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NON-SPHERICAL COLLAPSE OF A RADIATING STAR

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We study the junction conditions for non-spherical (plane symmetric) collapsing radiating star consisting of a shearing fluid undergoing radial heat flow with outgoing radiation. Radiation of the system is described by plane symmetric Vaidya solution. Physical quantities relating to the local conservation of momentum and surface red-shift are also obtained.

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1. Introduction

Gravitational collapse is one of the most thorny and important problem in classical general relativity. It has many interesting applications in astrophysics where the formation of compact stellar objects such as white dwarf and neutron star are usually preceded by a period of radiative collapse. In order to study gravitational collapse, it is necessary to describe adequately the geometry of interior and exterior regions and to give conditions which allow matching of them.

The pioneering work on gravitational collapse appeared in the famous paper of Oppenheimer and Snyder [1] in which they studied collapse of dust with a static Schwarzschild exterior whereas the interior space-time is represented by Friedman like solution. Since that time, many authors have added to a more realistic treatment of the collapse. The case with static exterior was studied by Misner and Sharp [2], for a perfect fluid in the interior. Outgoing radiation of the collapsing body has been considered by Vaidya [3]. It then become possible to model the radiating star by matching them to exterior Vaidya space-time [4,5]. The inclusion of the dissipation in the source by allowing radial heat flow while the body undergoes radiating collapse has been advanced by Santos and collaborators [5,6,7,8,9,10].

These studies were restricted to spherically symmetric space-times. On the other-hand, non-spherical collapse not so well understood. However, non-spherical collapse could occur in real astrophysical situation, and it is also important for a

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better understanding of both cosmic censorship conjecture [11] and hoop conjecture [12]. In-fact, collapse of cylindrical system led to the formulation of the hoop conjecture [12]. The plane symmetric models has also received significant attention owing to its close resemblance with spherically symmetric one. Exterior solutions to plane symmetric Einstien's field equations were obtained by Taub [13], while plane symmetric version of Vaidya solution were given by Dutta [14], and Carlson and Safko [15].

In ref. [6] the junction conditions for the spherically symmetric collapse of isotropic fluid undergoing radial heat flow with outgoing unpolarized radiation has been studied. The main objective to extend is this work of Santos[6] to plane symmetric solutions. The interior space-time $\mathcal{V}_I$ is modeled by shearing fluid undergoing radial heat flow with outgoing radiation in a plane symmetric space-time. The exterior space-time $\mathcal{V}_E$ is described by the plane symmetric Vaidya space-time [14,15], which represent a radial flow of unpolarized radiation. This is done in section III. In this section we also derive the formula for the total luminosity perceived at infinity and for the surface red-shift, which are of particular interest since they are observable quantities. In section II we give the field equations which govern the plane symmetric collapse of a radiating star with outgoing radiation. We conclude with some general remarks.

We have used the units which fix the speed of light and gravitational constant via $8\pi G = c = 1$.

2. Field Equations in Plane Symmetric Space-time

Let us consider a plane symmetric distribution of fluid undergoing dissipation in the form of heat flow. While the dissipative fluid collapses, it produces the unpolarized radiation.

We consider a plane surface with its motion described by a time-like 3-surface Σ, which divides space-times into interior and exterior manifolds $\mathcal{V}_I$ and $\mathcal{V}_E$. The interior space-time $\mathcal{V}_I$ is described by most general plane symmetric metric, which in comoving coordinates reads:

$$ds^2 = -A(r,t)^2dt^2 + B(r,t)^2dr^2 + C(r,t)^2(dx^2 + dy^2) \tag{1}$$

The exterior space-time is described by plane symmetric Vaidya metric [14,15], which represents an outgoing unpolarized radiation,

$$ds^2 = \frac{2m(v)}{\mathbf{r}}dv^2 - 2dvd\mathbf{r} + \mathbf{r}^2(dx^2 + dy^2) \tag{2}$$

The arbitrary function $m(v)$, represents the mass at retarded time v inside the boundary surface Σ. We assume that the source of Einstein field equations in the interior space-time is given by

$$G^-_{ab} = \kappa T_{ab} = \kappa[(\zeta + p)u_a u_b + p g_{ab} + q_a u_b + q_b u_a] \tag{3}$$

where ζ is the energy density of fluid, p denotes the isotropic pressure, u_a is 4-velocity, and q_a is radial heat flux satisfying $q_a u^a = 0$. Since we utilize comoving coordinates, we shall have

$$u^a = \frac{1}{A}\delta^a_0, \qquad q^a = q\delta^a_1 \tag{4}$$

The line element (1), in our plane symmetric case, yields the following Einstein's field equations

$$G^-_{00} = -\left(\frac{A}{B}\right)^2\left[2\frac{C''}{C} + \left(\frac{C'}{C}\right)^2 - 2\frac{C'B'}{CB}\right] + \left(\frac{\dot{C}}{C}\right)^2 + 2\frac{\dot{C}\dot{B}}{CB} = \kappa\zeta A^2 \tag{5}$$

$$G^-_{11} = \left(\frac{C'}{C}\right)^2 + 2\frac{A'C'}{AC} - \left(\frac{B}{A}\right)^2\left[2\frac{\ddot{C}}{C} + \left(\frac{\dot{C}}{C}\right)^2 - 2\frac{\dot{A}\dot{C}}{AC}\right] = \kappa p B^2 \tag{6}$$

$$\begin{aligned} G^-_{22} = & \left(\frac{C}{B}\right)^2\left[\frac{A''}{A} + \frac{C''}{C} + \frac{A'C'}{AC} - \frac{C'B'}{CB} - \frac{A'B'}{AB}\right] - \left(\frac{C}{A}\right)^2 \\ & \times\left[\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{C}\dot{B}}{CB} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{A}\dot{B}}{AB}\right] = \kappa p C^2 \end{aligned} \tag{7}$$

$$G^{2-}_2 = G^{3-}_3 \tag{8}$$

$$G^-_{01} = 2\left[\frac{\dot{C}'}{C} - \frac{\dot{B}C'}{BC} - \frac{A'\dot{C}}{AC}\right] = \kappa q A B^2 \tag{9}$$

where the dot and the prime stand respectively for differentiation with respect to t and r.

3. Junction Conditions

To study the junction conditions, we follow the approach of Santos [6]. Hence we have to demand when approaching Σ in $\mathcal{V}_I$ and $\mathcal{V}_E$

$$(ds^2_-)_\Sigma = (ds^2_+)_\Sigma = (ds^2)_\Sigma \tag{10}$$

where the subscript Σ means that the quantities are to be evaluated on Σ and let $K^\pm_{ij}$ is extrinsic curvature to Σ, defined by

$$K^\pm_{ij} = -n^\pm_\alpha \frac{\partial^2\chi^\alpha_\pm}{\partial\xi^i\partial\xi^j} - n^\pm_\alpha \Gamma^\alpha_{\beta\gamma}\frac{\partial\chi^\beta_\pm}{\partial\xi^i}\frac{\partial\chi^\gamma_\pm}{\partial\xi^j} \tag{11}$$

and where $\Gamma^\alpha_{\beta\gamma}$ are Christoffel symbols, $n^\pm_\alpha$ the unit normal vectors to Σ, χ^α are the coordinates of the interior and exterior space-time and ξ^i are the coordinates that defines Σ.

The intrinsic metric on the hypersurface $r = r_\Sigma$ is given by

$$ds^2 = -d\tau^2 + \mathcal{R}^2(\tau)(dx^2 + dy^2) \tag{12}$$

with coordinates $\xi^a = (\tau,\ x,\ y)$.

We use comoving coordinates and consider the interior of the space-time $\mathcal{V}_I$ is described by line element (1). In this coordinate the surface Σ, being the boundary of the matter distribution, will have the equation

$$f(r,t) = r - r_\Sigma = 0 \tag{13}$$

where r_Σ is a constant. The vector with component $\partial f/\partial \chi^a_-$ is orthogonal to Σ. Consequently the unit normal vector takes the form

$$n_a^- = B(r_\Sigma, t)\delta_a^1 \tag{14}$$

From the junction condition (10) we obtain

$$\frac{dt}{d\tau} = \frac{1}{A(r_\Sigma, t)} \tag{15}$$

$$C(r_\Sigma, t) = \mathcal{R}(\tau) \tag{16}$$

The non-vanishing components of extrinsic curvature K_{ij}^- of Σ can be calculated as in the spherical case (for details see Santos[6]) and the result is

$$K_{\tau\tau}^- = \left[-\frac{A'}{AB}\right]_\Sigma \tag{17}$$

$$K_{x\,x}^- = \left[\frac{C'C}{B}\right]_\Sigma \tag{18}$$

$$K_{y\,y}^- = K_{x\,x}^- \tag{19}$$

The equation for the surface Σ in $\mathcal{V}_E$ is

$$f(\mathbf{r}, v) = \mathbf{r} - \mathbf{r}_\Sigma = 0 \tag{20}$$

therefore

$$\frac{\partial f}{\partial \chi^a_+} = \left(-\frac{d\mathbf{r}_\Sigma}{dv}, 1, 0, ..., 0\right) \tag{21}$$

and unit normal to Σ is

$$n_a^+ = \left[-\frac{2m(v)}{\mathbf{r}} + 2\frac{d\mathbf{r}}{dv}\right]^{-1/2} \left(-\frac{d\mathbf{r}}{dv}, 1, 0, ..., 0\right) \tag{22}$$

The first junction condition (10) for the line element (2) and (12) yields the following relations

$$\mathbf{r}_\Sigma = \mathcal{R}(\tau) \tag{23}$$

$$\left(\frac{dv}{d\tau}\right)_\Sigma^{-2} = \left[-\frac{2m(v)}{\mathbf{r}} + 2\frac{d\mathbf{r}}{dv}\right]_\Sigma \tag{24}$$

With the help of (24) we can rewrite the normal vector as

$$n_a^+ = (-\dot{\mathbf{r}}, \dot{v}, 0, ..., 0) \tag{25}$$

The non-vanishing components of extrinsic curvature K^{+}_{ij} of Σ are given by

$$K^{+}_{\tau\tau} = \left[\frac{d^2v}{d\tau^2}\left(\frac{dv}{d\tau}\right)^{-1} - \left(\frac{dv}{d\tau}\right)\frac{m(v)}{\mathbf{r}^2}\right]_{\Sigma} \tag{26}$$

$$K^{+}_{x\,x} = \left[\mathbf{r}\frac{d\mathbf{r}}{d\tau} - 2m(v)\left(\frac{dv}{d\tau}\right)\right]_{\Sigma} \tag{27}$$

$$K^{+}_{y\,y} = K^{+}_{x\,x} \tag{28}$$

From Eqs. (18) and (27) we have

$$\left[-\frac{2m(v)}{\mathbf{r}} + 2\frac{d\mathbf{r}}{dv}\right]_{\Sigma} = \left[\frac{CC'}{B}\right]_{\Sigma} \tag{29}$$

With the help of Eqs. (15), (16) and (24), we can write Eq. (29) as

$$m(v) = \frac{\mathbf{r}}{2}\left[\frac{\dot{C}^2}{A^2} - \frac{C'^2}{B^2}\right] \tag{30}$$

which can be interpreted as the total energy entrapped within the surface Σ. This expression (30) is analogous to well known mass function, in spherically symmetry case, introduced by Cahill and McVittie [16]. From Eqs. (26) and (28), using (15), we have

$$\left[\frac{d^2v}{d\tau^2}\left(\frac{dv}{d\tau}\right)^{-1} - \left(\frac{dv}{d\tau}\right)\frac{m(v)}{\mathbf{r}^2}\right]_{\Sigma} = -\left(\frac{A'}{AB}\right)_{\Sigma} \tag{31}$$

Substituting Eqs. (15), (16) and (30) into (29), results to

$$\left(\frac{dv}{d\tau}\right)_{\Sigma} = \left[\frac{C'}{B} + \frac{\dot{C}}{A}\right]^{-1}_{\Sigma} \tag{32}$$

Differentiating (32) with respect to τ and using Eqs. (30), Eq. (31) can be cast as

$$\frac{1}{B^2}\left[\frac{C'^2}{C^2} + 2\frac{A'C'}{AC}\right] - \frac{1}{A^2}\left[2\frac{\ddot{C}}{C} + \left(\frac{\dot{C}}{C}\right)^2 - 2\frac{\dot{A}\dot{C}}{AC}\right] = \frac{2}{AB}\left[\frac{\dot{C}'}{C} - \frac{\dot{B}C'}{BC} - \frac{A'\dot{C}}{AC}\right] \tag{33}$$

Comparing (33) with (6) and (9), we can finally write

$$(p)_{\Sigma} = (qB)_{\Sigma} \tag{34}$$

which is equivalent to result obtained by Santos [6] for the spherically symmetry case. Eq. (34) shows that for a plane symmetric shearing distribution of a collapsing fluid, undergoing dissipation in the form of heat flow, the isotropic pressure on the surface of discontinuity Σ can not be zero. Clearly, if the fluid stops dissipation, i.e., q_{Σ} = 0, the pressure will vanish at the boundary which implies the radiation can not exist and exterior space-time $\mathcal{V}_E$ is a Taub space-time [13].

Differentiating partially (30) with respect to t and utilizing (6) and (9), leads to

$$\left(\frac{\partial m}{\partial t}\right)_{\Sigma} = \left[\frac{dm}{dv}\frac{dv}{d\tau}\left(\frac{dt}{d\tau}\right)^{-1}\right]_{\Sigma} = -\left[\frac{\mathbf{r}^2}{2}\left(\kappa p\dot{C} + \kappa q A C'\right)\right]_{\Sigma} \tag{35}$$

On using (15), (16), (32) and (34), we obtain that

$$\left[-\frac{2}{\mathbf{r}^2}\frac{dm}{dv}\left(\frac{dv}{d\tau}\right)^2\right]_{\Sigma} = [\kappa p]_{\Sigma} \tag{36}$$

Therefore, the total luminosity for an observer at rest at infinity is

$$L_{\infty} = \lim_{r\to\infty}\frac{\kappa}{2}\mathbf{r}^2\epsilon = -\left(\frac{dm}{dv}\right)_{\Sigma} = \left[\frac{1}{2}\,\kappa\,\mathbf{r}^2\,p\left(\frac{\dot{C}}{A} + \frac{C'}{B}\right)^2\right]_{\Sigma} \tag{37}$$

where $dm/dv \leq 0$ since $L_{\infty} > 0$. Let the observer with 4-velocity is considered to be on Σ, the radiation energy density that this observer measures on Σ is

$$\epsilon_{\Sigma} = \frac{2}{\kappa}\left[-\frac{1}{\mathbf{r}^2}\left(\frac{dv}{d\tau}\right)^2\frac{dm}{dv}\right]_{\Sigma} \tag{38}$$

Inspection of Eqs (36) and (38), reveals that

$$\epsilon_{\Sigma} = p_{\Sigma} \tag{39}$$

This result is also valid in the analogous study in spherical symmetric system [17] Eq. (36) expresses the local conservation of momentum if we consider the momentum of radiation flowing in $\mathcal{V}_E$ and the energy density as given by (38).

Defining luminosity observed on Σ as

$$L_{\Sigma} = \frac{\kappa}{2}\,\mathbf{r}^2\,\epsilon_{\Sigma} \tag{40}$$

The boundary red-shift can be used to determine the time of formation of the horizon. The boundary red-shift z_{Σ} of the radiation emitted by a star is given by

$$1 + z_{\Sigma} = \left(\frac{dv}{d\tau}\right)_{\Sigma} \tag{41}$$

and thus the total luminosity L_{Σ} perceived by an observer on Σ is related with L_{∞} by the formula

$$(1 + z_{\Sigma})^2 = \frac{L_{\Sigma}}{L_{\infty}} \tag{42}$$

The results are analogous to one obtained previously in spherical case [5,6,7,8].

For completeness we also calculate the shear for the metric (1) giving the expression of shear scalar as

$$\sigma^2 = \frac{2}{3A^2}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)^2 \tag{43}$$

4. Concluding Remarks

To sum up, this extends the previous studies of junctions conditions for a collapsing radiating star with outgoing radiation in the spherically symmetry to the plane symmetric one.

The results obtained here may be a necessary ingredient for a study in planer collapse of a radiating star in plane symmetric space-time.

The physical conditions necessary for a acceptable model of a planer collapse will be the subject of forthcoming paper.

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