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hep-ph/0210021 CERN–TH/2002–262 IFUP–TH/2002–39

Predictions of the most minimal see-saw model

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We derive the most minimal see-saw texture from an extra-dimensional dynamics. If LMA is the solution to the solar neutrino problem, it predicts $\theta_{13} = 0.07 \pm 0.02$ and $m_{ee} = 2.5 \pm 0.7$ meV. Assuming thermal leptogenesis, the sign of the CP-phase measurable in neutrino oscillations, together with the sign of baryon asymmetry, determines the order of heavy neutrino masses. Unless heavy neutrinos are almost degenerate, successful leptogenesis fixes the lightest mass. Depending on the sign of the neutrino CP-phase, the supersymmetric version of the model with universal soft terms at high scale predicts BR($\mu \to e\gamma$) or BR($\tau \to \mu\gamma$), and gives a lower bound on the other process.

Introduction

The minimal see-saw [1] texture that allows to explain the solar and atmospheric neutrino anomalies in terms of oscillations contains two heavy singlet neutrinos $N_{\rm atm}$ and $N_{\rm sun}$ coupled as [2]

$$\begin{aligned}\mathscr{L} \;=\; & \mathscr{L}_{\rm SM} + \frac{M_{\rm atm}^2}{2} N_{\rm atm}^2 + \frac{M_{\rm sun}^2}{2} N_{\rm sun}^2 \qquad (1)\\ & + \lambda_{\rm sun} H N_{\rm sun} \, (s L_e + c\, e^{-i\phi/2} L_\mu + 0\, L_\tau) \\ & + \lambda_{\rm atm} H N_{\rm atm} (0 L_e + s_{\rm atm} L_\mu + c_{\rm atm} L_\tau),\end{aligned}$$

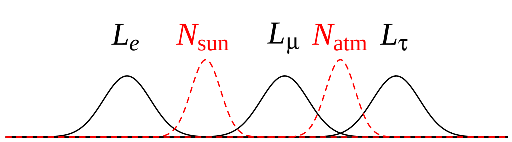
where M_i, λ_i, ϕ, s and $s_{\rm atm}$ are free parameters. We abbreviate $s_i = \sin\theta_i$, $c_i = \cos\theta_i$, $t_i = \tan\theta_i$. In this model the phase ϕ is the unique source of CP-violation in the lepton sector [3]. Possible connection [4, 5] between the sign of the observed baryon asymmetry of the universe and the CP-violation in neutrino oscillations via ϕ was the original motivation of the model [2, 5]. This texture is predictive if the zeros are replaced by sufficiently small numbers.

In this letter we motivate the model (1) and show in detail how it can be tested using low and high-energy observables. After deriving predictions for neutrino experiments, we clarify that the sign of the baryon asymmetry, together with the sign of the neutrino CP-violation in oscillations, determines a discrete ambiguity of the model: the order of $M_{\rm sun}$ and $M_{\rm atm}$. Successful leptogenesis [6] fixes the mass of the lightest heavy neutrino. In the supersymmetric version of the model, either BR($\mu \to e\gamma$) or BR($\tau \to \mu\gamma$) is predicted, depending on the sign of the neutrino CP-phase. The other process is a function of the heaviest singlet neutrino mass only, and has a lower bound.

At first sight the texture (1) looks quite artificial: e.g. we do not know how a U(1) flavour symmetry could justify it. However, (1) can be easily obtained from extra-dimensional models. Following [7] we consider a 5-dimensional fermion $\Psi(x)$ in presence of a domain wall $\varphi(x_5)$. The system is described by the action

$$S = \int d^5x\, \bar\Psi [i\partial\!\!\!/ + \lambda\varphi(x_5) - m]\Psi.$$

The Kaluza-Klein spectrum of Ψ contains a massless chiral mode localized around $x_5 = x_5^*$, where $\lambda\varphi(x_5^*) = m$. When $\varphi(x_5)$ can be approximated with a linear function, the chiral zero mode has a Gaussian profile in the extra dimension with λ-dependent width. Assuming that the Higgs H is not localized, small Yukawa couplings between N and L are naturally given by a small overlap between their wave functions as depicted in the figure below



This setup naturally generates the desired matrix of

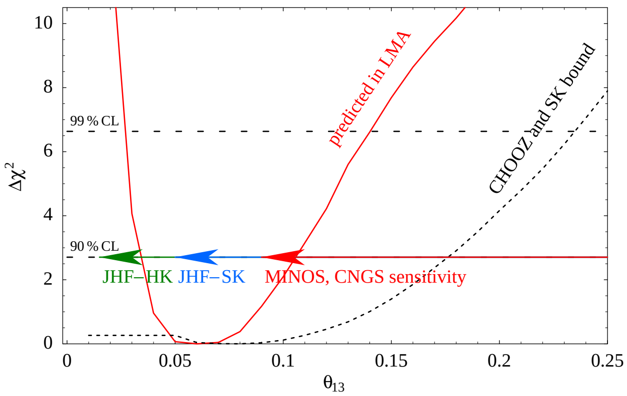


Figure 1: *Prediction for θ_{13} within LMA, and present bound from CHOOZ and SK. The arrows indicate the expected sensitivity of future experiments.*

Yukawa couplings

$$\begin{array}{c} \\ N_{\rm sun} \\ N_{\rm atm} \end{array} \begin{array}{c} \begin{array}{ccc} L_e & L_\mu & L_\tau \end{array} \\ \left(\begin{array}{ccc} \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon^{11.}) \\ \mathcal{O}(\epsilon^{14.}) & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) \end{array}\right) \end{array}$$

(where ϵ is a free parameter) and suppresses $N_{\rm sun}N_{\rm atm}$ mixing mass terms. While we do not gain any new insight proceeding along this route, we are motivated to study the implications of the model.

Neutrinos

The model (1) predicts the following Majorana mass matrix for the light neutrinos:

$$m_\nu = m_{\rm atm} \begin{pmatrix} \epsilon s^2 & \epsilon s c e^{-i\phi/2} & 0 \\ \epsilon s c e^{-i\phi/2} & s^2_{\rm atm} + \epsilon c^2 e^{-i\phi} & s_{\rm atm} c_{\rm atm} \\ 0 & s_{\rm atm} c_{\rm atm} & c^2_{\rm atm} \end{pmatrix},$$

where

$$m_{\rm atm} = \frac{\lambda^2_{\rm atm} v}{M_{\rm atm}}, \qquad \epsilon = \frac{\lambda^2_{\rm sun}/M_{\rm sun}}{\lambda^2_{\rm atm}/M_{\rm atm}}.$$

$N_{\rm atm}$ plays the rôle of 'dominant right-handed neutrino' [8]. Neutrinos have a hierarchical mass spectrum and the lightest neutrino is massless[1]. At leading order in ϵ, the oscillation parameters are

$$\Delta m^2_{\rm atm} = m^2_{\rm atm} > 0, \qquad \Delta m^2_{\rm sun} = R \Delta m^2_{\rm atm},$$

with $R = \epsilon^2 (s^2 + c^2 c^2_{\rm atm})^2$. We define $m_{\rm sun} \equiv \sqrt{\Delta m^2_{\rm sun}}$. The mixing angles in the standard notation are

$$\theta_{23} = \theta_{\rm atm}, \qquad \theta_{13} = \epsilon s c s_{\rm atm}, \qquad \tan\theta_{12} = \frac{s}{c c_{\rm atm}}.$$

The neutrino mixing matrix V relating the mass eigenstates ν_i to the flavour eigenstates, $\nu_\ell = V_{\ell i}\nu_i$, is

$$\begin{aligned} V &= \mathrm{diag}\,(1, e^{-i\frac{\phi}{2}}, e^{-i\frac{\phi}{2}}) \cdot R_{23}(\theta_{23}) \cdot \mathrm{diag}\,(1, e^{i\phi}, 1) \\ &\quad \cdot R_{13}(\theta_{13}) \cdot R_{12}(\theta_{12}) \cdot \mathrm{diag}\,(1, 1, e^{i\frac{\phi}{2}}), \end{aligned}$$

where $R_{ij}(\theta_{ij})$ represents a rotation by θ_{ij} in the ij plane. The first phase matrix in V is unphysical and can be absorbed into the phases of (L_e, L_μ, L_τ). The last phase matrix contains a practically unmeasurable Majorana phase. The phase matrix in the middle determines that the CP-violating phase in oscillations (observable in the planned experiments if LMA is the solution to the solar neutrino problem) is exactly the phase ϕ in (1). A 'positive' phase, $0 < \phi < \pi$ induces $P(\nu_e \to \nu_\mu) > P(\nu_\mu \to \nu_e) = P(\bar\nu_e \to \bar\nu_\mu)$ in vacuum oscillations with baseline $L < 2\pi E_\nu/\Delta m^2_{\rm sun}$.

Therefore this model *predicts* (see also [2])

$$\theta_{13} \simeq \frac{\sqrt{R}}{2} \sin 2\theta_{12} \tan\theta_{23}, \quad m_{ee} = m_{\rm sun} \sin^2\theta_{12}, \quad (2)$$

where m_{ee} is the ee element of the neutrino mixing matrix to be measured in neutrino-less double-beta ($0\nu2\beta$) decay experiments. Present atmospheric neutrino data indicate $\Delta m^2_{\rm atm} \approx 3\ 10^{-3}\,{\rm eV}^2$ and $t^2_{23} \approx 1$ [9]. At the moment the biggest uncertainty is associated with $\Delta m^2_{\rm sun}$ [10, 11]. If LMA is the true solution to the solar neutrino anomaly (as present data indicate), then $\Delta m^2_{\rm sun} \approx 6\ 10^{-5}\,{\rm eV}^2$ and $\tan^2\theta_{12} \approx 0.4$, while $\Delta m^2_{\rm sun} \approx 8\ 10^{-8}\,{\rm eV}^2$ and $\tan^2\theta_{12} \approx 0.6$ for the alternative LOW solution. Lower values of $\Delta m^2_{\rm sun}$ are possible in fine-tuned (Q)VO solutions. If the solar anomaly is due to neutrino oscillations, KamLand or Borexino experiments should settle the issue [12]. Combining present atmospheric and solar data[2], we obtain for the

[1] An alternative minimal texture, where $N_{\rm sun}$ couples to L_τ rather than to L_μ, is equally acceptable. Its predictions concerning neutrinos can be obtained exchanging in the equations below $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$. At the moment atmospheric data do not distinguish between them.

If the singlet neutrinos have a pseudo-Dirac mass term $M\,N_{\rm sun}N_{\rm atm}$, rather than the masses of eq. (1), one gets light neutrinos with inverted mass hierarchy. The resulting texture predicts $\theta_{12} \approx \pi/4$ which is disfavoured by the present data.

[2] A function $\chi^2(p)$, extracted from a global up-to-date fit of solar and atmospheric data, contains the present information on the parameters $p = \{\theta_{\rm sun}, \theta_{\rm atm}, \Delta m^2_{\rm sun}, \Delta m^2_{\rm atm}\}$. In Gaussian approximation, the present information on a parameter $a = f(p)$ is then extracted computing $\chi^2(a) = \min_{p: f(p)=a} \chi^2(p)$. $\chi^2(a)$ is 'distributed as χ^2 with 1 degree of freedom'. In Bayesian inference this means that the probability of different a values is $\propto e^{-\chi^2(a)/2}$.

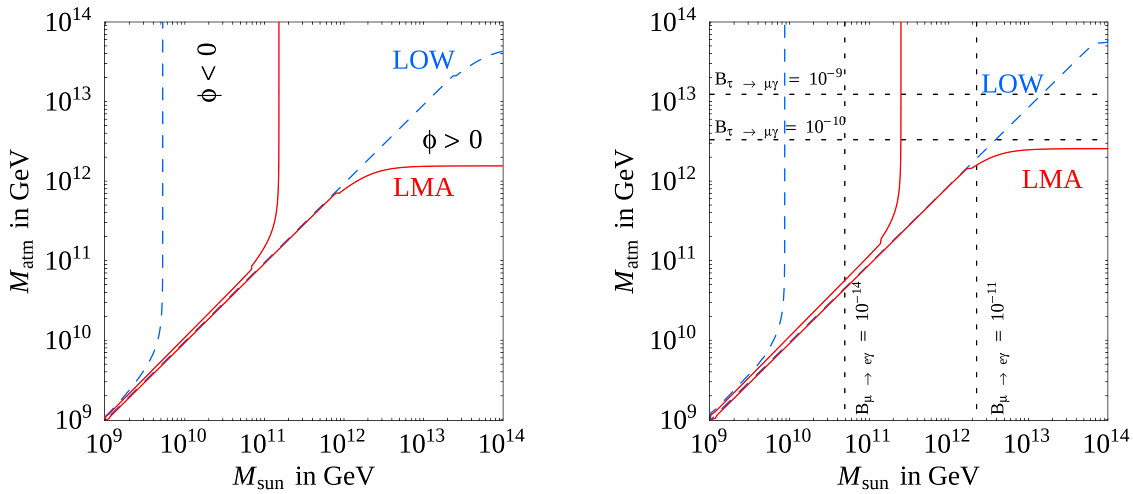


Figure 2: *Heavy neutrino masses* $M_{\rm sun}$ *and* $M_{\rm atm}$ *which imply successful leptogenesis for* $|\sin\phi| = 1$ *and for LMA (red continuous line) and LOW (blue dashed line) best-fit oscillations. The left (right) plot refers to the non-supersymmetric (supersymmetric) minimal see-saw model. In the supersymmetric case we also show the contour lines of* $\mathrm{BR}(\mu \to e\gamma)$ *and* $\mathrm{BR}(\tau \to \mu\gamma)$ *assuming* $m_0 = 100\,\mathrm{GeV}$, $M_{1/2} = 150\,\mathrm{GeV}$, $A_0 = 0$, $\tan\beta = 10$ *and LMA. In the LOW case* $\mathrm{BR}(\mu \to e\gamma)$ *is a factor* $1.6\ 10^{-3}$ *lower.*

LMA solution the predictions

$$\theta_{13} = 0.07 \pm 0.02, \qquad m_{ee} = 2.5 \pm 0.7 \text{ meV}. \qquad (3)$$

The predicted value of m_{ee} is below the sensitivity of the planned next-generation $0\nu2\beta$ experiments [13]. Therefore we focus on studying θ_{13}.

Fig. 1 shows the $\Delta\chi^2$ distribution for the predicted θ_{13}, compared with the present bound from CHOOZ [14] and SK [9] ($\theta_{13} < 10^\circ$ at 90% CL). If LMA is the true solution, KamLand will be able to measure the solar oscillation parameters with few % error [12]. Long baseline experiments will measure the atmospheric parameters with few % error [15], allowing to predict θ_{13} with $\sim$ 10% error. First-generation long-baseline experiments will be sensitive to $\theta_{13} \gtrsim 0.08$ [15]. The whole LMA predicted range for θ_{13} can be covered at second-generation experiments, such as JHF [15]. The LOW solution predicts $\theta_{13} \approx 0.002$, just below the sensitivity of the most optimistic neutrino factory projects [15].

So far we have discussed the predictions for the light-neutrino mass matrix m_ν. Within this model, oscillation experiments have already fixed it, except for the CP-violating phase. Future experiments will test the model. To get information on the heavy neutrino masses in (1), and to test the high-energy part of the model, we need an additional input from leptogenesis.

Leptogenesis

The decays of the lightest right-handed neutrino,

$$N_1 = N_{\rm sun} \quad \text{or} \quad N_{\rm atm},$$

generate a lepton asymmetry only in L_μ (see (1)). The generated lepton asymmetry is then converted into a baryon asymmetry by sphalerons [16]. The baryon-to-entropy ratio in the non-supersymmetric model is given by

$$\frac{n_B}{s} = (0.85 \pm 0.15) \times 10^{-10} = -\frac{3\epsilon_1\eta}{2183}, \qquad (4)$$

where ϵ_1 is the CP-asymmetry in N_1 decays and $\eta < 1$ is an efficiency factor, determined by solving the relevant set of Boltzmann equations [17].

In Fig. 2a we show the iso-curves of the predicted n_B/s in the $(M_{\rm sun}, M_{\rm atm})$ plane assuming the best-fit values of oscillation parameters. Unless the heavy neutrinos are extremely degenerate, which we regard as a fine tuning, Fig. 2 implies that the N_1 Yukawa couplings are sufficiently large that N_1 quickly reaches the thermal abundance and washes out the lepton asymmetry eventually generated by the heavier singlet neutrino.

The main features of leptogenesis in this model can be understood by simple analytic approximations as follows.

- If $M_{\rm sun} \ll M_{\rm atm}$,

$$\epsilon_1 = \frac{3}{16\pi}\frac{m_{\rm atm}M_{\rm sun}}{v^2}\frac{s_{23}^2}{1+c_{23}^2 t_{12}^2}\sin\phi.$$

For $M_1 \ll 10^{14}\,\text{GeV}$ only $\Delta L = 1$ washout scatterings contribute to the efficiency factor, and η is approximately given by [17]

$$\eta \approx 10^{-4}\,\text{eV}/\tilde{m},$$

where the effective mass $\tilde{m}$ is given only in terms of the L_μ interactions in (1). The texture predicts at the best-fit LMA point

$$\tilde{m} = m_{\rm sun}\frac{c_{12}^2}{c_{23}^2} \approx 0.01\,\text{eV}.$$

Thus $\eta \sim 0.01$. The observed baryon asymmetry is obtained for $\phi < 0$ and $M_{\rm sun} \approx 10^{11}\,\text{GeV}/|\sin\phi|$ independently of $M_{\rm atm}$.

- If $M_{\rm atm} \ll M_{\rm sun}$,

$$\begin{aligned}\epsilon_1 &= -\frac{3}{16\pi}\frac{m_{\rm sun}M_{\rm atm}}{v^2}\frac{t_{23}^2}{1+t_{12}^2}\sin\phi,\\ \tilde{m} &= s_{\rm atm}^2 m_{\rm atm} \approx 0.03\,\text{eV},\end{aligned}$$

thus $\eta \sim 0.003$. The observed baryon asymmetry is obtained for $\phi > 0$ and $M_{\rm atm} \approx 10^{12}\,\text{GeV}/|\sin\phi|$ independently of $M_{\rm sun}$.

- If $M_{\rm atm} \approx M_{\rm sun}$ the CP-asymmetry is enhanced [18] by $1/|M_{\rm atm} - M_{\rm sun}|$ and reaches a maximum $\epsilon \sim 1$ when the mass difference is comparable to the decay widths. The observed baryon asymmetry can be obtained for a large range of relatively low heavy neutrino masses. Its sign still depends on which singlet neutrino is heavier, and it does not fix the sign of CP-violation in oscillations.

To summarize, Fig. 2 implies that we need to know both the sign of ϕ and the sign of the baryon asymmetry to determine the discrete ambiguity of the model: the mass ordering of the heavy neutrinos. For hierarchical heavy neutrinos leptogenesis determines the mass of the lightest one, but does not test the model.

Supersymmetry and lepton flavour violation

If nature is supersymmetric, it could be possible to fix and test the high-energy part of the model. For leptogenesis and neutrino masses the presence of supersymmetry changes only few $\mathcal{O}(1)$ coefficients: (i) the vacuum expectation value v is replaced by $v\sin\beta$; (ii) the CP-asymmetry ϵ_1 becomes 2 times larger when $M_{\rm sun}$ and $M_{\rm atm}$ are hierarchical [19] (iii) numerically eq. (4) remains practically unchanged since the number of model degrees of freedom is about doubled; (iv) washout becomes more efficient [20]:

$$\eta \approx 0.3\ 10^{-4}\,\text{eV}/\tilde{m}.$$

The final result is shown in Fig. 2b which differs from Fig. 2a by a small factor. We observe a potential conflict between obtaining a successful thermal leptogenesis and avoiding overproduction of gravitinos [21] in this model. If gravitinos do exists, they either must be heavier than $m_{\tilde{G}} > 10\,\text{TeV}$ in order to allow the mass-scales of Fig. 2b, or, for $m_{\tilde{G}} \sim 1\,\text{TeV}$, one must have $M_1 < 10^8\,\text{GeV}$. The last condition is satisfied only when $M_{\rm sun}$ and $M_{\rm atm}$ are almost degenerate.

In supersymmetric extensions of the see-saw model, the renormalization effects due to the neutrino Yukawa couplings imprint lepton flavour violation in the slepton masses [22]. Assuming that soft terms are universal at the unification scale (a hypothesis that collider experiments can partly test), in a generic see-saw model

$$\mathscr{W} = \mathscr{W}_{\rm MSSM} + \frac{M_{ij}}{2}N_i N_j + \lambda_N^{ij}\,N^i L^j H_{\rm u},$$

the correction to the 3×3 mass matrix of left-handed sleptons is given by

$$\boldsymbol{m}_{\tilde{L}}^2 = m_0^2 - \frac{1}{(16\pi)^2}(3m_0^2 + A_0^2)\boldsymbol{\lambda}_N^\dagger \ln(\frac{M_{\rm GUT}^2}{\boldsymbol{M}\boldsymbol{M}^\dagger})\boldsymbol{\lambda}_N + \cdots.$$

In general see-saw models the presence of too many uncontrollable neutrino parameters does not allow to make real predictions on lepton flavour violation (LFV). The present model allows us to compute the $\mu \to e\gamma$ and $\tau \to \mu\gamma$ rates [23] (and related LFV processes [24]) in terms of the two high-energy parameters $M_{\rm sun}$ and $M_{\rm atm}$. Assuming that thermal leptogenesis generates the observed baryon asymmetry, we get predictions more sharp than what suggested by a naïve counting of the number of free parameters. Barring the case of almost degenerate singlet neutrinos $M_{\rm sun} \approx M_{\rm atm}$ (where only the ratio

$$\begin{aligned}\frac{\text{BR}\,(\mu\to e\gamma)}{\text{BR}\,(\tau\to\mu\gamma)} &= \frac{m_\mu^5\tau_\mu}{m_\tau^5\tau_\tau}\frac{\Delta m_{\rm sol}^2}{\Delta m_{\rm atm}^2}\frac{\sin^2 2\theta_{12}}{\sin^2 2\theta_{23}\cos^2\theta_{23}}\\ &\approx 0.2\ (\text{LMA}),\ \ 3\ 10^{-4}\ (\text{LOW})\qquad (5)\end{aligned}$$

can be predicted), leptogenesis fixes the mass of the lightest singlet neutrino allowing to compute its Yukawa couplings, and consequently, the LFV rates that it induces. The predictions depend on the sign of the CP-violating phase ϕ measurable in oscillations.

- If $\phi < 0$, N_1 is $N_{\rm sun}$, BR($\mu \to e\gamma$) can be predicted while BR($\tau \to \mu\gamma$) remains a function of a single unknown parameter, $M_{\rm atm}$. Since $M_{\rm atm} > M_{\rm sun}$ the model also predicts a *lower bound* on BR($\tau \to \mu\gamma$).
- If instead $\phi > 0$, N_1 is $N_{\rm atm}$, BR($\tau \to \mu\gamma$) can be predicted, together with a lower bound on BR($\mu \to e\gamma$). The latter is a function of the unknown $M_{\rm sun} > M_{\rm atm}$.

As usual, the predicted LFV rates depend on sparticle masses which can be measured at colliders. Taking into account naturalness considerations and experimental bounds, we give our numerical examples for $m_0 = 100\,{\rm GeV}$, $M_{1/2} = 150\,{\rm GeV}$, $A_0 = 0$ and $\tan\beta = 10$. In Fig. 2b we show the iso-curves of the LFV processes for this input, assuming the best-fit LMA oscillation parameters. The branching ratios are calculated by solving numerically the renormalization group equations and using exact formulæ in [23]. Both BR($\mu \to e\gamma$) and BR($\tau \to \mu\gamma$) can be in the reach of future experiments [25]. Their behavior is approximately given by

$$\begin{aligned} {\rm BR}(\mu \to e\gamma) &\approx 2.7\, r\ 10^{-12}\left(\frac{M_{\rm sun}}{10^{12}\,{\rm GeV}}\right)^2, \\ {\rm BR}(\tau \to \mu\gamma) &\approx 1.5\, r\ 10^{-11}\left(\frac{M_{\rm atm}}{10^{12}\,{\rm GeV}}\right)^2, \end{aligned}$$

where the logarithmic dependence on the heavy masses is neglected and we have introduced an approximate scaling factor

$$r \approx \left(\frac{\tan\beta}{10}\right)^2 \left(\frac{150\,{\rm GeV}}{m_{\rm SUSY}}\right)^4,$$

($r = 1$ at our reference point) in order to show the dominant dependence on supersymmetric model parameters. In particular, the branching ratios decouple as $1/m_{\rm SUSY}^4$ if sparticles are heavy. When sparticles masses will be measured, it will be possible to present more precise predictions.

For hierarchical heavy neutrinos, for $|\sin\phi| = 1$, and for the LMA best-fit oscillation parameters, the predictions are

$$\begin{matrix} {\rm BR}(\mu \to e\gamma) \approx 2\, r\ 10^{-13} \\ {\rm BR}(\tau \to \mu\gamma) \gtrsim 3\, r\ 10^{-12} \end{matrix} \qquad \text{for} \qquad \phi < 0,$$

and

$$\begin{matrix} {\rm BR}(\tau \to \mu\gamma) \approx 7\, r\ 10^{-11} \\ {\rm BR}(\mu \to e\gamma) \gtrsim r\ 10^{-11} \end{matrix} \qquad \text{for} \qquad \phi > 0.$$

These results imply that, if also $\tau \to \mu\gamma$ is observed for $\phi < 0$, or if $\mu \to e\gamma$ is observed for $\phi > 0$, all the model parameters in (1) can be entirely determined.

In this model the electron and muon electric dipole moments [26] and $\tau \to e\gamma$ are generated at a negligible level.

Conclusions

Unlike the general see-saw model [27], the most minimal see-saw model (1) allows to determine the low energy neutrino mass matrix entirely from neutrino oscillation experiments. If LMA is the solution to the solar neutrino anomaly, the model predicts eq.s (2), (3). The sign of the oscillation CP-phase, together with the sign of the baryon asymmetry, fixes the order of the two heavy neutrino masses. Unless they are almost degenerate, successful thermal leptogenesis determines the lightest of them. The supersymmetric version of the model predicts either BR($\mu \to e\gamma$) or BR($\tau \to \mu\gamma$), depending on the sign of ϕ. The other process remains a function of the heavier neutrino mass only, and has a lower bound on its branching ratio. If the heavy neutrinos are almost degenerate, the model predicts only the ratio BR($\mu \to e\gamma$)/BR($\tau \to \mu\gamma$) according to (5). Observation of the LFV processes allows in principle to fix all the model parameters in (1), and to test its high-energy part.

The KamLand experiment should soon tell if LMA oscillations generate the solar neutrino anomaly. We will update the hep-ph version of this paper when results will be available.

Acknowledgements. We thank J. Ellis and T. Yanagida for discussions. This work is partially supported by EU TMR contract No. HPMF-CT-2000-00460 and by ESF grant No. 5135.

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