SIMULTANEOUS MATCHING OF DISPERSION FUNCTION AND TWISS PARAMETERS IN A TRANSFER LINE

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Abstract

Dispersion matching in a beam transfer line is an important issue in order to avoid blow-up and luminosity reduction. This is the case for the LHC beam, due to its small emittance and relatively large momentum spread. The dispersion matching can be performed with quadrupoles, but one has to impose the additional constraint of leaving the Twiss parameters unchanged, to preserve the betatron matching.

A first order pertubative approach, using the *MICADO* solver, has been applied to the problem of simultaneous betatron and dispersion matching. A theoretical derivation of the correction matrix, as well as simulated and experimental results are presented.

1. Introduction

The performance of the new generation of circular machines heavily relies on the injector chain performance. In order to achieve the design luminosity all sources of emittance growth should be avoided. This means that the transfer lines between the various machines should be carefully tuned in order to match the beam parameters at the injection point of the next circular machine.

If one neglects the emittance dilution produced by injection oscillations, which can be cured by properly steering the beam using the injection elements, the other sources of mismatch are the dispersion mismatch and the betatron mismatch.

In the first case the dispersion or its derivative at the end of the transfer line do not match the values for the circular machine due either to dipolar or quadrupolar errors along the transfer line, or wrong initial values at the entrance of the beam line. The resulting emittance growth is quite sensible. This effect is in fact similar to an injection mismatch: particles with different energies enter into the machine at a wrong position and/or angle and perform betatron oscillations. In the second case, the Twiss parameters at the injection point do not agree. The injected beam ellipse will rotate in phase space to match the machine parameters, hence producing beam dilution. Also in this case the source of mismatch can be found in the transfer line quadrupoles, or the initial values.

The correct approach to this problem is to find a strategy to simultaneously correct both dispersion and betatron mismatch. The simple technique of reducing the betatron mismatch without controlling the dispersion is not enough. Such a combined approach is imposed by the large momentum spread foreseen for the LHC.

In the present note, the correction matrix is derived using a perturbative approach and assuming that only quadrupoles can be used to correct the mismatch. A careful analysis of the high order terms is carried out, although the first order is usually enough to achieve good results. Different techniques are applied to the problem of minimising the mismatch: a MICADO approach and a full minimisation algorithm. These techniques have been bench-marked by using a model of the TT2 transfer line and also by performing some real measurements using the 26 GeV/c LHC-like beam: an overall reduction of the mismatch could be achieved in all the cases considered.

2. Twiss matching

The starting point for the analysis of the Twiss matching is the study of the evolution of the Twiss parameters along a transfer line. It is well-known [1] that the optical parameters between two sections of beam line evolve according to the following rules

$$\begin{aligned} \beta_{\mathrm{m}} &= C^2\beta_{\mathrm{c}} - 2CS\alpha_{\mathrm{c}} + S^2\gamma_{\mathrm{c}} && (1)\\ \alpha_{\mathrm{m}} &= -C\,C'\beta_{\mathrm{c}} + (C\,S' + S\,C')\alpha_{\mathrm{c}} - S\,S'\gamma_{\mathrm{c}}, && (2) \end{aligned}$$

Here the transfer matrix between corrector and monitor is given in terms of the so-called cosine-like C and sine-like S functions and their derivatives

$$\mathcal{T}_{c\to m} = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}. \tag{3}$$

Furthermore, the parameter $\gamma = (1+\alpha^2)/\beta$ has been introduced.

To quantify the effect of a betatron mismatch and to determine the approach to compensate such a mismatch, it is common use to insert a thin lens quadrupolar element at the location of the corrector. This will generate a variation in the optical parameters downstream and such a variation can be measured by using a beam monitor device. In this case the transfer matrix between the corrector and the monitor can be obtained from Eq. (3) by multiplying by the thin quadrupole transfer matrix. The final result is

$$\overline{\mathcal{T}}_{\mathrm{c}\to\mathrm{m}} = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \Delta k & 1 \end{pmatrix} = \begin{pmatrix} C\ + S\,\Delta k & S \\ C' + S'\Delta k & S' \end{pmatrix}, \tag{4}$$

where Δk is the integrated gradient of the error. By using the matrix (4) in Eq. (2), one can find the resulting Twiss parameters at the monitor location, namely

$$\begin{aligned} \bar\beta_{\mathrm{m}} &= \beta_{\mathrm{m}} + 2\,S(C\beta_{\mathrm{c}} - S\alpha_{\mathrm{c}})\Delta k + S^2\beta_{\mathrm{c}}(\Delta k)^2 && (5)\\ \bar\alpha_{\mathrm{m}} &= \alpha_{\mathrm{m}} + [2\,S\,S'\alpha_{\mathrm{c}} - (C\,S' + S\,C')\beta_{\mathrm{c}}]\Delta k - S\,S'\beta_{\mathrm{c}}(\Delta k)^2. && (6) \end{aligned}$$

It is customary to use the following variables to evaluate the betatron mismatch:

$$\left(\frac{(\Delta\beta)_{\mathrm{m}}}{\beta_{\mathrm{m}}},\quad \alpha_{\mathrm{m}}\frac{(\Delta\beta)_{\mathrm{m}}}{\beta_{\mathrm{m}}} - (\Delta\alpha)_{\mathrm{m}},\right) \tag{7}$$

in which $(\Delta\beta)_{\mathrm{m}} = \bar\beta_{\mathrm{m}} - \beta_{\mathrm{m}}$ and $(\Delta\alpha)_{\mathrm{m}} = \bar\alpha_{\mathrm{m}} - \alpha_{\mathrm{m}}$. Using the standard parametrisation of the transfer matrix $\mathcal{T}_{\mathrm{c}\to\mathrm{m}}$

$$\mathcal{T}_{\mathrm{c}\to\mathrm{m}} = \begin{pmatrix} \sqrt{\dfrac{\beta_{\mathrm{m}}}{\beta_{\mathrm{c}}}}(\cos\Delta\mu + \alpha_{\mathrm{c}}\sin\Delta\mu) & \sqrt{\beta_{\mathrm{m}}\beta_{\mathrm{c}}}\sin\Delta\mu \\ \dfrac{-(1+\alpha_{\mathrm{c}}\alpha_{\mathrm{m}})\sin\Delta\mu + (\alpha_{\mathrm{c}} - \alpha_{\mathrm{m}})\cos\Delta\mu}{\sqrt{\beta_{\mathrm{m}}\beta_{\mathrm{c}}}} & \sqrt{\dfrac{\beta_{\mathrm{c}}}{\beta_{\mathrm{m}}}}(\cos\Delta\mu - \alpha_{\mathrm{m}}\sin\Delta\mu) \end{pmatrix}, \tag{8}$$

where $\Delta\mu$ represents the phase-advance between the corrector and the monitor, Eq. (8) together with the expression (6), allows to recast the mismatch vector in the following form:

$$\frac{(\Delta\beta)_\mathrm{m}}{\beta_\mathrm{m}} = \beta_\mathrm{c}\sin 2\Delta\mu \;\; \Delta k + \frac{{\beta_\mathrm{c}}^2}{2}(1-\cos 2\Delta\mu) \;\; (\Delta k)^2 \tag{9}$$

$$\alpha_\mathrm{m}\frac{(\Delta\beta)_\mathrm{m}}{\beta_\mathrm{m}} - (\Delta\alpha)_\mathrm{m} = \beta_\mathrm{c}\cos 2\Delta\mu \;\; \Delta k + \frac{{\beta_\mathrm{c}}^2}{2}\sin 2\Delta\mu \;\; (\Delta k)^2. \tag{10}$$

The expression (10) of the mismatch vector contains linear and non linear terms in the quadrupolar gradient Δk. The linear part represent the well-know contribution to the β- and α-functions [2]. The same equations hold true also for the other plane, provided the sign of Δk is changed.

The nonlinear terms are usually dropped as the whole approach is based on a linear approximation. In fact, one should compute the transfer matrix from the first quadrupolar corrector to some monitor, including all the correctors in between, namely

$$\overline{\mathcal{T}}_{\mathrm{c}\to\mathrm{m}} = \prod_{i=1}^{N_m}\begin{pmatrix} C_i + S_i\,\Delta k_i & S_i \\ C_i' + S_i'\Delta k_i & S_i' \end{pmatrix} = \begin{pmatrix} \hat{C} & \hat{S} \\ \hat{C}' & \hat{S}' \end{pmatrix}, \tag{11}$$

or

$$\overline{\mathcal{T}}_{\mathrm{c}\to\mathrm{m}} = \prod_{i=1}^{N_m}\begin{pmatrix} C_i & S_i \\ C_i' & S_i' \end{pmatrix} + \Delta k_i \begin{pmatrix} S_i & 0 \\ S_i' & 0 \end{pmatrix}, \tag{12}$$

where N_m is the number of correctors between the beginning of the beam line and the monitor m. It is quite easy to prove by induction that the quantities $\hat{C}, \hat{S}, \hat{C}'$ and $\hat{S}'$ are polynomial functions of the gradients Δk_i. More precisely one has:

$$\begin{aligned} \hat{C} &= \hat{C}_{\mathrm{N_m}}(\Delta k_1, \Delta k_2, \cdots \Delta k_{\mathrm{N_m}}) & \qquad \hat{S} &= \hat{S}_{\mathrm{N_m}-1}(\Delta k_2, \Delta k_3, \cdots \Delta k_{\mathrm{N_m}}) \\ \hat{C}' &= \hat{C}'_{\mathrm{N_m}}(\Delta k_1, \Delta k_2, \cdots \Delta k_{\mathrm{N_m}}) & \qquad \hat{S}' &= \hat{S}'_{\mathrm{N_m}-1}(\Delta k_2, \Delta k_3, \cdots \Delta k_{\mathrm{N_m}}), \end{aligned}$$

where $\hat{C}_\mathrm{j}(x_1, x_2, \cdots, x_n)$ is a polynomial of order j in the variables $x_1, x_2, \cdots, x_n$. To obtain the correct expression for the propagation of the β- and α-functions, taking into account the nonlinear terms in the gradients Δk_i, one should evaluate the structure of the polynomials $\hat{C}, \hat{S}, \hat{C}', \hat{S}'$. By using the symbol $\lfloor\cdot\rfloor_i$ to represent the homogeneous polynomial of order i in the quadrupolar gradients, it is possible to recast Eq. (12) in the following form

$$\begin{aligned} \lfloor\overline{\mathcal{T}}_{\mathrm{c}\to\mathrm{m}}\rfloor_0 &= \prod_{i=1}^{N_m}\begin{pmatrix} C_i & S_i \\ C_i' & S_i' \end{pmatrix} \\ \lfloor\overline{\mathcal{T}}_{\mathrm{c}\to\mathrm{m}}\rfloor_1 &= \sum_{i=1}^{N_m}\Delta k_i\;\mathcal{T}_{\mathrm{i+1}\to\mathrm{m}}\begin{pmatrix} S_i & 0 \\ S_i' & 0 \end{pmatrix}\mathcal{T}_{\mathrm{c}\to\mathrm{i}} \\ \lfloor\overline{\mathcal{T}}_{\mathrm{c}\to\mathrm{m}}\rfloor_2 &= \sum_{i,j=1\;j>i}^{N_m}\Delta k_i\;\Delta k_j\;\mathcal{T}_{\mathrm{j+1}\to\mathrm{m}}\begin{pmatrix} S_j & 0 \\ S_j' & 0 \end{pmatrix}\mathcal{T}_{\mathrm{i+1}\to\mathrm{j}}\begin{pmatrix} S_i & 0 \\ S_i' & 0 \end{pmatrix}\mathcal{T}_{\mathrm{c}\to\mathrm{i}} \\ \vdots\;&\vdots \end{aligned}$$

where $\mathcal{T}_{\mathrm{i}\to\mathrm{j}}$ represents the transfer matrix between corrector i and corrector j.

To find out the quantities $\lfloor\beta\rfloor_i$, $\lfloor\alpha\rfloor_i$ it is simply a matter of replacing C, S, C', S' with the quantities $\hat{C}, \hat{S}, \hat{C}', \hat{S}'$ in Eq. (2) and grouping terms of the same order in the gradients Δk_i, namely

$$\begin{aligned}\lfloor\beta_{\mathrm{m}}\rfloor_i &= \beta_{\mathrm{c}}\sum_{j=0}^{i}\lfloor\hat{C}\rfloor_j\lfloor\hat{C}\rfloor_{i-j} - 2\alpha_{\mathrm{c}}\sum_{j=0}^{i}\lfloor\hat{C}\rfloor_j\lfloor\hat{S}\rfloor_{i-j} + \gamma_{\mathrm{c}}\sum_{j=0}^{i}\lfloor\hat{S}\rfloor_j\lfloor\hat{S}\rfloor_{i-j} \\ \lfloor\alpha_{\mathrm{m}}\rfloor_i &= -\beta_{\mathrm{c}}\sum_{j=0}^{i}\lfloor\hat{C}\rfloor_j\lfloor\hat{C}'\rfloor_{i-j} + \alpha_{\mathrm{c}}\sum_{j=0}^{i}(\lfloor\hat{C}\rfloor_j\lfloor\hat{S}'\rfloor_{i-j} + \lfloor\hat{S}\rfloor_j\lfloor\hat{C}'\rfloor_{i-j}) - \gamma_{\mathrm{c}}\sum_{j=0}^{i}\lfloor\hat{S}\rfloor_j\lfloor\hat{S}'\rfloor_{i-j},\end{aligned} \tag{13}$$

where it has been used the following expression for the product of homogeneous polynomials $\hat{C}, \hat{S}$ of degree N, M respectively:

$$\hat{C}\hat{S} = \sum_{i=0}^{N}\sum_{j=0}^{M}\lfloor\hat{C}\rfloor_i\lfloor\hat{S}\rfloor_j = \sum_{i=0}^{NM}\sum_{j=0}^{i}\lfloor\hat{C}\rfloor_j\lfloor\hat{S}\rfloor_{i-j}. \tag{14}$$

3. Dispersion matching

When bending magnets are present in the transfer line, the evolution of the dispersion function should be included in the formalism used to correct a mismatch in the optical parameters. The approach is now based on 3×3 transfer matrices [3]. The propagation of the dispersion function from a corrector location to a monitor placed downstream can be written using the matrix

$$\mathcal{T}_{\mathrm{c}\to\mathrm{m}} = \begin{pmatrix} C & S & \xi \\ C' & S' & \xi' \\ 0 & 0 & 1 \end{pmatrix}. \tag{15}$$

The 2×2 sub-matrix represents the transfer matrix for the betatronic motion between corrector and monitor. The quantities ξ, ξ' are different from zero only when bending magnets are present in the transfer line.

It is customary to introduce a quantity $\mathcal{W}$ defined as

$$\mathcal{W} = \frac{1}{\beta}[D^2 + (\alpha D + \beta D')^2], \tag{16}$$

similar to the Courant-Snyder invariant. In a bending-free region of a transfer line $\mathcal{W}$ is invariant and it is called *Dispersion invariant* [1, 4].

It is possible to repeat what was done for the betatron matching. The dispersion at a given monitor is linked to the value at the location of an upstream corrector (a normal quadrupole) and the transfer matrix of the section in between. The presence of a quadrupolar error, simulated by a thin lens element, modifies the transfer matrix, according to the following

$$\overline{\mathcal{T}}_{\mathrm{c}\to\mathrm{m}} = \begin{pmatrix} C & S & \xi \\ C' & S' & \xi' \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ \Delta k & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} C + S\,\Delta k & S & \xi \\ C' + S'\Delta k & S' & \xi' \\ 0 & 0 & 1 \end{pmatrix}. \tag{17}$$

The modified dispersion at the monitor location is then given by

$$\begin{aligned}\bar{D}_{\,\mathrm{m}} &= D_{\,\mathrm{m}} + S\;\; D_{\mathrm{c}}\,\Delta k &(18)\\ \bar{D}'_{\mathrm{m}} &= D'_{\mathrm{m}} + S'\; D_{\mathrm{c}}\,\Delta k, &(19)\end{aligned}$$

where

$$\begin{aligned}D_{\,\mathrm{m}} &= C\;\; D_{\mathrm{c}} + S\;\; D'_{\mathrm{c}} + \xi &(20)\\ D'_{\mathrm{m}} &= C'\; D_{\mathrm{c}} + S'\; D'_{\mathrm{c}} + \xi'. &(21)\end{aligned}$$

In this case, the existence of the dispersion invariant suggests to define a dispersion mismatch vector as

$$\left(\frac{(\Delta D)_{\mathrm{m}}}{\sqrt{\beta_{\mathrm{m}}}},\quad \alpha_{\mathrm{m}}\frac{(\Delta D)_{\mathrm{m}}}{\sqrt{\beta_{\mathrm{m}}}} + \sqrt{\beta_{\mathrm{m}}}(\Delta D')_{\mathrm{m}}\right), \tag{22}$$

where $(\Delta D)_{\mathrm{m}} = \bar{D}_{\mathrm{m}} - D_{\mathrm{m}}$ and $(\Delta D')_{\mathrm{m}} = \bar{D}'_{\mathrm{m}} - D'_{\mathrm{m}}$.

Once again, by using the form (8) for the transfer matrix between the corrector and the monitor, it is possible to obtain the expression of the dispersion mismatch vector

$$\begin{aligned}\frac{(\Delta D)_{\mathrm{m}}}{\sqrt{\beta_{\mathrm{m}}}} &= \sqrt{\beta_{\mathrm{c}}}D_{\mathrm{c}}\sin\Delta\mu\;\;\Delta k &(23)\\ \alpha_{\mathrm{m}}\frac{(\Delta D)_{\mathrm{m}}}{\sqrt{\beta_{\mathrm{m}}}} + \sqrt{\beta_{\mathrm{m}}}(\Delta D')_{\mathrm{m}} &= \sqrt{\beta_{\mathrm{c}}}D_{\mathrm{c}}\cos\Delta\mu\;\;\Delta k. &(24)\end{aligned}$$

In the case of the dispersion mismatch, the vector is a linear function of the corrector strength. The same expressions hold for a dispersive transfer line.

The approach used in the computation of the nonlinear terms depending on the quadrupolar gradients in the expression of the Twiss parameters can be applied even for the dispersion function. In this case the knowledge of the polynomial functions $\hat{C}, \hat{S}, \hat{C}', \hat{S}'$ allows to derive the development of D, D' as a function of Δk_i, namely

$$\begin{aligned}\lfloor D_{\,\mathrm{m}}\rfloor_i &= D_{\mathrm{c}}\lfloor\hat{C}\rfloor_i + D'_{\mathrm{c}}\lfloor\hat{S}\rfloor_i + \delta_{i,0}\xi\\ \lfloor D'_{\mathrm{m}}\rfloor_i &= D_{\mathrm{c}}\lfloor\hat{C}'\rfloor_i + D'_{\mathrm{c}}\lfloor\hat{S}'\rfloor_i + \delta_{i,0}\xi',\end{aligned} \tag{25}$$

where $\delta_{i,0}$ is the Kronecker delta.

4. Correction strategy

The first approach consists in dropping the nonlinear terms in the Eqs. (14) and (26). Then one can build up a correction matrix in the standard way (see for instance Ref. [2])

$$\begin{pmatrix} \dfrac{(\Delta\beta)_1^H}{\beta_1^H} \\ \alpha_1^H \dfrac{(\Delta\beta)_1^H}{\beta_1^H} - (\Delta\alpha)_1^H \\ \dfrac{(\Delta D)_1^H}{\sqrt{\beta_1^H}} \\ \alpha_1^H \dfrac{(\Delta D)_1^H}{\sqrt{\beta_1^H}} + \sqrt{\beta_1^H}(\Delta D')_1^H \\ \vdots \\ \vdots \\ \dfrac{(\Delta D)_{\mathrm{N_m}}^V}{\sqrt{\beta_{\mathrm{N_m}}^V}} \\ \alpha_{\mathrm{N_m}}^V \dfrac{(\Delta D)_{\mathrm{N_m}}^V}{\sqrt{\beta_{\mathrm{N_m}}^V}} + \sqrt{\beta_{\mathrm{N_m}}^V}(\Delta D')_{\mathrm{N_m}}^V \end{pmatrix} = \mathcal{C} \begin{pmatrix} \Delta k_1 \\ \Delta k_2 \\ \vdots \\ \vdots \\ \Delta k_{\mathrm{N_c}} \end{pmatrix}, \tag{26}$$

where $\mathcal{C}$ is an $8\mathrm{N_m} \times \mathrm{N_c}$. Here $\mathrm{N_m}$ stands for the number of monitors, while $\mathrm{N_c}$ is the number of correctors. The matrix elements $\mathcal{C}_{i,j}$ can be written as

$$\begin{aligned} \mathcal{C}_{i,j} &= \beta_{\mathrm{j}}^H \sin 2\Delta\mu_{\mathrm{ij}}^H & \mathcal{C}_{i+4,j} &= \beta_{\mathrm{j}}^V \sin 2\Delta\mu_{\mathrm{ij}}^V \\ \mathcal{C}_{i+1,j} &= \beta_{\mathrm{j}}^H \cos 2\Delta\mu_{\mathrm{ij}}^H & \mathcal{C}_{i+5,j} &= \beta_{\mathrm{j}}^V \cos 2\Delta\mu_{\mathrm{ij}}^V \\ \mathcal{C}_{i+2,j} &= \sqrt{\beta_{\mathrm{j}}^H} D_{\mathrm{j}}^H \sin \Delta\mu_{\mathrm{ij}}^H & \mathcal{C}_{i+6,j} &= \sqrt{\beta_{\mathrm{j}}^V} D_{\mathrm{j}}^V \sin \Delta\mu_{\mathrm{ij}}^V \\ \mathcal{C}_{i+3,j} &= \sqrt{\beta_{\mathrm{j}}^H} D_{\mathrm{j}}^H \cos \Delta\mu_{\mathrm{ij}}^H & \mathcal{C}_{i+7,j} &= \sqrt{\beta_{\mathrm{j}}^V} D_{\mathrm{j}}^V \cos \Delta\mu_{\mathrm{ij}}^V, \end{aligned}$$

for $1 \le j \le \mathrm{N_c}$ and $i = 8(l-1)+1, 1 \le l \le \mathrm{N_m}$. Note that one should in general include a weight factor between the betatronic part of the matrix and the dispersion part. This is obvious since the betatronic matrix elements have unit m^{-2}, while the dispersion part has unit $m^{-3/4}$. For all the measurements and simulations presented in this note this weight factor was set to unity, which was found to work fine, but that is just one possible choice. It is likely that a bad choice of weight factor will cause the minimisation procedure to diverge.

5. MICADO and MINIMO

The corrector strengths can be computed using a number of different algorithms. The algorithms discussed in this note are the well-known *MICADO* [5], and a slower but more general algorithm, which has been named *MINIMO* [6].

In principle, assuming that the response matrix is non-singular the number of free parameters (corrector magnets) have to be the same as the number of constraints (monitors) for the linear problem to be exactly solved. However, in general a very good approximate solution can be achieved by using only a small subset of correctors, provided that the subset is cleverly chosen. *MICADO* and *MINIMO* are two algorithms that have been developed to choose such a subset.

For a given subset of correctors, the optimal solution can be defined, for example, as the least-square fit or a *SVD* fit with a certain tolerance. The least-square fit suppresses all null-space corrections, that is, linear combinations of individual corrections that in total gives no effect on the monitors. The *SVD* fit also suppresses near-to-null space corrections, which are combinations of individual corrections that give a very small effect on the monitors. The definition of what is a very small correction is given by the tolerance level. The correction is computed using the pseudo-inverse of the response matrix, where all the singular values smaller than the tolerance are set to zero.

MICADO starts out by testing all the possible subsets containing only one corrector and finding the best one. Then it tests all subsets that can be obtained by adding one more corrector to this subset. In each iteration, one corrector is thus added, and the time to find a correction using a subset of n correctors out of a total of N available is approximately proportional to $(N^2n - Nn^2)/2$.

The assumption that is made in the *MICADO* algorithm, that the optimal set containing k correctors is a subset of the optimal set containing $k+1$ correctors is not true in general. It is in fact easy to construct counter-examples. The assumption is however a rather good approximation in most cases, and it significantly speeds up the algorithm.

MINIMO on the other hand, is a brute-force method. It checks all the possible solutions, without assumptions. Since the number of possible subsets of a certain number of available correctors can be very large, this method is slow, and in some cases, utterly useless because of the combinatorial growth of computation time. In fact, the time needed to find a correction containing n out of N correctors is $N!/(n!(M-n)!)$. However, in the case of transverse matching, the number of available correctors is rather small (typically < 10). Thus the computation time is acceptable, and *MINIMO* can be considered as an option. An implementation of *MINIMO* in *Mathematica* [7] have thus been done for the purposes of these tests.

6. Simulation results

Since the validation measurements are time-consuming and at least semi-destructive, simulations have been performed to test the method, and to quantify the difference between *MICADO* and *MINIMO*. It is not obvious how to compare the convergence properties of the two methods, because in reality an operator would be supervising the minimisation process and change the free parameter (the number of correctors in each iteration) if necessary. The chosen strategy consists of making a relatively large number of simulations with random initial errors, binning the simulation results according to the size of the initial error and plotting for each bin the average residual error as a function of both the number of correctors used in each iteration, and the number of iterations. All the simulations were carried out on the model of the same transfer line used to perform the real measurements [8]. The results show no significant difference between the two algorithms for small and moderate initial errors (Figs. 1 and 2).

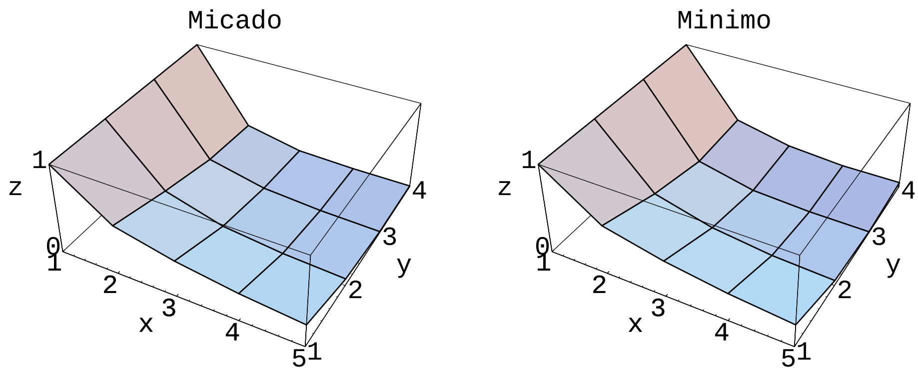

Fig. 1: Simulation results averaged over 45 seeds. The initial rms error belongs to the interval [0.2,0.4]. The vertical axes represents the average error, the x axes the iteration number, the y axes the number of correctors used in each iteration.

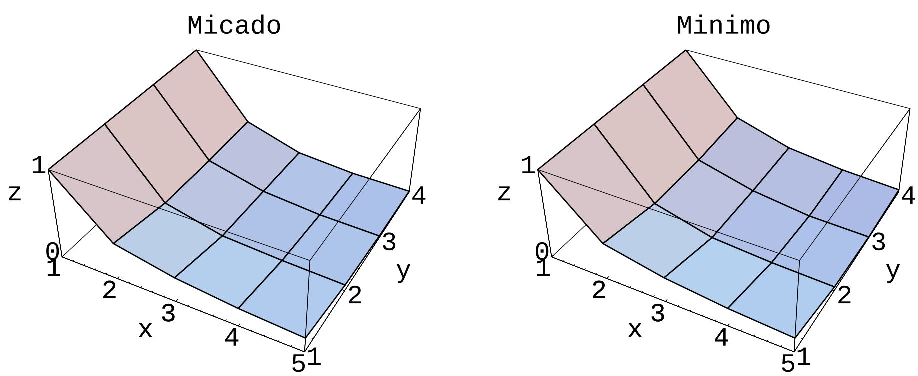

Fig. 2: Simulation results averaged over 17 seeds. The initial rms error belongs to the interval [1.0,1.2]. The axes are the same as in Fig. 1.

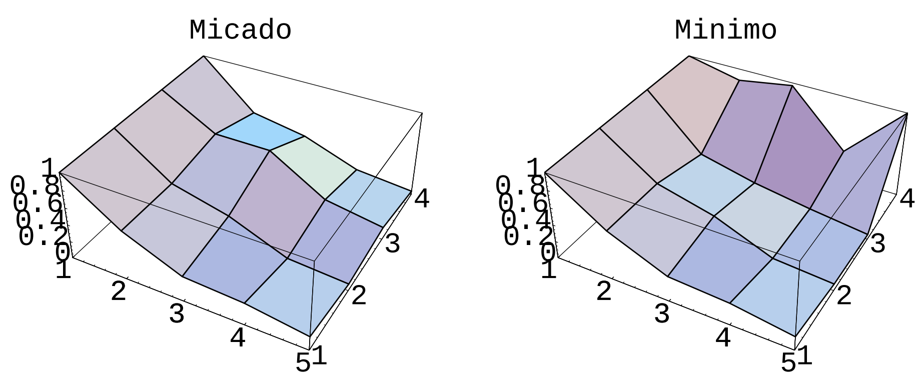

Fig. 3: Simulation results averaged over 2 seeds. The initial rms error belongs to the interval [1.8,2.0]. The axes are the same as in Fig. 1.

7. Experimental results

Tests of the method have been performed in the CPS-SPS transfer line. This line is divided into two parts: the TT2 line and TT10 line. TT2 transports the beam from the extraction point of the PS machine to the TT10 part, which, in turn, connects the transfer line to the SPS injection point. At the junction of the two lines, the beam is deflected about 81 mrad to the right. Due to the difference in height between the PS and SPS, a vertical deflection angle of about 60 mrad is imposed at the entrance of TT10 and then cancelled before injection in the SPS.

Three Secondary Emission Monitors are installed both in TT2 and in TT10 section. These two sets of monitors are routinely used to perform emittance and *Twiss* parameters measurement in both lines. For this purpose, it is used the standard method, with the dispersion measured by performing an energy shift.

A 26 GeV/c proton beam is extracted from the PS machine using a kicker magnet and delivered to the SPS through the TT2/TT10 line. The beam intensity is $\approx 1.1 \times 10^{12}$ ppp, distributed into 16 bunches 15 ns long with a momentum spread δ of about 0.6×10^{-3} at 2σ. The beam is extracted on a flat top in one turn (fast extraction) by the standard scheme based on bumper, septum and kicker. The nominal setting of these elements is reported in Table 1.

Element	Value
`PE.KFA71` [kV]	710
`PE.SMH16` [A]	28200
`PE.BSW16` [A]	1350
`PE.QKE16` [A]	1550
`PE.DHZ15` [A]	250

Table 1: Nominal setting of the extraction elements for the 26 GeV/c proton beam.

In Table 2 are listed the main parameters of the proton beam used during the matching studies.

p [GeV/c]	26
ε_H [μm] (normalised, rms)	3.0
ε_V [μm] (normalised, rms)	3.0
dp/p	10^{-3}
bunch length [ns] (4σ)	5-7
ε_l [eVs]	0.1

Table 2: Parameters of the proton beam used to study the simultaneous matching of betatron and dispersion functions.

The experimental validation of the optimisation scheme was performed in steps. In all cases the optical parameters was first measured, then an error was introduced on one or several quadrupoles, and *MICADO* or *MINIMO* was used to try to recover the initial values. First, the result of the proposed corrections was measured for different number of correctors, and compared to the linear prediction supplied by *MICADO* and *MINIMO*. The results are shown in Fig. 4 and Fig. 5 and show a fairly good agreement between measurement and prediction, with an apparent tendency for *MINIMO* to diverge when many correctors are used.

Then test were made to iteratively reduce the error down to zero. These results, using one corrector per iteration are shown in Fig. 6. In the case of one corrector per iteration, *MICADO* and *MINIMO* always give the same result. A test using three correctors per iteration was made to try to see a difference between the two algorithms, but no significant difference was found (see Figs. 7 and 8).

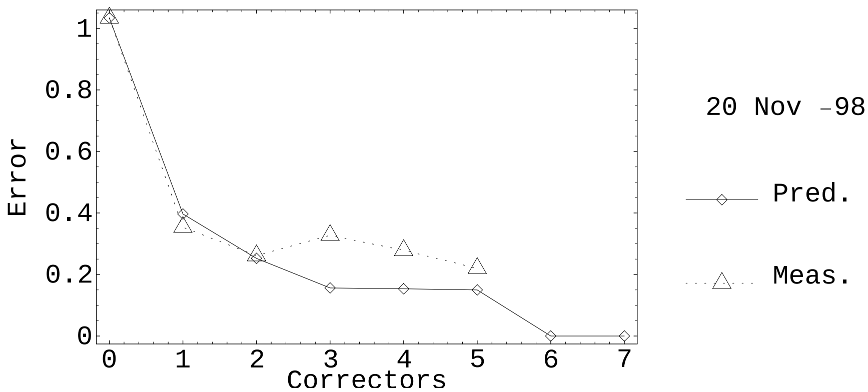

Fig. 4: The measured correction result and the *MICADO* prediction for a random initial error are shown as a function of the number of correctors used.

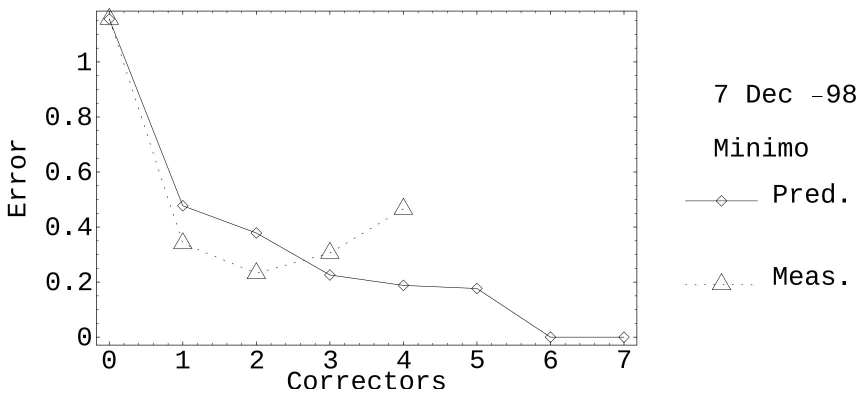

Fig. 5: The measured correction result and the *MINIMO* prediction for a random initial error are shown as a function of the number of correctors used.

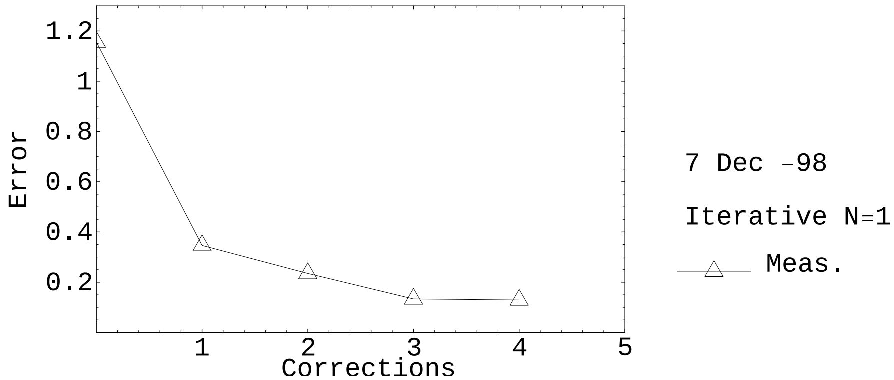

Fig. 6: The measured correction result and the *MICADO* prediction for a random initial error are shown as a function of the number of correctors used.

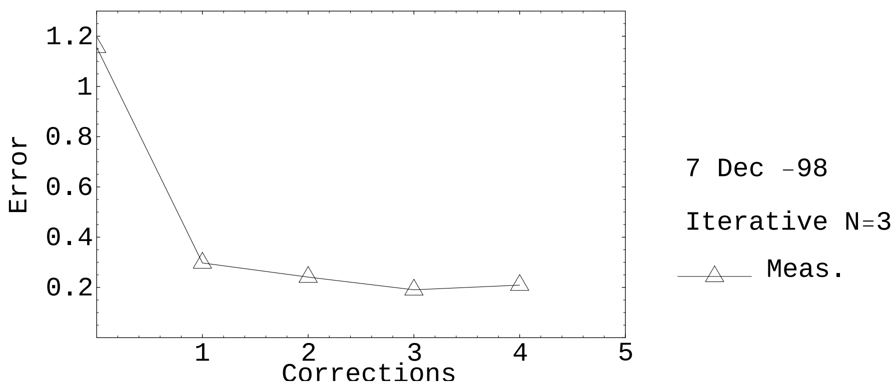

Fig. 7: The measured correction result and the *MICADO* prediction for a random initial error are shown as a function of the number of correctors used.

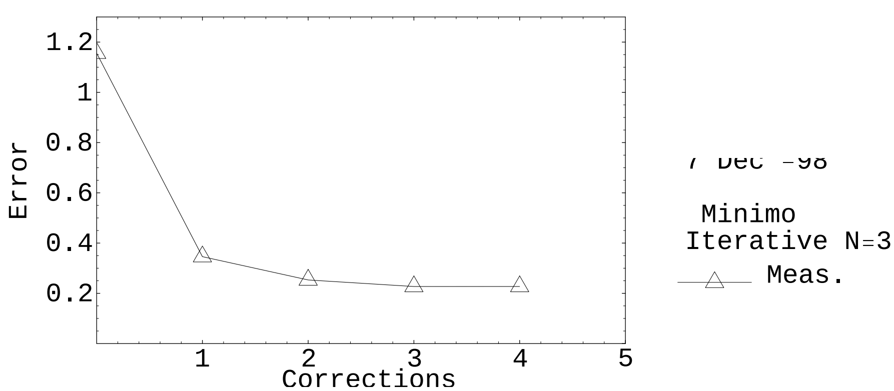

Fig. 8: The measured correction result and the *MINIMO* prediction for a random initial error are shown as a function of the number of correctors used.

8. Conclusions

We have found that both *MICADO* and *MINIMO* works well for combined dispersion and betatron matching, using the correction matrix derived in this note. A comparison between the two algorithms, show no major advantage of using *MINIMO* for this kind of correction, and since *MICADO* is faster it should be natural to choose this algorithm.

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