The ground state of three quarks [*]

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We measure the static three-quark potential in $SU(3)$ lattice gauge theory with improved accuracy, by using all available technical refinements, including Lüscher-Weisz exponential variance reduction. Together with insight gained from 3-state Potts model simulations, our results allow us to sort out the merits of the Δ- and Y-ansätze.

1. Introduction

The static $q\bar{q}$ potential has been studied extensively on the lattice. In the quenched theory, the elementary ansatz $V_{q\bar{q}}(r) = V_0 - \frac{\alpha}{r} + \sigma_{q\bar{q}}r$ turns out to be remarkably accurate. The next simplest system is one of 3 static quarks, from which one can gain phenomenological insight about the forces inside a baryon. Moreover, the 3 quarks must be connected by 3 glue strings to form a gauge-covariant object. These strings meet at a "gluon junction", which has been conjectured to be a non-perturbative excitation of the QCD vacuum [1], and might play an important role at the hadronization stage in heavy-ion collisions.

We extract the potential between 3 static quarks in $SU(3)$ gauge theory from the exponential decay with T of the expectation value of the baryonic Wilson loop Fig. 1. We aim at improving the accuracy of our earlier work [2] through several technical refinements. This new accuracy, augmented by insight from Potts model simulations, allows us to reach conclusions about the merits of the Δ-ansatz [3] and the Y-ansatz [4].

At short distances, the potential is described by perturbation theory, which gives it the form (constant + Coulomb). (*i*) The constant term is caused by UV divergences arising from the perimeter of the loop. For large T, the bary-
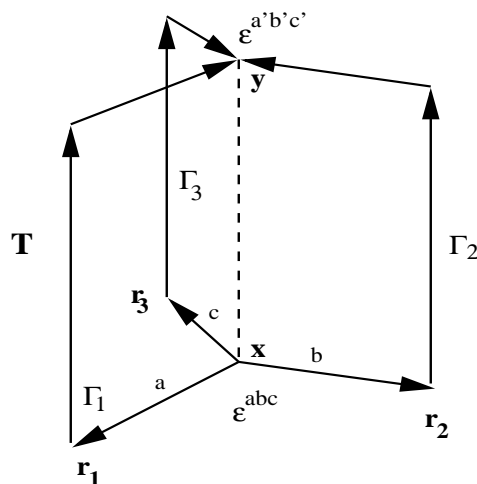
Figure 1. Baryonic Wilson loop with junctions at x and y: $W \equiv \frac{1}{6}\epsilon_{abc}\epsilon_{a'b'c'}\Gamma_1^{aa'}\Gamma_2^{bb'}\Gamma_3^{cc'}$.

onic loop perimeter is $\approx \frac{3}{2}$ that of a rectangular, mesonic loop, so that the constant term in the potential is multiplied by $\frac{3}{2}$ (for the same lattice spacing a). (*ii*) The exchange of one gluon between two of the quarks gives $\frac{1}{2}$ the one-gluon exchange term between a quark and antiquark at the same locations. Together, (*i*) and (*ii*) imply

$$V_{qqq}(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \frac{1}{2}\sum_{i<j} V_{q\bar{q}}(\vec{r}_i, \vec{r}_j) \equiv V_{qqq}^{\Delta}(\vec{r}_1, \vec{r}_2, \vec{r}_3) \quad (1)$$

At large distances, the Δ-ansatz predicts that the

potential grows linearly with the perimeter L_Δ of the quark triangle: $V_{qqq} \propto \sigma_{q\bar{q}} \frac{L_\Delta}{2}$, so that Eq.(1) still holds. It is derived from a model of confinement by center vortices using a beautiful topological argument [3]. The Y-ansatz predicts instead $V_{qqq} \propto \sigma_{q\bar{q}} L_Y$, where L_Y is the minimal length of the 3 flux tubes necessary to join the 3 quarks at the so-called Steiner point. It is derived from strong coupling arguments [4], and is consistent with the dual superconductivity confinement scenario [5]. Since $L_Y > \frac{L_\Delta}{2}$ for all 3-quark geometries, the Y-ansatz predicts a steeper potential

$$V_{qqq}^Y(\vec{r}_1, \vec{r}_2, \vec{r}_3) = V_{qqq}^\Delta(\vec{r}_1, \vec{r}_2, \vec{r}_3) + \sigma_{q\bar{q}}(L_Y - \frac{L_\Delta}{2})(2)$$

with V_{qqq}^Δ as per Eq.(1). Both ansätze are constrained to reproduce the diquark limit $\vec{r}_j \to \vec{r}_k$, $V_{qqq}(\vec{r}_i, \vec{r}_j, \vec{r}_k) \to V_{q\bar{q}}(\vec{r}_i, \vec{r}_j)$ exactly, and therefore contain no free parameter once $V_{q\bar{q}}$ is given. In this respect we differ from the analogous lattice study of [6], where $\sigma_{q\bar{q}}$ and σ_{qqq} are fitted separately and therefore not strictly equal.

2. Technical refinements

Because the difference between the Δ- and the Y-ansätze is very small ($1 \le \frac{L_Y}{L_\Delta/2} \le \frac{2}{\sqrt{3}}$), high accuracy in the determination of V_{qqq} is mandatory. The main difficulty at large quark separation is the contribution of excited qqq states. Besides smearing the spatial links as in [6], we use three additional techniques to control these systematic errors. (i) We form a variational basis with different junction locations (x, y in Fig. 1). (ii) We use multihit for the timelike links. (iii) We generalize the multilevel algorithm of [7], originally proposed for Polyakov loop correlators, to baryonic Wilson loops. This method provides a variance reduction exponential in T, which allows us to extract the potential from longer loops, with crucially improved filtering of excited states.

3. Results

A sample of current results based on 160 analyzed $16^3 \times 32$ configurations at $\beta = 5.8$ and 6.0 is shown in Fig. 2 (3 quarks in an equilateral triangle). They are compatible with our earlier measurements [2], but the reduced errors now clearly
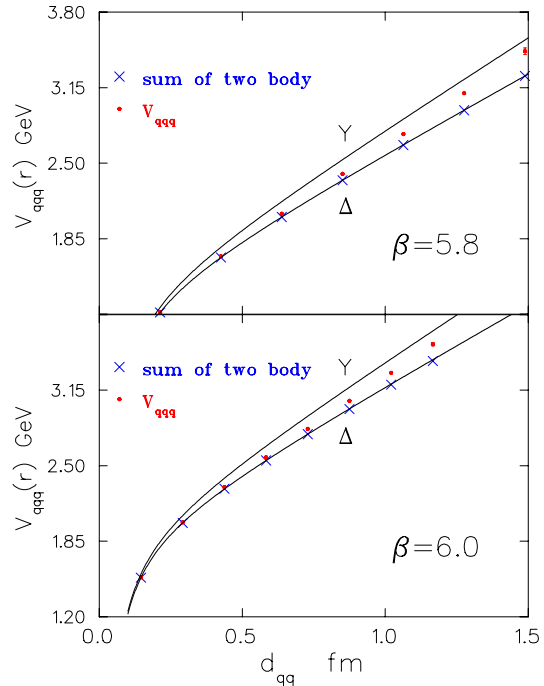

Figure 2. Static potential V_{qqq} vs quark separation at $\beta = 5.8$ and 6.0. Also shown are the Δ- and Y-predictions Eqs.(1) and (2).

show that neither ansatz gives a proper description of the potential. It approaches the Δ-ansatz at short distances as expected, but seems to rise faster, perhaps as fast as the Y-ansatz, at large distances. Furthermore, the larger the quark separation, the more our variational groundstate favors junctions located near the Steiner point.

To elucidate the asymptotics of the potential, we turned to the 3-state Potts model. This toy model preserves the center degrees of freedom of $SU(3)$ and is thus more likely to agree with center-vortex-based predictions of the Δ-ansatz. In this model, we measured the 3-spin correlation, after adjusting the coupling to match the $\beta_{SU(3)} = 6.0$ correlation length. High-precision cluster Monte Carlo results were obtained for multiple 3-spin geometries, in $2d$ and $3d$. In all cases, the 3-spin correlation behaved just like in $SU(3)$, falling "in-between" the Δ- and the Y-ansätze. But we could establish that the potential was rising asymptotically $\propto L_Y$. Large separations are required to see this. An example is
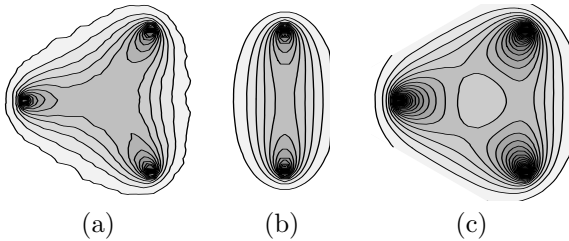
(a)   (b)   (c)

Figure 3. Action densities in the $2d$ Potts model:
(a) qqq, (b) $q\bar{q}$; (c) Δ prediction, i.e. superposi-
tion of 3 $q\bar{q}$ densities. The qq distance is ~ 2.8 fm.

shown in Fig. 3, where the change in action den-
sity caused by the 3 sources (a) is compared with
the superposition (c) of $q\bar{q}$ action densities (b), as
predicted by the Δ-ansatz. A Y string pattern is
visible, but the strings are "fat" (~ 0.8 fm) com-
pared to their length, even though the "quark"
separation is ~ 2.8 fm.

Remarkably, the approach to Y-asymptotia is
well described by the same ansatz in the $2d$-, $3d$-
Potts model and in $SU(3)$. As shown in Fig. 4,
the measured value of the 3-spin correlation (or
V_{qqq}) falls short of the Y-prediction by an amount

$$V_{qqq} - V_{qqq}^Y \approx c_0 + c_1 \exp(-L_\Delta/c_2) \qquad (3)$$

where c_0, c_1, c_2 depend on the qqq triangle geom-
etry. For the $SU(3)$ equilateral case, $c_2 \sim 2$ fm.

4. Conclusions

Our results show that the baryonic static po-
tential is neither of the Δ- nor of the Y-type. It
approaches the Δ-ansatz at short distances, but
rises like the Y-ansatz at large distances. For an
equilateral qqq arrangement, departure from the
Δ-ansatz is not significant until $d_{qq} \sim 0.7$ fm, so
that the Δ-ansatz may be the more relevant one
for quarks confined inside a hadron.

The delay in the onset of the Y-behavior is pre-
sumably caused by fluctuations in the location of
the junction. In the Potts model, locations x_J
away from the Steiner point are suppressed only if
the associated length $L(x_J)$ of the "glue" strings
exceeds the minimum L_Y by an amount compa-
rable to the correlation length ξ. For a junction
coinciding with one of the "quarks", for example,
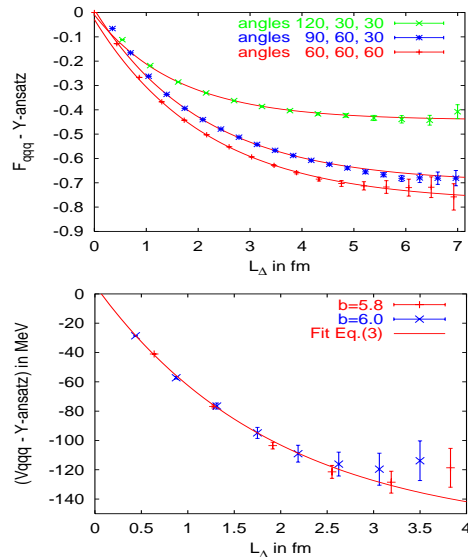the inequality $L(x_J) - L_Y \geq \xi$ becomes satisfied

Figure 4. Amount by which V_{qqq} misses the Y-
prediction vs perimeter of the quark triangle, for
the $3d$ Potts (top) and $SU(3)$ (bottom) cases.

only if $d_{qq} \geq 1.7$ fm.

Unfortunately, the Y-behavior at large dis-
tances does not help to rule out the center vortex
picture of confinement. Rather, the failure of the
Δ-type prediction of [3] appears to be due to ad-
ditional hidden assumptions about the indepen-
dence of certain linking numbers.

REFERENCES

1. D. Kharzeev, Phys. Lett. B **378**, 238 (1996).
2. C. Alexandrou, P. De Forcrand and
   A. Tsapalis, Phys. Rev. D **65**, 054503
   (2002); Nucl. Phys. Proc. Suppl. **106**, 403
   (2002); Nucl. Phys. Proc. Suppl. **109**, 153
   (2002).
3. J. M. Cornwall, Phys. Rev. D **54**, 6527 (1996).
4. N. Isgur and J. Paton, Phys. Rev. D **31**, 2910
   (1985).
5. M. N. Chernodub and D. A. Komarov, JETP
   Lett. **68**, 117 (1998); Y. Koma, E. M. Ilgen-
   fritz, T. Suzuki and H. Toki, Phys. Rev. D
   64, 014015 (2001).
6. T. T. Takahashi, H. Matsufuru, Y. Nemoto
   and H. Suganuma, Phys. Rev. Lett. **86**, 18
   (2001); Phys. Rev. D **65**, 114509 (2002).

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7. M. Lüscher and P. Weisz, JHEP **0109**, 010 (2001).