PARTICLE-IN-CELL SIMULATION OF BEAM-ELECTRON CLOUD INTERACTIONS

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Abstract

We study single bunch effects caused by an electron cloud using a simulation based on the particle-in-cell (PIC) method. The PIC method has been used for a strong-strong simulation of beam-beam effects. We apply the method to the electron cloud issues.

A head-tail effect has been discussed using analysis based on coherent modes, if anything. We can treat both of coherent and incoherent effects fairly in the PIC model. We study which or how phenomenon is important in the beam-electron cloud interactions.

1 INTRODUCTION

We discuss single bunch effect of beam-electron cloud interactions in KEKB-LER and CERN-SPS. An electron cloud with a density ρ_e of $\sim 10^{12} m^{-3}$ is considered. We study the beam-cloud effect by simulating the coulomb interaction using the Particle in Cell (PIC) method. Our discussion is limited to vertical motion in this paper.

The parameters are listed in Table 1.

Table 1: Basic parameters of the KEKB LER and CERN SPS

variable	KEKB-LER	SPS
particle type	e^+	p
circumference	3016 m	6900 m
beam energy	3.5 GeV	26 GeV
bunch population	3.3×10^{10}	7.5×10^{10}
bunch spacing	8 ns	—
rms beam sizes	0.42 mm	5 mm
	0.06 mm	3 mm
bunch length	4 mm	30 cm
rms energy spread	0.0007	0.0011
slippage factor	1.8×10^{-4}	5.78×10^{-4}
chromaticity	$4/8$	—
synchrotron tune	0.015	0.006
betatron tune	∼46	26.7
average beta function	15 m	40 m

2 ALGORITHM OF SIMULATION

We study beam-electron cloud interaction, in which the electron cloud is initialized uniformly in transverse plane with a size at beginning of collision. The interaction is described by two dimensional coulomb force because beam is relativistic. Magnetic field of beam is neglected, since it is smaller than the electric force by a factor v_e/c, where v_e is an electron velocity. Electric force of electron cloud itself is also neglected, since electron density is much smaller than beam density.

We use a lowest order integrator, which is combination of a thin lens kick and a drift in free space (or a lattice linear transformation), to integrate the motion in a time step. We first discuss the time step of the integrator. Electrons oscillate in the beam potential with an angular frequency [1, 2, 3]

$$\omega_{c,y}^2 = \frac{2\lambda_b r_e c^2}{(\sigma_x + \sigma_y)\sigma_y}, \tag{1}$$

where $\lambda_b (m^{-1})$ is line density of beam, and σ_x and σ_y are transverse beam size. Electrons interact with beam during its passage. In our parameter, rotation angles in the phase space of electron, $\omega\sigma_z/c$, are 2.8 and 1.2 for KEKB-LER and SPS, respectively. To integrate the electron motion correctly, a bunch should be sliced more than $\omega\sigma_z/c$. We sliced a bunch into 30 in both of KEKB-LER and SPS.

To integrate beam motion, tune shift due to electron cloud is index for decision of the time step of integrator. The tune shift is expressed by

$$\Delta\nu_y = \frac{r_e}{\gamma}\rho_e L\langle\beta_y\rangle \tag{2}$$

The tune shifts are 0.012 and 0.015 for KEKB-LER and SPS, respectively, for $\rho_e = 10^{12} m^{-3}$. The tune shift is not surprising, but is meaningful. We have to care a unphysical resonance behavior, which is caused by a rough time step for a large tune shift. It is smaller than beam-beam tune shift ($0.03 \sim 0.05$) of general e^+e^- circular colliders. [1] We use single interaction per revolution; that is electron cloud is localized at a position in a ring, in this paper. However, during the interaction, the electron cloud is focused at the beam center by its force, with the result that the tune shift of beam is larger [1, 3, 4]. The fact may require more interaction points. We will discuss this subject elsewhere.

We next discuss interactions of bunched beam and electron cloud localized at a position. The interactions are calculated by using a strong-strong model. The positron (proton) beam and electron cloud are expressed by macroparticles. We use the same algorithm as the strong-strong simulation of the beam-beam interactions [5]. The difference is that electrons in cloud are initialized in every interactions with beam and are non-relativistic. Electric potentials induced by beam and cloud are calculated, and macroparticles in beam (cloud) are kicked by the electric potential of electron cloud (beam).

[1] A resonant behavior for beam-beam effect is physical, because beams interact at a collision point(s).

We use Green function to calculate the electric potentials. The potential is expressed as an integral over the Green function,

$$\phi(\boldsymbol{r}) = \int d\boldsymbol{r}' G(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}') \qquad G(\boldsymbol{r}) = ln|\boldsymbol{r}|. \quad (3)$$

The green function guarantee an asymptotic boundary condition $\phi \to 0$ for $\boldsymbol{x} \to \infty$. We use the discrete Fast Fourier Transform (FFT) in two dimensional space to perform the integral. The transverse region containing the beam particle distribution is meshed ($n_x \times n_y$), and $G(\boldsymbol{x})$ and $\rho(\boldsymbol{x})$ are mapped on the mesh. The FFT's of $G(\boldsymbol{x})$ and $\rho(\boldsymbol{x})$ are evaluated, and their convolution gives the potential: i.e.

$$\hat{\rho}(\boldsymbol{k}) = \int \rho(\boldsymbol{x}) \exp(i\boldsymbol{k}\cdot\boldsymbol{x}) d\boldsymbol{x} \quad (4)$$

$$\hat{G}(\boldsymbol{k}) = \int G(\boldsymbol{x}) \exp(i\boldsymbol{k}\cdot\boldsymbol{x}) d\boldsymbol{x} \quad (5)$$

$$\phi(\boldsymbol{x}) = \frac{1}{(2\pi)^2} \int \hat{G}(\boldsymbol{k})\hat{\rho}(\boldsymbol{k}) \exp(-i\boldsymbol{k}\cdot\boldsymbol{x}) d\boldsymbol{k}. \quad (6)$$

FFT has a gain for speed of calculation compare than direct integration of Eq.(3). To evaluate Eq.(3) for non-periodic functions of the meshed space, $\rho(\boldsymbol{x})$ and $G(\boldsymbol{x})$, we use a $2n_x \times 2n_y$ mesh and put macro-particles in only an $n_x \times n_y$ region. The potential, which is given only on the mesh points, can be interpolated to other points using spline fitting.

The beam-cloud force is evaluated from the potential as follows,

$$\frac{d\boldsymbol{p}_p}{ds} = -\frac{2r_e}{\gamma}\frac{\partial\phi_e(\boldsymbol{r})}{\partial\boldsymbol{r}} \qquad \frac{d\boldsymbol{p}_e}{dt} = -2r_e c\frac{\partial\phi_p(\boldsymbol{r})}{\partial\boldsymbol{r}}. \quad (7)$$

3 SIMULATION RESULTS FOR KEKB-LER

We show simulation results for KEKB-LER obtained by following condition. 100,000 macro-particles for electron cloud is used. Electrons in the cloud are initialized to be an uniform distribution in transverse plane with a size of $3\sigma_x \times 7\sigma_y$ at the beginning of a collision with beam. Beam, which is sliced into 30, is represented by 300,000 macro-particles. Every slices include about 10,000 macro-particles. Transverse region containing macro-particles is meshed into $n_x \times n_y = 64 \times 256$. The vertical mesh is more than horizontal one because vertical instability is focused now. Electron cloud and beam sliced into 30 interact with each other one by one. Electrons drift between interactions of each slice.

After the collision of the electron cloud and 30 slices of beam, beam is transferred by the revolution matrix, and electron cloud is initialized again. We consider synchrotron oscillation. Beam is sorted into 30 slices longitudinally in every collisions. We do not take into account of the radiation damping of beam. We investigated the chromaticity dependence of the instability. The chromaticity is represented by a single kick

$$H_I = \frac{Q'}{2\pi}\left(\beta p_y^2 + y^2/\beta\right)\delta p/p. \quad (8)$$

3.1 Zero chromaticity

Figure 1 shows the beam size and dipole moment, and cloud dipole moment for $\nu_s = 0$ and $\nu_s = 0.015$. In the upper picture (a), which is for no-synchrotron oscillation, a beam break-up of tail part is observed. The actual beam size is integrating over Lower picture (b) is for $\nu_s = 0.015$. A complex head-tail mode, which is closed to 1 mode, is seen. Beam size blow-up along longitudinal direction $\langle y_b^2\rangle_z - \langle y_b\rangle_z^2$ is also seen in both case.

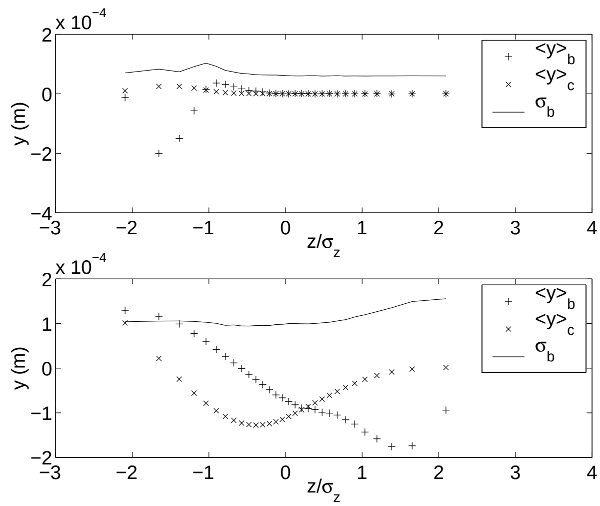

Figure 1: Beam dipole moment and size, and cloud dipole moment for without and with synchrotron oscillation, (a) $\nu_s = 0$ (b) $\nu_s = 0.015$. Head part of beam corresponds to positive z.

Figure 2 shows growth of beam size $\sigma_{y,b} = \sqrt{\langle y_b^2\rangle_{all} - \langle y_b\rangle_{all}^2}$ for $\nu_s = 0$ and $\nu_s = 0.015$. There is no threshold of the cloud density for $\nu_s = 0$, while there is a clear threshold for the finite synchrotron tune. In the figure the threshold is about $\rho_c = 5 \times 10^{11}$ for $\nu_s = 0.015$. We made sure the threshold depend on ν_s. This value is consistent with that obtained by a transverse rigid micro-bunch model [2]. The growth is exponential at small amplitude, but become slower for $\sigma_y = 2 \sim 3\sigma_{y0}$. Beam size for $\nu_s > 0$ is larger than that for $\nu_s = 0$ in the figure. The fact may be due to that the oscillation of beam for $\nu_s = 0$ is limited only its tail part, and electron cloud also shaken only interactions with the tail part of beam.

3.2 Finite chromaticity

We next discuss chromaticity dependence of the instability. In the theory of head-tail instability, 1-mode, which has an eigen frequency of $\nu_\beta - \nu_s$, becomes unstable for

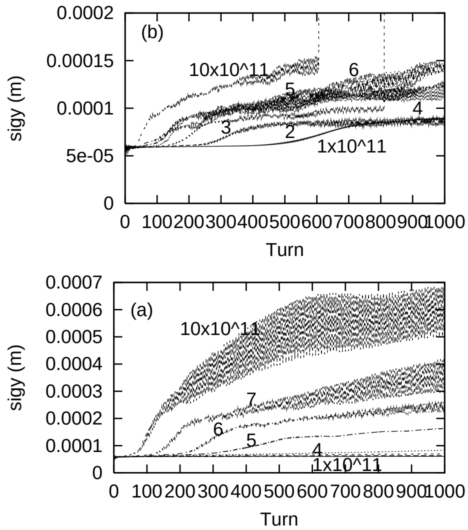

Figure 2: Beam size evolution for various cloud densities.

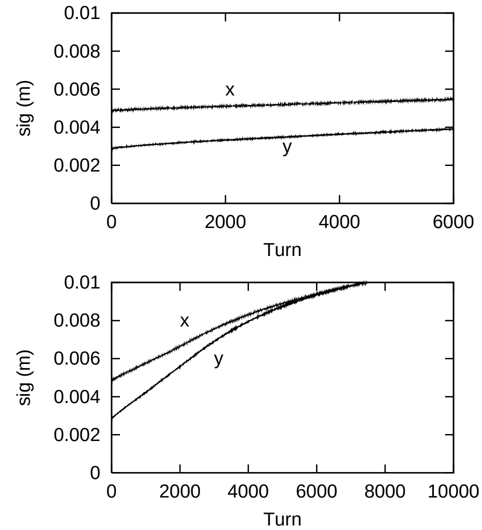

Figure 4: Beam size evolution of SPS instability.

a positive chromaticity [6]. The strong head-tail instability is suppressed by chromaticity in another approach [7, 8]. It is not obvious whether the positive chromaticity reduces instability. Rigid micro-bunch model [2] showed a growth of 1-mode for positive chromaticity. The fact is consistent with the head-tail theory [6]. Figure 3 shows chromaticity dependence of the beam size enlargement. The figure tells us that the enlargement is reduced for higher chromaticity as is discussed in Ref. [7, 8].

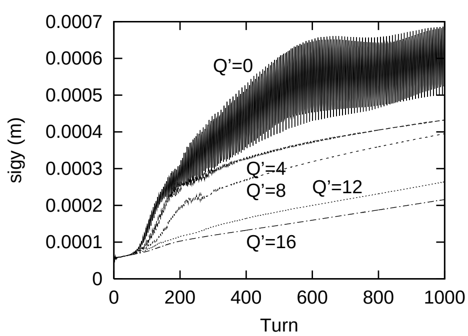

Figure 3: Chromaticity dependence of instability for $\rho = 10^{12} m^{-3}$.

4 SIMULATION RESULTS FOR SPS

This simulation was applied to a proton-electron cloud instability at SPS. Figure 4 shows preliminary results of beam size evolution. We still have some unclear results in the SPS simulation. Most important fact is that coherent oscillation signal is not obtained during the enlargement. We need more study for the SPS instability.

5 SUMMARY

We have studied single bunch instability caused by beam-electron cloud interactions using strong-strong model based on the Particle In Cell method. A coherent instability signal which correspond to strong head tail instability is obtained for KEKB-LER. The threshold density of electron cloud is consistent with that obtained by the previous rigid micro-bunch model [2] for zero chromaticity. Positive chromaticity works to suppress the instability against the previous model. For SPS instability, we continue to study more detail.

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