# Impact of $B$ physics on model building and vice versa: an example 

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#### Abstract

We motivate that the start-up of the $B$ factories has opened a new precision flavour physics era, with an important effect on model building. Using as an example a left-right model with spontaneous CP violation, we will show how the inclusion of the new experimental data on $B$ physics observables, together with the old observables coming from kaon physics, has significantly widened our capacity to strongly constrain the parameter space up to the point to exclude models. On the contrary, using certain hypotheses, mainly concerning isospin, we discuss how theory may help us to 'test' the data on charged, neutral and mixed $B \rightarrow \pi K$ decays once experimental errors will be reduced.


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In the post-LEP era, $B$ physics, following the example of the fantastic accuracy achieved in the precision tests of the Standard Model by the LEP experiments, could bring us the possibility to open a new era of precision: the precision flavour physics era. A huge experimental and theoretical effort will be necessary to achieve this goal.

On the experimental side, the start-up of $B$ factories is providing us with a cascade of new experimental data on $B$ meson decays: $B_{d} \rightarrow J / \psi K_{s}, B \rightarrow \pi \pi, B \rightarrow \pi K$, etc. Moreover, forthcoming hadronic machines (LHCb [1] and BTEV [2]) will collect data on $B_{s}$ decay modes such as $B_{s} \rightarrow J / \psi \phi$ and $B_{s} \rightarrow K K$, which may open new avenues in the search for new physics. On the theoretical side, we have two main tools to help us in this search: CP violation in $B$ and $K$ physics and FCNC rare decays, both inclusive and exclusive. Their study will offer precise and very valuable information on new interactions associated to the flavour sector of the fundamental theory that lies beyond the Standard Model.
$B$ physics provides us with a set of new CP-conserving and CP-violating observables in $B_{d}$ and $B_{s}$ meson decays (see reviews $[3,4]$ ): $\Delta m_{B q}, \Delta \Gamma_{q}$, and sides and angles $(\alpha, \beta, \gamma)$ of the Unitarity Triangle (UT). Some of these observables are related to the matrix element:

$$
\begin{equation*}
\left\langle M^{0}\right| \mathcal{H}_{\mathrm{eff}}^{|\Delta| F=2}\left|\bar{M}^{0}\right\rangle=2 m_{M}\left(M_{12}^{\mathrm{SM}}+M_{12}^{\mathrm{NP}}+M_{12}^{\mathrm{LD}}\right) \quad F=S(\text { kaons }), B(B \text { mesons }) \tag{1}
\end{equation*}
$$

where $m_{M}$ stands for the corresponding kaon or $B$ meson mass; $M_{12}^{S M}$ is the SM contribution, $M_{12}^{\mathrm{NP}}$ the new physics contribution and $M_{12}^{\mathrm{LD}}$ the $(\Delta F=1)^{2}$ contributions. Mass differences in the $B_{d, s}$ system are obtained from (1):

$$
\begin{equation*}
\Delta m_{B q}=2\left|M_{12}^{(q)}\right| \quad q=d, s \tag{2}
\end{equation*}
$$

In order to disentangle new physics effects in the mass difference [5], it is important to recall that this observable is afflicted by hadronic uncertainties coming from $f_{B q}^{2} B_{q}$. Concerning the weak mixing phase

$$
\begin{equation*}
\phi_{M}^{B_{q}}=\arg M_{12}^{B_{q}}, \tag{3}
\end{equation*}
$$

it measures the angle $2 \beta$ in the $B_{d}$ system and a very small angle $\delta \gamma$ in the $B_{s}$ system. The sine of $2 \beta$ can be obtained from the CP asymmetry in $B_{d}^{0} \rightarrow J / \psi K_{S}^{0}[6]$ or the sides of the UT, while the cosine, in particular its sign, is a very interesting future observable. The corresponding angle in the $B_{s}$ case is negligibly small in the SM, i.e. the corresponding CP asymmetries are small, so it is an excellent place to look for new physics.

Regarding the other two angles, $\gamma$ can be obtained from non-leptonic B decays such as $B \rightarrow \pi K[7]-[15]$ and the sides of the UT and, finally, $\alpha$ it is traditionally analysed using isospin in $B \rightarrow \pi \pi[16]$ or using $\mathrm{SU}(3)$ plus dynamical assumptions and factorization [17], although an interesting alternative [11] to control the hadronic penguin parameters is to use the decay modes $B_{d} \rightarrow \pi \pi$ and $B_{s} \rightarrow K K$ (or $B_{d} \rightarrow \pi K$ ) to extract, instead, $\gamma$ and together with $\beta$ obtain $\alpha[10,11,18]$.

The width difference $\Delta \Gamma_{q}$ in the $B_{d}$ system is expected to be too small to be measurable, but it could be non-negligible for the $B_{s}$ meson. Unfortunately, in the presence of new physics it can only decrease [19].

The second tool in the search for new physics is the analysis of rare $B$ meson decays (see [20] for a review). These are processes that are suppressed at tree level, i.e. new physics can compete on the same footing as the SM. They allow us to put constraints on the parameter space of models beyond the SM. One of the more important rare decay is the inclusive $B \rightarrow X_{s} \gamma$ that provides information on the magnitude of the Wilson coefficient $C_{7}$ and it has been evaluated very precisely in the SM [21] and supersymmetry [22] ${ }^{1}$. Other important rare modes are the inclusive and exclusive semileptonic decay modes driven by the quark transition $b \rightarrow s l^{+} l^{-}$, whose forward-backward asymmetry [25] ${ }^{2}$ provides information on the sign of $C_{7}$ and also on $C_{9}$ and $C_{10}$.

Here we will focus mainly on the first tool. In sections 1 to 3 , we will show an example of the impact that new experimental data coming from $B$ physics are having on model building. We will see how a specific type of left-right model with spontaneous CP violation [27] gets strongly constrained and could even be excluded, thanks to the new observables coming from $B$ physics and their combined analysis with the old $K$ physics observables. In section 4 we will try to argue the other way around and we will use theory together with certain reasonable assumptions to 'test' data on $B \rightarrow \pi K$ decays.

## 1 Description of the model: left-right model with spontaneous CP

This model is based on the gauge group $S U(2)_{L} \times S U(2)_{R} \times U(1)$ with the feature that CP violation is spontaneous originating from a phase in the vacuum expectation values. There is a lot of literature on left-right models [27]-[33] with and without spontaneous CP violation.

The Spontaneously Broken Left-Right Model (SB-LR) has the interesting properties of being fully testable and distinct from the SM. The particle content of this model, concerning the quark, gauge and scalar sectors, consist of left and right quark doublets:

$$
q_{L i}=\binom{U_{i}}{D_{i}}_{L} \sim(2,1,1 / 6) \quad q_{R i}=\binom{U_{i}}{D_{i}}_{R} \sim(1,2,1 / 6)
$$

that acquire their masses via a spontaneous breakdown of the symmetry such that the bidoublet $\phi$ acquires a vev

$$
\Phi=\left(\begin{array}{cc}
\phi_{0}^{1} & \phi_{1}^{+} \\
\phi_{2}^{-} & \phi_{0}^{2}
\end{array}\right) \sim(2, \overline{2}, 0) \quad \rightarrow \quad\langle\Phi\rangle=\frac{1}{\sqrt{2}}\left(\begin{array}{cc}
v & 0 \\
0 & w
\end{array}\right) .
$$

In order to complete the breakdown of the symmetry group to $U(1)_{e m}$ respecting the LR

[^0]symmetry, two other triplets (or doublets) are required:
\[

\chi_{L}=\left($$
\begin{array}{c}
\chi_{L}^{++} \\
\chi_{L}^{+} \\
\chi_{L}^{0}
\end{array}
$$\right) \sim(3,1,2), \quad \chi_{R}=\left($$
\begin{array}{c}
\chi_{R}^{++} \\
\chi_{R}^{+} \\
\chi_{R}^{0}
\end{array}
$$\right) \sim(1,3,2) \quad \rightarrow \quad\left\langle\chi_{L, R}\right\rangle=\frac{1}{\sqrt{2}}\left($$
\begin{array}{c}
0 \\
0 \\
v_{L, R}
\end{array}
$$\right)
\]

The scalar sector then contains a SM-like neutral scalar, a single charged scalar, and two neutral scalars, with flavour-changing couplings to quarks.

The relevant parameters for the rest of the discussion coming from the scalar sector will be the ratio of vev of the bidoublet $r=|v / w|$ and the phase $\alpha=\arg (v w)$. However, we will use a more convenient combination of them called $\beta$ and $\beta^{\prime 3}$ defined by [31]:

$$
\beta=\arctan \frac{2 r \sin \alpha}{1-r^{2}} \quad e^{i \beta^{\prime}}=\frac{1-r^{2} e^{-2 i \alpha}}{\left|1-r^{2} e^{-2 i \alpha}\right|} .
$$

Other parameters are the mass of the charged and flavour-changing scalars. However, since they are both taken to be heavy, the neutral FC scalars cannot mix with the light scalar and must be nearly degenerate [32], we take a common mass parameter for the charged and neutral FC scalars $M_{H}$.

Concerning the gauge sector, in addition to the usual charged $W_{L}^{ \pm}$and neutral $Z_{L}$ gauge bosons we have an extra charged $W_{R}^{ \pm}$and neutral $Z_{R}$ gauge boson. Only the charged gauge bosons will be relevant to our discussion. The spontaneous breakdown of the symmetry generates also the mass of the charged gauge bosons:

$$
M_{W^{ \pm}}^{2}=\left(\begin{array}{cc}
\frac{g_{L}^{2}}{4}\left(2 v_{L}^{2}+|v|^{2}+|w|^{2}\right) & -g_{L} g_{R} v^{*} w / 2 \\
-g_{L} g_{R} v w^{*} / 2 & \frac{g_{R}^{2}}{4}\left(2 v_{R}^{2}+|v|^{2}+|w|^{2}\right)
\end{array}\right) \equiv\left(\begin{array}{cc}
M_{L}^{2} & M_{L R}^{2} e^{-i \lambda} \\
M_{L R}^{2} e^{i \lambda} & M_{R}^{2}
\end{array}\right)
$$

The mixing between the two physical charged $W$ bosons is

$$
\binom{W_{1}^{+}}{W_{2}^{+}}=\left[\begin{array}{cc}
\cos \zeta & -e^{i \lambda} \sin \zeta \\
e^{-i \lambda} \sin \zeta & \cos \zeta
\end{array}\right]\binom{W_{L}^{+}}{W_{R}^{+}}
$$

where the $W_{L}-W_{R}$ mixing angle is defined as

$$
\tan 2 \zeta=-\frac{2 M_{L R}^{2}}{M_{R}^{2}-M_{L}^{2}}
$$

The charged current reads (with $g \equiv g_{L} \equiv g_{R}$ and without displaying unphysical scalars and charged Higgs contributions):

$$
\begin{aligned}
\mathcal{L}_{c c}= & -\frac{g}{\sqrt{2}} \bar{U}_{i}\left[\cos \zeta\left(V_{L}\right)_{i j} \gamma^{\mu} P_{L}-e^{-i \lambda} \sin \zeta\left(V_{R}\right)_{i j} \gamma^{\mu} P_{R}\right] D_{j} W_{1 \mu}^{+} \\
& -\frac{g}{\sqrt{2}} \bar{U}_{i}\left[e^{i \lambda} \sin \zeta\left(V_{L}\right)_{i j} \gamma^{\mu} P_{L}+\cos \zeta\left(V_{R}\right)_{i j} \gamma^{\mu} P_{R}\right] D_{j} W_{2 \mu}^{+}
\end{aligned}
$$

giving rise to two CKM matrices one left $\left(V_{L}\right)$ and one right $\left(V_{R}\right)$. The phase structure of these matrices will be explained in the next section.

[^1]
## 2 Quark mixing, phases and parameters

The combination of P invariance together with spontaneous CP violation imposes strong restrictions on the coupling matrices of the Yukawa interaction part of the lagrangian,

$$
\begin{equation*}
-\mathcal{L}_{Y}=\Gamma_{i j} \bar{q}_{L i} \Phi q_{R j}+\Delta_{i j} \bar{q}_{L i} \widetilde{\Phi} q_{R j}+\text { h.c. } \tag{4}
\end{equation*}
$$

Both coupling matrices $\Delta$ and $\Gamma$ are taken to be real and symmetric ${ }^{4}$. This is crucial, because it means that we can diagonalize the mass matrices by only two unitary matrices

$$
\begin{equation*}
M^{(u)}=U D^{(u)} U^{T}, \quad M^{(d)}=V D^{(d)} V^{T}, \tag{5}
\end{equation*}
$$

where $D^{(u, d)}$ are diagonal mass matrices. We have in this model two CKM matrices; in the basis where the coupling matrices are symmetric, these are related to one another by the following relation:

$$
K \equiv K_{L}=U^{\dagger} V=K_{R}^{*}
$$

Those models not fulfilling this constraint are not affected by the present analysis [33]. These CKM matrices can be written in a more standard form: $K_{L}$ in standard CKM form with a unique phase $\delta$ and $K_{R}$ containing the remaining 5 new phases: $\alpha_{1}, \alpha_{2}, \alpha_{3}, \epsilon_{1}, \epsilon_{2}$.

The important point to recall is that all phases (including $\delta$ ) can be expressed as an exact function of $m_{u, d}, r, \alpha, V_{i j}$.

Finally, we collect here all new parameters of the model that we have introduced up to now:

- $M_{2} \sim O(1 \mathrm{TeV})$, mass of right-handed gauge boson;
- $\zeta$, the mixing angle between $W_{R}$ and $W_{L}, \zeta \geq 0$;
- $g_{R}$, the coupling of $W_{R}$. We set here $g_{R}=g_{L}$.
- $0 \leq r \leq 1$ and $0 \leq \alpha \leq \pi$ (or better $\beta$ ), parametrizing the spontaneous breakdown of CP. We will indeed work in the so-called "natural" region for $r \sim O\left(m_{b} / m_{t}\right) \sim 0.02$ that explains the observed smallness of the CKM mixing angles. This value implies $\zeta \sim O\left(10^{-4}\right)$.
- degenerate extra Higgs masses $M_{H} \sim O(10 \mathrm{TeV})$, and we assume $M_{H}>M_{2}$.
- quark mass signs, $2^{5}=32$-fold ambiguity (mass ratios). Moreover, we distinguish two values of $\delta$ in case of no CP violation: $\delta=0$ (CLASS I solutions) and $\delta=\pi$ (CLASS II solutions). Notice that the sign of the quark masses is an observable in SB-LR models.

[^2]
## 3 Impact of SB-LR on observables

Before starting the discussion of the constraints that the model receives from $B$ and $K$ physics observables, it is important to discuss the behaviour of the model in two general regimes for the common Higgs mass $\left(M_{H}\right)$ and the extra charged gauge boson $\left(M_{2}\right)$ :

- Decoupling limit: this corresponds to $M_{2}, M_{H} \rightarrow \infty$. In this limit we observe that the CKM phase $\delta$ gets strongly restricted $\left|\delta^{S B L R}\right|<0.25$ for class I solutions and $\left|\delta^{S B L R}-\pi\right|<0.25$ for class II. However, a global fit [34] yields $\delta=1.0 \pm 0.2$. This implies that the SM limit of this model is inconsistent by $3.5 \sigma$ with current experiments.
- Finite $M_{2}$ and $M_{H}$ masses: the gauge boson mass $M_{2}$ entering through the mixing angle $\zeta$ or as a propagator cannot induce observable effects at tree level. However, at loop level, sizeable effects are expected, because the Inami-Lin functions in a LR model are larger roughly by a factor of 4 with respect to the SM and the suppression due to the mixing angle is compensated by large quark mass terms from spin-flips $\zeta \rightarrow \zeta m_{t} / m_{b}$ inside the loop in $b \rightarrow s \gamma$. Concerning the Higgs, their contribution is also heavily suppressed at tree level by factors $\left(m_{W} / M_{H}\right)^{2} \sim 10^{-4}$. We have the contributions of neutral FC Higgs to $\Delta F=2$ processes at tree level and the charged Higgs contributions enhanced by $m_{t} / m_{b}$ in b penguins. Finally, the gauge and Higgs contributions are similar in size in $K$ physics observables, while the Higgs contribution dominates in $B$ mixing.


### 3.1 Constraints from the $K$ system

The observables that we considered are $\Delta M_{K}, \epsilon_{K}$ and $\epsilon^{\prime}$. They are plagued by theoretical uncertainties. For example, the long distance contributions $M_{12}^{\mathrm{LD}}(1)$ in the $K$ system are not very well known, but they are expected to be sizeable. Then one is forced to make a reasonable assumption concerning $\Delta m_{K}$ : the LR contribution should at most saturate $\Delta m_{K}$, i.e. $2\left|M_{12}^{\mathrm{K}, \mathrm{LR}}\right|<\Delta m_{K}^{\exp }$. The second observable, $\epsilon$, provides us with information about the phase difference between $M_{12}$ and $\Gamma_{12}$. In order to check the usual formula used for $\epsilon$ we rederive it from

$$
\begin{equation*}
\epsilon=\frac{1}{2 \sqrt{2}} e^{i \pi / 4} \sin \left(\arg M_{12}+2 \arg a_{0}\right) \tag{6}
\end{equation*}
$$

avoiding approximations in the SM that need not be valid in the SB-LR. Following all the procedure [27] one can show that: a) phase redefinitions of $V_{L}$ and $V_{R}$ cancel in (6), b) from the experimental result $|\epsilon|^{\exp }=(2.280 \pm 0.013) \times 10^{-3}$, the following bound can be obtained: $\arg M_{12}+2 \arg a_{0}=(6.449 \pm 0.037) \times 10^{-3}$. Some remarks are in order here. First, since both terms are small and also $\operatorname{Re} M_{12} \approx\left|M_{12}\right|$ (6) can be reduced to the standard formula for $\epsilon$. Second, while in the SM $\arg a_{0}$ is neglected, in LR-SB this is no longer true and the $W_{R}$ contribution computed in [32] is: $2\left|\arg a_{0}^{\mathrm{LR}}\right|<0.005 \cdot\left(\frac{1 \mathrm{TeV}}{M_{2}}\right)^{2}$. However, given the uncertainties involved in the computation of this contribution and
the lack of Higgs contribution, we prefer to include this term twice as an uncertainty of $\arg M_{12}$ (assuming the Higgs contribution to be smaller than the extra charged gauge boson):

$$
\begin{align*}
6.375 \times 10^{-3}-0.01\left(\frac{1 \mathrm{TeV}}{M_{2}}\right)^{2}<\tilde{\theta}_{M} & <6.523 \times 10^{-3}+0.01\left(\frac{1 \mathrm{TeV}}{M_{2}}\right)^{2}  \tag{7}\\
\text { with } \quad \tilde{\theta}_{M} & =\left|\frac{2 \operatorname{Re} M_{12}}{\Delta m_{K}}\right| \arg M_{12}
\end{align*}
$$

Finally, concerning $\epsilon^{\prime}$, we have been extremely conservative and have only required the SB-LR model to predict correctly the sign of this observable.

The consequence of imposing the previous constraints on the parameter space of this model are mainly of two types [27]:
a) As expected from the previous discussion, in the decoupling limit $M_{2}, M_{H} \rightarrow \infty$, we obtain from the constraint on $\epsilon(7)$ :

$$
\tilde{\theta}_{M}<2.9 \times 10^{-3}
$$

which implies again that this limit is excluded by the smallness of the CKM phase $\delta$ in the SB-LR model.
b) In the finite mass case we get lower and upper bounds:

- Lower bounds on the extra boson masses:

$$
\begin{equation*}
M_{2}>1.85 \mathrm{TeV}, \quad M_{H}>5.2 \mathrm{TeV} \tag{8}
\end{equation*}
$$

The bound on $M_{2}$ is the usual one, while the bound on $M_{H}$ is lower, since we included not only the charm contribution but also the top quark contribution (destructive interference), which was usually neglected.

- Upper bounds on the extra boson masses, not very constraining:

$$
\begin{equation*}
M_{2}<73.5 \mathrm{TeV}, \quad M_{H}<230 \mathrm{TeV} \tag{9}
\end{equation*}
$$

These results are illustrated in Fig. 1. Interestingly, once these bounds are combined with those coming from $B$ physics, the allowed region of parameter space of this model gets strongly reduced, as we will show in the following sections.

## $3.2 \quad B$ physics constraints

In this section, we discuss the constraints coming from $B^{0}-\bar{B}^{0}$ Mixing. First those coming from the mass difference $\Delta m_{B_{d}}$ and $\Delta m_{B_{s}}$, and then the CP asymmetry $B_{d}^{0} \rightarrow J / \psi K_{S}^{0}$.


Figure 1: Allowed values for $M_{2}$ and $M_{H}$ from the $K$ physics constraints.

### 3.2.1 $\Delta m_{B_{d}}$ and $\Delta m_{B_{s}}$

In the SB-LR model, $M_{12}$ gets 3 types of new contributions:

$$
\begin{equation*}
M_{12}=M_{12}^{\mathrm{SM}}+M_{12}^{W_{1} W_{2}}+M_{12}^{S_{1} W_{2}}+M_{12}^{H} \equiv M_{12}^{\mathrm{SM}}+M_{12}^{\mathrm{LR}} . \tag{10}
\end{equation*}
$$

The new contributions are box diagrams including $W_{R}$ and unphysical scalars, and treelevel neutral Higgs exchanges. If we write the total $M_{12}$ in a more compact form [27]

$$
\begin{align*}
M_{12} & =M_{12}^{\mathrm{SM}}\left(1+\kappa e^{i \sigma_{q}}\right), \\
\text { with } \kappa & \equiv\left|\frac{M_{12}^{\mathrm{LR}}}{M_{12}^{\mathrm{SM}}}\right|, \\
\sigma_{q} & \equiv \arg \frac{M_{12}^{\mathrm{LR}}}{M_{12}^{\mathrm{SM}}}=\arg \left(-\frac{V_{t b}^{R} V_{t q}^{R *}}{V_{t b}^{L} V_{t q}^{L *}}\right) . \tag{11}
\end{align*}
$$

we find numerically for the quasi-spectator independent $\kappa$,

$$
\begin{equation*}
\kappa=\frac{B_{B}^{S}\left(m_{b}\right)}{B_{B}\left(m_{b}\right)}\left[\left(\frac{7 \mathrm{TeV}}{M_{H}}\right)^{2}+\eta_{2}^{L R}\left(m_{b}\right)\left(\frac{1.6 \mathrm{TeV}}{M_{2}}\right)^{2}\left\{0.051-0.013 \ln \left(\frac{1.6 \mathrm{TeV}}{M_{2}}\right)^{2}\right\}\right] \tag{12}
\end{equation*}
$$

where $\eta_{2}^{L R}\left(m_{b}\right) \approx 1.7$ is a LO short-distance correction and the ratio of bag factors evaluated from QCD sum rules or to leading order in $1 / N_{c}$ is $B_{B}^{S}\left(m_{b}\right) / B_{B}\left(m_{b}\right)=1.2 \pm 0.2$. The constraint on $\Delta m_{B_{d}}$ and the bound on $\Delta m_{B_{s}}$ translate into a constraint on $\kappa$. From the experimental results

$$
\begin{equation*}
\Delta m_{B_{d}}=(0.472 \pm 0.016) \mathrm{ps}^{-1} \quad \text { and } \quad \Delta m_{B_{s}}>12.4 \mathrm{ps}^{-1} \tag{13}
\end{equation*}
$$



Figure 2: First plot: Constraints from $\Delta m_{B_{d}}$. Dashed lines are the experimental result and theory errors. The lower curves are class I solutions, the upper curves are class II. Second plot: Correlation between $\Delta m_{B_{d}}$ and $\Delta m_{B_{s}}$. Short dashes denote experimental results and theory error for $\Delta m_{B_{d}}$, long dashes the lower bound on $\Delta m_{B_{s}}$. Left lines are class I and right ones class II.
and from the expression for $\Delta m_{B_{d}}$, we obtain

$$
\begin{equation*}
\left|\left(V_{t b}^{L} V_{t d}^{L *}\right)^{2}\left(1+\kappa e^{i \sigma_{d}}\right)\right|=(6.7 \pm 2.7) \times 10^{-5}, \tag{14}
\end{equation*}
$$

which translates into

$$
\kappa<3
$$

while the lower bound on $\Delta m_{B_{s}}$ implies

$$
\begin{equation*}
\left|\left(V_{t b}^{L} V_{t s}^{L *}\right)^{2}\left(1+\kappa e^{i \sigma_{s}}\right)\right|>9.6 \times 10^{-4} \tag{15}
\end{equation*}
$$

However, since the ratio between the mass differences seems to have a smaller theoretical error, owing to lattice results on this particular combination of hadronic parameters, it is interesting to use also the following bound

$$
\left|\frac{\left(V_{t b}^{L} V_{t s}^{L *}\right)^{2}\left(1+\kappa e^{i \sigma_{s}}\right)}{\left(V_{t b}^{L} V_{t d}^{L *}\right)^{2}\left(1+\kappa e^{i \sigma_{d}}\right)}\right|=\frac{\Delta m_{B_{s}}}{\Delta m_{B_{d}}} \frac{m_{B_{d}}}{m_{B_{s}}}\left(\frac{f_{B_{d}}}{f_{B_{s}}}\right)^{2} \frac{\hat{B}_{B_{d}}}{\hat{B}_{B_{s}}}>17.2 .
$$

The main implications of these constraints, illustrated in Fig. 2, are that an upper bound on $\kappa$ is obtained, that the decoupling limit that corresponds here to ( $\kappa \rightarrow 0$ ) is excluded, as can be seen in the first plot of Fig. 2. Also, class II solutions are excluded for $\beta \geq 0.021$ (the upper curves of the first plot of Fig. 2 go further up for these values of $\beta$ ) and, finally, class I solutions require $\kappa>0.52$ and class II $\kappa>0.42$, which are mainly driven by the Higgs contribution that become essential. The SB-LR can naturally accommodate any value of $\Delta m_{B_{s}}$ larger than the SM value, as illustrated in the second plot of Fig. 2.


Figure 3: $\sin 2 \beta_{\mathrm{CKM}}^{\mathrm{eff}}$ in the $\mathrm{SB}-\mathrm{LR}$ as a function of $\kappa$ for several values of $\beta$.

### 3.2.2 CP asymmetry $B_{d}^{0} \rightarrow J / \psi K_{S}^{0}$

The gold observable at present coming from $B$ physics is the time-dependent CP asymmetry in $B_{d}^{0} \rightarrow J / \psi K_{S}^{0}$, the experimental averaged number for this asymmetry is [35]:

$$
\begin{equation*}
a_{\mathrm{CP}}=\frac{\Gamma\left(\bar{B}_{d}^{0}(t) \rightarrow J / \psi K_{S}^{0}\right)-\Gamma\left(B_{d}^{0}(t) \rightarrow J / \psi K_{S}^{0}\right)}{\Gamma\left(B_{d}^{0}(t) \rightarrow J / \psi K_{S}^{0}\right)+\Gamma\left(\bar{B}_{d}^{0}(t) \rightarrow J / \psi K_{S}^{0}\right)}=(0.78 \pm 0.08) \sin \left(\Delta m_{B} t\right) \tag{16}
\end{equation*}
$$

where we assume vanishing direct CP violation. This observable can be written

$$
a_{\mathrm{CP}}=\operatorname{Im} \lambda \sin \left(\Delta m_{B} t\right),
$$

In the $\mathrm{SM}, \operatorname{Im} \lambda$ measures directly the angle $\beta$ of the Unitarity Triangle

$$
\begin{equation*}
\operatorname{Im} \lambda=\sin 2 \beta_{\mathrm{CKM}} \quad \beta_{\mathrm{CKM}}=\arg \left(-\frac{V_{c d}^{L} V_{c b}^{L *}}{V_{t d}^{L} V_{t b}^{L *}}\right) \tag{17}
\end{equation*}
$$

while in the SB-LR symmetric model this observable measures

$$
\begin{aligned}
\operatorname{Im} \lambda\left(B^{0} \rightarrow J / \psi K_{S}^{0}\right) & =\sin 2 \beta_{\mathrm{CKM}}^{\mathrm{eff}} \\
& =\sin \left[2 \beta_{\mathrm{CKM}}+\arg \left(1+\kappa e^{i \sigma_{d}}\right)-\arg \left(1+\frac{M_{12}^{\mathrm{K}, \mathrm{LR}}}{M_{12}^{\mathrm{K}, \mathrm{SM}}}\right)\right] .
\end{aligned}
$$

The constraints obtained from this observable are illustrated in Fig. 3: all negative values for $\sin 2 \beta$ are excluded, the SM expectation for $\sin 2 \beta_{\mathrm{CKM}}^{\mathrm{eff}} \approx 0.75$ can be easily accommodated, for $\beta<0.03$ there are two branches, one with small $\sin 2 \beta_{\mathrm{CKM}}^{\mathrm{eff}}<0.4$, the other with all possible values between 0 and 1 . Finally, if we impose $\sin 2 \beta_{\mathrm{CKM}}^{\mathrm{eff}}$ to be around its SM expectation this implies $\kappa \approx 0.6$ or $\kappa>1.2$. These are the independent constraints that the CP asymmetry in $B_{d}^{0} \rightarrow J / \psi K_{S}^{0}$ imposes in the parameter space. The final step will be to combine all of them.

### 3.3 Combining constraints: results and consequences

Up to this point we have seen, after considering the observables $\Delta m_{K}, \Delta m_{B_{d}}, \Delta m_{B_{s}}, \epsilon$, $\epsilon^{\prime}$ (only sign) and $\sin 2 \beta_{\mathrm{CKM}}^{\mathrm{eff}}$, that a SB-LR model based on the gauge group $S U(2)_{L} \times$ $S U(2)_{R} \times U(1)$ with spontaneous CP violation reproduces easily the experimental results for CP-conserving observables.


Figure 4: First plot: Allowed region in $\left(M_{2}, M_{H}\right)$, taking into account all constraints. Second plot: Allowed values for the CP-violating parameters $\epsilon$ and $\sin 2 \beta_{\mathrm{CKM}}^{\mathrm{eff}}$ after imposing the other constraints.

But, once we combine CP-violating observables from the old $K$ physics observables with the new $B$ physics observables, they become very restrictive: a strong anticorrelation is found between the signs of $\operatorname{Re} \epsilon$ and $\sin 2 \beta_{\mathrm{CKM}}^{\mathrm{eff}}$.

The combination of all these constraints yields the following results[27]:

- all but one quark mass signatures are excluded;
- predictions for $\Delta m_{B_{s}}$ are in the range ( $\left.0.6-1.1\right) \Delta m_{B_{s}}^{\mathrm{SM}, \exp }$, i.e. not much information expected from this observable if it falls in the SM range;
- the parameter space for gauge and scalar masses are strongly restricted when combining both $K$ and $B$ constraints.

$$
2.75 \mathrm{TeV}<M_{W_{R}}<13, \mathrm{TeV} \quad 10.2 \mathrm{TeV}<M_{H}<14.6 \mathrm{TeV}
$$

Finally, as the main conclusions, the SM limit of the $\mathrm{SB}-\mathrm{LR}$ model is excluded by more than $4 \sigma$ and the maximal value of $a_{\mathrm{CP}}\left(B \rightarrow J / \psi K_{S}\right) \equiv \sin 2 \beta_{\mathrm{CKM}}^{\mathrm{eff}}<0.1$ because of the anticorrelation mentioned above, is incompatible with the present experimental result by several $\sigma$, pointing to a possible ruling out of this model in its present form.

## 4 Vice versa: model independent sum rules to 'test' data on $B \rightarrow \pi K$

Here, we will try to argue the other way around, and show how theory together with some reasonable hypothesis can help us, in a model-independent way, to 'test', in a certain sense, data on $B \rightarrow \pi K$ decays. We will construct a set of relations or sum rules [9, 15, 37, 38] relating different observables of $B \rightarrow \pi K$ decays.

Rare $B$ decays based on the quark transition $\bar{b} \rightarrow \bar{s} q \bar{q}$ are described by the effective hamiltonian [36]:

$$
\mathcal{H}_{\mathrm{eff}}=\frac{G_{F}}{\sqrt{2}}\left\{\sum_{i=1,2} C_{i}\left(\lambda_{u} Q_{i}^{u}+\lambda_{c} Q_{i}^{c}\right)-\lambda_{t} \sum_{i=3}^{10} C_{i} Q_{i}\right\}+\text { h.c. }
$$

where $Q_{1,2}^{c}$ are current-current operators and $Q_{3-6}$ are QCD-penguin operators; both induce a change of isospin $(\Delta I=0)$, while $Q_{1,2}^{u}$ are current-current operators and $Q_{7-10}$ are electroweak operators generating a change of isospin $(\Delta I=0,1)$.

Since isospin plays a fundamental role in this discussion, it is helpful to recall a few points. An initial $B$ meson state $|B\rangle$ has isospin $I=1 / 2$ and a final state $|\pi K\rangle$ can have isospin $I=1 / 2,3 / 2$. As a consequence, the amplitude of a $B \rightarrow \pi K$ process can be decomposed in three pieces:

$$
\begin{equation*}
\mathcal{A}(B \rightarrow \pi K)=D_{\frac{1}{2} 0}+A_{\frac{1}{2} 1}+A_{\frac{3}{2} 1}, \tag{18}
\end{equation*}
$$

where the subindices $(D, A)_{I_{\pi K} \Delta I}$ stand for $I_{\pi K}$ final state of isospin and $\Delta I$ change of isospin of the b quark transition. $D$ refers to the dominant contribution coming from QCD penguins, and $A_{\frac{1}{2} 1}, A_{\frac{3}{2} 1}$ contain contributions from electroweak penguins and currentcurrent operators but not QCD penguins. New physics contributes in particular to $A_{\frac{1}{2} 1}$ and $A_{\frac{3}{2} 1}$, and they, therefore, are the interesting pieces to measure.
$B \rightarrow \pi K$ decays are described using CP-averaged branching ratios [8],[9]:

$$
\begin{align*}
& R=\left[\frac{\mathrm{BR}\left(B_{d}^{0} \rightarrow \pi^{-} K^{+}\right)+\mathrm{BR}\left(\overline{B_{d}^{0}} \rightarrow \pi^{+} K^{-}\right)}{\operatorname{BR}\left(B^{+} \rightarrow \pi^{+} K^{0}\right)+\mathrm{BR}\left(B^{-} \rightarrow \pi^{-} \overline{K^{0}}\right)}\right], \\
& R_{c}=2\left[\frac{\mathrm{BR}\left(B^{+} \rightarrow \pi^{0} K^{+}\right)+\mathrm{BR}\left(B^{-} \rightarrow \pi^{0} K^{-}\right)}{\operatorname{BR}\left(B^{+} \rightarrow \pi^{+} K^{0}\right)+\mathrm{BR}\left(B^{-} \rightarrow \pi^{-} \overline{K^{0}}\right)}\right], \\
& R_{0}=2\left[\frac{\mathrm{BR}\left(B_{d}^{0} \rightarrow \pi^{0} K^{0}\right)+\mathrm{BR}\left(\overline{B_{d}^{0}} \rightarrow \pi^{0} \overline{K^{0}}\right)}{\mathrm{BR}\left(B^{+} \rightarrow \pi^{+} K^{0}\right)+\mathrm{BR}\left(B^{-} \rightarrow \pi^{-} \overline{K^{0}}\right)}\right] . \tag{19}
\end{align*}
$$

We will use these definitions here, because, in those terms, the expressions for the sum rules become simpler. Other definitions for the charged and neutral channels that are used in the literature are $R_{*}=1 / R_{c}[9]$ and $R_{n}=R / R_{0}[8]$. CP asymmetries are the second type of observables:

$$
\begin{align*}
& \mathcal{A}_{\mathrm{CP}}^{0+}=\frac{\mathrm{BR}\left(B^{+} \rightarrow \pi^{0} K^{+}\right)-\mathrm{BR}\left(B^{-} \rightarrow \pi^{0} K^{-}\right)}{\operatorname{BR}\left(B^{+} \rightarrow \pi^{0} K^{+}\right)+\operatorname{BR}\left(B^{-} \rightarrow \pi^{0} K^{-}\right)}, \\
& \mathcal{A}_{\mathrm{CP}}^{+0}=\frac{\operatorname{BR}\left(B^{+} \rightarrow \pi^{+} K^{0}\right)-\operatorname{BR}\left(B^{-} \rightarrow \pi^{-} \overline{K^{0}}\right)}{\operatorname{BR}\left(B^{+} \rightarrow \pi^{+} K^{0}\right)+\operatorname{BR}\left(B^{-} \rightarrow \pi^{-} \overline{K^{0}}\right)}, \\
& \mathcal{A}_{\mathrm{CP}}^{-+}=\frac{\operatorname{BR}\left(B_{d}^{0} \rightarrow \pi^{-} K^{+}\right)-\operatorname{BR}\left(\overline{B_{d}^{0}} \rightarrow \pi^{+} K^{-}\right)}{\operatorname{BR}\left(B_{d}^{0} \rightarrow \pi^{-} K^{+}\right)+\operatorname{BR}\left(\overline{B_{d}^{0}} \rightarrow \pi^{+} K^{-}\right)}, \\
& \mathcal{A}_{\mathrm{CP}}^{00}=\frac{\mathrm{BR}\left(B_{d}^{0} \rightarrow \pi^{0} K^{0}\right)-\mathrm{BR}\left(\overline{B_{d}^{0}} \rightarrow \pi^{0} \overline{K^{0}}\right)}{\mathrm{BR}\left(B_{d}^{0} \rightarrow \pi^{0} K^{0}\right)+\mathrm{BR}\left(\overline{B_{d}^{0}} \rightarrow \pi^{0} \overline{K^{0}}\right)} . \tag{20}
\end{align*}
$$

Generically, the CP-averaged branching ratios can be written in the form $1+\alpha A_{\frac{1}{2} 1}+$ $\beta A_{\frac{3}{2} 1}$, where $\alpha$ and $\beta$ are constants that depend on the particular CP-averaged branching ratio that we are describing. Therefore, we can attach a physical meaning to these observables $R, R_{c}, R_{0}$, as a measure of the physics that violates isospin, either standard or new. In other words, if there was no isospin violation, $A_{\frac{1}{2} 1}=A_{\frac{3}{2} 1}=0$ and all CP-averaged branching ratios would measure 1. Consequently, we can write these observables in the following form [15]:

$$
\begin{align*}
R & =1+u_{+} \\
R_{c} & =1+z_{+} \\
R_{0} & =1+n_{+} \tag{21}
\end{align*}
$$

Using isospin decomposition we can show that

$$
\begin{align*}
& u_{+} \sim \mathcal{O}(r)+\mathcal{O}\left(r^{2}, r \rho, q_{C} r\right) \\
& z_{+} \sim \mathcal{O}\left(r_{c}\right)+\mathcal{O}\left(r_{c}^{2}, r_{c} \rho, q r_{c}\right) \tag{22}
\end{align*}
$$

If we assign a generic value $\epsilon(<1)$ to the small parameters $r, r_{c}$ and $q, q_{c}, \rho$ then $u_{+}$and $z_{+}$are quantities of $\mathcal{O}(\epsilon)$ in this sense. Moreover, it is possible to relate $n_{+}$with the other two parameters $u_{+}$and $z_{+}$up to a quantity of $\mathcal{O}\left(\epsilon^{2}\right)$ :

$$
\begin{equation*}
n_{+}=u_{+}-z_{+}+k_{1} \quad \text { with } \quad k_{1} \sim \mathcal{O}\left(r_{c} r, \ldots\right) \sim \mathcal{O}\left(\epsilon^{2}\right) \tag{23}
\end{equation*}
$$

Therefore, combining (22) and (23), one arrives at the well-known sum rule [9, 15, 38]

$$
\begin{equation*}
\text { I) } R_{0}-R+R_{c}-1=k_{1} \tag{24}
\end{equation*}
$$

There is, however, an interesting way of reading this sum rule, and it is the following: if we take old data [15] and only central values to make the point clearer, sum rule (24) will imply

$$
R_{0}=1+\underbrace{n_{+}}_{+0.21}=1+\underbrace{u_{+}}_{+0.00}-\underbrace{z_{+}}_{+0.41}+k_{1}
$$

This means that in order for $k_{1}$, a quantity of order $\epsilon^{2}$, could compensate for the values of $u_{+}$and $z_{+}$(quantities of order $\epsilon$ ) and reproduce the experimental value of $R_{0}=1+n_{+}$ ( $R_{0}=1.21 \pm 0.35$ with old data set), the value of $k_{1}\left(\right.$ of $\left.\mathcal{O}\left(\epsilon^{2}\right)\right)$ should be $k_{1} \sim 0.62$, which is indeed higher than the experimental values of the other quantities of order $\epsilon$ ( $u_{+} \sim 0.00$ and $z_{+} \sim 0.41$ ). Of course, this is too naive, since we are taking only central values and within errors (see Table 1) everything is compatible. However, this may tell us that the central values of these observables (CP-averaged branching ratios) are expected to change a lot. Indeed if one repeats the experiment with the new data of CLEO [39], BaBar [40, 41] and Belle [41], again all experiments are equally good, once errors are taken into account as they should. However, it is interesting to notice that Belle's central

| Obs. | Order | Old | CLEO | BaBar | Belle |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $u_{+}=R-1$ | $\epsilon$ | $+.00 \pm .18$ | $-.06 \pm .28$ | $+.02 \pm .15$ | $+.16 \pm .24$ |
| $z_{+}=R_{c}-1$ | $\epsilon$ | $+.41 \pm .29$ | $+.27 \pm .48$ | $+.27 \pm .24$ | $+.33 \pm .37$ |
| $n_{+}=R_{0}-1$ | $\epsilon$ | $+.21 \pm .35$ | $+.60 \pm .79$ | $-.06 \pm .37$ | $-.18 \pm .41$ |
| $k_{1}$ | $\epsilon^{2}$ | $+.62 \pm .45$ | $+.93 \pm .92$ | $+.19 \pm .43$ | $-.01 \pm .53$ |

Table 1: Sum rule parameters evaluated using old data [15] and CLEO, BaBar and Belle data [39]-[41]. Notice that, for simplicity reasons, we are neglecting the small phase-space difference ( $6 \%$ ), in the sum rule expressions, between $B^{ \pm} \rightarrow \pi^{ \pm} K^{0}$ and $B_{d}^{0} \rightarrow \pi^{ \pm} K^{\mp}, \pi^{0} K^{0}$ affecting $R$ and $R_{0}$.
values seem to follow the sum rule wonderfully. If errors get reduced with time we can start discriminating between the different experiments.

Moreover, it is easy to understand the physical meaning of this observable called $k_{1}$. From isospin one arrives at

$$
\begin{align*}
-\sqrt{2} A\left(B^{+} \rightarrow \pi^{0} K^{+}\right) & =A\left(B^{+} \rightarrow \pi^{+} K^{0}\right)+d_{1} \\
-A\left(B^{0} \rightarrow \pi^{-} K^{+}\right) & =A\left(B^{+} \rightarrow \pi^{+} K^{0}\right)+d_{2} \\
\sqrt{2} A\left(B^{0} \rightarrow \pi^{0} K^{0}\right) & =A\left(B^{+} \rightarrow \pi^{+} K^{0}\right)+d_{2}-d_{1} \tag{25}
\end{align*}
$$

where $d_{i}(i=1,2)$ are functions of $A_{\frac{1}{2} 1}=A_{\frac{3}{2} 1}$ and vanish if there is no isospin breaking. In general one can write

$$
\begin{equation*}
d_{i}=|P| \xi_{i} e^{i \theta_{i}}\left(e^{i \gamma}-a_{i} e^{i \phi_{a_{i}}}-i b_{i} e^{i \phi_{b_{i}}}\right), \tag{26}
\end{equation*}
$$

where $P$ contain all CP-conserving terms of the penguin contribution to $B^{+} \rightarrow \pi^{+} K^{0}$. The $\xi_{i}$ parametrize isospin breaking, and they are expected to be small parameters. $\theta_{i}$, $\phi_{a_{i}}, \phi_{b_{i}}$ are strong phases, and $\gamma$ and $i b_{i}$ parametrize weak phases that change sign under a CP transformation. We will follow the notation of [42, 15]. We show explicitly in (26) the dependence on $\gamma$, meaning that $b_{1}$ and $b_{2}$ can be non-zero only if there is new physics.

From the dependence of $k_{1}$ on the $d_{i}$ parameters

$$
\begin{equation*}
k_{1}=\frac{2}{x}\left(\left|d_{1}\right|^{2}+\left|\overline{d_{1}}\right|^{2}-\operatorname{Re}\left[d_{1} d_{2}^{*}\right]-\operatorname{Re}\left[\overline{d_{1}}{\overline{d_{2}}}^{*}\right]\right) \tag{27}
\end{equation*}
$$

it is easy to interpret $k_{1}$ as a measure of the misalignment between the isospin-breaking contributions to two channels: $\sqrt{2} A\left(B^{+} \rightarrow \pi^{0} K^{+}\right)$and $A\left(B^{0} \rightarrow \pi^{-} K^{+}\right)$. Even in the presence of isospin-breaking if the new contributions to these channels are equal, i.e. if $d_{1}=d_{2}$, then $k_{1}$ is exactly zero. On the contrary, if the isospin contribution to these channels has opposite sign then $k_{1}$ is maximal. We should look at data to discern which of the two scenarios is closer to the one realized in nature. It is also possible to write down a completely general expression for $k_{1}$, valid for any model (see [15]).


Figure 5: Sum rule I evaluated for the SM using NLO QCD factorization [13] for values of $\gamma$ in the first quadrant: (a) low uncertainty $\left(\varrho_{A}=1\right)$ from annihilation topologies, (b) large uncertainty $\left(\varrho_{A}=2\right)$ from annihilation topologies

Table 2: Strongly correlated observables associated to sum rules III-V

| III | $\mathcal{O}_{1}^{\mathrm{III}}=R$ | $\mathcal{O}_{2}^{\mathrm{III}}=R_{0} R_{c}$ |
| :--- | :--- | :--- |
| IV | $\mathcal{O}_{1}^{\mathrm{IV}}=R_{c}$ | $\mathcal{O}_{2}^{\mathrm{IV}}=-R_{0} / R+2$ |
| V | $\mathcal{O}_{1}^{\mathrm{V}}=R_{0}$ | $\mathcal{O}_{2}^{\mathrm{V}}=-R_{c} / R+2$ |

In order to have a reference value we show the prediction for $k_{1}$ using NLO QCD factorization in the Standard Model. The two plots of Fig. 5 correspond to two different estimates of the uncertainty coming from the annihilation topologies.

We can go beyond this sum rule and try to construct the simplest sets of observables strongly correlated by isospin (we will number them III-V to follow the notation of [15]). They can help us in guessing what we may expect from the data. The result is the following sum rules [15]:

$$
\begin{equation*}
\text { III) } R=R_{0} R_{c}+k_{3} \tag{28}
\end{equation*}
$$

with $k_{3}=z_{+}\left(z_{+}-u_{+}\right)-k_{1}-k_{1} z_{+}$;

$$
\begin{equation*}
\text { IV) } R_{c}=-\frac{R_{0}}{R}+2+k_{4} \tag{29}
\end{equation*}
$$

with $k_{4}=\left(u_{+} z_{+}+k_{1}\right) /\left(1+u_{+}\right) ;$and, finally,

$$
\begin{equation*}
\text { V) } R_{0}=-\frac{R_{c}}{R}+2+k_{5} \tag{30}
\end{equation*}
$$

| Observable | CLEO | BaBar | Belle |
| :---: | :---: | :---: | :---: |
| $k_{3}$ | $-1.10 \pm 1.31$ | $-0.17 \pm 0.51$ | $+0.07 \pm 0.60$ |
| $k_{4}$ | $+0.97 \pm 0.91$ | $+0.19 \pm 0.43$ | $+0.04 \pm 0.51$ |
| $k_{5}$ | $+0.95 \pm 0.90$ | $+0.18 \pm 0.43$ | $-0.03 \pm 0.50$ |

Table 3: Sum rules III-V evaluated using old data and CLEO, BaBar and Belle data


Figure 6: Sum rules III (a) and IV(b) evaluated for the SM using NLO QCD factorization [13] for values of $\gamma$ in the first quadrant in the large uncertainty case ( $\varrho_{A}=2$ ) from annihilation topologies
with $k_{5}=k_{1}+u_{+}\left(u_{+}-z_{+}\right) /\left(1+u_{+}\right)$. This sum rule can be related with one proposed in [9] (but with the inverse $R / R_{c}$ ) for the SM case and in an approximate form, i.e. keeping only the term $\xi_{i}^{2}$.

The associated observables to these sum rules are given in Table 2. The plots of associated observables (Fig. 6) have an interesting interpretation: in the absence of isospinbreaking, both observables should fall in the diagonal of Figs. 6, with $\mathcal{O}_{i}^{\alpha}=1$. If isospin breaking is small, $O_{1}^{\alpha}$ and $O_{2}^{\alpha}$ should stay near the diagonal. The deviation from 1 along the diagonal gives an idea of the isospin-breaking terms of order $\xi_{i}$ (remember that $R$, $R_{c}$ and $R_{0}$ measure isospin breaking of this size). This is useful to have an idea of the maximal size of this breaking. Notice that it also implies that each pair of observables ( $O_{1}^{\alpha}$, $O_{2}^{\alpha}$ ) is chosen in such a way as to present the same deviation of order $\xi_{i}$, independently of the model.

More interestingly, deviations from the diagonal would measure isospin-breaking contributions of order $\xi_{i}^{2}$. It implies that if the isospin is not badly broken, we can estimate that the deviations from the diagonal will be smaller than the square of the maximal deviation from 1 along the diagonal. For instance, in Fig. 6b the maximal deviation from 1 along the diagonal is approximately 0.5 ; the maximal expected deviation from the diagonal would then be 0.25 and, indeed, this is the case. This rule applies to

Table 4: Strongly correlated observables associated to sum rules VI and VII

$$
\begin{array}{|lll}
\hline \text { VI } \quad \mathcal{O}_{1}^{\mathrm{VI}}=\mathcal{A}_{\mathrm{CP}}^{-+} R \quad \mathcal{O}_{2}^{\mathrm{VI}}=A_{\mathrm{CP}}^{+0}-1+\left(1+\mathcal{A}_{\mathrm{CP}}^{00} R_{0}-\mathcal{A}_{\mathrm{CP}}^{+0}\right)\left(1+\mathcal{A}_{\mathrm{CP}}^{0+} R_{c}-\mathcal{A}_{\mathrm{CP}}^{+0}\right) \\
\hline \text { VII } \mathcal{O}_{1}^{\mathrm{VII}}=\mathcal{A}_{\mathrm{CP}}^{0+} R_{c} \quad \mathcal{O}_{2}^{\mathrm{VII}}=A_{\mathrm{CP}}^{+0}+\left(\mathcal{A}_{\mathrm{CP}}^{-+} R-\mathcal{A}_{\mathrm{CP}}^{00} R_{0}\right) /\left(1+\mathcal{A}_{\mathrm{CP}}^{-+} R-\mathcal{A}_{\mathrm{CP}}^{+0}\right) \\
\hline
\end{array}
$$

all figures evaluated using NLO QCD factorization.
As in the case of $k_{1}$ the numerical values of the observables $k_{3}, k_{4}$ and $k_{5}$ are compatible, but their central values differ significantly (see Table 3). Again Belle central values are in the expected ballpark. However experimental errors in these parameters are still too large to be conclusive.

In a similar way, one can also find a set of sum rules for the CP asymmetries, once the building blocks are identified:

$$
\begin{aligned}
\mathcal{A}_{\mathrm{CP}}^{-+} R & =\mathcal{A}_{\mathrm{CP}}^{+0}+u_{-} \\
\mathcal{A}_{\mathrm{CP}}^{0+} R_{c} & =\mathcal{A}_{\mathrm{CP}}^{+0}+z_{-} \\
\mathcal{A}_{\mathrm{CP}}^{00} R_{0} & =\mathcal{A}_{\mathrm{CP}}^{+0}+n_{-},
\end{aligned}
$$

where the asymmetries are as defined in (20). Using again isospin decomposition, we can demonstrate that:

$$
\begin{aligned}
& u_{-} \sim \mathcal{O}(r)+\mathcal{O}\left(r^{2}, r \rho, q_{C} r\right) \sim \mathcal{O}(\epsilon) \\
& z_{-} \sim \mathcal{O}\left(r_{c}\right)+\mathcal{O}\left(r_{c}^{2}, r_{c} \rho, q r_{c}\right) \sim \mathcal{O}(\epsilon)
\end{aligned}
$$

and that $n_{-}=u_{-}-z_{-}+k_{2}$ with $k_{2} \sim \mathcal{O}\left(r_{c} r, \ldots\right) \sim \mathcal{O}\left(\epsilon^{2}\right)$. From the expression of $n_{-}$ the sum rule follows automatically [15, 9]

$$
\text { II) } \mathcal{A}_{\mathrm{CP}}^{00} R_{0}-\mathcal{A}_{\mathrm{CP}}^{-+} R+\mathcal{A}_{\mathrm{CP}}^{0+} R_{c}-\mathcal{A}_{\mathrm{CP}}^{+0}=k_{2},
$$

where $k_{2}$ can be related to the contributions to the different channels

$$
k_{2}=\frac{2}{x}\left(\left|d_{1}\right|^{2}-\left|\overline{d_{1}}\right|^{2}-\operatorname{Re}\left[d_{1} d_{2}^{*}\right]+\operatorname{Re}\left[\overline{d_{1}}{\overline{d_{2}}}^{*}\right]\right)
$$

and it also admits a nice interpretation: $k_{2}$ measures the importance of weak phase differences between $d_{1}$ and its CP conjugate, and between $d_{2}$ and its CP conjugate. The same conditions that force $k_{1}$ to vanish also apply to $k_{2}$. But, in addition, $k_{2}$ also vanishes if $d_{1}=\bar{d}_{1}$ and $d_{2}=\bar{d}_{2}$.

We, also, show as in the case of the CP-averaged branching ratios, the prediction for the sum rule II evaluated using NLO QCD factorization in Fig. 7 and an example of strongly correlated observables using CP asymmetries. In Table 4 these observables are defined and their prediction using QCD NLO factorization in the SM is illustrated in Fig. 8 (see [15] for more details).


Figure 7: Sum rule II evaluated for the SM using NLO QCD factorization for values of $\gamma$ in the first quadrant: (a) low uncertainty $\left(\varrho_{A}=1\right)$ from annihilation topologies, (b) large uncertainty ( $\varrho_{A}=2$ ) from annihilation topologies.


Figure 8: Correlation between $O_{1}^{\mathrm{VI}, \mathrm{VII}}$ and $O_{2}^{\mathrm{VI}, \mathrm{VII}}$.

In conclusion we have shown that $B$-physics is becoming a powerful tool to discriminate and even exclude models. We have seen that a certain type of left-right models with spontaneous CP violation may be excluded by the new information obtained from $B$ physics. However, a last word about this specific model would still require, of course, to vary all input parameters (in particular CKM angles and quark masses). On the hand, we have also shown that theory with a few reasonable hypotheses may help us in some cases to 'test' data, for instance, in the sum rules for $B \rightarrow \pi K$ decays. Still more experimental precision is needed.

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## References

[1] P. Ball et al., CERN-TH-2000-101, "B decays at the LHC", hep-ph/0003238.
[2] K. Anikeev et al., FERMILAB-Pub-01/197, "B physics at the Tevatron: Run II and beyond", hep-ph/0201071.
[3] A.J. Buras, lectures at the Erice School, August 2000, TUM-HEP-402-01 [hepph/0101336]; Y. Nir, Lectures at 55th Scottish U. Summer School, 2001 [hepph/0109090].
[4] R. Fleischer, DESY-THESIS-2002-022, hep-ph/0207108, to be published in Physics Reports; and hep-ph/0208083 in these proceedings.
[5] A. Ali and D. London, Eur. Phys. J C9 (1999) 687; ibid. C18 (2001) 665.
[6] A.B. Carter and A.I. Sanda, Phys. Rev. Lett. 45 (1980) 952; Phys. Rev. D23 (1981) 1567; I.I. Bigi and A.I. Sanda, Nucl. Phys. B193 (1981) 85.
[7] M. Gronau, J.L. Rosner and D. London, Phys. Rev. Lett. 73 (1994) 21; R. Fleischer, Phys. Lett. B365 (1996) 399; R. Fleischer and T. Mannel, Phys. Rev. D57 (1998) 2752; M. Gronau and J.L. Rosner, Phys. Rev. D57 (1998) 6843; R. Fleischer, Eur. Phys. J. C6 (1999) 451; Phys. Lett. B435 (1998) 221; M. Neubert and J.L. Rosner, Phys. Lett. B441 (1998) 403; Phys. Rev. Lett. 81 (1998) 5076; M. Bargiotti et al., and Eur. Phys. J. C24 (2002) 361.
[8] A.J. Buras and R. Fleischer, Eur. Phys. J. C11 (1999) 93; R. Fleischer, Eur. Phys. J. C16 (2000) 97.
[9] M. Neubert, JHEP 9902 (1999) 014 [hep-ph/9812396].
[10] R. Fleischer and J. Matias, Phys. Rev. D61 (2000) 074004.
[11] R. Fleischer and J. Matias, Phys. Rev. D66 (2002) 054009. [hep-ph/0204101]
[12] M. Gronau and J.L. Rosner, Phys. Rev. D65 (2002) 013004 [E: D65 (2002) 079901].
[13] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914; Nucl. Phys. B606 (2001) 245.
[14] H.-n. Li and H.L. Yu, Phys. Rev. D53 (1996) 2480; Y.Y. Keum, H.-n. Li and A.I. Sanda, Phys. Lett. B504 (2001) 6; Y.Y. Keum and H.-n. Li, Phys. Rev. D63 (2001) 074006; Y. Y. Keum and A. I. Sanda, hep-ph/0209014; Y. Y. Keum, hep-ph/0209002 and hep-ph/0209208
[15] J. Matias, Phys. Lett. B520 (2001) 131.
[16] M. Gronau and D. London, Phys. Rev. Lett. 65 (1990) 3381.
[17] M. Gronau and J. L. Rosner, Phys. Rev. D65 (2002) 093012; hep-ph/0205323.
[18] R. Fleischer, Phys. Lett. B459 (1999) 306; Eur. Phys. J. C16 (2000) 87.
[19] Y. Grossman, Phys.Lett. B380 (1996) 99.
[20] G. Isidori, hep-ph/0110255
[21] C. Greub and T. Hurth, Phys. Rev. D56, 2934 (1997); K. G. Chetyrkin, M. Misiak and M. Munz, Phys. Lett. B400 (1997) 206 [Erratum-ibid. B425 (1998) 414].
[22] G. Degrassi, P. Gambino and G. F. Giudice, JHEP 0012 (2000) 009 [hepph/0009337]; M. Carena et al., Phys. Lett. B499 (2001) 141 [hep-ph/0010003].
[23] C. H. Chang, G. L. Lin and Y. P. Yao, Phys. Lett. B 415 (1997) 395 [hepph/9705345]; G. L. Lin, J. Liu and Y. P. Yao, Phys. Rev. Lett. 64, 1498 (1990); Phys. Rev. D42 (1990) 2314.
[24] S. Bertolini and J. Matias, Phys. Rev. D57 (1998) 4197 [hep-ph/9709330].
[25] A. Ali, G. F. Giudice and T. Mannel, Z. Phys. C67, 417 (1995) [hep-ph/9408213].
[26] M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. B612, 25 (2001) [hepph/0106067]; T. Feldmann, hep-ph/0108142.
[27] P. Ball, J. M. Frere and J. Matias, Nucl. Phys. B572 (2000) 3 [hep-ph/9910211].
[28] R. N. Mohapatra and J. C. Pati, Phys. Rev. D11, 566 (1975). G. Senjanovic, Nucl. Phys. B153, 334 (1979), Phys. Rev. D11, 2558 (1975); G. Senjanovic and R. N. Mohapatra, Phys. Rev. D12, 1502 (1975), Phys. Rev. Lett. 44, 912 (1980).
[29] A. Donini, F. Feruglio, J. Matias and F. Zwirner, Nucl. Phys. B507 (1997) 51 [hepph/9705450]; J. Matias, hep-ph/9705373; J. Matias and A. Vicini, Int. J. Mod. Phys. A15 (2000) 3369 [hep-ph/9803278].
[30] G. Barenboim, J. Bernabeu, J. Matias and M. Raidal, Phys. Rev. D60 (1999) 016003 [hep-ph/9901265]; J. Matias, hep-ph/9905363; P. Ball and R. Fleischer, Phys. Lett. B475, 111 (2000) [hep-ph/9912319].
[31] J. M. Frere, J. Galand, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D46 (1992) 337.
[32] G. Ecker and W. Grimus, Nucl. Phys. B258, 328 (1985).
[33] K. Kiers, J. Kolb, J. Lee, A. Soni and G. H. Wu, hep-ph/0205082.
[34] F. Parodi, P. Roudeau and A. Stocchi, Nuovo Cim. A112 (1999) 833 [hepex/9903063].
[35] BaBar Collaboration (B. Aubert et al.), BABAR-CONF-02/01, hep-ex/0203007; Talk by T. Karim (Belle Collaboration), XXXVIIth Rencontres de Moriond, Electroweak Interactions and Unified Theories, Les Arcs, France, 2002; CDF Collaboration (T. Affolder et al.), Phys. Rev. D61 (2000) 072005; ALEPH Collaboration (R. Barate et al.), Phys. Lett. B492 (2000) 259.
[36] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125 [hep-ph/9512380].
[37] H. J. Lipkin, [hep-ph/9809347].
[38] M. Gronau and J. L. Rosner, Phys. Rev. D59 (1999) 113002 [hep-ph/9809384].
[39] CLEO Collaboration (D. Cronin-Hennessy et al.), Phys. Rev. Lett. 85 (2000) 515; CLEO Collaboration (S. Chen et al.), Phys. Rev. Lett. 85 (2000) 525.
[40] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 87 (2001) 151802 [hepex/0105061];
[41] R. Bartoldus, talk at FPCP, Philadelphia, PA, USA, May 2002.
[42] Y. Grossman, M. Neubert and A. L. Kagan, JHEP 9910 (1999) 029, [hepph/9909297].


[^0]:    ${ }^{1}$ There is also an exclusive, very rare mode related to this: $B_{s} \rightarrow \gamma \gamma$. It has also been evaluated in the SM [23] and supersymmetry [24].
    ${ }^{2}$ Important progress with a full NLO calculation of this asymmetry has been reported in [26].

[^1]:    ${ }^{3}$ These combinations are useful to calculate the phases of the CKM matrices, because the dependence on $\beta^{\prime}$ becomes trivial. Moreover, for a natural choice of parameters $\beta^{\prime}$ is negligibly small.

[^2]:    ${ }^{4}$ There are special cases where this is not so, which are not discussed here.

