tilde{\chi}_1^+}m_Z^2(m_{\tilde{\ell}_1}^2-t) \\
&\quad +m_{\tilde{\chi}_j^+}(f_{1RL}(i)f_{1RL}(j)(m_Z^4+s(-m_{\tilde{\chi}_1^+}^2+t)-m_Z^2(s+t-u)) \\
&\quad +f_{1LR}(i)f_{1LR}(j)(m_Z^4+s(-m_{\tilde{\chi}_1^+}^2+t)-m_Z^2(s+t-u)))) \\
&\quad +f_{1LL}(i)(3(f_{1LR}(j)+f_{1RL}(j))m_{\tilde{\chi}_1^+}m_{\tilde{\chi}_j^+}m_Z^2(m_{\tilde{\ell}_1}^2-t) \\
&\quad -2f_{1LL}(j)(-m_{\tilde{\ell}_1}^4 t-m_Z^4 t-m_Z^2 st+m_Z^2t^2+m_Z^2tu-t^2u \\
&\quad +m_{\tilde{\ell}_1}^2(2m_Z^4-2m_Z^2t+st+t^2+tu)+m_{\tilde{\chi}_1^+}^2(m_{\tilde{\ell}_1}^4+tu-m_{\tilde{\ell}_1}^2(s+t+u))))) \\
&\quad /(m_Z^2(m_{\tilde{\chi}_i^+}^2-t)(-m_{\tilde{\chi}_j^+}^2+t)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= ((f_{2L}(i)f_{2L}(j)+f_{2R}(i)f_{2R}(j))(m_{\tilde{\chi}_1^+}^2-u)(m_{\tilde{\ell}_1}^4+(m_Z^2-u)^2 \\
&\quad -2m_{\tilde{\ell}_1}^2(m_Z^2+u)))/(m_Z^2(-m_{\tilde{\ell}_i}^2+u)(-m_{\tilde{\ell}_j}^2+u)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= (-2(f_{1LR}(j)f_{2L}(i)+f_{1RL}(j)f_{2R}(i))m_{\tilde{\chi}_1^+}m_{\tilde{\chi}_j^+}(m_Z^4+m_{\tilde{\ell}_1}^2(3m_Z^2-s)+su \\
&\quad -m_Z^2(s+2t+u))+f_{1LL}(j)(f_{2L}(i)+f_{2R}(i))(m_{\tilde{\ell}_1}^4(s-t-u) \\
&\quad +(m_Z^2-u)(-s^2+t^2+m_Z^2(s-t-u)+u^2) \\
&\quad +m_{\tilde{\ell}_1}^2(-s^2+t^2-su+tu+2u^2+2m_Z^2(s-t+u)) \\
&\quad +m_{\tilde{\chi}_1^+}^2(2m_{\tilde{\ell}_1}^4-(m_Z^2-u)(-s+t+u)-m_{\tilde{\ell}_1}^2(2m_Z^2-s+t+3u)))) \\
&\quad /(2m_Z^2(m_{\tilde{\chi}_j^+}^2-t)(m_{\tilde{\ell}_i}^2-u)) \\
|\mathcal{T}|^2 &= \sum_{i,j}(\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II}) \qquad \text{(C1)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\chi}_1^+ \longrightarrow \nu_\ell\gamma$

I. t-channel $\widetilde{\chi}_1$ exchange

II. u-channel $\widetilde{\ell}_1$ exchange

$$
\begin{aligned}
f_{1L} &= C^L_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_1^-}\; C_{\widetilde{\chi}_1^+-\widetilde{\chi}_1^+-\gamma} \\
f_{1R} &= C^R_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_1^-}\; C_{\widetilde{\chi}_1^+-\widetilde{\chi}_1^+-\gamma} \\
f_{2L} &= C_{\widetilde{\ell}_1-\widetilde{\ell}_1-\gamma}\; C^L_{\widetilde{\chi}_1^- -\nu_\ell-\widetilde{\ell}_i} \\
f_{2R} &= C_{\widetilde{\ell}_1-\widetilde{\ell}_1-\gamma}\; C^R_{\widetilde{\chi}_1^- -\nu_\ell-\widetilde{\ell}_i} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (2(f_{1L}^2+f_{1R}^2)(m_{\tilde{\chi}_1^+}^4-m_{\tilde{\ell}_1}^4-st-m_{\tilde{\chi}_1^+}^2(6m_{\tilde{\ell}_1}^2-5t+u)+m_{\tilde{\ell}_1}^2(s+t+u)))
\end{aligned}
$$

$$
\begin{aligned}
& /(m^2_{\tilde{\chi}^+_1} - t)^2 \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= -2(f^2_{2L} + f^2_{2R})(m^2_{\tilde{\chi}^+_1} - u)(m^2_{\tilde{\ell}_1} + u)/(m^2_{\tilde{\ell}_1} - u)^2 \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= ((f_{1L}f_{2L} + f_{1R}f_{2R})(-s^2 + t^2 + u^2 - m^2_{\tilde{\chi}^+_1}(6m^2_{\tilde{\ell}_1} - 3s - 3t + u) \\
& +m^2_{\tilde{\ell}_1}(s - t + 3u)))/(2(m^2_{\tilde{\chi}^+_1} - t)(m^2_{\tilde{\ell}_1} - u)) \\
|\mathcal{T}|^2 &= \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + \mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + 2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \qquad \text{(C2)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\chi}^+_1 \longrightarrow \nu_\ell h \quad [\nu_\ell H]$

I. t-channel $\widetilde{\chi}_{1,2}$ exchange

II. u-channel $\widetilde{\tau}_{1,2}$ exchange

$$
\begin{aligned}
f_{1LL}(i) &= C^L_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}^-_i}\, C^L_{\widetilde{\chi}^+_1-\widetilde{\chi}^+_i-h} \qquad [C^L_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}^-_i}\, C^L_{\widetilde{\chi}^+_1-\widetilde{\chi}^+_i-H}] \\
f_{1LR}(i) &= C^L_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}^-_i}\, C^R_{\widetilde{\chi}^+_1-\widetilde{\chi}^+_i-h} \qquad [C^L_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}^-_i}\, C^R_{\widetilde{\chi}^+_1-\widetilde{\chi}^+_i-H}] \\
f_{1RL}(i) &= C^R_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}^-_i}\, C^L_{\widetilde{\chi}^+_1-\widetilde{\chi}^+_i-h} \qquad [C^R_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}^-_i}\, C^L_{\widetilde{\chi}^+_1-\widetilde{\chi}^+_i-H}] \\
f_{1RR}(i) &= C^R_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}^-_i}\, C^R_{\widetilde{\chi}^+_1-\widetilde{\chi}^+_i-h} \qquad [C^R_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}^-_i}\, C^R_{\widetilde{\chi}^+_1-\widetilde{\chi}^+_i-H}] \\
f_{2L}(i) &= C_{\widetilde{\ell}_1-h-\widetilde{\ell}^-_i}\, C^L_{\widetilde{\chi}^-_1-\nu_\ell-\widetilde{\ell}_i} \qquad [C_{\widetilde{\ell}_1-H-\widetilde{\ell}^-_i}\, C^L_{\widetilde{\chi}^-_1-\nu_\ell-\widetilde{\ell}_i}] \\
f_{2R}(i) &= C_{\widetilde{\ell}_1-h-\widetilde{\ell}^-_i}\, C^R_{\widetilde{\chi}^-_1-\nu_\ell-\widetilde{\ell}_i} \qquad [C_{\widetilde{\ell}_1-H-\widetilde{\ell}^-_i}\, C^R_{\widetilde{\chi}^-_1-\nu_\ell-\widetilde{\ell}_i}] \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (m_{\tilde{\chi}^+_i}(f_{1LL}(i)(f_{1LR}(j)m_{\tilde{\chi}^+_1}(-m^2_{\tilde{\ell}_1} + t) + f_{1LL}(j)m_{\tilde{\chi}^+_j}(-m^2_{\tilde{\chi}^+_1} + u)) \\
& +f_{1RR}(i)(f_{1RL}(j)m_{\tilde{\chi}^+_1}(-m^2_{\tilde{\ell}_1} + t) + f_{1RR}(j)m_{\tilde{\chi}^+_j}(-m^2_{\tilde{\chi}^+_1} + u))) \\
& +f_{1LR}(i)(f_{1LL}(j)m_{\tilde{\chi}^+_1}m_{\tilde{\chi}^+_j}(-m^2_{\tilde{\ell}_1} + t) + f_{1LR}(j)(m^4_{\tilde{\ell}_1} + m^2_{\tilde{\chi}^+_1}(2m^2_{\tilde{\ell}_1} - t) \\
& +st - m^2_{\tilde{\ell}_1}(s + t + u))) + f_{1RL}(i)(f_{1RR}(j)m_{\tilde{\chi}^+_1}m_{\tilde{\chi}^+_j}(-m^2_{\tilde{\ell}_1} + t) \\
& +f_{1RL}(j)(m^4_{\tilde{\ell}_1} + m^2_{\tilde{\chi}^+_1}(2m^2_{\tilde{\ell}_1} - t) + st - m^2_{\tilde{\ell}_1}(s + t + u)))) \\
& /((m^2_{\tilde{\chi}^+_i} - t)(-m^2_{\tilde{\chi}^+_j} + t)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= (f_{2L}(i)f_{2L}(j) + f_{2R}(i)f_{2R}(j))(m^2_{\tilde{\chi}^+_1} - u)/(-m^2_{\tilde{\ell}_i} + u)(-m^2_{\tilde{\ell}_j} + u) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= (f_{1LR}(i)f_{2L}(j)m_{\tilde{\chi}^+_1}(m^2_{\tilde{\ell}_1} - t) + f_{1RL}(i)f_{2R}(j)m_{\tilde{\chi}^+_1}(m^2_{\tilde{\ell}_1} - t) \\
& +(f_{1LL}(i)f_{2L}(j) + f_{1RR}(i)f_{2R}(j))m_{\tilde{\chi}^+_i}(m^2_{\tilde{\chi}^+_1} - u))/((m^2_{\tilde{\chi}^+_i} - t)(m^2_{\tilde{\ell}_j} - u)) \\
|\mathcal{T}|^2 &= \sum_{i,j}(\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + \mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + 2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II}) \qquad \text{(C3)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\chi}_1^+ \longrightarrow \nu_\ell A$

I. t-channel $\widetilde{\chi}_{1,2}$ exchange

II. u-channel $\widetilde{\ell}_2$ exchange

$$
\begin{aligned}
f_{1LL}(i) &= C^L_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_i^-}\, C^L_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-A} \\
f_{1LR}(i) &= C^L_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_i^-}\, C^R_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-A} \\
f_{1RL}(i) &= C^R_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_i^-}\, C^L_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-A} \\
f_{1RR}(i) &= C^R_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_i^-}\, C^R_{\widetilde{\chi}_1^+-\widetilde{\chi}_i^+-A} \\
f_{2L} &= C_{\widetilde{\ell}_1-A-\widetilde{\ell}_2^-}\, C^L_{\widetilde{\chi}_1^--\nu_\ell-\widetilde{\ell}_2} \\
f_{2R} &= C_{\widetilde{\ell}_1-A-\widetilde{\ell}_2^-}\, C^R_{\widetilde{\chi}_1^--\nu_\ell-\widetilde{\ell}_2} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (m_{\tilde{\chi}_i^+}(f_{1LL}(i)(f_{1LR}(j)m_{\tilde{\chi}_1^+}(-m^2_{\tilde{\ell}_1}+t)+f_{1LL}(j)m_{\tilde{\chi}_j^+}(-m^2_{\tilde{\chi}_1^+}+u)) \\
&\quad +f_{1RR}(i)(f_{1RL}(j)m_{\tilde{\chi}_1^+}(-m^2_{\tilde{\ell}_1}+t)+f_{1RR}(j)m_{\tilde{\chi}_j^+}(-m^2_{\tilde{\chi}_1^+}+u))) \\
&\quad +f_{1LR}(i)(f_{1LL}(j)m_{\tilde{\chi}_1^+}m_{\tilde{\chi}_j^+}(-m^2_{\tilde{\ell}_1}+t)+f_{1LR}(j)(m^4_{\tilde{\ell}_1}+m^2_{\tilde{\chi}_1^+}(2m^2_{\tilde{\ell}_1}-t) \\
&\quad +st-m^2_{\tilde{\ell}_1}(s+t+u)))+f_{1RL}(i)(f_{1RR}(j)m_{\tilde{\chi}_1^+}m_{\tilde{\chi}_j^+}(-m^2_{\tilde{\ell}_1}+t) \\
&\quad +f_{1RL}(j)(m^4_{\tilde{\ell}_1}+m^2_{\tilde{\chi}_1^+}(2m^2_{\tilde{\ell}_1}-t)+st-m^2_{\tilde{\ell}_1}(s+t+u)))) \\
&\quad /((m^2_{\tilde{\chi}_i^+}-t)(-m^2_{\tilde{\chi}_j^+}+t)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= (f^2_{2L}+f^2_{2R})(m^2_{\tilde{\chi}_1^+}-u)/(m^2_{\tilde{\ell}_2}-u)^2 \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= (f_{1LR}(i)f_{2L}m_{\tilde{\chi}_1^+}(m^2_{\tilde{\ell}_1}-t)+f_{1RL}(i)f_{2R}m_{\tilde{\chi}_1^+}(m^2_{\tilde{\ell}_1}-t)+(f_{1LL}(i)f_{2L} \\
&\quad +f_{1RR}(i)f_{2R})m_{\tilde{\chi}_i^+}(m^2_{\tilde{\chi}_1^+}-u))/((m^2_{\tilde{\chi}_i^+}-t)(m^2_{\tilde{\ell}_2}-u)) \\
|\mathcal{T}|^2 &= \sum_{i,j}\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+2\sum_i\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \qquad \text{(C4)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\chi}_1^+ \longrightarrow \ell W^+$

I. s-channel ν_ℓ annihilation

II. t-channel $\widetilde{\chi}^0_{(1,2,3,4)}$ exchange

Note the L-R switch for the neutralino couplings below.

$$
\begin{aligned}
f_{1L} &= C^L_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_1^-}\, C_{\nu_\ell-\ell-W^+} \\
f_{1R} &= C^R_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_1^-}\, C_{\nu_\ell-\ell-W^+} \\
f_{2LL}(i) &= C^R_{\widetilde{\ell}_1-\ell-\widetilde{\chi}_i^0}\, C^L_{W-\widetilde{\chi}_1^--\widetilde{\chi}_i^0}
\end{aligned}
$$

$$
\begin{aligned}
f_{2LR}(i) &= C^R_{\tilde{\ell}_1-\ell-\tilde{\chi}^0_i}\ C^R_{W-\tilde{\chi}^-_1-\tilde{\chi}^0_i} \\
f_{2RL}(i) &= C^L_{\tilde{\ell}_1-\ell-\tilde{\chi}^0_i}\ C^L_{W-\tilde{\chi}^-_1-\tilde{\chi}^0_i} \\
f_{2RR}(i) &= C^L_{\tilde{\ell}_1-\ell-\tilde{\chi}^0_i}\ C^R_{W-\tilde{\chi}^-_1-\tilde{\chi}^0_i} \\
\mathcal{T}_{\mathrm{I}}\times\mathcal{T}_{\mathrm{I}} &= 1/(m^2_W s^2)(f^2_{1R}(m^4_{\tilde{\ell}_1}(m^2_\ell - s) + m^4_{\tilde{\chi}^+_1}(-m^2_\ell + s) + s(-m^4_W + (m^2_\ell - s)u \\
&\quad +m^2_W(s-t+u)) + m^2_{\tilde{\chi}^+_1}(-2m^4_W + 2m^2_W s + (m^2_\ell - s)(t+u)) \\
&\quad +m^2_{\tilde{\ell}_1}(2m^4_W - 2m^2_W s - (m^2_\ell - s)(s+t+u)))) \\
\mathcal{T}_{\mathrm{II}}\times\mathcal{T}_{\mathrm{II}} &= (m_{\tilde{\chi}^0_i}(f_{2RL}(i)(3f_{2RR}(j)m_{\tilde{\chi}^+_1}m^2_W(m^2_{\tilde{\ell}_1} - m^2_\ell - t) \\
&\quad +m_{\tilde{\chi}^0_j}(6f_{2LR}(j)m_{\tilde{\chi}^+_1}m_\ell m^2_W + f_{2RL}(j)(m^4_W + m^2_{\tilde{\chi}^+_1}(m^2_\ell - s) + (-m^2_\ell + s)t \\
&\quad -m^2_W(s+t-u)))) + f_{2LL}(j)(3f_{2LR}(i)m_{\tilde{\chi}^+_1}m^2_W(m^2_{\tilde{\ell}_1} - m^2_\ell - t) \\
&\quad +f_{2RL}(i)m_\ell(-2m^4_W + 2m^2_W t + t(m^2_{\tilde{\ell}_1} + m^2_\ell - s - u) + m^2_{\tilde{\chi}^+_1}(-m^2_{\tilde{\ell}_1} - m^2_\ell \\
&\quad +s+u))) + f_{2LR}(i)(m_{\tilde{\chi}^0_j}(6f_{2RL}(j)m_{\tilde{\chi}^+_1}m_\ell m^2_W + f_{2LR}(j)(m^4_W + m^2_{\tilde{\chi}^+_1}(m^2_\ell - s) \\
&\quad +(-m^2_\ell + s)t - m^2_W(s+t-u))) + f_{2RR}(j)m_\ell(-2m^4_W + 2m^2_W t + t(m^2_{\tilde{\ell}_1} + m^2_\ell \\
&\quad -s-u) + m^2_{\tilde{\chi}^+_1}(-m^2_{\tilde{\ell}_1} - m^2_\ell + s + u)))) + f_{2LL}(i)(3m_{\tilde{\chi}^+_1}m^2_W(f_{2LR}(j)m_{\tilde{\chi}^0_j}(m^2_{\tilde{\ell}_1} \\
&\quad -m^2_\ell - t) + 2f_{2RR}(j)m_\ell t) + f_{2RL}(j)m_\ell m_{\tilde{\chi}^0_j}(-2m^4_W + 2m^2_W t + t(m^2_{\tilde{\ell}_1} + m^2_\ell \\
&\quad -s-u) + m^2_{\tilde{\chi}^+_1}(-m^2_{\tilde{\ell}_1} - m^2_\ell + s + u)) + f_{2LL}(j)(2m^2_\ell m^4_W + m^4_{\tilde{\ell}_1}t - m^4_\ell t \\
&\quad -2m^2_\ell m^2_W t + m^4_W t + m^2_\ell st + m^2_W st - m^2_W t^2 + m^2_\ell tu - m^2_W tu + t^2 u \\
&\quad +m^2_{\tilde{\chi}^+_1}(-m^4_{\tilde{\ell}_1} + m^4_\ell - tu - m^2_\ell(s+u) + m^2_{\tilde{\ell}_1}(s+t+u)) - m^2_{\tilde{\ell}_1}(2m^4_W \\
&\quad -2m^2_W t + t(s+t+u)))) + f_{2RR}(i)(3m_{\tilde{\chi}^+_1}m^2_W(f_{2RL}(j)m_{\tilde{\chi}^0_j}(m^2_{\tilde{\ell}_1} - m^2_\ell - t) \\
&\quad +2f_{2LL}(j)m_\ell t) + f_{2LR}(j)m_\ell m_{\tilde{\chi}^0_j}(-2m^4_W + 2m^2_W t + t(m^2_{\tilde{\ell}_1} + m^2_\ell - s - u) \\
&\quad +m^2_{\tilde{\chi}^+_1}(-m^2_{\tilde{\ell}_1} - m^2_\ell + s + u)) + f_{2RR}(j)(2m^2_\ell m^4_W + m^4_{\tilde{\ell}_1}t - m^4_\ell t - 2m^2_\ell m^2_W t \\
&\quad +m^4_W t + m^2_\ell st + m^2_W st - m^2_W t^2 + m^2_\ell tu - m^2_W tu + t^2 u \\
&\quad +m^2_{\tilde{\chi}^+_1}(-m^4_{\tilde{\ell}_1} + m^4_\ell - tu - m^2_\ell(s+u) + m^2_{\tilde{\ell}_1}(s+t+u)) \\
&\quad -m^2_{\tilde{\ell}_1}(2m^4_W - 2m^2_W t + t(s+t+u))))) \\
&\quad /(m^2_W(m^2_{\tilde{\chi}^0_i} - t)(-m^2_{\tilde{\chi}^0_j} + t)) \\
\mathcal{T}_{\mathrm{I}}\times\mathcal{T}_{\mathrm{II}} &= (f_{1R}(-2f_{2LL}(j)m_{\tilde{\chi}^+_1}m_\ell(m^4_{\tilde{\ell}_1} + 4m^4_W - m^2_W s - m^2_\ell t - m^2_W t + st \\
&\quad +m^2_{\tilde{\ell}_1}(m^2_\ell + 4m^2_W - s - t - 2u) + m^2_{\tilde{\chi}^+_1}(m^2_{\tilde{\ell}_1} + m^2_\ell - s - u) - m^2_\ell u \\
&\quad -3m^2_W u + su + tu + u^2) + 2m_{\tilde{\chi}^0_j}(3f_{2LR}(j)m_\ell m^2_W(m^2_{\tilde{\chi}^+_1} - m^2_{\tilde{\ell}_1} + s)
\end{aligned}
$$

$$
\begin{aligned}
&+f_{2RL}(j)m_{\tilde{\chi}_1^+}(2m_W^4+m_{\tilde{\chi}_1^+}^2(m_\ell^2-s)+m_{\tilde{\ell}_1}^2(m_\ell^2-s)-2m_W^2s-m_\ell^2t+st\\
&-m_\ell^2u+su))+f_{2RR}(j)(-5m_\ell^2m_W^2s-m_W^4s+m_W^2s^2-m_\ell^2m_W^2t-m_W^4t\\
&-2m_W^2st+m_W^2t^2-m_{\tilde{\ell}_1}^4(s+t-u)+m_\ell^2m_W^2u+m_W^4u+m_\ell^2su+m_W^2su\\
&-s^2u+m_\ell^2tu+m_W^2tu-t^2u-m_\ell^2u^2-2m_W^2u^2+u^3+m_{\tilde{\chi}_1^+}^2((-2m_\ell^2+s\\
&+t-u)u+m_W^2(-s-5t+u)+m_{\tilde{\ell}_1}^2(4m_\ell^2+8m_W^2-3s-t+u))\\
&+m_{\tilde{\ell}_1}^2(4m_W^4+s^2+2st+t^2+su+tu-2u^2+m_\ell^2(8m_W^2-s-3t+u)\\
&-2m_W^2(s+t+u))))))/(2m_W^2s(m_{\tilde{\chi}_j^0}^2-t))
\end{aligned}
$$

$$
|\mathcal{T}|^2 = \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+\sum_{i,j}\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+2\sum_i\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \qquad \text{(C5)}
$$

$\widetilde{\ell}_1\widetilde{\chi}_1^+ \longrightarrow \ell H^+$

I. s-channel ν_ℓ annihilation

II. t-channel $\widetilde{\chi}^0_{(1,2,3,4)}$ exchange

Note the L-R switch for the neutralino couplings below.

$$
\begin{aligned}
f_{1L} &= C^L_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_1^-}\,C_{\nu_\ell-\ell-H^+}\\
f_{1R} &= C^R_{\widetilde{\ell}_1-\nu_\ell-\widetilde{\chi}_1^-}\,C_{\nu_\ell-\ell-H^+}\\
f_{2LL}(i) &= C^R_{\widetilde{\ell}_1-\ell-\widetilde{\chi}_i^0}\,C^L_{H^+-\widetilde{\chi}_1^--\widetilde{\chi}_i^0}\\
f_{2LR}(i) &= C^R_{\widetilde{\ell}_1-\ell-\widetilde{\chi}_i^0}\,C^R_{H^+-\widetilde{\chi}_1^--\widetilde{\chi}_i^0}\\
f_{2RL}(i) &= C^L_{\widetilde{\ell}_1-\ell-\widetilde{\chi}_i^0}\,C^L_{H^+-\widetilde{\chi}_1^--\widetilde{\chi}_i^0}\\
f_{2RR}(i) &= C^L_{\widetilde{\ell}_1-\ell-\widetilde{\chi}_i^0}\,C^R_{H^+-\widetilde{\chi}_1^--\widetilde{\chi}_i^0}\\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (f_{1R}^2(m_{\tilde{\chi}_1^+}^4-m_{\tilde{\ell}_1}^4+s(m_\ell^2-t)+m_{\tilde{\chi}_1^+}^2(2m_\ell^2-t-u)\\
&\quad+m_{\tilde{\ell}_1}^2(-2m_\ell^2+s+t+u)))/s^2\\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= (m_{\tilde{\chi}_i^0}(f_{2RR}(i)(f_{2RL}(j)m_{\tilde{\chi}_1^+}(-m_{\tilde{\ell}_1}^2+m_\ell^2+t)-m_{\tilde{\chi}_j^0}(2f_{2LL}(j)m_{\tilde{\chi}_1^+}m_\ell\\
&\quad+f_{2RR}(j)(m_{\tilde{\chi}_1^+}^2+m_\ell^2-u))+f_{2LR}(j)m_\ell(2m_{\tilde{\chi}_1^+}^2+m_{\tilde{\ell}_1}^2+m_\ell^2-s-u))\\
&\quad+f_{2LL}(i)(f_{2LR}(j)m_{\tilde{\chi}_1^+}(-m_{\tilde{\ell}_1}^2+m_\ell^2+t)-m_{\tilde{\chi}_j^0}(2f_{2RR}(j)m_{\tilde{\chi}_1^+}m_\ell\\
&\quad+f_{2LL}(j)(m_{\tilde{\chi}_1^+}^2+m_\ell^2-u))+f_{2RL}(j)m_\ell(2m_{\tilde{\chi}_1^+}^2+m_{\tilde{\ell}_1}^2+m_\ell^2-s-u)))\\
&\quad+f_{2LR}(i)(f_{2LL}(j)m_{\tilde{\chi}_1^+}m_{\tilde{\chi}_j^0}(-m_{\tilde{\ell}_1}^2+m_\ell^2+t)+m_\ell(-2f_{2RL}(j)m_{\tilde{\chi}_1^+}t\\
&\quad+f_{2RR}(j)m_{\tilde{\chi}_j^0}(2m_{\tilde{\chi}_1^+}^2+m_{\tilde{\ell}_1}^2+m_\ell^2-s-u))+f_{2LR}(j)(m_{\tilde{\ell}_1}^4-m_\ell^4+m_\ell^2s
\end{aligned}
$$

$$
\begin{aligned}
& +m_{\tilde{\chi}_1^+}^2(2m_{\tilde{\ell}_1}^2 - 2m_\ell^2 - t) + st + m_\ell^2 u - m_{\tilde{\ell}_1}^2(s+t+u))) \\
& +f_{2RL}(i)(f_{2RR}(j)m_{\tilde{\chi}_1^+}m_{\tilde{\chi}_j^0}(-m_{\tilde{\ell}_1}^2 + m_\ell^2 + t) + m_\ell(-2f_{2LR}(j)m_{\tilde{\chi}_1^+}t \\
& +f_{2LL}(j)m_{\tilde{\chi}_j^0}(2m_{\tilde{\chi}_1^+}^2 + m_{\tilde{\ell}_1}^2 + m_\ell^2 - s - u)) + f_{2RL}(j)(m_{\tilde{\ell}_1}^4 - m_\ell^4 + m_\ell^2 s \\
& +m_{\tilde{\chi}_1^+}^2(2m_{\tilde{\ell}_1}^2 - 2m_\ell^2 - t) + st + m_\ell^2 u - m_{\tilde{\ell}_1}^2(s+t+u)))) \\
& /((m_{\tilde{\chi}_i^0}^2 - t)(-m_{\tilde{\chi}_j^0}^2 + t))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= -(f_{1R}(m_{\tilde{\chi}_j^0}(f_{2RR}(j)m_\ell(m_{\tilde{\chi}_1^+}^2 - m_{\tilde{\ell}_1}^2 + s) + f_{2LL}(j)m_{\tilde{\chi}_1^+}(m_{\tilde{\chi}_1^+}^2 + m_{\tilde{\ell}_1}^2 \\
& +2m_\ell^2 - t - u)) - f_{2LR}(j)(m_{\tilde{\ell}_1}^4 + m_{\tilde{\chi}_1^+}^2(m_{\tilde{\ell}_1}^2 + m_\ell^2) + st \\
& +m_{\tilde{\ell}_1}^2(m_\ell^2 - s - t - u)) + f_{2RL}(j)m_{\tilde{\chi}_1^+}m_\ell(-2m_{\tilde{\chi}_1^+}^2 - 2m_\ell^2 + s + t + u))) \\
& /(s(m_{\tilde{\chi}_j^0}^2 - t)) \\
|\mathcal{T}|^2 &= \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + \sum_{i,j}\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + 2\sum_i \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \qquad \text{(C6)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\chi}_1^+ \longrightarrow W^-\ell^+$

I. t-channel $\widetilde{\nu}_\ell$ exchange

$$
\begin{aligned}
f_{1L} &= C^L_{\ell-\widetilde{\nu}_\ell-\widetilde{\chi}_1^-}\, C_{\widetilde{\nu}_\ell-\widetilde{\ell}_1-W} \\
f_{1R} &= C^R_{\ell-\widetilde{\nu}_\ell-\widetilde{\chi}_1^-}\, C_{\widetilde{\nu}_\ell-\widetilde{\ell}_1-W} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= ((-4f_{1L}f_{1R}m_{\tilde{\chi}_1^+}m_\ell + f_{1L}^2(m_{\tilde{\chi}_1^+}^2 + m_\ell^2 - t) + f_{1R}^2(m_{\tilde{\chi}_1^+}^2 + m_\ell^2 - t))(m_{\tilde{\ell}_1}^4 \\
& +(m_W^2 - t)^2 - 2m_{\tilde{\ell}_1}^2(m_W^2 + t)))/(m_W^2(m_{\tilde{\nu}_\ell}^2 - t)^2) \\
|\mathcal{T}|^2 &= \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} \qquad \text{(C7)}
\end{aligned}
$$

$\widetilde{\ell}_1\widetilde{\chi}_1^+ \longrightarrow H^-\ell^+$

I. t-channel $\widetilde{\nu}_\ell$ exchange

$$
\begin{aligned}
f_{1L} &= C^L_{\ell-\widetilde{\nu}_\ell-\widetilde{\chi}_1^-}\, C_{\widetilde{\nu}_\ell-\widetilde{\ell}_1-H^+} \\
f_{1R} &= C^R_{\ell-\widetilde{\nu}_\ell-\widetilde{\chi}_1^-}\, C_{\widetilde{\nu}_\ell-\widetilde{\ell}_1-H^+} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (-4f_{1L}f_{1R}m_{\tilde{\chi}_1^+}m_\ell + f_{1L}^2(m_{\tilde{\chi}_1^+}^2 + m_\ell^2 - t) + f_{1R}^2(m_{\tilde{\chi}_1^+}^2 + m_\ell^2 - t))/(t - m_{\tilde{\nu}_\ell}^2)^2 \\
|\mathcal{T}|^2 &= \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} \qquad \text{(C8)}
\end{aligned}
$$

$\widetilde{\ell}_1 \widetilde{\chi}_1^- \longrightarrow \ell W^-$

I. t-channel $\widetilde{\chi}^0_{(1,2,3,4)}$ exchange

II. u-channel ν_ℓ exchange

Note the L-R switch for the couplings below.

$$
\begin{aligned}
f_{1LL}(i) &= C^R_{\widetilde{\ell}_1-\ell-\widetilde{\chi}^0_i}\, C^L_{W-\widetilde{\chi}^-_1-\widetilde{\chi}^0_i} \\
f_{1LR}(i) &= C^R_{\widetilde{\ell}_1-\ell-\widetilde{\chi}^0_i}\, C^R_{W-\widetilde{\chi}^-_1-\widetilde{\chi}^0_i} \\
f_{1RL}(i) &= C^L_{\widetilde{\ell}_1-\ell-\widetilde{\chi}^0_i}\, C^L_{W-\widetilde{\chi}^-_1-\widetilde{\chi}^0_i} \\
f_{1RR}(i) &= C^L_{\widetilde{\ell}_1-\ell-\widetilde{\chi}^0_i}\, C^R_{W-\widetilde{\chi}^-_1-\widetilde{\chi}^0_i} \\
f_{2L} &= C^R_{\ell-\widetilde{\nu}_\ell-\widetilde{\chi}^-_1}\, C_{\widetilde{\nu}_\ell-\widetilde{\ell}_1-W} \\
f_{2R} &= C^L_{\ell-\widetilde{\nu}_\ell-\widetilde{\chi}^-_1}\, C_{\widetilde{\nu}_\ell-\widetilde{\ell}_1-W} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (m_{\tilde{\chi}^0_i}(f_{1RL}(i)(3f_{1RR}(j)m_{\tilde{\chi}^+_1}m_W^2(m^2_{\tilde{\ell}_1}-m_\ell^2-t) \\
&\quad +m_{\tilde{\chi}^0_j}(6f_{1LR}(j)m_{\tilde{\chi}^+_1}m_\ell m_W^2+f_{1RL}(j)(m_W^4+m^2_{\tilde{\chi}^+_1}(m_\ell^2-s) \\
&\quad +(-m_\ell^2+s)t-m_W^2(s+t-u))))+f_{1LL}(j)(3f_{1LR}(i)m_{\tilde{\chi}^+_1}m_W^2(m^2_{\tilde{\ell}_1}-m_\ell^2-t) \\
&\quad +f_{1RL}(i)m_\ell(-2m_W^4+2m_W^2t+t(m^2_{\tilde{\ell}_1}+m_\ell^2-s-u) \\
&\quad +m^2_{\tilde{\chi}^+_1}(-m^2_{\tilde{\ell}_1}-m_\ell^2+s+u)))+f_{1LR}(i)(m_{\tilde{\chi}^0_j}(6f_{1RL}(j)m_{\tilde{\chi}^+_1}m_\ell m_W^2 \\
&\quad +f_{1LR}(j)(m_W^4+m^2_{\tilde{\chi}^+_1}(m_\ell^2-s)+(-m_\ell^2+s)t-m_W^2(s+t-u))) \\
&\quad +f_{1RR}(j)m_\ell(-2m_W^4+2m_W^2t+t(m^2_{\tilde{\ell}_1}+m_\ell^2-s-u) \\
&\quad +m^2_{\tilde{\chi}^+_1}(-m^2_{\tilde{\ell}_1}-m_\ell^2+s+u))))+f_{1LL}(i)(3m_{\tilde{\chi}^+_1}m_W^2(f_{1LR}(j)m_{\tilde{\chi}^0_j}(m^2_{\tilde{\ell}_1}-m_\ell^2 \\
&\quad -t)+2f_{1RR}(j)m_\ell t)+f_{1RL}(j)m_\ell m_{\tilde{\chi}^0_j}(-2m_W^4+2m_W^2t+t(m^2_{\tilde{\ell}_1}+m_\ell^2-s-u) \\
&\quad +m^2_{\tilde{\chi}^+_1}(-m^2_{\tilde{\ell}_1}-m_\ell^2+s+u))+f_{1LL}(j)(2m_\ell^2m_W^4+m^4_{\tilde{\ell}_1}t-m_\ell^4t \\
&\quad -2m_\ell^2m_W^2t+m_W^4t+m_\ell^2st+m_W^2st-m_W^2t^2+m_\ell^2tu-m_W^2tu+t^2u \\
&\quad +m^2_{\tilde{\chi}^+_1}(-m^4_{\tilde{\ell}_1}+m_\ell^4-tu-m_\ell^2(s+u)+m^2_{\tilde{\ell}_1}(s+t+u))-m^2_{\tilde{\ell}_1}(2m_W^4 \\
&\quad -2m_W^2t+t(s+t+u))))+f_{1RR}(i)(3m_{\tilde{\chi}^+_1}m_W^2(f_{1RL}(j)m_{\tilde{\chi}^0_j}(m^2_{\tilde{\ell}_1}-m_\ell^2-t) \\
&\quad +2f_{1LL}(j)m_\ell t)+f_{1LR}(j)m_\ell m_{\tilde{\chi}^0_j}(-2m_W^4+2m_W^2t+t(m^2_{\tilde{\ell}_1}+m_\ell^2-s-u) \\
&\quad +m^2_{\tilde{\chi}^+_1}(-m^2_{\tilde{\ell}_1}-m_\ell^2+s+u))+f_{1RR}(j)(2m_\ell^2m_W^4+m^4_{\tilde{\ell}_1}t-m_\ell^4t \\
&\quad -2m_\ell^2m_W^2t+m_W^4t+m_\ell^2st+m_W^2st-m_W^2t^2+m_\ell^2tu-m_W^2tu+t^2u \\
&\quad +m^2_{\tilde{\chi}^+_1}(-m^4_{\tilde{\ell}_1}+m_\ell^4-tu-m_\ell^2(s+u)+m^2_{\tilde{\ell}_1}(s+t+u))-m^2_{\tilde{\ell}_1}(2m_W^4
\end{aligned}
$$

$$
\begin{aligned}
& -2m_W^2 t + t(s+t+u)))))/(m_W^2(m_{\tilde{\chi}_i^0}^2 - t)(-m_{\tilde{\chi}_j^0}^2 + t)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} \;=\; & ((4f_{2L}f_{2R}m_{\tilde{\chi}_1^+}m_\ell + f_{2L}^2(m_{\tilde{\chi}_1^+}^2 + m_\ell^2 - u) + f_{2R}^2(m_{\tilde{\chi}_1^+}^2 + m_\ell^2 - u))(m_{\tilde{\ell}_1}^4 \\
& +(m_W^2 - u)^2 - 2m_{\tilde{\ell}_1}^2(m_W^2 + u)))/(m_W^2(m_{\tilde{\nu}_\ell}^2 - u)^2) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \;=\; & (2m_{\tilde{\chi}_i^0}(f_{1LR}(i)(f_{2R}m_\ell(m_W^4 - 2m_W^2 s + m_{\tilde{\ell}_1}^2(3m_W^2 - t) - m_W^2 t \\
& +m_{\tilde{\chi}_1^+}^2(m_{\tilde{\ell}_1}^2 + 3m_W^2 - u) - m_W^2 u + tu) + f_{2L}m_{\tilde{\chi}_1^+}(-m_W^4 + m_W^2 s \\
& +m_{\tilde{\ell}_1}^2(-m_\ell^2 - 3m_W^2 + s) + 2m_W^2 t + m_W^2 u - su + m_\ell^2(-3m_W^2 + u))) \\
& +f_{1RL}(i)(f_{2L}m_\ell(m_W^4 - 2m_W^2 s + m_{\tilde{\ell}_1}^2(3m_W^2 - t) - m_W^2 t + m_{\tilde{\chi}_1^+}^2(m_{\tilde{\ell}_1}^2 + 3m_W^2 \\
& -u) - m_W^2 u + tu) + f_{2R}m_{\tilde{\chi}_1^+}(-m_W^4 + m_W^2 s + m_{\tilde{\ell}_1}^2(-m_\ell^2 - 3m_W^2 + s) \\
& +2m_W^2 t + m_W^2 u - su + m_\ell^2(-3m_W^2 + u)))) + f_{1LL}(i)(2f_{2R}m_{\tilde{\chi}_1^+}m_\ell(m_{\tilde{\ell}_1}^4 \\
& +2m_W^4 - m_W^2 s - 2m_W^2 t + m_{\tilde{\ell}_1}^2(m_\ell^2 + m_W^2 - s - 2u) + m_\ell^2(3m_W^2 - u) \\
& -3m_W^2 u + su + u^2) + f_{2L}(5m_\ell^2 m_W^2 s + m_W^4 s - m_W^2 s^2 + m_\ell^2 m_W^2 t - m_W^4 t \\
& +m_W^2 t^2 + m_{\tilde{\ell}_1}^4(s - t - u) - m_\ell^2 m_W^2 u - m_W^4 u - m_\ell^2 su - m_W^2 su + s^2 u \\
& -m_\ell^2 tu + m_W^2 tu - t^2 u + m_\ell^2 u^2 + 2m_W^2 u^2 - u^3 + m_{\tilde{\ell}_1}^2(-s^2 + t^2 \\
& +m_\ell^2(-8m_W^2 + s + t - u) - su + tu + 2u^2 + 2m_W^2(s - t + u)) \\
& +m_{\tilde{\chi}_1^+}^2(2m_{\tilde{\ell}_1}^4 - (m_W^2 - u)(-s + t + u) + m_\ell^2(-6m_W^2 + 2u) \\
& -m_{\tilde{\ell}_1}^2(2m_\ell^2 + 2m_W^2 - s + t + 3u)))) + f_{1RR}(i)(2f_{2L}m_{\tilde{\chi}_1^+}m_\ell(m_{\tilde{\ell}_1}^4 + 2m_W^4 \\
& -m_W^2 s - 2m_W^2 t + m_{\tilde{\ell}_1}^2(m_\ell^2 + m_W^2 - s - 2u) + m_\ell^2(3m_W^2 - u) - 3m_W^2 u + su \\
& +u^2) + f_{2R}(5m_\ell^2 m_W^2 s + m_W^4 s - m_W^2 s^2 + m_\ell^2 m_W^2 t - m_W^4 t + m_W^2 t^2 \\
& +m_{\tilde{\ell}_1}^4(s - t - u) - m_\ell^2 m_W^2 u - m_W^4 u - m_\ell^2 su - m_W^2 su + s^2 u - m_\ell^2 tu \\
& +m_W^2 tu - t^2 u + m_\ell^2 u^2 + 2m_W^2 u^2 - u^3 + m_{\tilde{\ell}_1}^2(-s^2 + t^2 \\
& +m_\ell^2(-8m_W^2 + s + t - u) - su + tu + 2u^2 + 2m_W^2(s - t + u)) \\
& +m_{\tilde{\chi}_1^+}^2(2m_{\tilde{\ell}_1}^4 - (m_W^2 - u)(-s + t + u) + m_\ell^2(-6m_W^2 + 2u) \\
& -m_{\tilde{\ell}_1}^2(2m_\ell^2 + 2m_W^2 - s + t + 3u)))))/(2m_W^2(m_{\tilde{\chi}_i^0}^2 - t)(m_{\tilde{\nu}_\ell}^2 - u)) \\
|\mathcal{T}|^2 \;=\; & \sum_{i,j}\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + \mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + 2\sum_i \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \qquad \text{(C9)}
\end{aligned}
$$

$\widetilde{\ell}_1 \widetilde{\chi}_1^- \longrightarrow \ell H^-$

I. t-channel $\widetilde{\chi}^0_{(1,2,3,4)}$ exchange

II. u-channel ν_ℓ exchange

Note the L-R switch for the couplings below.

$$
\begin{aligned}
f_{1LL}(i) &= C^R_{\widetilde{\ell}_1-\ell-\widetilde{\chi}^0_i}\, C^L_{H^+-\widetilde{\chi}^-_1-\widetilde{\chi}^0_i} \\
f_{1LR}(i) &= C^R_{\widetilde{\ell}_1-\ell-\widetilde{\chi}^0_i}\, C^R_{H^+-\widetilde{\chi}^-_1-\widetilde{\chi}^0_i} \\
f_{1RL}(i) &= C^L_{\widetilde{\ell}_1-\ell-\widetilde{\chi}^0_i}\, C^L_{H^+-\widetilde{\chi}^-_1-\widetilde{\chi}^0_i} \\
f_{1RR}(i) &= C^L_{\widetilde{\ell}_1-\ell-\widetilde{\chi}^0_i}\, C^R_{H^+-\widetilde{\chi}^-_1-\widetilde{\chi}^0_i} \\
f_{2L} &= C^R_{\ell-\widetilde{\nu}_\ell-\widetilde{\chi}^-_1}\, C_{\widetilde{\nu}_\ell-\widetilde{\ell}_1-H^+} \\
f_{2R} &= C^L_{\ell-\widetilde{\nu}_\ell-\widetilde{\chi}^-_1}\, C_{\widetilde{\nu}_\ell-\widetilde{\ell}_1-H^+} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (m_{\tilde{\chi}^0_i}(-f_{1RL}(i)(f_{1RR}(j)m_{\tilde{\chi}^+_1}(m^2_{\tilde{\ell}_1}-m^2_\ell-t)+m_{\tilde{\chi}^0_j}(2f_{1LR}(j)m_{\tilde{\chi}^+_1}m_\ell \\
&\quad +f_{1RL}(j)(m^2_{\tilde{\chi}^+_1}+m^2_\ell-u)))+f_{1LL}(j)(f_{1LR}(i)m_{\tilde{\chi}^+_1}(-m^2_{\tilde{\ell}_1}+m^2_\ell+t) \\
&\quad +f_{1RL}(i)m_\ell(2m^2_{\tilde{\chi}^+_1}+m^2_{\tilde{\ell}_1}+m^2_\ell-s-u))-f_{1LR}(i)(m_{\tilde{\chi}^0_j}(2f_{1RL}(j)m_{\tilde{\chi}^+_1}m_\ell \\
&\quad +f_{1LR}(j)(m^2_{\tilde{\chi}^+_1}+m^2_\ell-u))-f_{1RR}(j)m_\ell(2m^2_{\tilde{\chi}^+_1}+m^2_{\tilde{\ell}_1}+m^2_\ell-s-u))) \\
&\quad +f_{1LL}(i)(f_{1LR}(j)m_{\tilde{\chi}^+_1}m_{\tilde{\chi}^0_j}(-m^2_{\tilde{\ell}_1}+m^2_\ell+t)+m_\ell(-2f_{1RR}(j)m_{\tilde{\chi}^+_1}t \\
&\quad +f_{1RL}(j)m_{\tilde{\chi}^0_j}(2m^2_{\tilde{\chi}^+_1}+m^2_{\tilde{\ell}_1}+m^2_\ell-s-u))+f_{1LL}(j)(m^4_{\tilde{\ell}_1}-m^4_\ell+m^2_\ell s \\
&\quad +m^2_{\tilde{\chi}^+_1}(2m^2_{\tilde{\ell}_1}-2m^2_\ell-t)+st+m^2_\ell u-m^2_{\tilde{\ell}_1}(s+t+u))) \\
&\quad +f_{1RR}(i)(f_{1RL}(j)m_{\tilde{\chi}^+_1}m_{\tilde{\chi}^0_j}(-m^2_{\tilde{\ell}_1}+m^2_\ell+t)+m_\ell(-2f_{1LL}(j)m_{\tilde{\chi}^+_1}t \\
&\quad +f_{1LR}(j)m_{\tilde{\chi}^0_j}(2m^2_{\tilde{\chi}^+_1}+m^2_{\tilde{\ell}_1}+m^2_\ell-s-u))+f_{1RR}(j)(m^4_{\tilde{\ell}_1}-m^4_\ell+m^2_\ell s \\
&\quad +m^2_{\tilde{\chi}^+_1}(2m^2_{\tilde{\ell}_1}-2m^2_\ell-t)+st+m^2_\ell u-m^2_{\tilde{\ell}_1}(s+t+u)))) \\
&\quad /((m^2_{\tilde{\chi}^0_i}-t)(-m^2_{\tilde{\chi}^0_j}+t)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= (4f_{2L}f_{2R}m_{\tilde{\chi}^+_1}m_\ell+f^2_{2L}(m^2_{\tilde{\chi}^+_1}+m^2_\ell-u)+f^2_{2R}(m^2_{\tilde{\chi}^+_1}+m^2_\ell-u))/(m^2_{\tilde{\nu}_\ell}-u)^2 \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= (m_{\tilde{\chi}^0_i}(f_{1LR}(i)(2f_{2R}m_{\tilde{\chi}^+_1}m_\ell+f_{2L}(m^2_{\tilde{\chi}^+_1}+m^2_\ell-u))+f_{1RL}(i)(2f_{2L}m_{\tilde{\chi}^+_1}m_\ell \\
&\quad +f_{2R}(m^2_{\tilde{\chi}^+_1}+m^2_\ell-u)))+f_{1RR}(i)(f_{2R}m_{\tilde{\chi}^+_1}(m^2_{\tilde{\ell}_1}-m^2_\ell-t) \\
&\quad +f_{2L}m_\ell(-2m^2_{\tilde{\chi}^+_1}-m^2_{\tilde{\ell}_1}-m^2_\ell+s+u))+f_{1LL}(i)(f_{2L}m_{\tilde{\chi}^+_1}(m^2_{\tilde{\ell}_1}-m^2_\ell-t) \\
&\quad +f_{2R}m_\ell(-2m^2_{\tilde{\chi}^+_1}-m^2_{\tilde{\ell}_1}-m^2_\ell+s+u))) \\
&\quad /((m^2_{\tilde{\chi}^0_i}-t)(m^2_{\tilde{\nu}_\ell}-u))
\end{aligned}
$$

$$|\mathcal{T}|^2 = \sum_{i,j} \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + \mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + 2\sum_i \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \tag{C10}$$

Appendix D: Neutralino-Sneutrino Coannihilation

Below is the list of the amplitudes squared for neutralino-sneutrino coannihilation. Note that, for identical-particle final states, one needs to divide them by two when performing the momentum integrations. Below, $\widetilde{\nu}$ refers to $\widetilde{\nu}_{e,\mu}$.

$\widetilde{\nu}\widetilde{\nu}^* \longrightarrow WW$

I. s-channel H annihilation
II. s-channel h annihilation
III. u-channel $\widetilde{e}_L$ exchange
IV. point interaction
V. s-channel Z annihilation

$$\begin{aligned}
f_1 &= C_{H-W-W}\, C_{\widetilde{\nu}-\widetilde{\nu}-H} \\
f_2 &= C_{h-W-W}\, C_{\widetilde{\nu}-\widetilde{\nu}-h} \\
f_3 &= (C_{\widetilde{\nu}-\widetilde{e}-W})^2 \\
f_4 &= C_{\widetilde{\nu}-\widetilde{\nu}-W-W} \\
f_5 &= C_{Z-W-W}\, C_{\widetilde{\nu}-\widetilde{\nu}-Z} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (12m_W^4 - 4m_W^2 s + s^2)/(4m_W^4(s-m_H^2)^2) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= (12m_W^4 - 4m_W^2 s + s^2)/(4m_W^4(s-m_h^2)^2) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= (m_{\widetilde{\nu}}^4 + (m_W^2-u)^2 - 2m_{\widetilde{\nu}}^2(m_W^2+u))^2/(m_W^4(u-m_{\widetilde{e}_L}^2)^2) \\
\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} &= (12m_W^4 - 4m_W^2 s + s^2)/(4m_W^4) \\
\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V} &= (-32m_{\widetilde{\nu}}^6 m_W^2 - 24m_W^6 s + (t^2-u^2)^2 - 8m_W^4(s^2 - 2(t-u)^2 - s(t+u)) \\
&\quad +2m_W^2(s^3 - 2s^2(t+u) - 2(t-u)^2(t+u) - s(t^2-6tu+u^2)) \\
&\quad +4m_{\widetilde{\nu}}^4(16m_W^4 + (t-u)^2 + m_W^2(-6s+8(t+u))) \\
&\quad +4m_{\widetilde{\nu}}^2(24m_W^6 - (t-u)^2(t+u) + 4m_W^4(s-2(t+u)) + 2m_W^2(-4tu \\
&\quad +s(t+u))))/(4m_W^4(s-m_Z^2)^2)
\end{aligned}$$

$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} = (12m_W^4 - 4m_W^2 s + s^2)/(4m_W^4(s-m_H^2)(s-m_h^2))$$

$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} = (6m_W^6 + m_{\tilde\nu}^4(6m_W^2 - s) - su^2 + 2m_W^2 u(s+2t+u) - m_W^4(5s+4t+8u) + 2m_{\tilde\nu}^2(10m_W^4 + su - m_W^2(s+2t+4u)))/(2m_W^4(s-m_H^2)(u-m_{\tilde e_L}^2))$$

$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} = -(12m_W^4 - 4m_W^2 s + s^2)/(4m_W^4(s-m_H^2))$$

$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V} = -((t-u)(16m_W^4 + m_{\tilde\nu}^2(4m_W^2 - 2s) + s(t+u) - 2m_W^2(2s+t+u))) /(4m_W^4(s-m_H^2)(s-m_Z^2))$$

$$\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} = (6m_W^6 + m_{\tilde\nu}^4(6m_W^2 - s) - su^2 + 2m_W^2 u(s+2t+u) - m_W^4(5s+4t+8u) + 2m_{\tilde\nu}^2(10m_W^4 + su - m_W^2(s+2t+4u)))/(2m_W^4(s-m_h^2)(u-m_{\tilde e_L}^2))$$

$$\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} = -(12m_W^4 - 4m_W^2 s + s^2)/(4m_W^4(s-m_h^2))$$

$$\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V} = -((t-u)(16m_W^4 + m_{\tilde\nu}^2(4m_W^2 - 2s) + s(t+u) - 2m_W^2(2s+t+u))) /(4m_W^4(s-m_h^2)(s-m_Z^2))$$

$$\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} = -(6m_W^6 + m_{\tilde\nu}^4(6m_W^2 - s) - su^2 + 2m_W^2 u(s+2t+u) - m_W^4(5s+4t+8u) + 2m_{\tilde\nu}^2(10m_W^4 + su - m_W^2(s+2t+4u)))/(2m_W^4(u-m_{\tilde e_L}^2))$$

$$\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V} = (t^2u^2 - u^4 + 2m_{\tilde\nu}^6(8m_W^2 - t + u) + 4m_W^6(s-t+u) + m_W^4(2s^2 + 3t^2 + 4s(t-u) + 4tu - 7u^2) + 2m_W^2 u(s^2 - 2st - 2t^2 + 2u^2) - 2m_{\tilde\nu}^2(8m_W^6 + m_W^4(4s+5t-5u) + u(t^2+tu-2u^2) + m_W^2(s^2 - 2st - 2t^2 + 2su - 8tu + 2u^2)) + m_{\tilde\nu}^4(t^2 + 4tu - 5u^2 + 4m_W^2(s - 4(t+u)))) /(2m_W^4(s-m_Z^2)(u-m_{\tilde e_L}^2))$$

$$\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} = ((t-u)(16m_W^4 + m_{\tilde\nu}^2(4m_W^2 - 2s) + s(t+u) - 2m_W^2(2s+t+u))) /(4m_W^4(s-m_Z^2))$$

$$|\mathcal{T}|^2 = f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + f_4^2\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} + f_5^2\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V} + 2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} + 2f_1f_4\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} + 2f_1f_5\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V} + 2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} + 2f_2f_4\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} + 2f_2f_5\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V} + 2f_3f_4\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} + 2f_3f_5\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V} + 2f_4f_5\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} \quad \text{(D1)}$$

$\tilde\nu\tilde\nu^* \longrightarrow ZZ$

I. s-channel H annihilation

II. s-channel h annihilation

III. u-channel $\tilde\nu$ exchange

IV. t-channel $\widetilde{\nu}$ exchange

V. point interaction

$$
\begin{aligned}
f_1 &= C_{H-Z-Z}\, C_{\widetilde{\nu}-\widetilde{\nu}-H} \\
f_2 &= C_{h-Z-Z}\, C_{\widetilde{\nu}-\widetilde{\nu}-h} \\
f_3 &= (C_{\widetilde{\nu}-\widetilde{\nu}-Z})^2 \\
f_4 &= (C_{\widetilde{\nu}-\widetilde{\nu}-Z})^2 \\
f_5 &= C_{\widetilde{\nu}-\widetilde{\nu}-Z-Z} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (12m_Z^4 - 4m_Z^2 s + s^2)/(4m_Z^4(s-m_H^2)^2) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= (12m_Z^4 - 4m_Z^2 s + s^2)/(4m_Z^4(s-m_h^2)^2) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= (m_{\tilde\nu}^4 + (m_Z^2-u)^2 - 2m_{\tilde\nu}^2(m_Z^2+u))^2/(m_Z^4(u-m_{\tilde\nu}^2)^2) \\
\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} &= (m_{\tilde\nu}^4 + (m_Z^2-t)^2 - 2m_{\tilde\nu}^2(m_Z^2+t))^2/(m_Z^4(t-m_{\tilde\nu}^2)^2) \\
\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V} &= (12m_Z^4 - 4m_Z^2 s + s^2)/(4m_Z^4) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= (12m_Z^4 - 4m_Z^2 s + s^2)/(4m_Z^4(s-m_H^2)(s-m_h^2)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} &= (6m_Z^6 + m_{\tilde\nu}^4(6m_Z^2-s) - su^2 \\
&\quad +2m_Z^2u(s+2t+u) - m_Z^4(5s+4t+8u) + 2m_{\tilde\nu}^2(10m_Z^4 + su - m_Z^2(s+2t \\
&\quad +4u)))/(2m_Z^4(s-m_H^2)(u-m_{\tilde\nu}^2)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV} &= (6m_Z^6 + m_{\tilde\nu}^4(6m_Z^2-s) - st^2 + 2m_Z^2t(s+2u+t) - m_Z^4(5s+4u+8t) \\
&\quad +2m_{\tilde\nu}^2(10m_Z^4 + st - m_Z^2(s+2u+4t)))/(2m_Z^4(s-m_H^2)(t-m_{\tilde\nu}^2)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V} &= -(12m_Z^4 - 4m_Z^2 s + s^2)/(4m_Z^4(s-m_H^2)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} &= (6m_Z^6 + m_{\tilde\nu}^4(6m_Z^2-s) - su^2 + 2m_Z^2u(s+2t+u) - m_Z^4(5s+4t+8u) \\
&\quad +2m_{\tilde\nu}^2(10m_Z^4 + su - m_Z^2(s+2t+4u)))/(2m_Z^4(s-m_h^2)(u-m_{\tilde\nu}^2)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV} &= (6m_Z^6 + m_{\tilde\nu}^4(6m_Z^2-s) - st^2 + 2m_Z^2t(s+2u+t) - m_Z^4(5s+4u+8t) \\
&\quad +2m_{\tilde\nu}^2(10m_Z^4 + st - m_Z^2(s+2u+4t)))/(2m_Z^4(s-m_h^2)(t-m_{\tilde\nu}^2)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V} &= -(12m_Z^4 - 4m_Z^2 s + s^2)/(4m_Z^4(s-m_h^2)) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} &= (m_{\tilde\nu}^4 + m_Z^4 + m_{\tilde\nu}^2(6m_Z^2-t-u) + tu - m_Z^2(2s+t+u))^2 \\
&\quad /(m_Z^4(t-m_{\tilde\nu}^2)(u-m_{\tilde\nu}^2)) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V} &= -(6m_Z^6 + m_{\tilde\nu}^4(6m_Z^2-s) - su^2 + 2m_Z^2u(s+2t+u) - m_Z^4(5s+4t+8u)
\end{aligned}
$$

$$+2m_{\tilde\nu}^2(10m_Z^4+su-m_Z^2(s+2t+4u)))/(2m_Z^4(u-m_{\tilde\nu}^2))$$

$$\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} = -(6m_Z^6+m_{\tilde\nu}^4(6m_Z^2-s)-st^2+2m_Z^2t(s+2u+t)-m_Z^4(5s+4u+8t)$$
$$+2m_{\tilde\nu}^2(10m_Z^4+st-m_Z^2(s+2u+4t)))/(2m_Z^4(t-m_{\tilde\nu}^2))$$

$$|\mathcal{T}|^2 = f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III}+f_4^2\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV}+f_5^2\mathcal{T}_{\rm V}\times\mathcal{T}_{\rm V}+2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II}$$
$$+2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III}+2f_1f_4\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV}+2f_1f_5\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm V}+2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III}+2f_2f_4\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV}$$
$$+2f_2f_5\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm V}+2f_3f_4\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV}+2f_3f_5\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm V}+2f_4f_5\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm V} \qquad \text{(D2)}$$

$\tilde\nu\tilde\nu^* \longrightarrow Zh \quad [ZH]$

I. s-channel Z annihilation
II. t-channel $\tilde\nu$ exchange
III. u-channel $\tilde\nu$ exchange

$$f_1 = C_{\tilde\nu-\tilde\nu-Z}\,C_{h-Z-Z} \qquad [C_{\tilde\nu-\tilde\nu-Z}\,C_{H-Z-Z}]$$
$$f_2 = C_{\tilde\nu-\tilde\nu-h}\,C_{\tilde\nu-\tilde\nu-Z} \qquad [C_{\tilde\nu-\tilde\nu-H}\,C_{\tilde\nu-\tilde\nu-Z}]$$
$$f_3 = C_{\tilde\nu-\tilde\nu-h}\,C_{\tilde\nu-\tilde\nu-Z} \qquad [C_{\tilde\nu-\tilde\nu-H}\,C_{\tilde\nu-\tilde\nu-Z}]$$
$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} = (-16m_{\tilde\nu}^2m_Z^2+4m_Z^2s+(t-u)^2)/(4m_Z^2(s-m_Z^2)^2)$$
$$\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} = (m_{\tilde\nu}^4+(m_Z^2-t)^2-2m_{\tilde\nu}^2(m_Z^2+t))/(m_Z^2(t-m_{\tilde\nu}^2)^2)$$
$$\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} = (m_{\tilde\nu}^4+(m_Z^2-u)^2-2m_{\tilde\nu}^2(m_Z^2+u))/(m_Z^2(u-m_{\tilde\nu}^2)^2)$$
$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} = -(t(t-u)+m_{\tilde\nu}^2(-8m_Z^2-t+u)+m_Z^2(2s-t+u))/(2m_Z^2(s-m_Z^2)(t-m_{\tilde\nu}^2))$$
$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} = -(u(u-t)+m_{\tilde\nu}^2(-8m_Z^2-u+t)+m_Z^2(2s-u+t))/(2m_Z^2(s-m_Z^2)(u-m_{\tilde\nu}^2))$$
$$\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} = (-m_{\tilde\nu}^4+m_Z^4+tu-m_{\tilde\nu}^2(-6m_Z^2+t+u)-m_Z^2(2s+t+u))$$
$$/(m_Z^2(u-m_{\tilde\nu}^2)(t-m_{\tilde\nu}^2))$$
$$|\mathcal{T}|^2 = f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III}+2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II}+2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III}$$
$$+2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} \qquad \text{(D3)}$$

$\tilde\nu\tilde\nu^* \longrightarrow Ah \quad [AH]$

I. s-channel Z annihilation

$$f_1 = C_{\tilde\nu-\tilde\nu-Z}\,C_{Z-h-A} \qquad [C_{\tilde\nu-\tilde\nu-Z}\,C_{Z-H-A}]$$
$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} = (t-u)^2/(s-m_Z^2)^2$$

$$|\mathcal{T}|^2 = f_1^2 \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} \tag{D4}$$

$\widetilde{\nu}\widetilde{\nu}^* \longrightarrow f\bar{f}$

I. s-channel Z annihilation

$$\begin{aligned} f_L &= C_{\widetilde{\nu}-\widetilde{\nu}-Z}\, C^L_{Z-f-f} \\ f_R &= C_{\widetilde{\nu}-\widetilde{\nu}-Z}\, C^R_{Z-f-f} \\ \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= -(f_L^2+f_R^2)(4m_{\widetilde{\nu}}^2 s - s^2 + (t-u)^2)/(s-m_z^2)^2 \\ |\mathcal{T}|^2 &= \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} \end{aligned} \tag{D5}$$

For quarks $|\mathcal{T}|^2$ is multiplied by 3 for color.

$\widetilde{\nu}\widetilde{\nu}^* \longrightarrow t\bar{t}$

I. s-channel Z annihilation

II. s-channel h annihilation

III. s-channel H annihilation

$$\begin{aligned} f_{1L} &= C_{\widetilde{\nu}-\widetilde{\nu}-Z}\, C^L_{Z-t-t} \\ f_{1R} &= C_{\widetilde{\nu}-\widetilde{\nu}-Z}\, C^R_{Z-t-t} \\ f_2 &= C_{\widetilde{\nu}-\widetilde{\nu}-h}\, C_{h-t-t} \\ f_3 &= C_{\widetilde{\nu}-\widetilde{\nu}-H}\, C_{H-t-t} \\ \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (-2m_t^2(f_{1L}-f_{1R})^2 s + 4m_{\widetilde{\nu}}^2(2m_t^2(f_{1L}-f_{1R})^2 - (f_{1L}^2+f_{1R}^2)s) \\ &\quad +(f_{1L}^2+f_{1R}^2)(s^2-(t-u)^2))/(s-m_z^2)^2 \\ \mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= 2(s-4m_t^2)/(s-m_h^2)^2 \\ \mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= 2(s-4m_t^2)/(s-m_H^2)^2 \\ \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= -((f_{1L}+f_{1R})2m_t(t-u))/((s-m_z^2)(s-m_h^2)) \\ \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} &= -((f_{1L}+f_{1R})2m_t(t-u))/((s-m_z^2)(s-m_H^2)) \\ \mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} &= 2(s-4m_t^2)/((s-m_H^2)(s-m_h^2)) \\ |\mathcal{T}|^2 &= 3(\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + 2f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} \\ &\quad +2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III}) \end{aligned} \tag{D6}$$

$\widetilde{\nu}\widetilde{\nu}^* \longrightarrow e\bar{e}$

I. s-channel Z annihilation

II. t-channel charginos exchange

$$
\begin{aligned}
f_{1L} &= C_{\widetilde{\nu}-\widetilde{\nu}-Z}\, C^{L}_{Z-e-e} \\
f_{1R} &= C_{\widetilde{\nu}-\widetilde{\nu}-Z}\, C^{R}_{Z-e-e} \\
f_2(i) &= (C_{\widetilde{\nu}-\widetilde{\chi}^+_i-e})^2 \\
\mathcal{T}_{\mathrm{I}}\times\mathcal{T}_{\mathrm{I}} &= (-(f_{1L}^2+f_{1R}^2)(4m_{\widetilde{\nu}}^2 s - s^2 + (t-u)^2))/(s-m_Z^2)^2 \\
\mathcal{T}_{\mathrm{II}}\times\mathcal{T}_{\mathrm{II}} &= f_2(i)f_2(j)(tu - m_{\widetilde{\nu}}^4)/((t-m^2_{\widetilde{\chi}^+_i})(t-m^2_{\widetilde{\chi}^+_j})) \\
\mathcal{T}_{\mathrm{I}}\times\mathcal{T}_{\mathrm{II}} &= ((1/2)f_2(j)f_{1L}(4m_{\widetilde{\nu}}^2 s - s^2 + (t-u)^2))/((s-m_Z^2)(t-m^2_{\widetilde{\chi}^+_j})) \\
|\mathcal{T}|^2 &= \mathcal{T}_{\mathrm{I}}\times\mathcal{T}_{\mathrm{I}} + \sum_{i,j}\mathcal{T}_{\mathrm{II}}\times\mathcal{T}_{\mathrm{II}} + 2\sum_j \mathcal{T}_{\mathrm{I}}\times\mathcal{T}_{\mathrm{II}}
\end{aligned} \tag{D7}
$$

$\widetilde{\nu}\widetilde{\nu}^* \longrightarrow \nu\bar{\nu}$

I. s-channel Z annihilation

II. t-channel neutralinos exchange

$$
\begin{aligned}
f_1 &= C_{\widetilde{\nu}-\widetilde{\nu}-Z}\, C_{Z-\nu-\nu} \\
f_2(i) &= (C_{\widetilde{\nu}-\widetilde{\chi}^0_i-\nu})^2 \\
\mathcal{T}_{\mathrm{I}}\times\mathcal{T}_{\mathrm{I}} &= (-4m_{\widetilde{\nu}}^2 s + s^2 - (t-u)^2)/(s-m_Z^2)^2 \\
\mathcal{T}_{\mathrm{II}}\times\mathcal{T}_{\mathrm{II}} &= (-2m_{\widetilde{\nu}}^4 + 2tu)/((t-m^2_{\widetilde{\chi}^0_i})(t-m^2_{\widetilde{\chi}^0_j})) \\
\mathcal{T}_{\mathrm{I}}\times\mathcal{T}_{\mathrm{II}} &= -(1/2)(-4m_{\widetilde{\nu}}^2 s + s^2 - (t-u)^2)/((s-m_Z^2)(t-m^2_{\widetilde{\chi}^0_i})) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\mathrm{I}}\times\mathcal{T}_{\mathrm{I}} + \sum_{i,j} f_2(i)f_2(j)\mathcal{T}_{\mathrm{II}}\times\mathcal{T}_{\mathrm{II}} + 2\sum_i f_1 f_2(i)\mathcal{T}_{\mathrm{I}}\times\mathcal{T}_{\mathrm{II}}
\end{aligned} \tag{D8}
$$

$\widetilde{\nu}\widetilde{\nu}^* \longrightarrow W^+H^-$

I. s-channel H annihilation

II. s-channel h annihilation

III. t-channel $\widetilde{e}_L$ exchange

$$
\begin{aligned}
f_1 &= C_{H-W^+-H^-}\, C_{\widetilde{\nu}-\widetilde{\nu}-H} \\
f_2 &= C_{h-W^+-H^-}\, C_{\widetilde{\nu}-\widetilde{\nu}-h}
\end{aligned}
$$

$$
\begin{aligned}
f_3 &= C_{\tilde{\nu}-\tilde{e}_L-W}\, C_{\tilde{\nu}-\tilde{e}_L-H^+} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (m_{H^+}^4 - 2m_{H^+}^2(2m_{\tilde{\nu}}^2 + 7m_W^2 + s - t - u) + (-2m_{\tilde{\nu}}^2 + m_W^2 - s + t + u)^2) \\
&\quad /(4m_W^2(s - m_H^2)^2) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= (m_{H^+}^4 - 2m_{H^+}^2(2m_{\tilde{\nu}}^2 + 7m_W^2 + s - t - u) + (-2m_{\tilde{\nu}_\ell}^2 + m_W^2 - s + t + u)^2) \\
&\quad /(4m_W^2(s - m_h^2)^2) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= (m_{\tilde{\nu}}^4 + (m_W^2 - t)^2 - 2m_{\tilde{\nu}}^2(m_W^2 + t))/(m_W^2(t - m_{\tilde{e}_L}^2)^2) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= (m_{H^+}^4 - 2m_{H^+}^2(2m_{\tilde{\nu}}^2 + 7m_W^2 + s - t - u) + (-2m_{\tilde{\nu}}^2 + m_W^2 - s + t + u)^2) \\
&\quad /(4m_W^2(s - m_H^2)(s - m_h^2)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} &= (2m_{\tilde{\nu}}^4 + m_W^4 - m_W^2 s - 2m_W^2 t - st + t^2 + m_{H^+}^2(-m_{\tilde{\nu}}^2 - 3m_W^2 + t) \\
&\quad + m_{\tilde{\nu}}^2(m_W^2 + s - 3t - u) + m_W^2 u + tu)/(2m_W^2(s - m_H^2)(t - m_{\tilde{e}_L}^2)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} &= (2m_{\tilde{\nu}}^4 + m_W^4 - m_W^2 s - 2m_W^2 t - st + t^2 + m_{H^+}^2(-m_{\tilde{\nu}}^2 - 3m_W^2 + t) \\
&\quad + m_{\tilde{\nu}}^2(m_W^2 + s - 3t - u) + m_W^2 u + tu)/(2m_W^2(s - m_h^2)(t - m_{\tilde{e}_L}^2)) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + 2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} + 2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} \\
&\quad + 2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} \qquad \text{(D9)}
\end{aligned}
$$

$\tilde{\nu}\tilde{\nu}^* \longrightarrow H^+H^-$

I. s-channel H annihilation

II. s-channel h annihilation

III. t-channel $\tilde{e}_L$ exchange

IV. point interaction

$$
\begin{aligned}
f_1 &= C_{H-H^+-H^-}\, C_{\tilde{\nu}-\tilde{\nu}-H} \\
f_2 &= C_{h-H^+-H^-}\, C_{\tilde{\nu}-\tilde{\nu}-h} \\
f_3 &= (C_{\tilde{\nu}-\tilde{e}_L-H^+})^2 \\
f_4 &= C_{\tilde{\nu}-\tilde{\nu}-H^+-H^-} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= 1/(s - m_H^2)^2 \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= 1/(s - m_h^2)^2 \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= 1/(t - m_{\tilde{e}_L}^2)^2 \\
\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV} &= 1
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm I}{\times}\mathcal{T}_{\rm II} &= 1/((s-m_H^2)(s-m_h^2)) \\
\mathcal{T}_{\rm I}{\times}\mathcal{T}_{\rm III} &= 1/((s-m_H^2)(t-m_{\tilde{e}_L}^2)) \\
\mathcal{T}_{\rm I}{\times}\mathcal{T}_{\rm IV} &= -1/(s-m_H^2) \\
\mathcal{T}_{\rm II}{\times}\mathcal{T}_{\rm III} &= 1/((s-m_h^2)(t-m_{\tilde{e}_L}^2)) \\
\mathcal{T}_{\rm II}{\times}\mathcal{T}_{\rm IV} &= -1/(s-m_h^2) \\
\mathcal{T}_{\rm III}{\times}\mathcal{T}_{\rm IV} &= -1/(t-m_{\tilde{e}_L}^2) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}{\times}\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}{\times}\mathcal{T}_{\rm II} + f_3^2\mathcal{T}_{\rm III}{\times}\mathcal{T}_{\rm III} + f_4^2\mathcal{T}_{\rm IV}{\times}\mathcal{T}_{\rm IV} + 2f_1f_2\mathcal{T}_{\rm I}{\times}\mathcal{T}_{\rm II} \\
&\quad +2f_1f_3\mathcal{T}_{\rm I}{\times}\mathcal{T}_{\rm III} + 2f_1f_4\mathcal{T}_{\rm I}{\times}\mathcal{T}_{\rm IV} + 2f_2f_3\mathcal{T}_{\rm II}{\times}\mathcal{T}_{\rm III} + 2f_2f_4\mathcal{T}_{\rm II}{\times}\mathcal{T}_{\rm IV} \\
&\quad +2f_3f_4\mathcal{T}_{\rm III}{\times}\mathcal{T}_{\rm IV} \qquad \text{(D10)}
\end{aligned}
$$

$\widetilde{\nu}\widetilde{\nu}^* \longrightarrow HH \quad [hh] \quad [hH]$

I. s-channel H annihilation

II. s-channel h annihilation

III. t-channel $\widetilde{\nu}$ exchange

IV. point interaction

$$
\begin{aligned}
f_1 &= C_{H-H-H}\,C_{\widetilde{\nu}-\widetilde{\nu}-H} \qquad [C_{H-h-h}\,C_{\widetilde{\nu}-\widetilde{\nu}-H}] \quad [C_{H-H-h}\,C_{\widetilde{\nu}-\widetilde{\nu}-H}] \\
f_2 &= C_{H-H-h}\,C_{\widetilde{\nu}-\widetilde{\nu}-h} \qquad [C_{h-h-h}\,C_{\widetilde{\nu}-\widetilde{\nu}-h}] \quad [C_{H-h-h}\,C_{\widetilde{\nu}-\widetilde{\nu}-h}] \\
f_3 &= (C_{\widetilde{\nu}-\widetilde{\nu}-H})^2 \qquad [(C_{\widetilde{\nu}-\widetilde{\nu}-h})^2] \quad [C_{\widetilde{\nu}-\widetilde{\nu}-h}\,C_{\widetilde{\nu}-\widetilde{\nu}-H}] \\
f_4 &= C_{\widetilde{\nu}-\widetilde{\nu}-H-H} \qquad [C_{\widetilde{\nu}-\widetilde{\nu}-h-h}] \quad [C_{\widetilde{\nu}-\widetilde{\nu}-H-h}] \\
\mathcal{T}_{\rm I}{\times}\mathcal{T}_{\rm I} &= 1/(s-m_H^2)^2 \\
\mathcal{T}_{\rm II}{\times}\mathcal{T}_{\rm II} &= 1/(s-m_h^2)^2 \\
\mathcal{T}_{\rm III}{\times}\mathcal{T}_{\rm III} &= 1/(t-m_{\tilde{\nu}}^2)^2 \\
\mathcal{T}_{\rm IV}{\times}\mathcal{T}_{\rm IV} &= 1 \\
\mathcal{T}_{\rm I}{\times}\mathcal{T}_{\rm II} &= 1/((s-m_H^2)(s-m_h^2)) \\
\mathcal{T}_{\rm I}{\times}\mathcal{T}_{\rm III} &= 1/((s-m_H^2)(t-m_{\tilde{\nu}}^2)) \\
\mathcal{T}_{\rm I}{\times}\mathcal{T}_{\rm IV} &= -1/(s-m_H^2) \\
\mathcal{T}_{\rm II}{\times}\mathcal{T}_{\rm III} &= 1/((s-m_h^2)(t-m_{\tilde{\nu}}^2)) \\
\mathcal{T}_{\rm II}{\times}\mathcal{T}_{\rm IV} &= -1/(s-m_h^2)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} &= -1/(t-m_{\tilde\nu}^2)\\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III}+f_4^2\mathcal{T}_{\rm IV}\times\mathcal{T}_{\rm IV}+2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II}\\
&\quad +2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III}+2f_1f_4\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm IV}+2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III}+2f_2f_4\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm IV}\\
&\quad +2f_3f_4\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm IV} \qquad \text{(D11)}
\end{aligned}
$$

$\widetilde{\nu}\widetilde{\nu}^* \longrightarrow AA$

I. s-channel H annihilation

II. s-channel h annihilation

III. point interaction

$$
\begin{aligned}
f_1 &= C_{\widetilde{\nu}-\widetilde{\nu}-H}\,C_{H-A-A}\\
f_2 &= C_{\widetilde{\nu}-\widetilde{\nu}-h}\,C_{h-A-A}\\
f_3 &= C_{\widetilde{\nu}-\widetilde{\nu}-A-A}\\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= 1/(s-m_H^2)^2\\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= 1/(s-m_h^2)^2\\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= 1\\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= 1/((s-m_H^2)(s-m_h^2))\\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} &= -1/(s-m_H^2)\\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} &= -1/(s-m_h^2)\\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+f_3^2\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III}+2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II}+2f_1f_3\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III}\\
&\quad +2f_2f_3\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} \qquad \text{(D12)}
\end{aligned}
$$

$\widetilde{\nu}\widetilde{\nu}^* \longrightarrow AZ$

I. s-channel H annihilation

II. s-channel h annihilation

$$
\begin{aligned}
f_1 &= C_{\widetilde{\nu}-\widetilde{\nu}-H}\,C_{H-Z-A}\\
f_2 &= C_{\widetilde{\nu}-\widetilde{\nu}-h}\,C_{h-Z-A}\\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (m_A^4-2m_A^2(2m_{\tilde\nu}^2+7m_Z^2+s-t-u)+(-2m_{\tilde\nu}^2+m_Z^2-s+t+u)^2)\\
&\quad /(4m_Z^2(s-m_H^2)^2)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= (m_A^4 - 2m_A^2(2m_{\tilde\nu}^2 + 7m_Z^2 + s - t - u) + (-2m_{\tilde\nu}^2 + m_Z^2 - s + t + u)^2) \\
&\quad /(4m_Z^2(s - m_h^2)^2) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= (m_A^4 - 2m_A^2(2m_{\tilde\nu}^2 + 7m_Z^2 + s - t - u) + (-2m_{\tilde\nu}^2 + m_Z^2 - s + t + u)^2) \\
&\quad /(4m_Z^2(s - m_H^2)(s - m_h^2)) \\
|\mathcal{T}|^2 &= f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + 2f_1f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \qquad \text{(D13)}
\end{aligned}
$$

$\widetilde{\nu}\widetilde{\nu} \longrightarrow \nu\nu$

I. t-channel neutralino exchange

II. u-channel neutralino exchange

$$
\begin{aligned}
f(i) &= (C_{\widetilde{\nu}-\widetilde{\chi}_i^0-\nu})^2 \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (2s\, m_{\tilde\chi_i^0} m_{\tilde\chi_j^0})/((t - m_{\tilde\chi_i^0}^2)(t - m_{\tilde\chi_j^0}^2)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= (2s\, m_{\tilde\chi_i^0} m_{\tilde\chi_j^0})/((u - m_{\tilde\chi_i^0}^2)(u - m_{\tilde\chi_j^0}^2)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= (2s\, m_{\tilde\chi_i^0} m_{\tilde\chi_j^0})/((t - m_{\tilde\chi_i^0}^2)(u - m_{\tilde\chi_j^0}^2)) \\
|\mathcal{T}|^2 &= \sum_{i,j} f(i)f(j)\,(\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + \mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + 2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II}) \qquad \text{(D14)}
\end{aligned}
$$

$\chi\widetilde{\nu} \longrightarrow \nu Z$

I. s-channel ν annihilation

II. t-channel $\widetilde{\nu}$ exchange

III. u-channel neutralino exchange

$$
\begin{aligned}
f_1 &= C_{\widetilde{\nu}-\widetilde{\chi}_1^0-\nu}\, C_{Z-\nu-\nu} \\
f_2 &= C_{\widetilde{\nu}-\widetilde{\nu}-Z}\, C_{\widetilde{\nu}-\widetilde{\chi}_1^0-\nu} \\
f_{3L}(i) &= C_{\widetilde{\nu}-\widetilde{\chi}_i^0-\nu}\, C^L_{\widetilde{\chi}_1^0-\widetilde{\chi}_i^0-Z} \\
f_{3R}(i) &= C_{\widetilde{\nu}-\widetilde{\chi}_i^0-\nu}\, C^R_{\widetilde{\chi}_1^0-\widetilde{\chi}_i^0-Z} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (1/2)(1/m_Z^2)(m_{\tilde\chi}^4 s - m_{\tilde\nu}^4 s + s(-m_Z^4 - st + m_Z^2(s + t - u)) \\
&\quad - m_{\tilde\chi}^2(2m_Z^4 - 2m_Z^2 s + s(t + u)) + m_{\tilde\nu}^2(2m_Z^4 - 2m_Z^2 s + s(s + t + u)))/(s)^2 \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= (1/2)(m_{\tilde\chi}^2 - t)(m_{\tilde\nu}^4 + (m_Z^2 - t)^2 - 2m_{\tilde\nu}^2(m_Z^2 + t))/(m_Z^2(t - m_{\tilde\nu}^2)^2) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= (1/2)(-1)(m_{\tilde\chi_i^0} f_{3L}(i)(3(-m_{\tilde\chi})m_Z^2 f_{3R}(j)(-m_{\tilde\chi}^2 - m_Z^2 + s + t)
\end{aligned}
$$

$$
\begin{aligned}
& +m_{\tilde{\chi}_j^0} f_{3L}(j)(m_Z^4 + s(-m_{\tilde{\chi}}^2 + u) - m_Z^2(s - t + u))) \\
& +f_{3R}(i)(-3(-m_{\tilde{\chi}}^3) m_{\tilde{\chi}_j^0} m_Z^2 f_{3L}(j) - m_{\tilde{\chi}}^6 f_{3R}(j) \\
& +3(-m_{\tilde{\chi}}) m_{\tilde{\chi}_j^0} m_Z^2 f_{3L}(j)(-m_Z^2 + s + t) + m_{\tilde{\chi}}^4 f_{3R}(j)(-2m_Z^2 + s + t + 2u) \\
& +f_{3R}(j)(2m_Z^6 + 2m_Z^2 su + tu^2 - 2m_Z^4(s + t + u)) \\
& +m_{\tilde{\chi}}^2 f_{3R}(j)(m_Z^4 + m_Z^2(s + t + u) - u(s + 2t + u)))) \\
& /(m_Z^2(u - m_{\tilde{\chi}_i^0}^2)(u - m_{\tilde{\chi}_j^0}^2)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \;=\; & (1/2)(-1/(2m_Z^2))(m_{\tilde{\nu}}^4(s + t - u) + (m_Z^2 - t)(-s^2 - t^2 + m_Z^2(s + t - u) + u^2) \\
& +m_{\tilde{\nu}}^2(-s^2 - st - 2t^2 + 2m_Z^2(s - t - u) + tu + u^2) + m_{\tilde{\chi}}^2(m_Z^2(-s + t - 5u) \\
& +m_{\tilde{\nu}}^2(8m_Z^2 - s + t - u) + t(s - t + u)))/(s(t - m_{\tilde{\nu}}^2)) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} \;=\; & (1/2)(1/(2m_Z^2))(-2(-m_{\tilde{\chi}}^3) m_{\tilde{\chi}_j^0} f_{3L}(j) s - m_{\tilde{\chi}}^4 f_{3R}(j)(s + t - u) \\
& +2(-m_{\tilde{\chi}}) m_{\tilde{\chi}_j^0} f_{3L}(j)(2m_Z^4 - 2m_Z^2 s + s(-m_{\tilde{\nu}}^2 + t + u)) \\
& +m_{\tilde{\chi}}^2 f_{3R}(j)(s^2 + 2st + t^2 + su + tu - 2u^2 + m_{\tilde{\nu}}^2(8m_Z^2 - 3s - t + u) \\
& -m_Z^2(5s + t + 3u)) + f_{3R}(j)(-4m_Z^4 s + m_{\tilde{\nu}}^2(4m_Z^4 - 4m_Z^2(s + t) + (s + t - u)u) \\
& -u(s^2 + t^2 - u^2) + m_Z^2(4s^2 + (t - u)u + s(4t + u)))))/(s(u - m_{\tilde{\chi}_j^0}^2)) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} \;=\; & (1/2)(1/m_Z^2)(m_{\tilde{\chi}}^4 f_{3R}(j)(m_{\tilde{\nu}}^2 + 3m_Z^2 - t) + (-m_{\tilde{\chi}}) m_{\tilde{\chi}_j^0} f_{3L}(j)(m_Z^4 + m_{\tilde{\nu}}^2(3m_Z^2 - s) \\
& +st - m_Z^2(s + t + 2u)) + m_{\tilde{\chi}}^2 f_{3R}(j)(3m_Z^4 + m_{\tilde{\nu}}^2(5m_Z^2 - s - t - u) \\
& +t(s + t + u) - m_Z^2(4s + 5t + 2u)) + f_{3R}(j)(-m_Z^4(s + t - u) - t^2 u \\
& +m_Z^2(s^2 + 2st + t^2 + tu - u^2) + m_{\tilde{\nu}}^2(tu + m_Z^2(-s - 3t + u)))) \\
& /((t - m_{\tilde{\nu}}^2)(u - m_{\tilde{\chi}_j^0}^2)) \\
|\mathcal{T}|^2 \;=\; & f_1^2 \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2 \mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + \sum_{i,j} \mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} + 2 f_1 f_2 \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \\
& +2\sum_j (f_1 \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} + f_2 \mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III}) \qquad \text{(D15)}
\end{aligned}
$$

$\chi\tilde{\nu} \longrightarrow eW^+$

I. s-channel ν annihilation

II. t-channel $\tilde{e}$ exchange

III. u-channel chargino exchange

$$f_1 = C_{\nu-e-W}\, C_{\tilde{\nu}-\tilde{\chi}_1^0-\nu}$$

$$
\begin{aligned}
f_{2L}(i) &= C_{\widetilde{\nu}-\widetilde{e}-W}\, C^{L}_{\widetilde{e}_i-\widetilde{\chi}^0_1-e} \\
f_{2R}(i) &= C_{\widetilde{\nu}-\widetilde{e}-W}\, C^{R}_{\widetilde{e}_i-\widetilde{\chi}^0_1-e} \\
f_{3L}(i) &= C_{\widetilde{\nu}-\widetilde{\chi}^+_i-e}\, C^{L}_{\widetilde{\chi}^0_1-\widetilde{\chi}^+_i-W} \\
f_{3R}(i) &= C_{\widetilde{\nu}-\widetilde{\chi}^+_i-e}\, C^{R}_{\widetilde{\chi}^0_1-\widetilde{\chi}^+_i-W} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (1/2)(m^4_{\tilde{\chi}}s - m^4_{\tilde{\nu}}s + s(-m^4_W - st + m^2_W(s+t-u)) - m^2_{\tilde{\chi}}(2m^4_W - 2m^2_W s \\
&\quad +s(t+u)) + m^2_{\tilde{\nu}}(2m^4_W - 2m^2_W s + s(s+t+u)))/(m^2_W s^2) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= (1/2)((f_{2R}(i)f_{2R}(j) + f_{2L}(i)f_{2L}(j))(m^2_{\tilde{\chi}} - t)(m^4_{\tilde{\nu}} + (m^2_W - t)^2 \\
&\quad -2m^2_{\tilde{\nu}}(m^2_W + t)))/(m^2_W(t - m^2_{\tilde{e}_i})(t - m^2_{\tilde{e}_j})) \\
\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III} &= (-1/2)((f_{3L}(i)m_{\tilde{\chi}^+_i}(3(-m_{\tilde{\chi}})m^2_W f_{3R}(j)(-m^2_{\tilde{\chi}} - m^2_W + s + t) \\
&\quad +m_{\tilde{\chi}^+_j}f_{3L}(j)(m^4_W + s(-m^2_{\tilde{\chi}} + u) - m^2_W(s - t + u))) \\
&\quad +f_{3R}(i)(-3(-m_{\tilde{\chi}})^3 m_{\tilde{\chi}^+_j}m^2_W f_{3L}(j) - m^6_{\tilde{\chi}}f_{3R}(j) \\
&\quad +3(-m_{\tilde{\chi}})m_{\tilde{\chi}^+_j}m^2_W f_{3L}(j)(-m^2_W + s + t) + m^4_{\tilde{\chi}}f_{3R}(j)(-2m^2_W + s + t + 2u) \\
&\quad +f_{3R}(j)(2m^6_W + 2m^2_W su + tu^2 - 2m^4_W(s+t+u)) \\
&\quad +m^2_{\tilde{\chi}}f_{3R}(j)(m^4_W + m^2_W(s+t+u) - u(s+2t+u))))) \\
&\quad /(m^2_W(u - m^2_{\tilde{\chi}^+_i})(u - m^2_{\tilde{\chi}^+_j})) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= (-1/2)(f_{2R}(j)(m^4_{\tilde{\nu}}(s+t-u) + (m^2_W - t)(-s^2 - t^2 + m^2_W(s+t-u) + u^2) \\
&\quad +m^2_{\tilde{\nu}}(-s^2 - st - 2t^2 + 2m^2_W(s-t-u) + tu + u^2) + m^2_{\tilde{\chi}}(m^2_W(-s+t-5u) \\
&\quad +m^2_{\tilde{\nu}}(8m^2_W - s + t - u) + t(s - t + u)))))/(2m^2_W s(t - m^2_{\tilde{e}_j})) \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III} &= (1/2)((-2(-m_{\tilde{\chi}})^3 m_{\tilde{\chi}^+_j}f_{3L}(j)s - m^4_{\tilde{\chi}}f_{3R}(j)(s+t-u) \\
&\quad +2(-m_{\tilde{\chi}})m_{\tilde{\chi}^+_j}f_{3L}(j)(2m^4_W - 2m^2_W s + s(-m^2_{\tilde{\nu}} + t + u)) \\
&\quad +m^2_{\tilde{\chi}}f_{3R}(j)(s^2 + 2st + t^2 + su + tu - 2u^2 + m^2_{\tilde{\nu}}(8m^2_W - 3s - t + u) \\
&\quad -m^2_W(5s + t + 3u)) + fprj(-4m^4_W s + m^2_{\tilde{\nu}}(4m^4_W - 4m^2_W(s+t) \\
&\quad +(s+t-u)u) - u(s^2 + t^2 - u^2) + m^2_W(4s^2 + (t-u)u + s(4t+u))))) \\
&\quad /(2m^2_W s(u - m^2_{\tilde{\chi}^+_j})) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} &= (1/2)(f_{2R}(i)(m^4_{\tilde{\chi}}f_{3R}(j)(m^2_{\tilde{\nu}} + 3m^2_W - t) + (-m_{\tilde{\chi}})m_{\tilde{\chi}^+_j}f_{3L}(j)(m^4_W + m^2_{\tilde{\nu}}(3m^2_W \\
&\quad -s) + st - m^2_W(s + t + 2u)) + m^2_{\tilde{\chi}}f_{3R}(j)(3m^4_W + m^2_{\tilde{\nu}}(5m^2_W - s - t - u) \\
&\quad +t(s+t+u) - m^2_W(4s + 5t + 2u)) + f_{3R}(j)(-m^4_W(s+t-u) - t^2 u
\end{aligned}
$$

$$+m_W^2(s^2+2st+t^2+tu-u^2)+m_{\tilde{\nu}}^2(tu+m_W^2(-s-3t+u)))))$$
$$/(m_W^2(t-m_{\tilde{e}_i}^2)(u-m_{\tilde{\chi}_j^+}^2))$$
$$|\mathcal{T}|^2 = f_1^2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+\sum_{i,j}\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+\sum_{i,j}\mathcal{T}_{\rm III}\times\mathcal{T}_{\rm III}+2f_1\sum_j\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II}+2f_1\sum_j\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm III}$$
$$+2\sum_{i,j}\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm III} \quad \text{(D16)}$$

$\chi\tilde{\nu} \longrightarrow h\nu \quad [H\nu]$

I. t-channel neutralino exchange

II. u-channel $\widetilde{\nu}$ exchange

$$f_{1L}(i) = C_{\widetilde{\nu}-\widetilde{\chi}_1^0-\nu}\,C^L_{\widetilde{\chi}_1^0-\widetilde{\chi}_i^0-h} \qquad [C_{\widetilde{\nu}-\widetilde{\chi}_1^0-\nu}\,C^L_{\widetilde{\chi}_1^0-\widetilde{\chi}_i^0-H}]$$
$$f_{1R}(i) = C_{\widetilde{\nu}-\widetilde{\chi}_1^0-\nu}\,C^R_{\widetilde{\chi}_1^0-\widetilde{\chi}_i^0-h} \qquad [C_{\widetilde{\nu}-\widetilde{\chi}_1^0-\nu}\,C^R_{\widetilde{\chi}_1^0-\widetilde{\chi}_i^0-H}]$$
$$f_2 = C_{\widetilde{\nu}-\widetilde{\chi}_1^0-\nu}\,C_{\widetilde{\nu}-\widetilde{\nu}-h} \qquad [C_{\widetilde{\nu}-\widetilde{\chi}_1^0-\nu}\,C_{\widetilde{\nu}-\widetilde{\nu}-H}]$$
$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} = (1/2)(f_{1R}(i)m_{\tilde{\chi}_i^0}(f_{1R}(j)m_{\tilde{\chi}_j^0}(m_{\tilde{\chi}}^2-u)+f_{1L}(j)(-m_{\tilde{\chi}})(-m_{\tilde{\chi}}^2-m_{h[H]}^2+s+u))$$
$$+f_{1L}(i)(f_{1R}(j)(-m_{\tilde{\chi}})m_{\tilde{\chi}_j^0}(-m_{\tilde{\chi}}^2-m_{h[H]}^2+s+u)$$
$$+f_{1L}(j)(m_{\tilde{\chi}}^4-m_{h[H]}^4-st-m_{\tilde{\chi}}^2(s+u)+m_{h[H]}^2(s+t+u))))$$
$$/((t-m_{\tilde{\chi}_i^0}^2)(t-m_{\tilde{\chi}_j^0}^2))$$
$$\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} = (1/2)(m_{\tilde{\chi}}^2-u)/(u-m_{\tilde{\nu}}^2)^2$$
$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} = (1/2)(f_{1R}(i)m_{\tilde{\chi}_i^0}(m_{\tilde{\chi}}^2-u)+f_{1L}(i)(-m_{\tilde{\chi}})(-m_{\tilde{\chi}}^2-m_{h[H]}^2+s+u))$$
$$/((t-m_{\tilde{\chi}_i^0}^2)(u-m_{\tilde{\nu}}^2))$$
$$|\mathcal{T}|^2 = \sum_{i,j}\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}+f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II}+2\sum_i f_2\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \quad \text{(D17)}$$

$\chi\tilde{\nu} \longrightarrow A\nu$

I. t-channel neutralino exchange

$$f_L(i) = C_{\widetilde{\nu}-\widetilde{\chi}_1^0-\nu}\,C^L_{\widetilde{\chi}_1^0-\widetilde{\chi}_i^0-A}$$
$$f_R(i) = C_{\widetilde{\nu}-\widetilde{\chi}_1^0-\nu}\,C^R_{\widetilde{\chi}_1^0-\widetilde{\chi}_i^0-A}$$
$$\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} = (1/2)(f_R(i)m_{\tilde{\chi}_i^0}(f_R(j)m_{\tilde{\chi}_j^0}(m_{\tilde{\chi}}^2-u)+f_L(j)(-m_{\tilde{\chi}})(-m_{\tilde{\chi}}^2-m_A^2+s+u))$$
$$+f_L(i)(f_R(j)(-m_{\tilde{\chi}})m_{\tilde{\chi}_j^0}(-m_{\tilde{\chi}}^2-m_A^2+s+u)+f_L(j)(m_{\tilde{\chi}}^4-m_A^4-st-m_{\tilde{\chi}}^2(s+u)$$
$$+m_A^2(s+t+u))))/((t-m_{\tilde{\chi}_i^0}^2)(t-m_{\tilde{\chi}_j^0}^2))$$

$$|\mathcal{T}|^2 = \sum_{i,j} \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} \qquad \text{(D18)}$$

$\chi\widetilde{\nu} \longrightarrow eH^+$

I. t-channel chargino exchange

II. u-channel $\widetilde{e}_L$ exchange

$$\begin{aligned}
f_{1aL}(i) &= C^L_{\widetilde{\chi}^0_1-\widetilde{\chi}^-_i-H^+} \\
f_{1aR}(i) &= C^R_{\widetilde{\chi}^0_1-\widetilde{\chi}^-_i-H^+} \\
f_{1bL}(i) &= C^L_{\widetilde{\nu}-\widetilde{\chi}^+_i-e} \\
f_{1bR}(i) &= C^R_{\widetilde{\nu}-\widetilde{\chi}^+_i-e} \\
f_2 &= C_{\widetilde{e}-\widetilde{\chi}^0_1-e} C_{\widetilde{\nu}-\widetilde{e}-H^+} \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= (1/2)(f_{1bR}(i)f_{1bR}(j)(m^4_{\tilde{\chi}}f_{1aL}(i)f_{1aL}(j) - m^4_{H^+}f_{1aL}(i)f_{1aL}(j) \\
&\quad -(-m_{\tilde{\chi}})^3(m_{\tilde{\chi}^+_j}f_{1aL}(i)f_{1aR}(j) + m_{\tilde{\chi}^+_i}f_{1aL}(j)f_{1aR}(i)) - f_{1aL}(i)f_{1aL}(j)st \\
&\quad -(-m_{\tilde{\chi}})(m_{\tilde{\chi}^+_j}f_{1aL}(i)f_{1aR}(j) + m_{\tilde{\chi}^+_i}f_{1aL}(j)f_{1aR}(i))(m^2_{H^+} - s - u) \\
&\quad -m_{\tilde{\chi}^+_i}m_{\tilde{\chi}^+_j}f_{1aR}(j)f_{1aR}(i)u + m^2_{H^+}f_{1aL}(i)f_{1aL}(j)(s+t+u) \\
&\quad +m^2_{\tilde{\chi}}(m_{\tilde{\chi}^+_i}m_{\tilde{\chi}^+_j}f_{1aR}(j)f_{1aR}(i) - f_{1aL}(i)f_{1aL}(j)(s+u))))/((t-m^2_{\tilde{\chi}^+_i})(t-m^2_{\tilde{\chi}^+_j})) \\
\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} &= (1/2)(m^2_{\tilde{\chi}} - u)/(u - m^2_{\tilde{e}_L})^2 \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} &= (1/2)(f_{1bR}(i)(-(-m_{\tilde{\chi}})^3f_{1aL}(i) + m^2_{\tilde{\chi}}m_{\tilde{\chi}^+_i}f_{1aR}(i) - m_{\tilde{\chi}^+_i}f_{1aR}(i)u \\
&\quad +(-m_{\tilde{\chi}})f_{1aL}(i)(-m^2_{H^+} + s + u)))/((t-m^2_{\tilde{\chi}^+_i})(u-m^2_{\tilde{e}_L})) \\
|\mathcal{T}|^2 &= \sum_{i,j}\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} + f_2^2\mathcal{T}_{\rm II}\times\mathcal{T}_{\rm II} + 2f_2\sum_i \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm II} \qquad \text{(D19)}
\end{aligned}$$

$\widetilde{\nu}_e\widetilde{\nu}^*_\mu \longrightarrow \nu_e\bar{\nu}_\mu$

I. t-channel neutralino exchange

$$\begin{aligned}
f(i) &= (C_{\widetilde{\nu}-\widetilde{\chi}^0_i-\nu})^2 \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= f(i)f(j)(-2m^4_{\tilde{\nu}} + 2tu)/((t-m^2_{\tilde{\chi}^0_i})(t-m^2_{\tilde{\chi}^0_j})) \\
|\mathcal{T}|^2 &= \sum_{i,j}\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} \qquad \text{(D20)}
\end{aligned}$$

$\widetilde{\nu}_e \widetilde{\nu}_\mu^* \longrightarrow e\bar{\mu}$

I. t-channel charginos exchange

$$
\begin{aligned}
f(i) &= (C_{\widetilde{\nu}-\widetilde{\chi}_i^+-e})^2 \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= f(i)f(j)(tu - m_{\tilde{\nu}}^4)/((t - m_{\tilde{\chi}_i^+}^2)(t - m_{\tilde{\chi}_j^+}^2)) \\
|\mathcal{T}|^2 &= \sum_{i,j} \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}
\end{aligned} \tag{D21}
$$

$\widetilde{\nu}_e \widetilde{\nu}_\mu \longrightarrow \nu_e \nu_\mu$

I. t-channel neutralino exchange

$$
\begin{aligned}
f(i) &= (C_{\widetilde{\nu}-\widetilde{\chi}_i^0-\nu})^2 \\
\mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I} &= f(i)f(j)(2s\, m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0})/((t - m_{\tilde{\chi}_i^0}^2)(t - m_{\tilde{\chi}_j^0}^2)) \\
|\mathcal{T}|^2 &= \mathcal{T}_{\rm I}\times\mathcal{T}_{\rm I}
\end{aligned} \tag{D22}
$$