Qualitative Analysis of Electron cloud effects in the NLC damping ring*

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Abstract

The qualitative analysis of the electron cloud formation is presented. Results are compared with simulations for the NLC damping ring [1].

1 INTRODUCTION

Since the discovery of instability at KEK photon factory [2], it was realized that the electron cloud can drive the fast multi-bunch [3] and, later, the single bunch instabilities [4] in the positron storage rings. The instabilities affect performance of the B-factories and design of the future linear colliders.

Effects of the e-cloud on the beam dynamics is conveniently described by the effective wake field [5] which can be calculated [6] given the density of the e-cloud. The estimate of the density is the main difficulty of the problem. The e-cloud is neither static in time nor uniform in space and depends on the bunch population N_b, bunch spacing s_b, geometry of the beam pipe, the flux of the synchrotron radiation (SR) photons, and the yield of secondary electrons. Due to these difficulties, the density is usually determined either by elaborate simulations or considered as a fitting parameter. Nevertheless, it is highly desirable to have some analytic estimate of the density to interpret the results of simulations and for scaling of these results with machine parameters. The goal of the paper is to provide such an estimate. Results of the analysis are applied to the NLC main damping ring and compared with the simulations for the NLC [1]. The relevant parameters of the ring are listed in Table.

The electron cloud where electrons moves randomly and can be characterized by some quasi-steady equilibrium distribution can exist only in the case of small currents. That is true for both practically important cases where electrons are generated by synchrotron radiation or are result of the beam induced multipactoring.

The paper is organized as following. We start with a simple case of the coasting beam where electrons oscillate in the self-consistent potential well and can be described by the Boltzmann distribution. Then, to define the temperature of the distribution, we need to consider bunched beam. The temperature is defined by the equilibrium of the energy losses. The next step is to take into account the multipactoring. It is shown that the space-charge potential of the secondary electrons generates a potential bump at the wall which defined by the equilibrium of the average number of electrons in the cloud. Effect of the finite bunch length in Section 6. Wherever it is possible, our results are compared with simulations [1].

* Work supported by Department of Energy contract DE–AC03–76SF00515.

2 STEADY-STATE: COASTING BEAM, NO SR

Let us start with a coasting beam with the average linear density N_b/s_b in a round beam pipe. Electrons of the cloud oscillate in the steady-state potential $U = U_{beam} + USC$ of the relativistic beam (in units of mc^2)

$$U_{beam} = -\frac{N_b r_e}{s_b} \ln[\frac{b^2}{r^2 + \sigma_y(\sigma_x + \sigma_y)}], \qquad (1)$$

plus the space-charge potential of the cloud

$$U_{SC} = 4\pi r_e [\int_0^b r' dr' n(r') \ln\frac{b}{r'} - \int_0^r r' dr' n(r') \ln\frac{r}{r'}]. \qquad (2)$$

The steady-state density corresponds to the condition that the total radial field $\propto (dU/dr)_{r=b}$ at the wall is zero. This condition defines the average density in the steady-state

$$n_0 = \frac{2\pi}{\pi b^2} \int r dr n(r) = \frac{N_b}{\pi s_b b^2}. \qquad (3)$$

This is the well known condition of neutrality which is, actually, independent of the form of the distribution $n(r)$.

For the NLC parameters, $n_0 = 2.2\, 10^7\ cm^3$. This agrees quite well with the results of simulations [1] which give the average in time density at saturation $3.0\, 10^7\ cm^{-3}$ at low level SR.

The average over time distribution function of electrons trapped in this potential well can be taken as Boltzmann distribution

$$\rho(r, v) = |N| e^{-\frac{1}{T}[(1/2)(v/c)^2 + U(r)]}, \qquad (4)$$

where T is temperature in units of mc^2, $|N|$ is the normalization factor related to the average density n_0 is $\int 2\pi r dr dv \rho(r, v) = \pi b^2 n_0$. The density of the cloud

$$n_{cl}(r) = \int dv \rho = |N| c \sqrt{2\pi T} e^{-U/T} = n_0 \frac{b^2}{2} \frac{e^{-U/T}}{\int r dr e^{-U(r)/T}}. \qquad (5)$$

The potential U in Eq. (5) is the total potential $U = U_b + U_{cl}$ of the beam and the cloud. The later is defined by the Poisson equation with the right-hand-side (RHS) proportional to $n_{cl}(r)$.

Let us define dimensionless $x = r/b$ and measure all potentials in units of T, introducing $V(x) = (U(r)/T)_{r=bx}$. Then, for a cylindrically symmetric beam pipe, $V(x) = V_{cl} - g \ln(1/x)$ where

$$g = \frac{2 N_b r_e}{T s_b}, \quad \hat{g} = \frac{2\pi r_e b^2}{T} n_0. \qquad (6)$$

The Poisson equation for V_{cl} takes the form

$$\frac{1}{x}\frac{\partial}{\partial x}x\frac{\partial V_{cl}}{\partial x} = -\hat{g}\frac{e^{-V(x)}}{\int xdxe^{-V(x)}}. \tag{7}$$

In the stationary case, the total potential $U(r)$ and the force $dU(r)/dr$ are zero at $r = b$. That gives the boundary conditions $V(1) = 0$, $(dV/dx)_{x=1} = 0$ or, for the space-charge potential, or

$$V_{cl}(1) = 0, \quad (\frac{dV_{cl}}{dx})_{x=1} = -g. \tag{8}$$

The space-charge potential is finite at $x = 0$. Integration of Eq. (7) with the weight x gives $(dV_{cl}/dx)_{x=1} = -\hat{g}$. Comparison of this result with Eq. (8) gives $\hat{g} = g$ and defines the average density

$$n_0 = \frac{N_b}{\pi s_b b^2}, \tag{9}$$

reproducing the density given by the condition of neutrality. Note, that the average density n_0 is independent of the shape of the density $n_{cl}(r)$ and temperature T.

Potentials $V(x)$, $V_{cl}(x)$, and

$$n_{cl} = \frac{n_0}{2}\frac{e^{-V(x)}}{\int_0^1 xdxe^{-V(x)}} \tag{10}$$

depend only on one parameter g. It is defined in the next section.

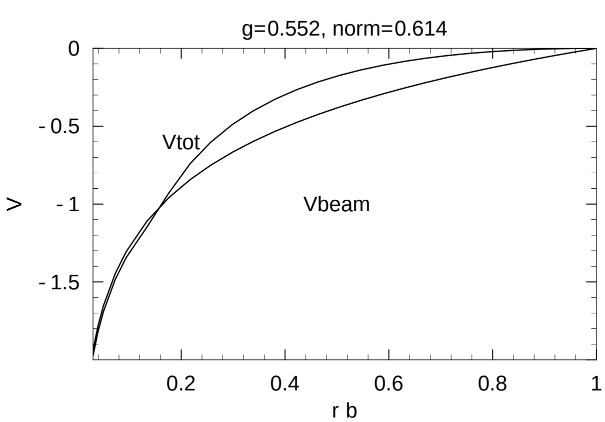

Figure 1: *Total self-consistent potential $V(x)$ and the beam potential $V_b = -g\ln(1/x)$ vs $x = r/b$. Parameter g is found from Eq. (16).*

3 STATIONARY DISTRIBUTION, BUNCHED BEAM

In the approximation of the averaged beam potential, electrons have regular motion oscillating in the self-consistent potential well. The averaging of the beam potential is a standard trick used for the similar problem of the ion instability. For the e-cloud this approximation require justification due to high frequency of the electron oscillations. For example, for the NLC DR, the linear frequency of oscillations

$$\Omega_{0y} = c0\sqrt{\frac{2N_b r_e}{s_b(\sigma_x+\sigma_y)\sigma_y}} \tag{11}$$

is equal to $\bar{\Omega}_{0,y}/2\pi = 31.7$ GHz and the number of oscillations between bunches $\bar{\Omega}_{0,y}s_b/(2\pi c) >> 1$. Obviously, the beam potential cannot be approximated by a potential of the coasting beam.

Nevertheless, an electron moves between bunches only by the distance small compared to beam pipe radius. Hence, before an electron can reach the wall, it is kicked by $v/c = 2N_b r_e/r$ several times. Electrons move changing direction and the motion is similar to a random walk. We can estimate the number of kicks n_{pass} an electron gets before it can reach the wall from

$$n_{pass} < (\frac{2N_b r_e s_b}{r})^2 >= b^2, \tag{12}$$

what defines n_{pass}. It is clear again that it makes sense to speak about e-cloud only for $\kappa \equiv 2N_b r_e s_b/b^2 << 1$. For the NLC parameters, $n_{pass} \simeq 3-4$ in agreement with simulations.

In the previous section, the temperature T remains undefined. Now we take into account the beam bunching considering bunches as point-like macro particles. The goal is to define the temperature T and the average over time density of the cloud.

The bunching of the beam has several implications. First, an electron in the beam pipe experiences periodic kicks. Neglecting the space-charge potential, we can write a simplectic map $M(x,v)$ giving transformation of the electron coordinates per bunch spacing $[x,v]-\ >[\bar{x},\bar{v}] = M(x,v)[x,v]$. The eigen values of the Jacobian $D[M[x,v],\{x,v\}]$ are real only for $x < \sigma_\perp/b$, i.e. in the region of the linear motion.

Elsewhere the motion is chaotic and the average in time distribution function can be taken in the form of Eq. (4) although the approximation of the coasting beam is not valid. That is possible due to the other effects of the bunched beam: heating of the cloud caused by the kicks balanced by the cooling of the cloud due to the loss of electrons.

A kick from a bunch increases the average energy of the e-cloud by

$$\Delta E_{gain} = 2\pi\int rdrdv\rho(r,v)(\frac{2N_b r_e}{r})^2, \tag{13}$$

where integration is over the phase space of the cloud.

The electrons in the vicinity of the beam are kicked to the wall and are replaced with the low energy secondary electrons. The later process produces cooling. To be lost, an electron has to reach the wall before the next bunch arrives. The trajectory of an electron between bunches can be estimated as following. Consider an electron with the initial conditions $r, v/c$ just before a bunch arrives. A bunch changes $\beta = v/c$ to $\beta_0 = v/c - 2N_b r_e/r$. After that, an electron moves in the field of the space charge. Let us assume, for a moment, a uniform density of the cloud, $n_{cl}(r) = n_0$. Then, the space-charge force is $2\pi r_e n_0 r$ and the electron is at $\bar{r} = r\cosh(\Omega_{pl} s_b/c) + (c\beta_0/\Omega_{pl})\sinh(\Omega_{pl} s_b/c)$ at the time of arrival of the next bunch. Here $(\Omega_{pl}/c)^2 = 2\pi n_0 r_e$. A quasi-stationary cloud can exist only if $(\Omega_{pl} s_b/c)^2 << 1$. For the NLC parameters, $n_0 = 2.2\,10^7\ cm^{-3}$, and $(\Omega_{pl} s_b/c)^2 = 0.277$. In the case of small (Ω_{pl}/c), $\bar{r} = r + (v/c - 2N_b r_e/r)s_b$ and is independent on n_0. The electron hits the wall if $|\bar{r}| > b$, or

$$\frac{v}{c} > \frac{b-r}{s_b} + \frac{2N_b r_e}{r}, \quad \text{or} \quad \frac{v}{c} < -\frac{b+r}{s_b} + \frac{2N_b r_e}{r}. \tag{14}$$

All electrons within this part of the phase space get lost and are replaced by the electrons from the cloud. The energy loss is equal to the energy of the lost particles before they were kicked to the wall:

$$\Delta E_{loss} = 2\pi \int r dr dv \rho(r,v)[\frac{1}{2}\frac{v^2}{c^2} + U(r)], \tag{15}$$

where integration is restricted by the condition Eq. (14) and $0 < r < b$. Here we neglected the energy brought to the cloud by the low energy secondary electrons coming in from the wall.

The balance of energies Eq. (13) and Eq. (15) gives the following equation:

$$\begin{aligned} & g\kappa \int_0^1 \frac{dx}{x} e^{-V(x)} F(x) = \\ & \int_0^1 x dx e^{-V(x)} [(\frac{1}{2} + V(x))(1 - F(x)) \\ + & \frac{1}{2\sqrt{\pi}}(z_+ e^{-z_+^2} + z_- e^{-z_-^2})], \end{aligned} \tag{16}$$

where

$$F(x) = (1/2)(Erf[z_+] + Erf[z_-]), \quad \kappa = 2N_b r_e s_b/b^2, \tag{17}$$

and

$$z_+ = \sqrt{\frac{g}{2p}}(1 - x + \frac{p}{x}), \quad z_- = \sqrt{\frac{g}{2p}}(1 + x - \frac{p}{x}). \tag{18}$$

Let us remind that, given κ, $V(x)$ depends only on g. Eq. (16) defines g, i.e. the temperature T. It is plausible to expect that $g \simeq 1/\ln(1/\kappa)$. The solution of Eq. (7) and Eq. (16) can be obtained numerically. Calculations for the NLC parameter $\kappa = 0.277$ define $g = 0.552$, what is close to the estimate above, $1/\ln(1/\kappa) = 0.780$. The temperature in units of mc^2 is $T = g(2N_b r_e/s_b)$, or $T = 92.4$ eV. The potential $V(x)$ is shown in Fig. 5. At small distances it goes as beam potential but at large distances is flatter due to the space charge contribution. The density profile $n(x)/n_0$, Eq. (10), for the same parameters is shown in Fig. 2. The

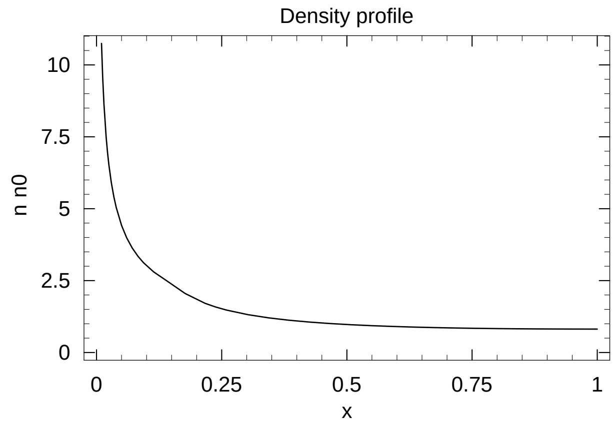

Figure 2: *The density $n(r)/n_0$, $n_0 = 2.2\,10^7\ cm^{-3}$ vs $x = r/b$ for the NLC parameter $2N_b r_e s_b/b^2 = 0.277$.*

density at the beam line (at the moment of a bunch arrival) is substantially larger that the average density n_0.

The number of electrons with the energy E hitting the wall of the drift chamber with the length L_d is

$$\frac{dN(E)}{dE} = 2\pi L_d \int r dr dv \rho(r,v) \delta[\frac{1}{2}(\frac{v}{c} - \frac{2N_b r_e}{r})^2 + U(r) - E], \tag{19}$$

where integration is taken over the region $v/c > \sqrt{2T} z_+$ and $v/c < -\sqrt{2T} z_-$, and ρ is the distribution function at the moment of bunch arrival.

The result of calculations is shown in Fig. 3. Parameters are the same as in Fig. 2.

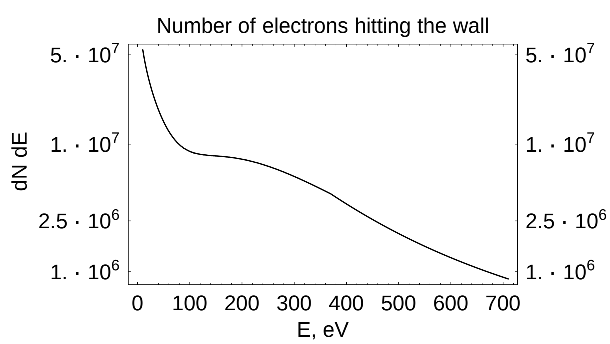

Figure 3: *Number of electrons per bunch dN/dE $1/eV$ accelerated from the e-cloud and hitting the wall with energy E.*

Finally, the number of electrons hitting the wall per passing bunch is given by the integral

$$N_{loss} = 2\pi L_d \int r dr dv \rho(r,v) \quad (20)$$

where the integration is over the region $v/c > \sqrt{2T} z_+$ and $v/c < -\sqrt{2T} z_-$. Calculation gives $N_{loss} = 5.53\,10^9$, 31% of the total $N_{tot} = \pi b^2 L_d n_0 = 1.74\,10^{10}$ electrons in the cloud in the drift with length L_d. This result may be compared with the simple estimate which assumes that all particles within radius r, where $(2N_b r_e/r)s_b > b$ are lost. If the density would be constant $n_0 = 2.2\,10^7\ 1/cm^3$, then $N_{loss} = 1.37\,10^9$. The actual number is higher because the density at the beam line is higher than the average density n_0.

The total energy loss is given by the integral

$$\frac{E_{loss}}{T} = \frac{\pi b^2 n_0}{\int_0^1 x dx Exp[-V(x)]} \int x dx e^{-V(x)}$$
$$\int du e^{-u^2} [(u - \frac{1}{x}\sqrt{\frac{\kappa g}{2}})^2 + V(x)]. \quad (21)$$

Here the variable $u = (v/c)/\sqrt{2T}$, and the integral is taken over $|x + u\sqrt{2\kappa/g} - \kappa/x| > 1$. Numeric integration gives power loss $(c/s_b)E_{loss} = 101\ W/m$.

4 JETS

Another effect of the bunched beam is production of jets of electrons.

Simulations show that, at the high level of the SR, the average electron density is higher than at the low level of the SR by a factor of two. (It is worth noting that a round beam pipe without the ante-chamber was used in simulations). For large SR, the primary photo-electrons move as a compact jet toward the beam line getting a kick

$$\Delta(\frac{v}{c}) = -\frac{2N_b r_e}{r} \quad (22)$$

from the parent bunch.

The density of a jet may be higher than that given by the condition of neutrality and depends on the yield η of the secondary electron emission, number of jets k_{jets} within the beam pipe, and the volume of a jet. The density averaged over the length L_d of the drift section where SR is absorbed and over the beam pipe cross-section,

$$< n_{e\gamma} >= Y \frac{N_\gamma N_b}{\pi b^2 L_d} k_{jets}, \quad (23)$$

is proportional to the number of photons

$$N_\gamma = \frac{5\alpha_0 \gamma}{2\sqrt{3}} \frac{L_b}{R} \quad (24)$$

radiated by a positron in the bend with radius R and length L_b per pass.

For the NLC parameters and $Y = 0.2$, $k_{jets} = 2$ and the average density $< n_{e\gamma} >= 5.5\,10^7\ 1/cm^3$ is very close to the result of simulations $6.\,10^7\ cm^{-3}$ with the large yield Y of the primary photo-electrons.

The jets may also explain why the electron density at the beam pipe line in simulations is much higher than the average electron density.

Initial energy spread of the primary photo-electrons leads to the difference in the distances of electrons in the jet from the beam line. Interaction with the bunch translates this difference in the energy spread of the electrons hitting the beam pipe wall. If the shortest distance of the jet centroid from the beam line is d, then

$$\frac{dN}{dE} = \frac{Y N_b N_\gamma}{l_{jet}} (\frac{2N_b r_e}{mc^2}) (\frac{mc^2}{2E})^{3/2}. \quad (25)$$

The distribution is shown in Fig. 4 for $Y = 0.2$.

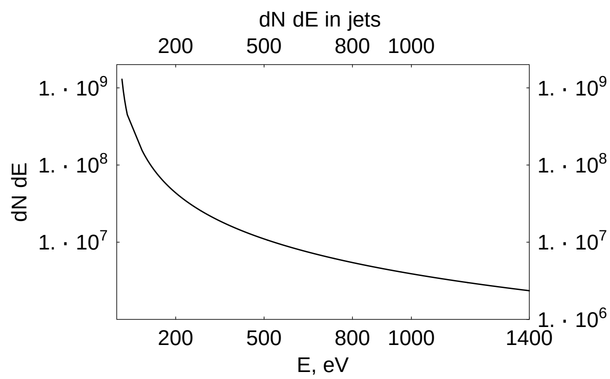

Figure 4: *Number of electrons dN/dE hitting the wall per bunch. Electrons are accelerated by the beam while a jet crosses the beam line. $Y = 0.2$*

5 SATURATION

High energy electrons hitting the wall produce secondary electrons which, after thermalization, may increase the density of the cloud in the avalanche-like way. Let us estimate the number of bunches m needed to reach saturation of the cloud density $n_0 = 2.2\,10^7\ 1/cm^3$. At the low level of the photo-electric yield $Y = 0.002$ taken in simulations [1], the SR adds to the average density $n_{SR} = 5.5\,10^5\ 1/cm^3$ per bunch (see Eq. (23)). Most of these electrons go wall-to-wall and only $(\eta - 1)n_{SR}$ of the secondary electrons remain in the cloud. Due to the multipactoring the density increases exponentially:

$$\frac{dn}{dm} = -\xi n + \eta \xi n + (\eta - 1) n_{SR}, \quad n = \frac{n_{SR}}{\xi}[e^{\xi(\eta-1)m} - 1]. \quad (26)$$

Here we introduced parameter $\xi = N_{loss}/N_{tot}$ defining the fraction of the cloud participating in multipactoring. The estimate of the previous section gives $\xi = 0.3$ and the density reaches saturation after

$$m = \frac{1}{\xi(\eta - 1)} \ln[\frac{n_0}{n_{SR}}\xi + 1] \qquad (27)$$

passes. For the NLC DR, $m = 19$ for $\eta = 1.45$. At the high SR photon flux, where $n_{SR} \simeq n_0$, the number of passes to reach saturation os of the order of $[\xi(\eta - 1)]^{-1} \simeq 7$. These estimates are in reasonable agreement with the simulations.

6 EFFECT OF THE MULTIPACTORING

It was mentioned above, that, for $\kappa > 1$, there are two region of distances from the beam line: in the vicinity of the beam, where electrons are wiped out by each passing bunch, and another one close to the wall.

The multipactoring adds the third region. Generally, there is a bump of the potential well in the vicinity of the wall which defines how many of the secondary electrons can go to the central regions. Such a sheath works as a virtual cathode. The density in the sheath near the wall depends on the balance of the number of electrons kicked to the wall from the central region and the number of electrons produced at the wall by the SR and multipactoring.

In the equilibrium, the number of lost particles is equal to the particles coming to the cloud from the wall. If the yield of secondary electrons is high, to sustain the equilibrium, the total potential changes to stop the back flow of the secondary electrons.

The distribution function $\rho(r, v)$ satisfies the Liouville equation with the source S,

$$\frac{\partial\rho}{\partial t} + v\frac{\partial\rho}{\partial r} - c^2\frac{\partial U(r)}{\partial r}\frac{\partial\rho}{\partial v} = S\frac{\delta(r - b)}{2\pi r}f(v). \qquad (28)$$

Here $f(v)$ is normalized distribution of the secondary electrons over velocity,

$$f(v) = \frac{v}{c^2 T_w}e^{-\frac{v^2}{2c^2 T_w}}, \quad \int_{-\infty}^{0} dv f(v) = 1. \qquad (29)$$

The temperature T_w is equal to the average energy of secondary electrons E_0 in units of mc^2, $T_w = \int dv(v^2/2c^2)f(v)$. In the estimate we assume $E_0 = 2$ eV, $T_w = 4.0\,10^{-6}$. The source S_{cl}, the number of secondary electrons ejected from the wall per unit time and unit length of the beam pipe, is given by the number of lost electrons dN_{loss}/ds and the yield of the secondary electrons η, $S_{cl} = (\eta - 1)(c/s_b)dN_{loss}/ds$. (More exactly, S_{cl} is given only by the lost particles with the sufficiently high energy, $E > 50$ eV). If there is the SR flux, it adds S_{SR}, $S = S_{cl} + S_{SR}$,

$$S_{SR} = Y N_\gamma N_b \frac{c}{s_b L_d}. \qquad (30)$$

We imply here that electrons generated at the wall are thermalized and are added to the e-cloud. This process works as a sink for the generated electrons and allows us to consider the average in time electron density $\rho(r, v) = \rho_{cl}(H) + \rho_s(r, v)$, where $H = v^2/2c^2 + U(r)$. Here the first term is the distribution function of the cloud and the second term describes secondary electrons,

$$\rho_s(r, v) = \frac{S}{2\pi b}\frac{f(c\sqrt{2H})}{c\sqrt{2H}}\Theta(b - r). \qquad (31)$$

The density of the secondary electrons $n_s = \int dv\rho_s$ at the wall is

$$n_s(r) = \frac{S}{2bc\sqrt{2\pi T_w}}. \qquad (32)$$

The total potential at the wall $V(1) = 0$, and in the vicinity of the wall can be expanded in series $V(x) = (1 - x)V_1 + (1 - x)^2(V_2/2) + ...$ To have maximum at $x_{max} < 1$, V_2 has to be negative. The potential is maximum $V_{max} = -V_1^2/(2V_2)$ at the distance $\Delta = (1 - x_{max}) = -V_1/V_2$ from the wall. Hence, $V_1 > 0$. The Poisson equation at $x \to 1$ relates the coefficients V_1, and V_2, $V_2 - V_1 = -G$, where

$$G = S\frac{r_e b}{cT}\sqrt{\frac{2\pi}{T_w}} + \frac{2\pi r_e b^2 n_0}{T\int_0^1 xdxexp[-V(x)]}. \qquad (33)$$

The second term in the right-hand-side is due to the density of the cloud.

To stop secondary electrons to go into the beam pipe, the maximum of the potential V_{max} has to be of the order of T_w/T. V_{max} can be estimated equating the number of particles returning to the cloud to dN_{loss}/ds. Electrons that go back into the beam pipe have to have energy $v^2/(2c^2) > TV_{max}$,

$$(\frac{dN}{ds})_{back} = \frac{s_b}{c}\int_{v/c<-\sqrt{2TV_{max}}} 2\pi r dr dv S f(v)\frac{\delta(r - b)}{2\pi r}$$
$$= \frac{s_b}{c} S e^{-V_{max}T/T_w}. \qquad (34)$$

Substituting S and equating that to $(dN/ds)_{loss} = N_{loss}/L_d$ defined by Eq. (**??**), we get

$$V_{max} = \frac{T_w}{T}\ln[\eta + \frac{Y N_\gamma N_b}{N_{loss}}]. \qquad (35)$$

This defines $V_1 = V_2 + G$ and $\Delta = -V_1/V_2$,

$$\Delta = \frac{-V_{max} + \sqrt{V_{max}^2 + 2GV_{max}}}{V_{max} + G - \sqrt{V_{max}^2 + 2GV_{max}}}. \qquad (36)$$

This result has meaning only if $\Delta << 1$, i.e. for the large enough density of the cloud. Otherwise, the height of the potential barrier can not reach T_w and the density keeps building up.

If $G << V_{max}$,

$$\Delta = \frac{2V_{max}}{G}. \qquad (37)$$

For the NLC parameters and $\eta = 1.45$, $\Delta = 0.082$ and $V_{max} = 0.032$ or 2.95 eV.

Although the height of the potential bump at the $r_{max} = b(1-\Delta)$ is small, of the order of T_w, it changes the equilibrium density of the cloud. To see the effect on the average density, let us again integrate the Poisson equation

$$\frac{1}{r}\frac{d}{dr}r\frac{dU_{cl}}{dr} = -4\pi r_e n_{cl}(r) \qquad (38)$$

over r with the weight r in the interval $0 < r < r_{max}$. Because U_{cl} is finite at $r = 0$, we get for the average density

$$< n_{cl} > \equiv \frac{2}{r_{max}^2}\int_0^{r_{max}} n_{cl}(r) r dr = -\frac{1}{2\pi r_{max}}(\frac{dU_{cl}}{dr})_{r=r_{max}}. \qquad (39)$$

The total potential $U(r) = U_{cl} - gT\ln(b/r)$ is maximum at $r = r_{max}$. Therefore, $(\frac{dU_{cl}}{dr})_{r=r_{max}} = -gT/r_{max}$, and

$$< n_{cl} > = \frac{gT}{2\pi}(\frac{1}{r_{max}})^2. \qquad (40)$$

Substitution of g from Eq. (6) and $r_{max} = b(1-\Delta)$ gives

$$< n_{cl} > = n_0(\frac{1}{(1-\Delta)})^2. \qquad (41)$$

The average density is higher than that given by the condition of neutrality but the difference is small provided $\Delta << 1$.

It is worth noting that, without the potential barrier, primary photo-electrons with positive energy go above the potential well. They add to the average density of electrons but their space charge reduces the density of the cloud in such a way that the total average density is still given by the condition of neutrality.

Electrons reflected by the potential barrier hit the wall again increasing the power deposited to the wall. The power deposited by this mechanism depends on the yields,

$$\frac{dP}{ds} = \frac{c}{s_b}T_w[(\eta-1)\pi b^2 n_0\frac{N_{loss}}{N_{tot}} + YN_\gamma\frac{N_b}{L_d}][1-(1+V_{max}\frac{T}{T_w})e^{-V_{max}T/T_w}]. \qquad (42)$$

For the NLC DR this contribution is negligible, less than W/m.

Another effect of the secondary electrons trapped at the wall is the introduction of a small azimuthal asymmetry of the potential well for the beam particles. The dipole component of such perturbation may cause an orbit distortion and the quadrupole component leads to the asymmetric dependence of the tune on the beam current. The estimate shows, however, that these effects are small.

7 EFFECT OF THE FINITE BUNCH LENGTH

We assumed everywhere above that a bunch can be described as a point-like macro particle. The finite bunch length may substantially change the number of lost particles from the region near the beam. As it was mentioned in Section 2, the number of oscillations within the bunch length for such electrons is large. (It may be not true for the electrons far away from the beam because the frequency of oscillations decreases with amplitude). The field of a bunch at a given location around the ring varies slowly compared to the period of oscillations and can be considered as an adiabatic perturbation. As it is well known, the amplitude of oscillations in this case returns to the initial value when the perturbation is turned off. It means, that an electron may decrease the amplitude of oscillations while bunch is passing by, but retains the initial velocity and position after the bunch goes away. These arguments mean that the number of the high energy electrons hitting the wall and power deposition are smaller for the larger bunch length. On the other hand, low energy electrons in vicinity of the beam can live there for a long time what would mean larger density at the beam line. From this point of view, it is preferable to have short bunches but with a large bunch current to be in the regime where electrons go wall-to-wall in one pass.

One of implications of the finite bunch length is the betatron tune variation along the bunch. The kick from the head of a bunch causes motion of the e-cloud electrons toward the beam line and increases density of e-cloud in the tail of the bunch. The tune spread is of the order of the tune shift:

$$\Delta Q = \frac{2\pi r_e n_0 < R >^2}{\gamma Q}, \qquad (43)$$

where $< R >$ is the average machine radius. The tune spread for the NLC is large, $\Delta Q = 0.0207$ at $n_0 = 2.22\,10^7\ 1/cm^{-3}$. The interaction with the dense jets can change tune of the bunches in the head of the bunch train differently than for the rest of the bunches causing tune variation along the bunch train.

8 EFFECT ON THE WAKE FIELD

The wake field of the cloud with the average density n_0 can be estimated analytically [5, 6]. For a long bunch, the short-range wake per unit length has the form of a single mode

$$W_{bunch}(z) = W_m\frac{2n(\kappa)l_b}{N_b}(\frac{\Omega_B}{c})\sin(\mu\xi)e^{-\frac{\mu\xi}{2Q}}, \qquad (44)$$

where the e-cloud density is taken at $r_{min} = b\kappa$ to take into account that the density at the beam line is different than the average density, $\Omega_B/2\pi$ is the linear bunch frequency of oscillations,

$$(\frac{\Omega_B}{c})^2 = \frac{2N_b r_e}{\sigma_y(\sigma_x+\sigma_y)\sigma_z\sqrt{2\pi}}, \qquad (45)$$

$\xi = \Omega_B z/c$, $l_b = \sigma_z\sqrt{2\pi}$, and W_m, μ and Q are characteristics of the wake with weak dependence on the aspect ratio σ_y/σ_x and the beam pipe aperture. They were calculated in the reference [6]: $W_m = 1.2$, $\mu = 0.9$, and $Q = 5$.

The bunch shunt impedance R_s per turn

$$\frac{R_s^{bunch}}{Q} = 2\pi R \frac{Z_0}{4\pi} \frac{2n(\kappa) l_b}{N_b} W_m \tag{46}$$

is $R_s \simeq 2.3$ MOhm/m.

8.1 Transverse coupled bunch instability

For a single bunch stability, $\Omega_B/c = 53\ 1/cm$ and $W_{max} = 4\,10^3\ cm^{-2}$.

To consider the CB instability, the long-range (LR) wake has to be scaled from the short-range wake Eq. (44) replacing the bunch length by s_b and, secondly, using the average density n_0. The maximum value of the LR wake is:

$$W_{LR}(z) = W_m \frac{n_0 s_b}{N_b} (\frac{\Omega_{beam}}{c}), \tag{47}$$

where

$$(\frac{\Omega_{beam}}{c})^2 = \frac{2 N_b r_e}{s_b r_{min}^2}. \tag{48}$$

Here $r_{min} \simeq N_b r_e s_b / b$ estimates the range of distances $r_{min} < r < b$ where electrons survive after a bunch pass.

The LR shunt impedance R_s^{beam} per turn

$$\frac{R_s^{beam}}{Q} = 2\pi R \frac{Z_0}{4\pi} \frac{n_0 s_b}{N_b} W_m \tag{49}$$

is $R_s^{beam} \simeq 134$ MOhm/m for the NLC DR nominal parameters, by a factor of two larger than in the simulations [1].

The maximum growth rate of the transverse CB

$$\frac{1}{\tau} = \frac{I_{beam} R_s^{beam}}{(E/e)} \frac{c_0 \beta_y}{4\pi R} e^{-(\Omega/c)_{beam}\sigma_z)^2} \tag{50}$$

is $\tau = 0.01$ ms.

9 SUMMARY

At high currents, electrons may go wall-to-wall between bunches and electron cloud, in the usual sense, does not exist.

Thermalization of electrons, takes place at a moderate current within some distances from the beam. Even if the number of the linear oscillations per bunch is large, such electrons can be described by the Boltzmann distribution due to randomness of the electron motion.

The jets of primary and secondary electrons may have high density and explain the high energy tail in the distribution of electrons hitting the wall.

A simple model of the e-cloud formations allows us to reproduce main results obtained in simulations explaining the level of the density at saturation and it dependence on the $\gamma - e$ yield. The temperature of the distribution is defined by the condition of the energy equilibrium. The multipactoring does not change the temperature much but rather affects the distribution of electrons in the vicinity of the wall. That explains why the average density of the cloud is close to that given by the condition of neutrality. The final bunch length may change the power deposited to the wall and the density of electrons at the beam line. Interaction with the cloud can cause the tune variation along the bunch train. Transverse CB instability requires strong feedback.

Acknowledgments

I thank M. Pivi for providing us with the results of his simulations and T. Raubenheimer for initiating this study.

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11 APPENDIX: RESULTS FOR THE NLC MAIN DR

Parameter	Description	Value
E, (GeV)	beam energy	1.98
C, m	circumference	299.792
β_x, m	horizontal	3.64
β_y, m	vertical	7.06
ν_x, m	horizontal tune	27.261
ν_y, m	vertical tune	11.136
ν_s	synch.tune	0.0035
b, cm	beam pipe radius	1.6
B, T	dipole field	1.2
L_b, m	bend length	0.96
L_d, m	drift length	0.975
E_0, eV	peak of secondary electrons	5.0
E_T, eV	energy spread of secondary electrons	2.0
Y,	photo-electric yield (low/high SR)	$0.002/0.2$
η,	secondary emission yield,	1.45
s_b, m	bunch spacing	0.84
N_b 10^{10}	bunch population,	1.5
$e_{x,N}$, mm mrad	norm. x-emitt.	3.86
$e_{Y,N}$, mm mrad	norm y-emitt.	0.018
σ_z, mm	rms bunch length	3.6
$\delta\,10^{-3}$	relat. energy spread	0.909

Table 1: Global parameters for the NLC main damping ring

Parameter	Description	Simul.	Analytic	
I_{beam}, Amp	aver.beam current	0.86	0.86	
n_0, $10^{13}\ m^{-3}$	average density	3.0	2.2	
n_{eff}, $10^{13}\ m^{-3}$,	effective density		3.11	
f_{beam}, MHz	LR wake frequency		152	
f_{beam}/f_{rev}		100-200	152	
W_{LR} period in s_b		4	4.7	
$W^y_{bunch}\ m^{-2}$	short range W_{max}		$4.E7$	
$W^y_{beam}\ 10^6\ m^{-2}$	LR W_{max}	0.60	1.4	
R^y_s MOhm/m	SR shunt		2.3	
R^y_s MOhm/m	LR shunt		134	
τ_x ms	LR growth time		0.018	
τ_y ms	LR growth time	0.1	0.01	
$\Delta\nu_y$,	incoher. tune spread		0.021	
T	temperature, eV		92.2	
N_{loss}/N_{tot}	lost per bunch		0.32	
Number of passes to saturat.	(high/low) SR	8/25	7/18	
P_{wall} W/m	power to the wall	80.	87.	
κ	parameter		0.277	0.0694
g	parameter		0.5529	0.743
norm	parameter $\int xdxe^{-V}$		0.614	
V_{max}	potent. bump, eV		0.8	
Δ	parameter		0.067	

Table 2: Comparison of the calculations with simulations [1].