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Non-perturbative scale evolution of four-fermion operators*

ALPHA Collaboration
M. Guagnelli[a], J. Heitger[b], C. Pena[a], S. Sint[c] and A. Vladikas[a†]

[a]I.N.F.N., Sezione di Roma 2, c/o Dipartimento di Fisica, Università di Roma "Tor Vergata", Via della Ricerca Scientifica 1, I-00133 Rome, Italy

[b]Westfälische Wilhelms-Universität Münster, Institut für Theoretische Physik, Wilhelm-Klemm-Strasse 9, D-48149 Münster, Germany

[c]CERN, Theory Division, CH-1211 Geneva 23, Switzerland

We apply the Schrödinger Functional (SF) formalism to determine the renormalisation group running of four-fermion operators which appear in the effective weak Hamiltonian of the Standard Model. Our calculations are done using Wilson fermions and the parity-odd components of the operators. Preliminary results are presented for the operator $O_{VA} = (\bar{s}\gamma_\mu d)\ (\bar{s}\gamma_\mu\gamma_5 d)$.

1. Introduction

Weak matrix elements (WMEs) of parity-odd four-fermion operators are crucial in many Standard Model processes (e.g. $\Delta I = 1/2$ enhancement, $\Delta S = 1$ non-leptonic Kaon decays). The non-perturbative aspects of these decays are expressed as WMEs $\langle \pi\pi|O_k|K\rangle$ of parity-odd operators O_k (where, in standard notation, $k = VA \pm AV, SP \pm PS, T\tilde{T}$ denotes the various Dirac structures). On the other hand, the non-perturbative aspects of $K^0 - \bar{K}^0$ oscillations (i.e. $\Delta S = 2$ transitions) are studied through WMEs of parity-even operators $\langle \bar{K}^0|O_{VV+AA}|K^0\rangle$. Even in this case, however, it is possible to map the problem into the computation of parity-odd matrix elements, either through the use of Ward Identities [1] or by implementing the twisted mass QCD formalism [2].

Unlike their parity-even counterparts, all parity-odd operators of interest display renormalisation patterns which are unaffected by the breaking of chiral symmetry on the lattice (i.e. the Wilson term) [3]. Our program consists in computing non-perturbatively the anomalous dimension (AD) matrix of the complete basis of parity-odd dimension-six operators with Wilson fermions. Our renormalisation scheme of choice is the SF, which is mass independent. This has two implications. First, our results also apply to those operators which characterise $\Delta I = 1/2$ transitions. Although the complete removal of divergences in this case requires complicated power subtractions, these, being proportional to quark masses, do not affect the operator AD. Second, our results also apply to operators of heavy flavour transitions (e.g $\Delta C, \Delta B = 2,\ \Delta B = 1$).

The renormalisation of any four-fermion operator (up to power subtractions) can be mapped to that of an operator with the same colour-Dirac structure but four distinct flavours. For example, once the operators

$$O^{\pm}_{VA+AV} = \frac{1}{2}\big[\big(\bar{\psi}_1\gamma_\mu\psi_2\big)\big(\bar{\psi}_3\gamma_\mu\gamma_5\psi_4\big)+ +\big(\bar{\psi}_1\gamma_\mu\gamma_5\psi_2\big)\big(\bar{\psi}_3\gamma_\mu\psi_4\big) \pm (2\leftrightarrow 4)\big] \quad (1)$$

have been properly renormalised, we can map the four distinct flavours to the physical flavours concerning the process of interest. Here we will only present results for the operator $Q \equiv O^{+}_{VA+AV}$.

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2. The step scaling function (SSF)

Our work consists in a straightforward generalisation of the computation of the SSF of the quark mass, performed by the Alpha collaboration in [4]. The operator anomalous dimension γ is defined through

$$\mu \ \frac{\partial \bar{Q}}{\partial \mu} = \gamma(\bar{g}) \ \bar{Q} \tag{2}$$

($\bar{Q}$ and $\bar{g}$ are renormalised quantities). Its perturbative expansion is given by

$$\gamma(g) \overset{g \to 0}{\sim} -g^2 \left(\gamma_0 + \gamma_1 g^2 + \gamma_2 g^4 + \ldots\right) \ , \tag{3}$$

with $\gamma_0 = 1/(4\pi^2)$ a universal coefficient. The (scheme-indepedent) renormalisation group invariant (RGI) operator is obtained upon integrating eq. (2)

$$Q^{RGI} = \bar{Q}(\mu) \ [2b_0\bar{g}^2]^{(-\gamma_0/2b_0)} \times \exp\left\{ - \int_0^{\bar{g}} dg \left[\frac{\gamma(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g}\right] \right\} \tag{4}$$

(b_0 is the LO coefficient of the Callan-Symanzik β-function). Once the scale dependence of the renormalised coupling $\bar{g}$ in the SF scheme is known [5], and proceeding along the line of ref. [4], three elements are required for the complete NP determination of a WME of the form $\langle f|Q^{RGI}|i\rangle$: (i) the lattice bare matrix elements at several bare couplings (work in progress); (ii) the value of the renormalisation constant Z_Q at the same bare couplings and a fixed low-energy reference scale of the order of Λ_{QCD}; (iii) the operator SSF from this non-perturbative low-energy scale to high ones, where $\bar{g}(\mu)$ is safely in the perturbative region and the integral in eq. (4) can be performed analytically to determine the ratio $Q^{RGI}/\bar{Q}(\mu)$.

We regularise the theory on a lattice of physical size L^4 with standard SF boundary conditions [4]. Renormalisation is performed in the chiral limit, at a scale equal to the IR cutoff L^{-1}. Since the operator renormalises multiplicatively,

$$\bar{Q}(L^{-1}) = \lim_{a \to 0} \ Z_Q(g_0, L/a) \ Q(a) \ , \tag{5}$$

the UV divergence is removed by a single renormalisation condition, namely by setting the expression

$$f_1^{-3/2}(g_0, L/a) \ Z_Q(g_0, L/a) \ h_Q(x_0) \tag{6}$$

at $x_0 = L/2$ to its tree level value. The correlator h_Q is as shown in Fig. 1; f_1 serves to cancel the quark boundary field renormalisation (see ref. [4]). For more details and an explicit notation, see ref. [6]. Different choices of boundary operators correspond to different renormalisation schemes. Parity conservation and the properties of SF boundary quark fields limit us to five non-trivial independent choices,

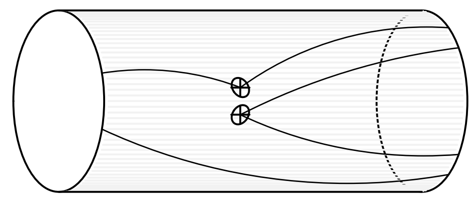

Figure 1. The SF correlation function h_Q. Note that the "spectator" quark line connecting the two time boundaries corresponds to a fifth flavour.

$$\begin{array}{ll} \gamma_5 \leftrightarrow \gamma_5 \ \gamma_5 & \gamma_5 \leftrightarrow \gamma_k \ \gamma_k \\ \gamma_k \leftrightarrow \gamma_k \ \gamma_5 & \gamma_k \leftrightarrow \gamma_5 \ \gamma_k \\ \multicolumn{2}{c}{\gamma_i \leftrightarrow \gamma_j \ \gamma_k \ \epsilon_{ijk} \ ,} \end{array} \tag{7}$$

plus their time-reversed counterparts (in eq. (7) the γ-matrix on the lhs. of the symbol $\leftrightarrow$ corresponds to the Dirac structure at time $x_0 = 0$, whereas the two γ-matrices on its rhs. correspond to the Dirac structures at time $x_0 = L$).

The SSF is defined as

$$\sigma_Q(u) \equiv \lim_{a \to 0} \Sigma_Q(u, a/L) \equiv \lim_{a \to 0} \frac{Z_Q(g_0, 2L/a)}{Z_Q(g_0, L/a)} \tag{8}$$

($u = \bar{g}^2(L)$) and, taking into account the running coupling from [5], can be used to run renormalised matrix elements along a broad range of scales through the recursion

$$\bar{Q}((2L)^{-1}) = \sigma_Q(u)\bar{Q}(L^{-1}) \ . \tag{9}$$

Our quenched simulations have been perfomed at the parameters (volumes, couplings and hopping parameters) of ref. [4]. The most difficult numerical task consists in the extrapolation of our data

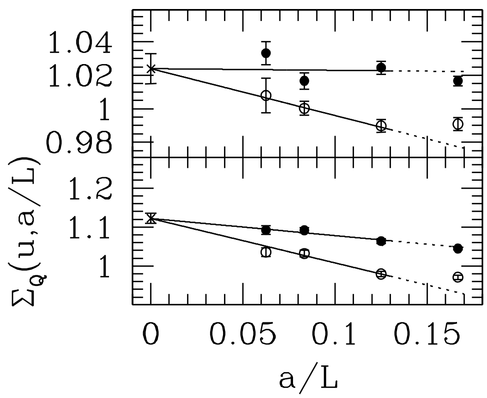

Figure 2. Continuum limit extrapolation of the SSF at renormalised couplings $u = 1.2430$ (upper plot) and $u = 2.770$ (lower plot). Solid (open) points are data corresponding to simulations with an improved Clover (Wilson) action. The leftmost point in each plot is the result of a combined linear extrapolation to the continuum limit of both data sets, with slopes indicated by the two lines. In both cases, the data at largest lattice spacing have not been included in the fit.

to the continuum limit: although our action is the non-perturbatively improved Clover one (i.e. cutoff effects are $\mathcal{O}(a^2)$ from the bulk, and $\mathcal{O}(g_0^4 a)$ from the time boundaries), our operator is unimproved, which gives rise to $\mathcal{O}(a)$ effects. Thus, for some renormalised couplings the continuum limit extrapolation appears to be stable, whereas for others it is rather unreliable. In order to improve on this we are currently repeating our computation, in the spirit of ref. [7] with an unimproved Wilson action. Since the SSF corresponding to the same renormalised coupling has the same continuum limit, combined fits of the Clover and Wilson SSF should stabilise the results. To show that this is actually the case, we present in Fig. 2 two of our first combined extrapolations at two different values of the renormalised coupling. Until this analysis is completed, we shall refrain from quoting an estimate for the quantity $Q^{RGI}/\bar{Q}(\mu)$.

In Fig. 3 we show our present results with an improved action for the SSF associated with the first renormalisation condition in eq. (7). The

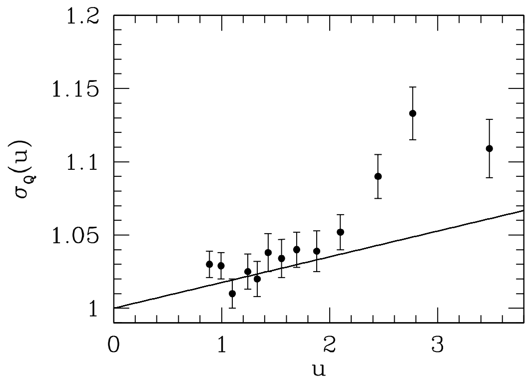

Figure 3. Step scaling function σ_Q in the continuum limit. The solid line is the LO perturbative prediction for σ_Q.

step scaling functions associated with the other choices of renormalisation scheme are qualitatively similar. Our results need to be supplemented by the NLO calculation of the SSF, which will enable us to estimate the importance of NP contributions to this quantity in the SF scheme. It will also provide the necessary information for the matching between SF and $\overline{\rm MS}$ schemes. This calculation is well under way.

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