

Electric and Magnetic Fluxes in $SU(2)$ Yang-Mills Theory

Lorenz von Smekal,^{a*} with Philippe de Forcrand^b

^a Institut für Theoretische Physik III, Universität Erlangen-Nürnberg, D-91058 Erlangen, Germany

^b ETH-Zürich, CH-8093 Zürich and CERN Theory Division, CH-1211 Geneva 23, Switzerland

We measure the free energies in $SU(2)$ of static fundamental charges and center monopoles. Dual to temporal center fluxes, the former provide a well-defined (dis)order parameter for deconfinement. In contrast, the monopole free energies vanish in the thermodynamic limit at all temperatures and are thus irrelevant for the transition.

1. Introduction

For pure $SU(N)$ gauge theory without quarks 't Hooft's gauge invariant electric and magnetic fluxes [1] describe, respectively, the effect of a static fundamental color charge and a center monopole in a finite volume. The partition function of a certain amount of electric/magnetic flux yields the free energy of a static electric/magnetic charge with boundary conditions to imitate the presence of its 'mirror' (anti)charge in a neighboring box along the direction of the flux.

Herein, we extend our previous study [2] of the purely electric fluxes in $SU(2)$ by measuring also the magnetic ones and combinations of the two.

2. Confinement: Twist vs. Electric Flux

Imposing 't Hooft's twisted b.c.'s fixes the total numbers modulo N of \mathbf{Z}_N -vortices through the 6 planes of the 4-dimensional Euclidean $1/T \times L^3$ box. In $SU(2)$ for example, twist in one plane corresponds to an ensemble with an odd number of \mathbf{Z}_2 -vortices through that plane. It differs by at least one from the periodic ensemble with an even number; and their free-energy difference is what it costs to add one such vortex to the system.

Intuitively it may help to assume that vortices can lower their free energy by spreading. At finite temperature $T > 0$ it is then clear that we need to distinguish between the twists of two types:

Magnetic twist is defined in a purely spatial plane in which the vortex can spread independent

of T . It fixes the conserved, \mathbf{Z}_N -valued and gauge-invariant magnetic flux \vec{m} through the plane. Correspondingly, its free energy, or that of a static center monopole, will vanish for $L \rightarrow \infty$ at all T as we demonstrate in the next section.

Twist in a temporal plane is classified by a vector $\vec{k} \in \mathbf{Z}_N^3$ parallel to its spatial edge. The other edge being of finite length $1/T$, the vortices are squeezed in such a plane more and more with increasing temperature. They can no-longer spread arbitrarily and this is what drives the confinement phase transition. In the thermodynamic limit, their free energy approaches zero (infinity) for T below (above) T_c [2–4]. This is the reversed behavior of a static fundamental charge.

As shown by 't Hooft, the partition functions of fixed units of electric and magnetic fluxes, $\vec{e}, \vec{m} \in \mathbf{Z}_N^3$, which we denote by $Z_e(\vec{e}, \vec{m})$, are obtained as 3-dimensional \mathbf{Z}_N -Fourier transforms, w.r.t. the temporal \vec{k} -twist, of those with twisted b.c.'s, $Z_k(\vec{k}, \vec{m})$. With the free energy of a purely electric flux one measures that of static fundamental charge in a perfectly well-defined (UV-regular) way [2]. This follows from the gauge-invariant definition of the Polyakov loop $P(\vec{x})$ in presence of temporal twist which entails,

$$Z_e(\vec{e}, 0) = \langle P(\vec{x}) P^\dagger(\vec{x} + L\vec{e}) \rangle_{L,T}, \quad (1)$$

relative to the no-flux ensemble (*i.e.*, the expectation value is taken therein and the l.h.s. is normalized such that $Z_e(0, 0) = 1$). In Ref. [2] we measured the ratios of partition functions $Z_k(\vec{k}, 0)$ with different \vec{k} -twists and the $Z_e(\vec{e}, 0)$ for the

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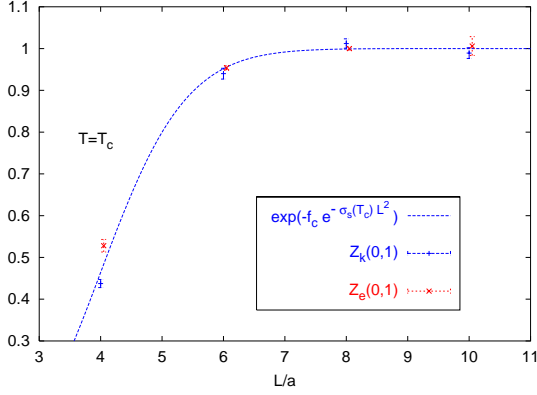


Figure 1. Finite volume partition functions of one magnetic flux at T_c , $Z_k(0,1)$ and $Z_e(0,1)$ relative to the periodic and no-flux ensemble, respectively.

various electric fluxes in $SU(2)$. We explicitly demonstrated the Kramers-Wannier duality, between the temporal center fluxes and the static charges, of the general pattern

$$Z_e(\vec{e}, 0) \xrightarrow{L \rightarrow \infty} \begin{cases} 0, & Z_k(\vec{k}, 0) \rightarrow 1, & T < T_c \\ 1, & Z_k(\vec{k}, 0) \rightarrow 0, & T > T_c \end{cases} \quad (2)$$

reflecting the different realization of the 3-dim. (electric) center symmetry in both phases. As expected from universality, this duality is the analogue in $SU(2)$ of that between the Wilson loops of the 3d \mathbf{Z}_2 -gauge theory and the 3d-Ising spins.

3. Magnetic Flux

There is no analogue in the 3d-spin systems to the magnetic fluxes, however. These are temporal 't Hooft loops of maximal size winding on the dual lattice around the temporal $1/T \times L$ planes. The magnetic current along the 't Hooft loop in this case creates a pair of static center monopoles at a distance L . Magnetic flux \vec{m} thus corresponds to one static center monopole inside the L^3 box with mirror image in a neighboring box along the direction of \vec{m} – the magnetic analogue of the Polyakov loop correlator on the r.h.s. of Eq. (1).

In agreement with this electric-magnetic duality, the temporal 't Hooft loops in $SU(2)$ show a screening behavior in both phases, for $T < T_c$ and for $T > T_c$ [3,5], just as spatial Wilson loops exhibit an area law in either case. One can thus

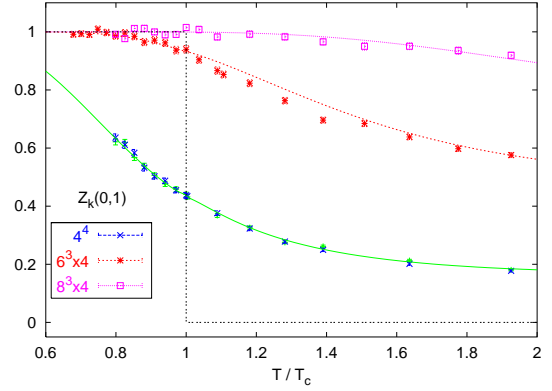


Figure 2. Magnetic flux versus temperature.

anticipate likewise that the monopole free energy in both phases must vanish in the thermodynamic limit. This would not exclude that it approaches a finite value at $T = T_c$ as in the case of electric charges and temporal center fluxes. We obtained in Ref. [2], *e.g.*, with $\vec{k} = (0, 0, 1)$ for one unit of the latter, $Z_k(\vec{k}, 0) = 0.54(1)$ at T_c in agreement with the universally related ratio in the Ising model. In particular, if the massless phase which the system passes through at T_c was selfdual, one would also expect a finite monopole free energy at $T = T_c$. This is, however, not the case, as shown in Fig. 1. The free energies of the magnetic fluxes in $SU(2)$ at T_c are well described by an exponential decrease $\propto \exp\{-\sigma_s(T_c)L^2\}$, where σ_s is the spatial string tension. As a check, our fit then yields for the spatial string tension at T_c ,

$$\sigma_s = (2.2 \pm 0.2) T_c^2, \text{ or } T_c/\sqrt{\sigma_s} = 0.675 \pm 0.03,$$

consistent with published values for the zero temperature $SU(2)$ string tension [6]. This screening of the temporal 't Hooft loops is similar to that of spatial ones in the low temperature phase:

In [2] we demonstrated that $-\ln Z_k(\vec{k}, 0) \propto \exp\{-\sigma(T)L/T\}$ for temporal \vec{k} -twist at $T < T_c$ and sufficiently large L . The important difference is the temperature dependence of the standard string tension $\sigma(T)$, as compared to the practically constant spatial one, σ_s here. As a result, the dominant behavior of the monopole free energies cannot be described by finite size scaling.

That they indeed vanish for all other T also, is seen in Fig. 2 where we plot the partition function

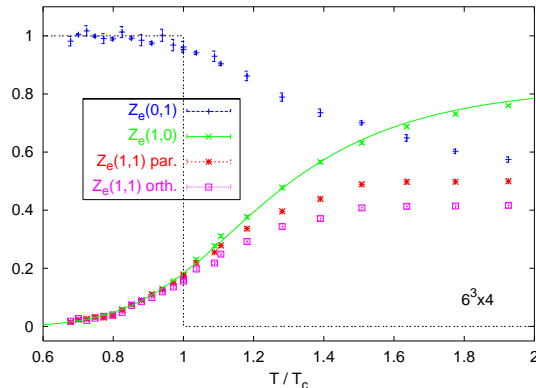


Figure 3. Electric and magnetic fluxes on the $6^3 \times 4$ lattice versus temperature.

$Z_k(0, \vec{m})$ of one magnetic twist (e.g., $\vec{m} = (0, 0, 1)$) versus temperature. While it is indistinguishable from a temporal twist on the symmetric 4^4 lattice as expected, with increasing spatial lattice size it rapidly and smoothly approaches unity over the whole temperature range of our simulations.

Our measurements of the flux partition functions $Z_e(\vec{e}, \vec{m})$ from two different lattice sizes are shown for various combinations of magnetic with electric fluxes in Figs. 3 and 4. The differences between parallel ($\vec{e} \cdot \vec{m}$ odd) and orthogonal ($\vec{e} \cdot \vec{m}$ even) fluxes on the smaller volumes have previously been observed at $T=0$ also [7]. They were then found in good agreement with semiclassical predictions. As the purely magnetic free energy vanishes, or $Z_e(0, \vec{m}) \rightarrow 1$ with increasing spatial size, these differences disappear and the partition functions approach those of purely electric fluxes for which we include the fit from [2] in the figures. Similarly, our data for all other combinations of twists and fluxes in $SU(2)$ indicate, at all T ,

$$Z_e(\vec{e}, \vec{m}) \xrightarrow{L \rightarrow \infty} Z_e(\vec{e}, 0), \quad Z_k(\vec{k}, \vec{m}) \xrightarrow{L \rightarrow \infty} Z_k(\vec{k}, 0).$$

Magnetic fluxes are irrelevant for the phase transition, and center monopoles always 'condense'. The corresponding 3-dim. magnetic center symmetry remains unbroken at all temperatures.

4. Conclusions

To summarize, changing the spatial twist is a gauge-invariant way of introducing one more

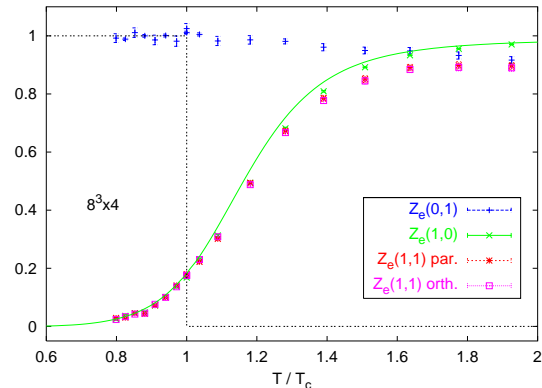


Figure 4. Same as in Fig. 3 for the $8^3 \times 4$ lattice.

static center monopole. The monopole free energy in the 3d Georgi-Glashow model was recently studied in an analogous way [8]. As therein, we observe for $SU(2)$ that the monopole free energy is zero, in the thermodynamic limit, at all temperatures. In contrast to the crossover behavior of the Georgi-Glashow model, however, in $SU(2)$ there is no indication of a plateau with finite monopole mass at intermediate volumes either.

Simulations were performed on the SGI Origin systems at RRZE, Erlangen, and ZHR, Dresden.

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