A Unified Description of Quark and Lepton Mass Matrices in a Universal Seesaw Model

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In the democratic universal seesaw model, the mass matrices are given by $\overline{f}_L m_L F_R + \overline{F}_L m_R f_R + \overline{F}_L M_F F_R$ (f: quarks and leptons; F: hypothetical heavy fermions), m_L and m_R are universal for up- and down-fermions, and M_F has a structure $(\mathbf{1}+b_f X)$ (b_f is a flavour-dependent parameter, and X is a democratic matrix). The model can successfully explain the quark masses and CKM mixing parameters in terms of the charged lepton masses by adjusting only one parameter, b_f. However, so far, the model has not been able to give the observed bimaximal mixing for the neutrino sector. In the present paper, we consider that M_F in the quark sectors are still "fully" democratic, while M_F in the lepton sectors are partially democratic. Then, the revised model can reasonably give a nearly bimaximal mixing without spoiling the previous success in the quark sectors.

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I. INTRODUCTION

A. What is the universal seesaw model?

Stimulated by the recent progress of neutrino experiments, there has been considerable interest in a unified description of the quark and lepton mass matrices. As one of such unified models, a non-standard model, the so-called "universal seesaw model" (USM) [1] is well known. The model describes not only the neutrino mass matrix M_ν but also the quark mass matrices M_u and M_d and the charged lepton mass matrix M_e by seesaw-type matrices, universally: the model has hypothetical fermions F_i ($F = U, D, N, E$; $i = 1, 2, 3$) in addition to the conventional quarks and leptons f_i ($f = u, d, \nu, e$; $i = 1, 2, 3$), and these fermions are assigned to $f_L = (2, 1)$, $f_R = (1, 2)$, $F_L = (1, 1)$ and $F_R = (1, 1)$ of SU(2)$_L\times$ SU(2)$_R$. The 6×6 mass matrix that is sandwiched between the fields $(\overline{f}_L, \overline{F}_L)$ and (f_R, F_R) is given by

$$M^{6\times 6} = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} , \tag{1.1}$$

where m_L and m_R are universal for all fermion sectors ($f = u, d, \nu, e$) and only M_F have structures dependent on the fermion sectors $F = U, D, N, E$. For $\Lambda_L < \Lambda_R \ll \Lambda_S$, where $\Lambda_L = O(m_L)$, $\Lambda_R = O(m_R)$ and $\Lambda_S = O(M_F)$, the 3×3 mass matrix M_f for the fermions f is given by the well-known seesaw expression

$$M_f \simeq -m_L M_F^{-1} m_R . \tag{1.2}$$

Thus, the model answers the question why the masses of quarks (except for top quark) and charged leptons are so small with respect to the electroweak scale Λ_L ($\sim 10^2$ GeV). On the other hand, the top quark mass enhancement is understood from the additional condition $\det M_F = 0$ for the up-quark sector ($F = U$) [2–4]. Since the seesaw mechanism does not work for the third family fermions, the top quark has a mass of the order of $m_L \sim \Lambda_L$.

For the neutrino sector, the mass matrix is given as

$$\left(\overline{\nu}_L\ \overline{\nu}^c_R\ \overline{N}_L\ \overline{N}^c_R\right) \begin{pmatrix} 0 & 0 & 0 & m_L \\ 0 & 0 & m_R^T & 0 \\ 0 & m_R & M_L & M_N \\ m_L^T & 0 & M_N^T & M_R \end{pmatrix} \begin{pmatrix} \nu^c_L \\ \nu_R \\ N^c_L \\ N_R \end{pmatrix} , \tag{1.3}$$

where $\nu^c_R \equiv (\nu_R)^c \equiv C\overline{\nu}^T_R$. Since $O(M_N) \sim O(M_L) \sim O(M_R) \gg O(m_R) \gg O(m_L)$, we obtain the mass matrix M_ν for the active neutrinos: ν_L

$$M_\nu \simeq -m_L M_R^{-1} m_L^T . \tag{1.4}$$

If we take the ratio $O(m_L)/O(m_R)$ suitably small, we can understand the smallness of the observed neutrino masses reasonably.

For an embedding of the model into a grand unification scenario, for example, see Ref. [5], where a possibility of SO(10)×SO(10) has been discussed.

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B. What is the democratic universal seesaw model?

As an extended version of the USM, the "democratic" USM [2,3] is also well known. The model has successfully given the quark masses and the Cabibbo–Kobayashi–Maskawa (CKM) [6] matrix parameters in terms of the charged lepton masses. The outline of the model is as follows:
(i) The mass matrices m_L and m_R have the same structure, except for their phase factors

$$m_L^f = m_R^f/\kappa = m_0 Z_f \ , \tag{1.5}$$

where κ is a constant with $\kappa \gg 1$ and Z_f are given by

$$Z_f = P(\delta_f) Z \ , \tag{1.6}$$

$$P(\delta_f) = \mathrm{diag}(e^{i\delta_1^f}, e^{i\delta_2^f}, e^{i\delta_3^f}) \ , \tag{1.7}$$

$$Z = \mathrm{diag}\,(z_1, z_2, z_3) \ , \tag{1.8}$$

with $z_1^2 + z_2^2 + z_3^2 = 1$.
(ii) In the basis on which the matrices m_L^f and m_R^f are diagonal, the mass matrices M_F are given by the form

$$M_F = m_0 \lambda (\mathbf{1} + 3 b_f X), \tag{1.9}$$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \ , \quad X = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \ . \tag{1.10}$$

(iii) The parameter b_f for the charged lepton sector is given by $b_e = 0$, so that in the limit of $\kappa/\lambda \ll 1$, the parameters z_i are given by

$$\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}} \ . \tag{1.11}$$

Then, the up- and down-quark masses are successfully given [2,3] by the choice of $b_u = -1/3$ and $b_d = -e^{i\beta_d}$ ($\beta_d = 18°$), respectively. Here, note that the choice $b_u = -1/3$ gives $\mathrm{det} M_U = 0$, so that the case with $b_u = -1/3$ gives $m_t \sim O(m_L)$. Another motivation for the choice $b_u = -1/3$ is that the model with $b_e = 0$ and $b_u = -1/3$ leads to the successful relation [7,2] $m_u/m_c \simeq (3/4)(m_e/m_\mu)$, which is almost independent of the value of the seesaw suppression factor κ/λ. For the choice of $b_u = -1/3$ and $b_d = -e^{i\beta_d}$ ($\beta_d = 18°$), the CKM matrix parameters are successfully given [2,3] by taking

$$\delta_1^u - \delta_1^d = \delta_2^u - \delta_2^d = 0 \ , \quad \delta_3^u - \delta_3^d \simeq \pi \ . \tag{1.12}$$

A more detailed formulation (including the renormalization group equation effects) is found in Ref. [8].

C. What is the problem?

It seems that the model is successful as far as the quark mass phenomenology is concerned, so that the future task is only to give a more reliable theoretical base to the model. However, the democratic USM has a serious problem in the neutrino phenomenology: In the previous model, the parameters z_i are fixed by the observed charged lepton masses as shown in (1.11), and the only adjustable parameter is b_ν defined by (1.9). For $b_\nu \simeq -1/2$ ($b_\nu \simeq -1$), we can obtain the maximal mixing between ν_μ and ν_τ (ν_e and ν_μ) [9], while we cannot give the nearly bimaximal mixing, which is suggested by the observed atmospheric [10] and solar [11,12] neutrino data.

This suggests that the previous model with the universal structure of M_F is too tight. Therefore, in the next section, we assume that for the lepton sectors, the democratic matrix X in (1.9) will be changed by a "partially" democratic matrix, which is given by a rotation R_X from the fully democratic matrix in the quark sector. Then, we can obtain the observed nearly-bimaximal mixing. However, generally speaking, the success is not so remarkable because we have three additional parameters in the rotation matrix R_X. The problem is whether the rotation R_X has a physical meaning or not.

In Sec. II, we will investigate a rotation matrix R_X that leads to the observed nearly bimaximal mixing and suggests an interesting relation between quarks and leptons. In Sec. III, the numerical results are given and neutrino phenomenology is discussed. In Sec. IV, the mysterious characteristics of the rotation matrix R_X are discussed. Finally, Sec. V is devoted to the conclusions.

II. S$_2$ SYMMETRY VERSUS S$_3$ SYMMETRY

A. Basic assumption

For the quark sectors, the model is essentially unchanged from the previous model, i.e. the mass terms are given by

$$m_0 \sum_{f=u,d} \left[\overline{f}_L Z F_R + \kappa \overline{F}_L Z f_R \right.$$

$$\left. + \lambda \overline{F}_L P^\dagger(\delta_f)(\mathbf{1} + 3b_f X) P(\delta_f) F_R \right] + \mathrm{h.c.} \ , \tag{2.1}$$

where we have changed the place of the phase matrix P from Z to M_F, so that m_L and m_R are completely flavour-independent. On this basis the mass matrices m_L and m_R are diagonal, the mass matrix M_F is invariant under the permutation symmetry S$_3$ except for the phase factors. As investigated in Refs. [2,3], in order to give reasonable values of the CKM matrix parameters, it was required to choose

$$P(\delta_u) P^\dagger(\delta_d) = P(\delta_u - \delta_d) \simeq \mathrm{diag}(1, 1, -1) \ , \tag{2.2}$$

although the origin of such a phase inversion is still an open question. In this paper, we assume

$$P(\delta_u) = \mathrm{diag}(1,1,-1)\ , \quad P(\delta_d) = \mathrm{diag}(1,1,1)\ . \tag{2.3}$$

For the lepton sectors, we assume

$$m_0 \sum_{f=e,\nu} \Big[\overline{f}_L Z F_R + \kappa \overline{F}_L Z f_R + \lambda \overline{F}'_L P^\dagger(\delta_f)(\mathbf{1}+3b_f X)P(\delta_f) F'_R \Big] + \text{h.c.}\ , \tag{2.4}$$

where, for convenience, we have dropped the Majorana mass terms $\overline{N}_L M_L N_L^c + \overline{N}_R^c M_R N_R$ from the expression (2.4), since we always assume that the Majorana mass matrices M_L and M_R have the same structure as the Dirac mass matrix $M_N = \lambda m_0 P^\dagger(\delta_\nu)(\mathbf{1}+3b_\nu X)P(\delta_\nu)$. In (2.4), we have defined

$$F' = R_X^T F\ . \tag{2.5}$$

Here, we have tacitly assumed symmetries $\mathrm{SU(2)}'_L \times \mathrm{SU(2)}'_R$ for the heavy fermions F_L and F_R in addition to the symmetries $\mathrm{SU(2)}_L \times \mathrm{SU(2)}_R$ for f_L and f_R, so that we have required the same rotation R_X for the heavy leptons $(N_i, E_i)_L$ (and $(N_i, E_i)_R$). Then, the heavy lepton mass terms in (2.4) can be rewritten as

$$m_0 \lambda \sum_{f=e,\nu} \overline{F}_L (\mathbf{1}+3b_f X_f) F_R + \text{h.c.}\ , \tag{2.6}$$

where

$$X_f = R_X P^\dagger(\delta_f) X P(\delta_f) R_X^T\ . \tag{2.7}$$

We take the phase matrices in the lepton sectors as

$$P(\delta_\nu) = P(\delta_u) = \mathrm{diag}(1,1,-1)\ ,$$

$$P(\delta_e) = P(\delta_d) = \mathrm{diag}(1,1,1)\ , \tag{2.8}$$

corresponding to (2.3). Then, the effective charged lepton and neutrino mass matrices are given by

$$M_e \simeq -m_0 \frac{\kappa}{\lambda} Z R_X (\mathbf{1}+3a_e X) R_X^T Z \equiv m_0^e Z(1+3a_e X_e) Z\ , \tag{2.9}$$

$$M_\nu \simeq -m_0 \frac{1}{\lambda} Z R_X P^\dagger(\delta_\nu)(\mathbf{1}+3a_\nu X)P(\delta_\nu) R_X^T Z \equiv m_0^\nu Z(1+3a_\nu X_\nu) Z\ , \tag{2.10}$$

where $m_0^e = -m_0(\kappa/\lambda)$, $m_0^\nu = -m_0/\lambda$, $X_e = R_X X R_X^T$ and $X_\nu = R_X P^\dagger(\delta_\nu) X P(\delta_\nu) R_X^T$, and we have used

$$(\mathbf{1}+3b_f X)^{-1} = \mathbf{1} + 3a_f X\ , \tag{2.11}$$

$$a_f = -b_f/(1+3b_f)\ . \tag{2.12}$$

The rotation R_X is between the basis in the quark sectors and that in the lepton sectors. Our interests are as follows: What rotation R_X can give reasonable neutrino masses and mixings? What relation does it suggest between quarks and leptons?

B. A special form of R_X

In the heavy down-quark mass matrix M_D, we have considered that the matrix X_d is completely democratic, i.e. $X_d = X$ defined by (1.10). Hereafter, we define the "fully" democratic matrix X defined in (1.10) as $X_3 \equiv X$. The matrix X_f is a rank-1 matrix, which satisfies the relation $(X_f)^2 = X_f$. We suppose that the matrices X_f $(f = e, \nu)$ in the heavy lepton sectors will not be "fully" democratic, but "partially" democratic. The simplest expression of the partially democratic matrix is

$$X_2 \equiv \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\ . \tag{2.13}$$

We identify X_e as $X_e = X_2$. The rotation R_X, which transforms X_3 into X_2, i.e.

$$R_X X_3 R_X^T = X_2\ , \tag{2.14}$$

is given by

$$R_X = R_3(-\frac{\pi}{4}) \cdot T \cdot R_3(\theta) \cdot (-P_3) \cdot A\ , \tag{2.15}$$

$$R_3(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\ , \tag{2.16}$$

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}\ , \tag{2.17}$$

$$T = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\ , \quad P_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\ . \tag{2.18}$$

The matrix A transforms the fully democratic matrix X_3 to the diagonal form

$$A X_3 A^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv Z_3\ . \tag{2.19}$$

The matrix Z_3 is invariant under the rotation $R_3(\theta)$ with an arbitrary θ. The transformation T has been introduced in order to transform Z_3 to $Z_1 \equiv \mathrm{diag}(1,0,0)$. Finally, the rotation $R_3(-\pi/4)$ transforms Z_1 to X_2. In the definition of R_X, (2.15), we have inserted the matrix $-P_3$ on the left-hand side of the matrix A. The matrix $-P_3$ does not have any effect on the matrix Z_3. In the numerical study in the next section, we are interested in the case where $(R_X)_{13}$ takes a small positive value, so that the matrix $-P_3$ has been introduced to make the the numerical search easier.

For further convenience, we express the rotation $R_3(\theta)$ by a new angle parameter $\varepsilon = \theta - \pi/4$. Then, the explicit form of R_X is given by

$$R_X = \begin{pmatrix} x_3 & x_2 & x_1 \\ \sqrt{\frac{2}{3}} - x_3 & \sqrt{\frac{2}{3}} - x_2 & \sqrt{\frac{2}{3}} - x_1 \\ \sqrt{\frac{2}{3}}(x_1 - x_2) & \sqrt{\frac{2}{3}}(x_3 - x_1) & \sqrt{\frac{2}{3}}(x_2 - x_3) \end{pmatrix} , \tag{2.20}$$

where x_i are given by

$$x_1 = \frac{1}{\sqrt{6}} - \frac{c-s}{\sqrt{6}} ,$$

$$x_2 = \frac{1}{\sqrt{6}} + \frac{c-s}{2\sqrt{6}} - \frac{c+s}{2\sqrt{2}} , \tag{2.21}$$

$$x_3 = \frac{1}{\sqrt{6}} + \frac{c-s}{2\sqrt{6}} + \frac{c+s}{2\sqrt{2}} ,$$

($s = \sin\varepsilon$ and $c = \cos\varepsilon$) and they satisfy the relations

$$x_1^2 + x_2^2 + x_3^2 = 1 , \tag{2.22}$$

$$x_1 + x_2 + x_3 = \sqrt{\frac{3}{2}} . \tag{2.23}$$

Since we have assumed the inversion $P(\delta_u)$, (2.3), the heavy up-quark mass matrix M_U (therefore, the matrix $P^\dagger(\delta_u) X_3 P(\delta_u)$) is not invariant under the permutation symmetry S_3, although it is still invariant under the permutation symmetry S_2 for the fields u_1 and u_2, because of the form

$$X_u = P^\dagger(\delta_u) X_3 P(\delta_u) = \frac{1}{3}\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} \equiv X_3' . \tag{2.24}$$

Since the matrix X_3' is not invariant under the permutation symmetry S_3, the neutral heavy lepton mass matrix M_N has a somewhat complicated form: the rank-1 matrix X_ν is generally given by

$$X_\nu = \begin{pmatrix} y_1^2 & y_1 y_2 & y_1 y_3 \\ y_1 y_2 & y_2^2 & y_2 y_3 \\ y_1 y_3 & y_2 y_3 & y_3^2 \end{pmatrix} , \tag{2.25}$$

where y_i satisfy the normalization $y_1^2 + y_2^2 + y_3^2 = 1$. By comparing the result $R_X X_3' R_X^T$ from (2.20) with the expression (2.25), we find

$$y_1 = \frac{1}{3\sqrt{2}} + \frac{\sqrt{2}}{3}(c-s) ,$$

$$y_2 = \frac{1}{3\sqrt{2}} - \frac{\sqrt{2}}{3}(c-s) , \tag{2.26}$$

$$y_3 = \frac{2}{3}(c+s) .$$

In the next section, we will investigate the neutrino mass matrix (2.10) numerically. The expression (2.25) is not always S_2-invariant. Therefore, in the next section, we will require the matrix X_ν to have also an S_2-invariant form. Then, the parameter ε is fixed, so that the model can again reduce to a one parameter model with only b_ν.

III. NUMERICAL STUDY OF THE NEUTRINO MASS MATRIX

In order to find the numerical study of the neutrino mass matrix (2.10) without spoiling the previous success in the quark sectors, we evaluate (2.9) in the limit of $b_e \to 0$. Then, the values of the parameters z_i are still given by (1.11). Therefore, the numerical success in the quark sectors [2,3] is unchanged. The matrix U_ν by which the mass matrix (2.10) is diagonalized as

$$U_\nu^\dagger M_\nu U_\nu^* = D_\nu \equiv \mathrm{diag}(m_1^\nu, m_2^\nu, m_3^\nu) , \tag{3.1}$$

is the so-called Maki–Nakagawa–Sakata–Pontecorvo (MNSP) [13] matrix. Hereafter, we will simply call U_ν the lepton mixing matrix.

The neutrino mass matrix M_ν has two parameters, b_ν and ε. First, we try to require that the matrix X_ν be invariant under a permutation symmetry S_2. Although, as suggested from the form $X_e = X_2$ in (2.13), the case with $y_1 = y_2$ is very interesting, regrettably it cannot give the observed nearly-bimaximal mixing for any value of b_ν. Of the possible cases $y_1 = y_2$, $y_2 = y_3$ and $y_3 = y_1$, only the case $y_3 = y_1$ has a solution that gives reasonable mixing and mass values. The case with $y_1 = y_3$ fixes the parameters x_i and ε as

$$y_1 = y_3 = 0.6900 , \quad y_2 = -0.2186 , \tag{3.2}$$

$$x_1 = 0.014811 , \quad x_2 = 0.23904 , \quad x_3 = 0.970890 , \tag{3.3}$$

$$\varepsilon = 2.043^\circ . \tag{3.4}$$

As we defined in (2.22) and (2.23), the parameters x_i satisfy the relation

$$x_1^2 + x_2^2 + x_3^2 = \frac{2}{3}(x_1 + x_2 + x_3)^2 \ . \tag{3.5}$$

On the other hand, it is well known that the observed charged lepton masses satisfy the relation [14]

$$m_e + m_\mu + m_\tau = \frac{2}{3}\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2 \ , \tag{3.6}$$

i.e.

$$z_1^2 + z_2^2 + z_3^2 = \frac{2}{3}(z_1 + z_2 + z_3)^2 \ . \tag{3.7}$$

In fact, from relation (3.6), the observed charged lepton masses m_e and m_μ predict $m_\tau^{theor} = 1776.97$ MeV, which is in excellent agreement with the observed value $m_\tau^{obs} = 1776.99^{+0.29}_{-0.26}$ MeV, together with the parameter values of z_i for $b_e = 0$:

$$z_1 = 0.016473 \ , \quad z_2 = 0.23687 \ , \quad z_3 = 0.97140 \ , \tag{3.8}$$

which correspond to

$$\varepsilon = 2.268^\circ \ . \tag{3.9}$$

It should be noted that the values (3.3) [and (3.4)] are very near to the values (3.8) [and (3.9)]. We may consider that the parameters z_i are identical with the x_i, which gives $y_3 = y_1$ at a unification scale $\mu = M_X$.

In the numerical search, the value of the parameter b_ν is determined as the prediction $R = \Delta m^2_{21}/\Delta m^2_{32}$ gives the observed value [10,12]

$$R_{obs} \simeq \frac{5.0 \times 10^{-5}\mathrm{eV}^2}{2.5 \times 10^{-3}\mathrm{eV}^2} = 2.0 \times 10^{-2} \ . \tag{3.10}$$

In Table I, we list the numerical results of b_ν, m_i^ν, Δm^2_{21}, Δm^2_{32}, $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, and $|(U_\nu)_{13}|^2$ as Case A. Here, for simplicity, we have used the values $4|(U_\nu)_{11}|^2|(U_\nu)_{12}|^2$ and $4|(U_\nu)_{23}|^2|(U_\nu)_{33}|^2$ as the values of $\sin^2 2\theta_{12}$ and $\sin^2 2\theta_{23}$, respectively, because $R \ll 1$. For reference, in Table I, we also list a case with $x_i = z_i = \sqrt{m_i^e/(m_e + m_\mu + m_\tau)}$ as Case B. In this case, the scenario is that the partially democratic form of X_ν with $y_3 = y_1$ is slightly broken at $\mu = m_Z$, still keeping $x_i = z_i$. From the numerical point of view, there is no essential difference between the two cases.

The predicted value of $\sin^2 2\theta_{12}$ [$\tan^2\theta_{12}$],

$$\sin^2 2\theta_{12} = 0.80 \quad [\tan^2\theta_{12} = 0.38] \ , \tag{3.11}$$

is in good agreement with the present best fit value [12] $\tan^2\theta_{solar} = 0.34$ [$\sin^2 2\theta_{solar} = 0.76$]. It should be noted that the predicted value (3.11) gives a suitable deviation from $\sin^2 2\theta_{12} = 1.0$, although the Zee-type model cannot give such a sizeable deviation from $\sin^2 2\theta_{12} = 1.0$ [15].

It is also worth while noting that in Table I the value of b_ν is very near to $b_\nu = -2/3$. The results $b_e = 0$, $b_u = -1/3$, $b_\nu \simeq -2/3$ and $b_d \simeq -1$ may suggest the existence of some unified rule for b_f.

Finally, we must excuse ourselves for taking the parameter b_e as $b_e \to 0$ in the numerical calculations. We have assumed that the heavy charged lepton mass matrix M_E is given by $M_E = \lambda m_0(\mathbf{1} + 3b_e X_2)$ on the basis of F (not F'), i.e. M_E has the partially democratic form. However, the choice $b_e = 0$ makes this assumption nonsense. We consider that the value of the parameter b_e is $b_e \simeq 0$, but it is not $b_e = 0$. In fact, although the relation (3.6) has given, for the observed charged lepton mass values m_e and m_μ, the excellent prediction of the tau lepton mass m_τ, however, for the values [16] of m_e and m_μ at $\mu = m_Z$ we obtain the predicted value $m_\tau(m_Z) = 1724.99$ MeV, which slightly deviates from the observed value $m_\tau(m_Z) = 1746.69^{+0.30}_{-0.27}$ MeV [16]. This deviation can be adjusted by taking a small deviation of b_e from zero.

IV. MEANINGS OF THE ROTATION R_X

In the previous section, we have found that the values of the parameters x_i with the requirement $y_1 = y_3$ are very close to the values of z_i, which are evaluated from the observed charged lepton masses. It should be noted that only for such a case with $x_i \simeq z_i$ we obtain a solution of the value of the parameter b_ν that gives reasonable masses and mixings. In other words, even if we do not require the condition $y_1 = y_3$, the phenomenological two-parameter study with ε and b_ν can find a reasonable solution only when $x_i \simeq z_i$. This suggests that the rotation R_X has a special meaning not only for the neutrino mass matrix, but also for the charged lepton mass parameter matrix Z. We consider that the coincidence $x_i \simeq z_i$ is not accidental.

The rotation R_X has the following property:

$$R_X \begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \ , \tag{4.1}$$

in addition to the property (2.14). Therefore, it means that the parameters z_i can be obtained from the vector $(1,0,0)$ by the following rotation:

$$\begin{pmatrix} z_3 \\ z_2 \\ z_1 \end{pmatrix} = (R_X)^T_{x_i=z_i} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \ . \tag{4.2}$$

If we define a rotation matrix $\widetilde{R}_X$ as

$$\widetilde{R}_X = T R_X T \ , \tag{4.3}$$

where T is defined by (2.18), the relations become more intuitive:

$$\widetilde{R}_X X_3 \widetilde{R}_X^T = \widetilde{X}_2 \equiv \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} , \quad (4.4)$$

$$(\widetilde{R}_X)_{x_i=z_i}\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} , \quad (4.5)$$

$$(\widetilde{R}_X)_{x_i=z_i}\cdot Z\cdot(3X_3)\cdot Z\cdot(\widetilde{R}_X)^T_{x_i=z_i} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \quad (4.6)$$

However, in order to obtain the same numerical results as those in the previous section, we must change the assumption $X_\nu = R_X P_3 X_3 P_3 R_X^T$ to the following assumption

$$X_\nu = \widetilde{R}_X P_1 X_3 P_1 \widetilde{R}_X^T , \quad (4.7)$$

where

$$P_1 = \mathrm{diag}(-1, 1, 1) . \quad (4.8)$$

Then, the parameters y_i in the expression (2.26) are given by the same relations with the exchange between y_1 and y_3. Since we require $y_1 = y_3$, the numerical results are exactly identical with those in the previous section. In the previous scenario we have assumed, with the rotation R_X, that the heavy up fermions take the same phase matrix $P_3 = P(\delta_u) = P(\delta_\nu)$. In this case with $\widetilde{R}_X$, we must assume that $P(\delta_u) = P_3$, but $P(\delta_\nu) = P_1$. Although the scenario with $\widetilde{R}_X$ is more intuitive, we cannot at present answer the question why quarks require the inversion P_3 and why leptons require the inversion P_1.

In any case, it is essential that the parameter values (z_1, z_2, z_3) [or (z_3, z_2, z_1)] come from $(0, 0, 1)$ [or $(1, 0, 0)$] by the rotation $\widetilde{R}_X$ [or R_X]. Especially, it is noted that the parameters z_i satisfy the relation (2.23) [therefore (3.7)], which leads to the charged lepton mass relation (3.6). Thus, the rotation R_X has special meanings not only as a rotation between the heavy quarks (U, D) and (N, E), but also as a rotation that determines the charged lepton mass parameters z_i.

V. CONCLUSIONS

We have proposed an improved version of the democratic universal seesaw model in order to extend the success of the unified description of the quark and charged lepton mass matrices to the neutrino mass matrix. In the original model, the mass matrices m_L and m_R were given by a universal structure Z, independently of the fermion sectors $f = u, d, e, \nu$, and the hypothetical heavy fermion mass matrices M_F have the same structure, "a unit matrix plus a democratic matrix", which includes only one flavour-dependent complex parameter b_f. The constraint was too tight, so that the model could not give the observed nearly-bimaximal neutrino mixing. In the improved model, the mass matrices m_L^f (also m_R^f) are still flavour-independent, while the heavy fermion mass matrices have different structures between quark and lepton sectors, i.e. in the quark sectors, M_F still have democratic forms, while in the lepton sector, M_F have only "partially" democratic forms. If we take a special rotation R_X, which transforms the 3×3 democratic matrix X_3 to the 2×2 democratic matrix X_2 as (2.14) and if we take the parameters x_i as $x_i \simeq z_i \propto \sqrt{m_i^e}$ and $b_\nu \simeq -2/3$, we can obtain reasonable values of neutrino masses and mixings.

For the quark and charged lepton sectors, in the original democratic universal seesaw model [2,3], we have already obtained reasonable values of the masses and mixings by taking $b_e = 0$, $b_u = -1/3$, and $b_d \simeq -1$. Those values of b_f are unchanged in the present revised model and, moreover, in order to explain the observed nearly bimaximal neutrino mixing, the value $b_\nu \simeq -2/3$ is required. What is meaning of these parameter values

$$b_e = 0 , \quad b_u = -1/3 , \quad b_\nu \simeq -2/3 , \quad b_d \simeq -1 \ ? \quad (5.1)$$

This is a future task for us.

We have also numerically searched a rotation matrix $R(\theta_{12}, \theta_{23}, \theta_{31})$ that can give reasonable values for the observed neutrino mixings and masses, without requiring the constraint (2.14). We found that the only solution is the rotation R_X with $x_i \simeq z_i$ (the values z_i are given by (1.11)) for $b_\nu \simeq -2/3$. The solution R_X transforms the "fully" democratic matrix X_3 into the partially democratic matrix X_2 and the parameters x_i satisfy the relation (3.5), which leads to the charged lepton mass formula (3.6). The rotation R_X with $x_i \simeq z_i$ also transforms the matrix X_3' (2.24) into a partially democratic matrix X_ν (2.25) with $y_1 = y_3$. These mean that the observed neutrino data require not a mere numerical solution of $R(\theta_{12}, \theta_{23}, \theta_{31})$, but the special solution R_X with $x_i = z_i$. The observed charged lepton masses, which are proportional to z_i^2, are closely related to the rotation R_X with $x_i = z_i$, for example as (4.2), (3.7), and so on. These facts give us a sufficient motivation for the rotation R_X with $x_i = z_i$ be taken seriously. However, at present, the theoretical origin of the rotation is not clear. This is also a future task to us.

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	Case A with $x_i \neq z_i$	Case B with $x_i = z_i$
b_ν	-0.680	-0.684
a_ν	-0.654	-0.650
m_1^ν (eV)	2.39×10^3	2.43×10^3
m_2^ν (eV)	7.46×10^3	7.48×10^3
m_3^ν (eV)	5.06×10^2	5.06×10^2
Δm_{21}^2 (eV^2)	5.00×10^{-5}	5.01×10^{-5}
Δm_{32}^2 (eV^2)	2.50×10^{-3}	2.50×10^{-3}
$\Delta m_{21}^2/\Delta m_{32}^2$	2.00×10^{-2}	2.00×10^{-2}
$\sin^2 2\theta_{12}$	0.796	0.801
$(\tan^2 \theta_{12})$	(0.377)	(0.383)
$\sin^2 2\theta_{23}$	0.978	0.979
$\lvert(U_\nu)_{13}\rvert^2$	6.65×10^{-3}	6.68×10^{-3}

TABLE I. Predictions of the neutrino masses and mixing parameters. For the predictions Δm_{ij}^2 and m_i^ν, we have used the value $\Delta m_{32}^2 = 2.5 \times 10^{-3}$ eV^2 from the atmospheric neutrino data [10]. In case A, the values x_i are determined from the requirement $y_1 = y_3$, and the values z_i are obtained from the relation (3.7) and the observed values of m_e and m_μ. In case B, the values x_i are taken as $x_i = z_i$, where z_i are obtained as in case A.