

Neutrino Spectrum Distortion Due to Oscillations and its BBN Effect

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Abstract

We study the distortion of electron neutrino energy spectrum due to oscillations with the sterile neutrino $\nu_e \leftrightarrow \nu_s$, for different initial populations of the sterile state δN_s at the onset of oscillations. The influence of this spectrum distortion on Big Bang Nucleosynthesis is analyzed. Only the case of an initially empty sterile state was studied in previous publications.

The primordial abundance of He-4 is calculated for all possible δN_s : $0 \leq \delta N_s \leq 1$ in the model of oscillations, effective after electron neutrino decoupling, for which the spectrum distortion effects on the neutron–proton transitions are the strongest.

It is found that the spectrum distortion effect may be the dominant one not only in the case of small δN_s , but also in the case of big initial population of ν_s . For example, in the resonant case it may play a considerable role even for very big $\delta N_s \sim 0.8$.

keywords: cosmology, primordial nucleosynthesis, neutrino oscillations

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Introduction

Sterile neutrinos ν_s may be present at the onset of the nucleosynthesis epoch. There may be different reasons for their production — they are naturally produced in GUT models [1], in models with large extra dimensions [2] and Manyfold Universe models [3], in mirror matter models [4]. They may be produced also in $\nu_{\mu,\tau} \leftrightarrow \nu_s$ oscillations in the preceding epoch ¹ according to the most popular 4-neutrino mixing schemes [5].

The degree of population of ν_s , δN_s , may be different depending on the concrete model of ν_s production. Hence, we will consider further on δN_s as a parameter.

In recent years, strong constraints on the sterile neutrino impact in oscillations, explaining atmospheric and solar neutrino anomalies, were obtained from the analysis of experimental oscillations data [6, 7, 8]. And although a pure sterile channel solution is excluded for any of the neutrino anomalies, the analysis of neutrino oscillation experimental data allows a certain, small fraction of sterile neutrino to participate into these oscillations. In the analysis [9] and [10] it was suggested that a small fraction of ν_s is not only allowed but even desirable for solar neutrino data. Recent measurements of the cross section of ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction lead to a predicted total ${}^8\text{B}$ neutrino flux by 13% larger than measured by SNO, which also may be providing evidence for sterile neutrino [11].

There also exist stringent cosmological constraints on ν_s produced in oscillations, based on oscillations influence on BBN nucleosynthesis of ${}^4\text{He}$. For the case $\nu_{\mu,\tau} \leftrightarrow \nu_s$ see for example refs. [12, 13] and for the $\nu_e \leftrightarrow \nu_s$ see refs. [14, 13]. The constraints on electron–sterile neutrino oscillations [15, 16], excluded the active–sterile LOW solution to the solar neutrino puzzle, in addition to the already excluded in pioneer works (see for example ref.[17]) sterile LMA solution and sterile solution to the atmospheric neutrino anomaly. (For more detail about constraints on neutrino oscillations from cosmology, see refs. [18, 12].)

Most of the cosmological constraints on active-sterile mixing were obtained in simple two-neutrino mixing schemes ², for the case when the sterile neutrino state was initially

¹For example, atmospheric and LSND neutrino data require oscillations with maximal mixings and mass differences $\delta m_{atm}^2 \sim 10^{-3} \text{ eV}^2$ and $\delta m_{LSND}^2 \sim 10^{-1} \text{ eV}^2$, which are effective before the BBN epoch. So, in many schemes owing to $\nu_\mu \leftrightarrow \nu_s$ or $\nu_\tau \leftrightarrow \nu_s$ oscillations, ν_s state may be partially thermalized before the nucleosynthesis epoch.

²For discussion and calculation of cosmological constraints in a specific 4-neutrino mixing schemes see ref. [19, 20, 13]

empty at the epoch before oscillations became effective in the Universe evolution, $\delta N_s = N_\nu - 3 = 0$. N_ν is the number of neutrino species in equilibrium. Since the presence of a non-empty sterile state before oscillations was not considered in previous analysis of oscillations effects on the neutrino spectrum distortion and on BBN, in this work we address this question. We omit the assumption $\delta N_s = 0$ and explore the general case $\delta N_s \neq 0$.

In the $\nu_{\mu,\tau} \leftrightarrow \nu_s$ oscillation case, $\delta N_s \neq 0$ present initially just leads to an earlier increase of the total energy density of the Universe, and it is straightforward to re-scale the existing constraints. In the $\nu_e \leftrightarrow \nu_s$ oscillations case, however, the presence of ν_s at the onset of oscillations influences also the kinetic effects of $\nu_e \leftrightarrow \nu_s$ on BBN. Therefore, we chose $\nu_e \leftrightarrow \nu_s$ oscillations case for exploring electron neutrino distortion, caused by oscillations, and its influence on nucleons freezing and on primordial ${}^4\text{He}$, Y_p for different δN_s values in the range $0 \leq \delta N_s \leq 1$.

The spectrum distortion of the electron neutrinos

We analyze the case of oscillations effective after neutrino decoupling, therefore further on we denote by δN_s the degree of population of the sterile neutrino state at active neutrino decoupling ($T \sim 2$ MeV). Here we explore ν_e spectrum distortion, considering the degree of population of the sterile neutrino state δN_s as a free parameter and, varying its value in the range $[0, 1]$ with a step 0.1. In the next section we calculate the kinetic effect of oscillations on primordial abundance of ${}^4\text{He}$ for different δN_s .

The mixing of electron neutrino with ν_s has the following two types of effects on BBN: (a) it leads to *an increase of the energy density* of the Universe [21], and (b) it *changes the nucleons kinetics* essential for n/p -freezing, through the depletion of electron neutrino number density, distortion of the equilibrium spectrum of ν_e , and production of asymmetry between neutrinos and anti-neutrinos, all due to oscillations [18]. We will parametrize these kinetic effects and denote them δN_{kin} further on.

(a) The first effect is usually described by an increase of the effective number of the energy density degrees of freedom $g_{eff} = (30/\pi^2)(\rho/T^4)$. At the BBN epoch $g_{eff} = 10.75 + 7/4\delta N_s(T_\nu/T)^4$. Hence, this effect leads to a faster expansion rate $H \sim g_{eff}^{1/2}$ and higher freezing temperature for nucleons $T_f \sim g_{eff}^{1/6}$, when nucleons were more abundant:

$$n/p \sim \exp[-\Delta m/T_f]$$

This reflects into an overproduction of ${}^4\text{He}$, since it strongly depends on the n/p -freezing ratio: $Y_p \sim 2 \exp(-\Delta m/T_f)/[1 + \exp(-\Delta m/T_f)]$, where $\Delta m = m_n - m_p \sim 1.3$

MeV is the neutron–proton mass difference. This effect of g_{eff} increase on BBN is well known [22]. The approximate fit to the exact calculations is: $\delta Y_p \sim 0.013\delta N_s$. The maximum helium overproduction corresponding to $\delta N_s = 1$ is $\sim 5\%$.

(b) The influence of the kinetic effects of oscillations on BBN is quite obvious, having in mind that: (i) oscillations take place between equilibrium electron neutrino and less populated sterile neutrino ensemble, that (ii) the oscillations probability is inversely proportional to the energy of neutrinos $P \sim \delta m^2/E$, so that neutrinos with different momenta start oscillating at different cosmic times, and that (iii) the proton density is bigger than the neutron one. Due to that, the neutrino energy spectrum $n_\nu(E)$ may strongly deviate from its equilibrium form [23, 18].

In case oscillations proceed after the decoupling of active neutrinos, a strong spectrum distortion for both the electron neutrino and the anti-neutrino is possible. This spectrum distortion affects the kinetics of nucleons freezing - it leads to an earlier n/p -freezing and an overproduction of ${}^4\text{He}$ yield.

The effect can be easily understood having in mind that the distortion leads both to a *depletion of the active neutrino number densities* in favor of the sterile ones N_ν :

$$N_\nu \sim \int dE E^2 n_\nu(E)$$

and to a *decrease of the mean neutrino energy*.³ This lowers the weak rates, governing nucleons transitions during neutrons freezing, with respect to their values in the standard BBN model, $\Gamma_{weak} \sim N_{\nu_e} \bar{E}_\nu^2$, and hence reflects into earlier freezing when neutrons were more abundant. So, helium is over-produced.

The generation of neutrino-antineutrino asymmetry in the resonant oscillations has a subdominant effect on ${}^4\text{He}$. It slightly suppresses oscillations at small mass differences, leading to a decrease of helium overproduction.

δN_{kin} depend strongly on the initial population of the sterile neutrino at BBN. Larger δN_s decreases the kinetic effects, because the element of initial non-equilibrium between the active and the sterile states is less expressed. Hence, for any specific value of δN_s it is necessary to provide a separate analysis.

In the case $\delta N_s = 1$ ν_s are in equilibrium (the sterile state is as abundant as the electron one), and hence the n - p kinetics does not feel the oscillations, $\delta N_{kin} = 0$. The final effect is only due to the energy increase, i.e. $\delta N_{tot} = 1$.

³The decrease of the electron neutrino energy due to oscillations into low temperature sterile neutrinos, has also an additional effect: Due to the threshold of the reaction converting protons into neutrons, when neutrinos have lower energy than the threshold one, protons are preferably produced, which may lead to an under-production of ${}^4\text{He}$ [24]. However, this turns to be a minor effect in the discussed oscillations model.

In the case $\delta N_s = 0$ the kinetic effect of oscillations was studied numerically for both the resonant [15] and non-resonant [16] oscillation cases. In this case δN_{kin} for given fixed mixing parameters reach their highest value, δN_{kin}^{max} , as far as the non-equilibrium element — the difference between the sterile and active neutrino number densities at the beginning of oscillations — is the greatest. The overproduction of ${}^4\text{He}$ may be enormous: up to 13.2% in the non-resonant oscillation case and up to 31.8% in the resonant one [25]. This corresponds effectively to a little more than 6 additional neutrino states $\delta N_{kin}^{max} \sim 6$. For the case $\delta N_s = 0$ the kinetic effects were carefully studied and stringent cosmological constraints on the oscillation parameters were obtained on the basis of helium-overproduction, caused by the kinetic effects of oscillations [14].

In this work, accounting for all the effects (a) and (b), we calculate $Y_p(\delta N_s, \delta m^2, \sin^2 2\theta)$ for $0 < \delta N_s \leq 1$ values, and reveal the dependence of δN_{kin} on δN_s .

We have analyzed the self-consistent evolution of the oscillating neutrino and the nucleons from the neutrino decoupling epoch at ~ 2 MeV till the freezing of nucleons. We have followed the line of work described in detail in ref. [15], omitting the assumption for negligible density of the sterile neutrinos at the onset of $\nu_e \leftrightarrow \nu_s$ oscillations.

It is hardly possible to describe analytically, without some radical approximations, the non-equilibrium picture of active–sterile neutrino oscillations, producing non-equilibrium neutrino number densities and distorting the neutrino spectrum. Satisfactory precise analytical description was found only for the case of relatively fast oscillations proceeding before neutrino freezing, with $\delta m^2 > 10^{-6}$ eV² and small mixing angles [26, 13]. Therefore, we have provided a self-consistent numerical analysis of the evolution of the nucleons number densities n_n and the ones of the oscillating neutrinos ρ and $\bar{\rho}$ in the high-temperature Universe, using the following coupled integro-differential equations, the first equation describing the kinetics of the neutrino ensembles in terms of the density matrix of neutrino ρ and anti-neutrino $\bar{\rho}$, the second equation – the kinetic evolution of the neutrons.

$$\begin{aligned} \frac{\partial \rho(t)}{\partial t} = & H p_\nu \frac{\partial \rho(t)}{\partial p_\nu} + \\ & + i [\mathcal{H}_o, \rho(t)] + i \sqrt{2} G_F \left(\pm \mathcal{L} - Q/M_W^2 \right) N_\gamma [\alpha, \rho(t)] + \text{O} \left(G_F^2 \right), \end{aligned} \quad (1)$$

$$\begin{aligned} (\partial n_n / \partial t) = & H p_n (\partial n_n / \partial p_n) + \\ & + \int d\Omega(e^-, p, \nu) |\mathcal{A}(e^- p \rightarrow \nu n)|^2 [n_{e^-} n_p (1 - \rho_{LL}) - n_n \rho_{LL} (1 - n_{e^-})] \\ & - \int d\Omega(e^+, p, \tilde{\nu}) |\mathcal{A}(e^+ n \rightarrow p \tilde{\nu})|^2 [n_{e^+} n_n (1 - \bar{\rho}_{LL}) - n_p \bar{\rho}_{LL} (1 - n_{e^+})]. \end{aligned} \quad (2)$$

where $\alpha_{ij} = U_{ie}^* U_{je}$, p_ν is the momentum of electron neutrino, n stands for the number density of the interacting particles, $d\Omega(i, j, k)$ is a phase-space factor, and \mathcal{A} is the amplitude of the corresponding process. The plus sign in front of \mathcal{L} corresponds to the neutrino ensemble, the minus sign - to the anti-neutrino ensemble.

Mixing just in the electron sector is assumed: $\nu_i = U_{il} \nu_l$ ($l = e, s$). The initial condition for the neutrino ensembles in the interaction basis is of the form:

$$\rho = n_\nu^{eq} \begin{pmatrix} 1 & 0 \\ 0 & S \end{pmatrix},$$

where $n_\nu^{eq} = \exp(-E_\nu/T)/(1 + \exp(-E_\nu/T))$, while S measures the degree of population of the sterile state.

\mathcal{H}_o is the free neutrino Hamiltonian. The ‘non-local’ term Q arises as a W/Z propagator effect, $Q \sim E_\nu T$. \mathcal{L} is proportional to the fermion asymmetry of the plasma and is essentially expressed through the neutrino asymmetries $\mathcal{L} \sim 2L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau}$, where $L_{\mu,\tau} \sim (N_{\mu,\tau} - N_{\bar{\mu},\bar{\tau}})/N_\gamma$ and $L_{\nu_e} \sim \int d^3p(\rho_{LL} - \bar{\rho}_{LL})/N_\gamma$.

The equations are for the neutrino and neutron number densities *in momentum space*, which allows to describe precisely the kinetic effects: spectrum distortion and neutrino-antineutrino asymmetry growth due to oscillations. For the description of the spectrum 1000 bins were used in the nonresonant oscillations case, and at least 5000 in the resonant one. These equations provide a simultaneous account of the different competing processes, namely: neutrino oscillations, Universe expansion, neutrino forward scattering, nucleons transformations.

The analysis was performed for all mixing angles ϑ and mass differences $\delta m^2 \leq 10^{-7}$ eV². The analyzed temperature interval was [2.0, 0.3] MeV, because at temperatures higher than 2 MeV the deviations from the standard BBN model without oscillations are negligible in the discussed model of oscillations.

As expected, the spectrum distortion is less expressed when increasing the degree of population of the sterile neutrino state δN_s (Fig.1.). Correspondingly, the kinetic effect on primordial nucleosynthesis should decrease. The results of our numerical analysis on spectrum distortion at different δN_s are illustrated in the Figs. 1a-c, where the dependence of the energy spectrum distortion of the electron neutrino on the initial population of the sterile state is shown.

At each temperature we have plotted the spectrum for three different levels of initial population of the sterile neutrino, namely $\delta N_s = 0.0, 0.5, 0.8$.

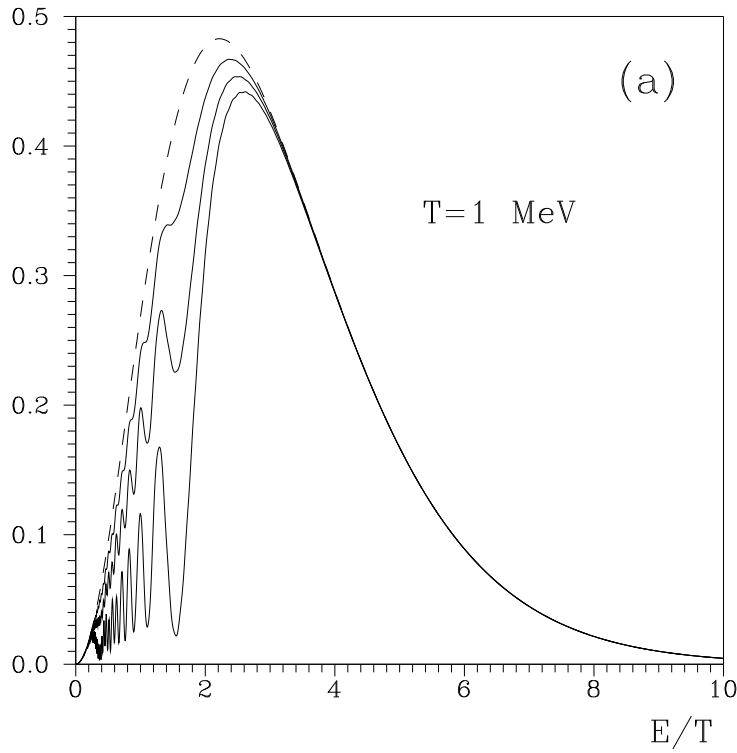
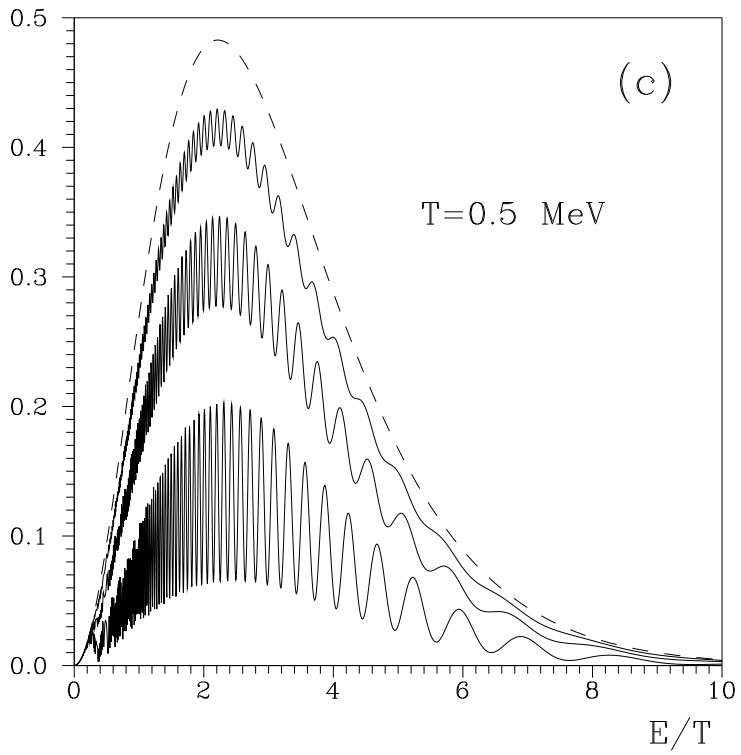
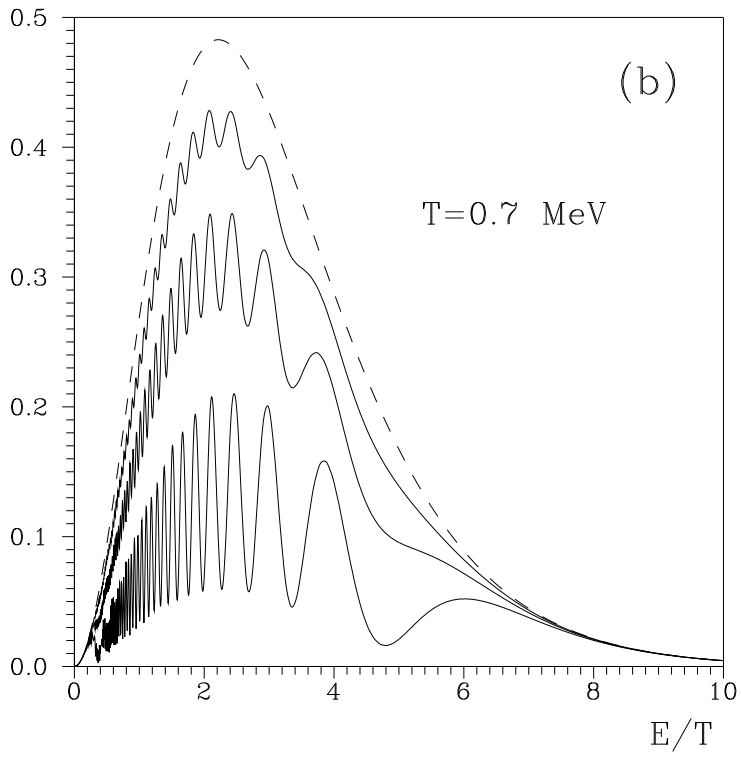


Figure 1a: The figure illustrates the degree of distortion of the electron neutrino energy spectrum $x^2\rho_{LL}(x)$, where $x = E/T$ at a characteristic temperature 1 MeV, caused by resonant oscillations with a mass difference $\delta m^2 = 10^{-7}$ eV² and $\sin^2 2\vartheta = 0.1$ for different initial sterile neutrino populations, correspondingly $\delta N_s = 0$ (the lower curve), $\delta N_s = 0.5$ and $\delta N_s = 0.8$ (the upper curve). The dashed curve gives the equilibrium neutrino spectrum for comparison.



Figures 1b,c: Distortion of the electron neutrino energy spectrum at a temperature 0.7 MeV (Fig.1b) and 0.5 MeV (Fig.1c) for the same parameters as for Fig.1a.

The oscillations parameters are $|\delta m^2| = 10^{-7} \text{ eV}^2$ and $\sin^2 2\vartheta = 0.1$. For illustrating the evolution of the spectrum distortion we have presented it at characteristic temperatures 1 (Fig.1a), 0.7 (Fig.1b) and 0.5 MeV (Fig.1c). At each δN_s the characteristic behavior of the spectrum distortion due to oscillations is observed. Namely, since oscillation rate is energy dependent $\Gamma \sim \delta m^2/E$ the low energy part of the spectrum is distorted first (as far as low energy neutrinos start to oscillate first) and later the distortion penetrates noticeably into the more energetic part of the spectrum.

We have found that the neutrino energy spectrum $n_\nu(E)$ may strongly deviate from its equilibrium form during all the period of interest (2 MeV – 0.3 MeV) even for considerably large δN_s , and hence the spectrum distortion may constitute the dominant effect on the overproduction of ${}^4\text{He}$.

The kinetic effect

For different δN_s we calculate precisely the n/p -freezing, essential for the production of helium, down to temperature 0.3 MeV. Then we calculate Y_p , accounting adiabatically for the following decay of neutrons till the start of nuclear reactions, at about 0.1 MeV.

We have found that *neutrino spectrum distortion effect on BBN is very strong even when there is a considerable population of the sterile neutrino state before the beginning of the electron–sterile oscillations*. It always gives positive δN_{kin} , which for a large range of initial sterile population values, are bigger than 1. The kinetic effects are the strongest for $\delta N_s = 0$: $Y_p^{max}(\delta N_s, \delta m^2, \sin^2 2\vartheta) = Y_p(0, \delta m^2, \sin^2 2\vartheta)$. They disappear for $\delta N_s = 1$, when ν_e and ν_s states are in equilibrium, and the total effect reduces to the SBBN with an additional neutrino.

In Fig.2 we present the frozen neutron number density relative to nucleons $X_n^f = N_n^f/N_{nuc}$ as a function of the sterile neutrino content at neutrino decoupling for a resonant and a nonresonant oscillation case. The oscillation parameters are $\delta m^2 = 10^{-7} \text{ eV}^2$ and $\delta m^2 = -10^{-7} \text{ eV}^2$ and $\sin^2 2\theta = 10^{-1}$. As far as $\delta Y_p/Y_p = \delta X_n^f/X_n^f$, the results are representative for the overproduction of primordially produced helium.

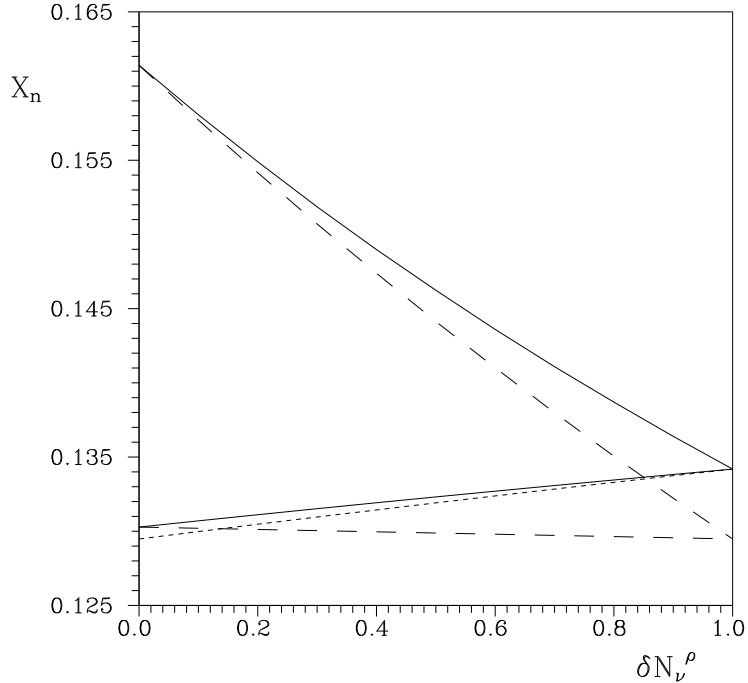


Figure 2: The solid curves present frozen neutron number density relative to nucleons $X_n^f = N_n^f/N_{nuc}$ as a function of the sterile neutrino initial population. The dashed curves present only the kinetic effect, while the dotted curve presents the effect due to the energy density increase. The upper two curves (dashed and solid) correspond to the resonant case, the lower dashed and solid curves - to the nonresonant one.

The dotted curve presents only the effect (a), due to the energy density increase $X_n^f = f(\delta N_s)$, the dashed curves present the pure kinetic effects (b) $X_n^f = f(\delta N_{kin})$, while the solid lines give the total effect. The upper dashed and solid curves correspond to the resonant case, the lower ones to the non-resonant one.

The analysis for these concrete oscillation parameters, shows that the overproduction of helium is strongly suppressed with the increase of δN_s for the resonant case, while in the non-resonant case it increases with δN_s . This is a result of the fact that, in the resonant case, the kinetic effects (b) due to the spectrum distortion are the dominant contribution to the overproduction of helium, even for very large degree of population of the sterile state, while in the non-resonant case the main contribution comes from the increase of degrees of freedom already at very small δN_s . An empirical approximation formula is:

$$\delta Y_p = 0.013[\delta N_{kin}^{max}(1 - \delta N_s) + \delta N_s],$$

where δN_{kin}^{max} is the value calculated in the case of oscillations with an initially empty sterile state, i.e. $\delta N_{tot} = \delta N_{kin}^{max}(1 - \delta N_s) + \delta N_s$. It is a good approximation for the

non-resonant case and a rather rough one for the resonant case: the deviation from the exactly calculated helium given in Fig. 2 may be up to $\delta Y_p/Y_p \sim 0.8\%$. Still, it can give some idea of $\delta Y_p/Y_p$ dependence on δN_s .

However, for other mixing parameters, the kinetic oscillation effects in the non-resonant case can be also considerable, as shown in ref.[25], the kinetic effect can be as high as $\delta N_{kin} \sim 3$ for initially empty sterile state. Hence, in the non-resonant case the spectrum distortion effects may be the dominant one even for much larger δN_s than in the case illustrated in Fig.2.

For each concrete δN_s value a detailed numerical analysis is necessary to reveal the interplay of effects (a) and (b) and their influence on primordial production of ${}^4\text{He}$.

In a forthcoming paper we apply the results obtained here to define the isohelium contours corresponding to 3% overproduction of ${}^4\text{He}$ for different δN_s , and we present the cosmological constraints for nonzero δN_s .

Conclusions

The presence of a *non-empty* sterile state before $\nu_e \leftrightarrow \nu_s$ oscillations was not considered in previous analysis of $\nu_e \leftrightarrow \nu_s$ oscillation effects on the neutrino spectrum distortion and on BBN. In this work we have studied the kinetic effects due to $\nu_e \leftrightarrow \nu_s$ oscillations in the general case $0 \leq \delta N_s \leq 1$.

We have provided a numerical analysis, investigating how the presence of the sterile neutrino state, partially populated before oscillations, will influence the production of ${}^4\text{He}$ in the model of BBN with electron–sterile oscillations effective after electron neutrino decoupling.

We have found that the effect of the neutrino spectrum distortion due to oscillations may be very strong, even for a considerable initial population of the sterile neutrino state. Correspondingly, the kinetic effect of oscillations remain the dominant one even for big δN_s .

The results of this analysis may be applied for different models generating sterile neutrino, like GUT models, mirror models, extra-dimensions models, etc., as far as the initial value of population of the sterile state δN_s depends on the concrete model of its production. These results may be of interest also for mixing schemes in which a portion of ν_s have been brought into equilibrium before neutrino decoupling, due to $\nu_\mu \leftrightarrow \nu_s$ or $\nu_\tau \leftrightarrow \nu_s$ oscillations. In case the ν_s presence is due to the much earlier (at atmospheric mass difference scale, or LSND) oscillations of $\nu_{\mu,\tau} \leftrightarrow \nu_s$, δN_s may be directly connected with the available constraints on the sterile neutrino fraction, deduced from

the neutrino oscillations experimental data analysis. So, we hope that the results may be indicative and helpful for choosing among the different possibilities for the sterile fraction in the subdominant active-sterile oscillations used in the oscillation analysis of neutrino anomalies.

A more general study of kinetic oscillations effects on BBN for non-empty initially sterile state in the framework of 4-neutrino mixing schemes seems appropriate, although much more complicated.

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