

PRIMORDIAL MAGNETIC FIELDS

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Large scale magnetic fields represent a triple point where cosmology, high-energy physics and astrophysics meet for different but related purposes. After reviewing the implications of large scale magnetic fields in these different areas, the rôle of primordial magnetic fields is discussed in various physical processes occurring prior to the decoupling epoch with particular attention to the big bang nucleosynthesis (BBN) epoch and to the electroweak (EW) epoch. The generation of matter–antimatter isocurvature fluctuations, induced by hypermagnetic fields, is analyzed in light of a possible increase of extra-relativistic species at BBN. It is argued that stochastic GW backgrounds can be generated by hypermagnetic fields at the LISA frequency. The problem of the origin of large scale magnetic fields is also scrutinized.

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1 A triple point

Through the last fifty years, the possible existence and implications of primordial magnetic fields became a very useful cross-disciplinary area at the interface of cosmology, astrophysics and high-energy physics [1, 2, 3, 4, 5, 6, 7]. From astrophysical observations, we do know that planets, stars, the interstellar medium and the intergalactic medium are all magnetized. The magnetic fields in these environments have values ranging from the μ G of the intra-cluster medium, to the G (in the case of the earth) up to presumably 10^{12} G, the typical magnetic fields of neutron stars. In principle, observations of magnetic fields in galaxies (and clusters of galaxies) could discriminate between a direct primordial origin of the large-scale fields and a primordial origin mediated by a dynamo amplification. None of the two options are, at the moment, supported by clear observational evidence. Furthermore, in both approaches there are various theoretical assumptions which have been (but still need to be) carefully scrutinized.

In high-energy physics, the possible existence of intergalactic magnetic fields is one of the crucial unknowns in the analysis of ultra-high energy cosmic rays above the GZK cut-off. The interplay of high-energy physics and astrophysics is indeed present since the origin of this subject. In 1949 the scientific argument between Fermi [8], on one side, and Alfvén [9, 10], Richtmyer and Teller [11], on the other, concerned exactly the possible existence of galactic magnetic fields. Fermi was convinced that high-energy cosmic rays are in equilibrium with the whole galaxy while Alfvén was supporting the idea that high energy cosmic rays are in equilibrium with stars. In order to make his argument consistent, Fermi postulated (rather than demonstrated) the existence of a μ G galactic magnetic field. Fermi thought that the origin of this field was primordial.

In cosmology the possible existence of magnetic fields prior to decoupling can influence virtually all the moments in the thermodynamical history of the Universe. Big-bang nucleosynthesis (BBN), electroweak phase transition (EWPT), decoupling time are all influenced by the existence of magnetic fields at the corresponding epochs. If magnetic fields were originated in the past history of the Universe, their birth should be related, in some way, to the interplay of gravitational and gauge interactions. Superstring theories and higher-dimensional theories, formulated through the past thirty years, pretend to give us some hints on the possible form of such an interplay and their implications may be useful to consider.

The present paper is organized as follows. In Section II the basic ideas on large scale magnetic field structure and observations will be briefly outlined. Section III collects some considerations on the evolution of magnetic fields. In Section IV the problem of the origin will be illustrated with particular attention to models where there is an effective evolution of the gauge coupling. Section V deals with the possible implications of hypermagnetic fields for the EW physics and for the generation of the BAU. In Section VI it will be shown that if hypermagnetic fields are present at the EW epoch, matter–antimatter fluctuations are likely to be produced at BBN. In Section VII the implications of hypermagnetic fields

for the GW backgrounds at the LISA and VIRGO/LIGO frequencies will be discussed. In Section VIII some speculations on the possible Faraday rotation of the CMB polarization will be presented. Section IX contains some concluding remarks.

2 A primer on magnetic fields observations

There already excellent reviews on the measurements of large scale magnetic fields in diffuse astrophysical plasmas [1, 2, 3, 4, 5]. Here we want just to give an elementary primer on the main ingredients of the detection strategies and focus the attention on recent observations.

2.1 Local and global observables

In order to measure large scale magnetic fields, one of the first effects coming to mind, is the Zeeman splitting. The energy levels of an hydrogen atom in the background of a magnetic field are not degenerate. The presence of a magnetic field produces a well known splitting of the spectral lines:

$$\Delta\nu_Z = \frac{eB_{\parallel}}{2\pi m_e}. \quad (2.1)$$

From the estimate of the splitting, the magnetic field intensity can be deduced. Now, the most common element in the interstellar medium is neutral hydrogen, emitting the famous 21-cm line (corresponding to a frequency of 1420 MHz). Suppose that a magnetic field of μ G strength is present in the interstellar medium. From Eq. (2.1), the induced splitting, $\Delta\nu_Z \sim 3\text{Hz}$, can be estimated. Hence, Zeeman splitting of the 21-cm line generates two oppositely circular polarized spectral lines whose apparent splitting is however sub-leading if compared to the Doppler broadening. In fact, the atoms and molecules in the interstellar medium suffer thermal motion and the amount of induced Doppler broadening is roughly given by

$$\Delta\nu_{\text{Dop}} \sim \left(\frac{v_{\text{th}}}{c}\right)\nu, \quad (2.2)$$

where v_{th} is the thermal velocity $\propto \sqrt{T/m}$ where m is the mass of the atom or molecule. The amount of Doppler broadening is $\Delta\nu_{\text{Dop}} \sim 30$ kHz which is much larger than the Zeeman splitting we ought to detect. Zeeman splitting of molecules and recombination lines should however be detectable if the magnetic field strength gets larger with the density. Indeed in the interstellar medium there are molecules with an unpaired electron spin. In these cases a Zeeman splitting comparable with the one of the 21-cm line can be foreseen. These molecules include OH, CN, CH and some other. In the past, OH clouds were used in order to estimate the magnetic field (see [3] and references therein). The possible caveat with this type of estimates is that the measurements can only be very *local*. The above mentioned molecules are much less common than neutral hydrogen and are localized in specific regions of the interstellar medium.

The first experimental evidence of the existence of large scale magnetic fields in galaxies came from the synchrotron emission. The emissivity formula for the synchrotron depends upon B_{\perp} and upon the relativistic electron density. The synchrotron has an intrinsic polarization which can give the orientation of the magnetic field, but not the specific sign of the orientation vector. The relativistic electron density is sometimes estimated using equipartition, i.e. the idea that magnetic and kinetic energy densities may be, after all, comparable. Equipartition is not an experimental evidence, it is a working hypothesis which may or may not be realized in the system under observation. For instance equipartition probably holds for the Milky Way but it does not seem to be valid in the Magellanic Clouds [13]. The average equipartition field strengths in galaxies ranges from the 4μ G of M33 up to the 19μ G of NGC2276 [14].

In order to infer the magnitude of the magnetic field strength Faraday effect has been widely used. When a polarized radio wave passes through a region of space containing a plasma with a magnetic field the polarization plane of the wave gets rotated by an amount which is directly proportional to the square of the plasma frequency (and hence to the electron density) and to the Larmor frequency (and hence to the magnetic field intensity). Calling ϕ the shift in the polarization plane of the wave, a linear regression, connecting the shift in the polarization plane and the square of the wavelength of observation, can be obtained:

$$\phi = \phi_0 + \text{RM}\lambda^2. \quad (2.3)$$

By measuring this relation for two (or more) separate (but close) wavelengths, the angular coefficient of the regression can be obtained and it turns out to be

$$\frac{\Delta\phi}{\Delta\lambda^2} = 811.9 \int \left(\frac{n_e}{\text{cm}^{-3}} \right) \left(\frac{B_{\parallel}}{\mu\text{G}} \right) d \left(\frac{\ell}{\text{kpc}} \right), \quad (2.4)$$

in units of rad/m^2 when all the quantities of the integrand are measured in the above units. As reminded, RM should be performed at sufficiently close wave-lengths. Typically the angles should be determined with an accuracy greater than $\delta\phi \sim \pm\pi$. Otherwise ambiguities may arise in the determination of the angular coefficient appearing in the linear regression of Eq. (2.3) [1, 3].

It should be appreciated that the RM contains not only the magnetic field (which should be observationally estimated), but also the column density of electrons. From the radio-astronomical observations, different techniques can be used in order to determine the column density of electrons. One possibility is to notice that in the observed Universe there are pulsars. Pulsars are astrophysical objects emitting regular pulses of electromagnetic radiation with periods ranging from few milliseconds to few seconds. By comparing the arrival times of different radio pulses at different radio wave-lengths, it is found that signals are slightly delayed as they pass through the interstellar medium exactly because electromagnetic waves travel faster in the vacuum than in an ionized medium. Hence, from pulsars the column density of electrons can be obtained in the form of the dispersion

measure, i.e. $DM \propto \int n_e dl$. Dividing the RM by DM, an estimate of the magnetic field can be obtained.

Already this simple-minded account of the main experimental techniques used for the detection of large scales magnetic fields shows that there may be problems in the determination of magnetic fields right outside galaxies. There magnetic fields are *assumed* to be often of nG strength. However, due to the lack of sources for the determination of the column density of electrons, it is hard to turn the assumption into an experimental evidence.

Finally, an interesting source of observational informations on galactic magnetic field structure is the polarization of synchrotron emission. Magnetic fields in external galaxies have an uniform and a random component. Synchrotron polarization at high radio frequencies (where Faraday rotation is small) can be used in order to estimate the relative weight of the mean and random parts of a given field since the polarization essentially depends upon the ratio between the uniform and the total (i.e. random plus uniform) magnetic field [7].

Since various theoretical speculations suggest that also clusters are magnetized, it would be interesting to know if regular Abell clusters possess large scale magnetic fields. Different results in this direction have been reported [15, 16, 17, 18]. Some studies during the past decade [15, 16] dealt mainly with the case of a single cluster (more specifically the Coma cluster). The idea was to target (with Faraday rotation measurements) radio sources inside the cluster. However, it was also soon realized that the study of many radio sources inside different clusters may lead to experimental problems due to the sensitivity limitations of radio-astronomical facilities. The strategy is currently to study a sample of clusters each with one or two bright radio-sources inside.

In the past it was shown that regular clusters have cores with a detectable component of RM [17, 18]. Recent results suggest that μ Gauss magnetic fields are indeed detected inside regular clusters [19]. Inside the cluster means in the intra-cluster medium. Therefore, these magnetic fields cannot be associated with individual galaxies.

Regular Abell clusters with strong x-ray emission were studied using a twofold technique [19, 20]. From the ROSAT * [21] full sky survey, the electron density has been determined. Faraday RM (for the same set of 16 Abell clusters) has been estimated through observations at the VLA †. The amusing result (confirming previous claims based only on one cluster [15, 16]) is that x-ray bright Abell clusters possess a magnetic field of μ Gauss strength. The clusters have been selected in order to show similar morphological features. All the 16 clusters monitored with this technique are at low red-shift ($z < 0.1$) and at high galactic latitude ($|b| > 20^\circ$).

These recent developments are rather promising and establish a clear connection be-

*The ROetgen SATellite was flying from June 1991 to February 1999. ROSAT provided a map of the x-ray sky in the range 0.1–2.5 keV. For the ROSAT catalog of X-ray bright Abell clusters see [22].

†The Very Large Array telescope is a radio-astronomical facility consisting of 27 parabolic antennas spread 20km^2 in the New Mexico desert.

tween radio-astronomical techniques and the improvements in the knowledge of x-ray sky. There are various satellite missions mapping the x-ray sky at low energies (ASCA, CHANDRA, NEWTON [‡]). There is the hope that a more precise knowledge of the surface brightness of regular clusters will help in the experimental determination of large scale magnetic fields between galaxies.

It is interesting to notice that intra-cluster magnetic fields of μ G strength can induce Faraday rotation on CMB polarization. By combining informations from Sunyaev-Zeldovich effect and X-ray emission from the same clusters, it has been recently suggested that a richer information concerning electron column density can be obtained [23]. In Fig. 1 the results reported in [19] are summarized. In Fig. 1 the RM of the sample of x-ray bright Abell clusters is reported after the subtraction of the RM of the galaxy. At high galactic latitude (where all the observed clusters are) the galactic contribution is rather small and of the order of 9.5rad/m^2 . In Fig. 1 the open points represent sources

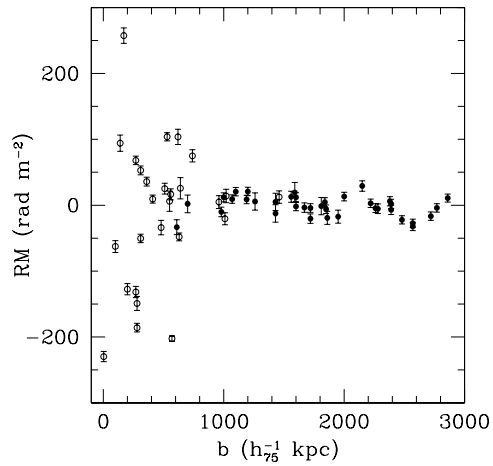


Figure 1: From Ref. [19] the RM deduced from a sample of 16 X-ray bright Abell clusters is reported as a function of the source impact parameter.

viewed through the thermal cluster gas, whereas the full points represent control sources at impact parameters larger than the cluster gas. The excess in RM attributed to clusters sources is clearly visible.

Using the described techniques large scale magnetic fields can be observed and studied in external galaxies, in clusters and also in our own galaxy. While the study of external galaxies and clusters may provide a global picture of magnetic fields, the galactic observations performed within the Milky Way are more sensitive to the local spatial variations of the magnetic field. For this reasons local and global observations are complementary.

[‡]ASCA is operating between 0.4 AND 10 keV and it is flying since February 1993. CHANDRA (NASA mission) and NEWTON (ESA mission) have an energy range comparable with the one of ASCA and were launched, almost simultaneously, in 1999.

The flipped side of the coin, as we will see in the second part of the present Section, is that the global structure of the magnetic field of our galaxy is not known directly and to high precision but it is deduced from (or corroborated by) the global knowledge of other spiral galaxies.

2.2 Geometrical models

Since the early seventies [1, 24] the magnetic field of the Milky way was shown to be parallel to the galactic plane. RM derived from pulsars allow consistent determinations of the magnetic field direction and intensity [28, 29]. In the Milky Way, the uniform component of the magnetic field is thought to lie in the plane of the galactic disk and it is thought to be directed approximately along the spiral arm. There is, though, a slight difference between the northern and southern hemisphere. While in the southern hemisphere the magnetic field is roughly $2\mu\text{G}$, the magnetic field in the northern hemisphere is three times smaller than the one of the southern hemisphere. Differently from other spirals, the Milky Way has also a large radio halo. Finally RM data seem to suggest that the magnetic field flips its direction from one spiral arm to the other. As far as the stochastic component of the galactic magnetic field is concerned, the situation seems to be, according to the reported results, still unclear [25, 26, 27]. It is, at present, unclear if the stochastic component of the galactic magnetic field is much smaller than (or of the same order of) the related homogeneous part. The geometries of large scale magnetic fields in spiral galaxies can be divided into two classes. We can have axysymmetric spirals (ASS) and bisymmetric spirals (BSS). This classification scheme refers, respectively, to even and odd parity with respect to rotation by an angle π around the galactic center. Speculations concerning the origin of galactic fields prefer to associate a primordial field with a BSS configuration while the ASS would be more associated with a field produced through a strong dynamo activity.

In the case of the Milky way, as we saw, the magnetic field flips its direction from one spiral arm to the other and then, as pointed out by Sofue and Fujimoto (SF) [29] the galactic magnetic field should probably be associated with a BSS model. In the SF model the radial and azimuthal components of the magnetic field in a bisymmetric logarithmic spiral configuration is given through

$$\begin{aligned} B_r &= f(r) \cos\left(\theta - \beta \ln \frac{r}{r_0}\right) \sin p \\ B_\theta &= f(r) \cos\left(\theta - \beta \ln \frac{r}{r_0}\right) \cos p \end{aligned} \quad (2.5)$$

where $r_0 \sim 10.5$ kpc is the galactocentric distance of the maximum of the field in our spiral arm, $\beta = 1/\tan p$ and $p = 10^\circ$ is the pitch angle of the spiral. The smooth profile $f(r)$ can be chosen in different ways. A motivated choice is [30, 31]

$$f(r) = 3 \frac{r_1}{r} \tanh^3\left(\frac{r}{r_2}\right) \mu\text{G} \quad (2.6)$$

where $r_1 = 8.5$ kpc is the distance of the Sun to the galactic center and $r_2 = 2$ kpc. The original model of SF does not have dependence in the z direction, however, the z dependence can be included and also more complicated models can be built [30]. Typically, along the z axis, magnetic fields are exponentially suppressed as $\exp[-z/z_0]$ with $z_0 \sim 4$ kpc. The structure of magnetic fields can be relevant when investigating the propagation of high-energy protons [34, 35] as noticed already long ago [36] (see also [28, 29, 30]).

3 Magnetic field evolution(s)

The galaxy is a gravitationally bound system formed by fluid of charged particles which is globally neutral for scales larger than the Debye sphere. In the interstellar medium, where the electron density is approximately $3 \times 10^{-2} \text{ cm}^{-3}$, the Debye sphere has a radius of roughly 10 m. Moreover, the galaxy is rotating with a typical rotation period of 3×10^8 yrs. Two complementary descriptions of the plasma can then be adopted. The first possibility is to study full kinetic system (the Vlasov-Landau equations [38, 39]). The second (complementary) description relies on the magnetohydrodynamical (MHD) treatment. Among the discussions of MHD effects, dynamo theory is particularly relevant. A full account of the various aspects and debates concerning the dynamo theory can be found in excellent textbooks [1, 2] and reviews [37]. After some elements of the Vlasov-Landau description, the the basic ideas concerning dynamo theory will be introduced.

3.1 Elements of a kinetic discussion

Already in flat space [40], and, a fortiori, in curved space [41], the kinetic approach is important once we deal with electric fields dissipation, charge and current density fluctuations and, in more general terms, with all the high frequency and small length scale phenomena in the plasma [42, 43]. The few elements of kinetic description will be given directly in curved spaces since they may be relevant for some applications [44].

Consider a conformally flat Friedmann-Robertson-Walker (FRW) metric written using the conformal time coordinate

$$ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2]. \quad (3.1)$$

Furthermore, consider an equilibrium homogeneous and isotropic conducting plasma, characterized by a distribution function $f_0(p)$ common for both positively and negatively charged ultrarelativistic particles (for example, electrons and positrons) . Suppose now that this plasma is slightly perturbed, so that the distribution functions are

$$f_+(\vec{x}, \vec{p}, \eta) = f_0(p) + \delta f_+(\vec{x}, \vec{p}, \eta), \quad f_-(\vec{x}, \vec{p}, \eta) = f_0(p) + \delta f_-(\vec{x}, \vec{p}, \eta), \quad (3.2)$$

where $+$ refers to positrons and $-$ to electrons, and \vec{p} is the conformal momentum. The Vlasov equation defining the curved-space evolution of the perturbed distributions can be

written as [44]

$$\frac{\partial f_+}{\partial \eta} + \vec{v} \cdot \frac{\partial f_+}{\partial \vec{x}} + e(\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f_+}{\partial \vec{p}} = \left(\frac{\partial f_+}{\partial \eta} \right)_{\text{coll}}, \quad (3.3)$$

$$\frac{\partial f_-}{\partial \eta} + \vec{v} \cdot \frac{\partial f_-}{\partial \vec{x}} - e(\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f_-}{\partial \vec{p}} = \left(\frac{\partial f_-}{\partial \eta} \right)_{\text{coll}}, \quad (3.4)$$

where the two terms appearing at the right hand side of each equation are the collision terms. The electric and magnetic fields are rescaled by the second power of the scale factor. This system of equation represents the curved space extension of the Vlasov-Landau approach to plasma fluctuations [38, 39]. All particle number densities here are related to the comoving volume. By subtracting Eqs. (3.3) and (3.4) we obtain the equations relating the fluctuations of the distributions functions of the charged particles present in the plasma to the induced gauge field fluctuations:

$$\begin{aligned} \frac{\partial}{\partial \eta} f(\vec{x}, \vec{p}, t) + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} f(\vec{x}, \vec{p}, t) + 2e\vec{E} \cdot \frac{\partial f_0}{\partial \vec{p}} &= -\nu(p)f, \\ \vec{\nabla} \cdot \vec{E} &= e \int d^3 p f(\vec{x}, \vec{p}, \eta), \\ \vec{\nabla} \times \vec{E} + \vec{B}' &= 0, \\ \vec{\nabla} \cdot \vec{B} &= 0, \\ \vec{\nabla} \times \vec{B} - \vec{E}' &= \int d^3 p \vec{v} f(\vec{x}, \vec{p}, \eta), \end{aligned} \quad (3.5)$$

where $f(\vec{x}, \vec{p}, \eta) = \delta f_+(\vec{x}, \vec{p}, \eta) - \delta f_-(\vec{x}, \vec{p}, \eta)$ and $\nu(p)$ is a typical frequency of collisions [41].

Now, if $\delta f_{\pm}(\vec{x}, \vec{p}, \eta) \neq 0$ at the beginning of the radiation dominated epoch η_0 and $E(\vec{x}, \eta_0) \simeq B(\vec{x}, \eta_0) = 0$ initially, the magnetic field at later times can be found from Eqs. (3.5) [40]. Various useful generalizations of the Vlasov-Landau system to curved spaces is given in [45, 46, 47].

3.2 Effective (MHD) description and dynamo instability

For scales sufficiently large compared with the Debye sphere and for frequencies sufficiently small compared with the plasma frequency, the spectrum of plasma excitations obtained from the kinetic theory matches the spectrum obtained from an (effective) MHD description [42]. Furthermore, since the galaxy is rotating and since the conditions of validity of the MHD approximation are met, it is possible to use the so-called dynamo instability in order to amplify a small magnetic inhomogeneity up to the observed value. This is, at least, the hope [37].

The pioneering attempts towards a MHD description of interstellar plasma, go back to the works of Alfvén [9] and of Fermi and Chandrasekar [12]. Since the work of Parker [48], on the so-called $\alpha - \Omega$ theory, the dynamo effect has been used in order to explain

(or, to ease) the problem of the origin of galactic magnetic fields. The standard dynamo theory has been questioned in different ways. Piddington [49, 50] pointed out that small-scale magnetic fields can grow large enough (until equipartition is reached) to swamp the dynamo action. The quenching of the dynamo action has been numerically shown by Kulsrud and Anderson [51]. More recently, it has been argued that if the large-scale magnetic field reaches the critical value $Re_M^{-1/2}v$ § the dynamo action could also be quenched [52, 53].

MHD equations can be derived from a microscopic (kinetic) approach and also from a macroscopic approach where the displacement current is neglected [42]. If the displacement current is neglected the electric field can be expressed using the Ohm law and the magnetic diffusivity equation is obtained

$$\frac{\partial \vec{B}}{\partial \eta} = \vec{\nabla} \times (\vec{V} \times \vec{B}) + \frac{1}{\sigma} \nabla^2 \vec{B}. \quad (3.6)$$

The first term at the r.h.s. of Eq. (3.6) is related to the dynamo term, while the second term is the diffusivity term. The conductivity σ appearing in Eq. (3.6) is a global quantity which can be computed in a kinetic approach [42] during a given phase of evolution of the background geometry [44]. If the plasma is non-relativistic $\sigma \propto T^{3/2}$. If the plasma is relativistic $\sigma \propto T$. In Eq. (3.6) the contribution containing the conductivity is usually called magnetic diffusivity term. The magnetic diffusivity scale is defined as

$$L_\sigma = \sqrt{\frac{\tau_U}{\sigma}}, \quad (3.7)$$

where τ_U is the age of the Universe at the corresponding epoch. Typical galactic values are $1/\sigma \sim 10^{25} \text{ cm}^2/\text{sec}$, $H_0 \sim \tau_U^{-1} \sim 10^{-18} \text{ Hz}$, $L_\sigma \sim \text{A.U.}$.

Eq. (3.6) is exact, in the sense that both \vec{V} and \vec{B} contain long and short wavelength modes. The aim of the various attempts of the dynamo theory is to get an equation describing only the “mean value” of the magnetic field. To this end the first step is to separate the exact magnetic and velocity fields as

$$\begin{aligned} \vec{B} &= \langle \vec{B} \rangle + \vec{b}, \\ \vec{V} &= \langle \vec{V} \rangle + \vec{v}, \end{aligned} \quad (3.8)$$

where $\langle \vec{B} \rangle$ and $\langle \vec{V} \rangle$ are the averages over an ensemble of many realizations of the velocity field \vec{V} . In order to derive the standard form of the dynamo equations few important assumptions should be made. These assumptions are :

§ Re_M is the magnetic Reynolds number [43], i.e. , approximately, the ratio of the first (the dynamo contribution) over the second term (the magnetic diffusivity term) appearing at the right hand side of Eq. (3.6); v is the velocity field at the outer scale of turbulence.

- - The scale of variation of the turbulent motion \vec{v} should be smaller than the typical scale of variation of $\langle \vec{B} \rangle$. In the galactic problem $\langle \vec{V} \rangle$ is the differential rotation of the galaxy, while \vec{v} is the turbulent motion generated by stars and supernovae. Typically the scale of variation of \vec{v} is less than 100 pc while the interesting scales for $\langle \vec{B} \rangle$ are larger than the kpc.
- - The field \vec{b} is such that $|\vec{b}| \ll |\langle \vec{B} \rangle|$.
- - It should happen that $\langle \vec{v} \cdot \vec{\nabla} \times \vec{v} \rangle \neq 0$.
- - Magnetic flux is frozen into the plasma (i.e. magnetic flux is conserved).

From the magnetic diffusivity equation (3.6), and using the listed assumptions, it is possible to derive the typical structure of the dynamo term by carefully averaging over the velocity field according to the procedure outlined in [1, 2, 37]. Inserting Eq. (3.8) into (3.6) and breaking the equation into a mean part and a random part, two separate induction equations can be obtained for the mean and random parts of the magnetic field

$$\frac{\partial \langle \vec{B} \rangle}{\partial \eta} = \vec{\nabla} \times \left(\langle \vec{V} \rangle \times \langle \vec{B} \rangle \right) + \vec{\nabla} \times \langle \vec{v} \times \vec{b} \rangle, \quad (3.9)$$

$$\frac{\partial \vec{b}}{\partial \eta} = \vec{\nabla} \times (\vec{v} \times \langle \vec{B} \rangle) + \vec{\nabla} \times (\langle \vec{V} \rangle \times \vec{b}) + \vec{\nabla} \times (\vec{v} \times \vec{b}) - \vec{\nabla} \times \langle \vec{v} \times \vec{b} \rangle, \quad (3.10)$$

where the (magnetic) diffusivity terms have been neglected. In Eq. (3.9), $\langle \vec{v} \times \vec{b} \rangle$ is called “turbulent emf” and it is the average of the cross product of the small-scale velocity field \vec{v} and of the small scale magnetic field \vec{b} over a scale much smaller than the scale of $\langle \vec{B} \rangle$ but much larger than the scale of turbulence. Sometimes, the calculation of the effect of $\langle \vec{v} \times \vec{b} \rangle$ is done in the case of incompressible and isotropic turbulence. In this case $\langle \vec{v} \times \vec{b} \rangle = 0$. This estimate is, however, not realistic since $\langle \vec{B} \rangle$ is not isotropic. More correctly [37], $\langle \vec{v} \times \vec{b} \rangle$ should be evaluated by using Eq. (3.10) which is usually written in a simplified form

$$\frac{\partial \vec{b}}{\partial \eta} = \vec{\nabla} \times (\vec{v} \times \langle \vec{B} \rangle), \quad (3.11)$$

where all but the first term of Eq. (3.10) have been neglected. To neglect the term $\vec{\nabla} \times (\langle \vec{V} \rangle \times \vec{b})$ does not pose any problem since it corresponds to choose a reference frame where $\langle \vec{V} \rangle$ is constant. However, the other terms, neglected in Eq. (3.11), are dropped because it is assumed that $|\vec{b}| \ll |\langle \vec{B} \rangle|$. This assumption may not be valid all the time and for all the scales. The validity of Eq. (3.11) seems to require that $1/\sigma$ is very large so that magnetic diffusivity can keep always \vec{b} small [54]. On the other hand [37] one can argue that \vec{b} is only present over very small scales (smaller than 100 pc) and in this case the approximate form of eq. (3.11) seems to be more justified.

From Eqs. (3.9)–(3.11) it is possible to get to the final result for the evolution equation of $\langle \vec{B} \rangle$ [37] as it is usually quoted

$$\frac{\partial \langle \vec{B} \rangle}{\partial \eta} = \vec{\nabla} \times (\alpha \langle \vec{B} \rangle) + \beta \nabla^2 \langle \vec{B} \rangle + \vec{\nabla} \times \left(\langle \vec{V} \rangle \times \langle \vec{B} \rangle \right), \quad (3.12)$$

where

$$\alpha = -\frac{\tau_0}{3} \langle \vec{v} \cdot \vec{\nabla} \times \vec{v} \rangle, \quad (3.13)$$

$$\beta = \frac{\tau_0}{3} \langle \vec{v}^2 \rangle, \quad (3.14)$$

where α is the dynamo term, β is the diffusion term and τ is the typical correlation time of the velocity field. The term α is, in general, space-dependent.

It is interesting to point out [2] that the dynamo term in Eq. (3.12) has a simple electro-dynamical meaning (when α is constant), namely, it can be interpreted as a mean ohmic current directed along the magnetic field:

$$\vec{J} = -\alpha \vec{B}. \quad (3.15)$$

This equation tells us that an ensemble of screw-like vortices with zero mean helicity is able to generate loops in the magnetic flux tubes in a plane orthogonal to the one of the original field.

If the velocity field is parity-invariant (i.e. no vorticity for scales comparable with the correlation length of the magnetic field), then the dynamics of the infrared modes is decoupled from the velocity field since, over those scales, $\alpha = 0$. When the (averaged) dynamo term dominates in Eq. (3.12), magnetic fields can be exponentially amplified. The standard lore is that the dynamo action stops when the value of the magnetic field reaches the equipartition value (i.e. when the magnetic and kinetic energy of the plasma are comparable). At this point the dynamo “saturates”. This statement means that, in more dynamical terms [1], back-reaction effects cannot be neglected anymore and Eq. (3.12) should then be supplemented with non-linear terms (of the order of \vec{B}^2), whose effect is to stabilize the amplification of the magnetic field.

It is sometimes useful to recall that the full MHD equations can be studied in two different limits : the ideal (or superconducting) approximation where the conductivity is assumed to be very high and the real (or resistive) limit where the conductivity takes a finite value. In the ideal limit both the magnetic flux and the magnetic helicity are conserved. This means, formally [55],

$$\frac{d}{d\eta} \int_{\Sigma} \vec{B} \cdot d\vec{\Sigma} = -\frac{1}{\sigma} \int_{\Sigma} \vec{\nabla} \times \vec{\nabla} \times \vec{B} \cdot d\vec{\Sigma}, \quad (3.16)$$

where Σ is an arbitrary closed surface which moves with the plasma. If we are in the inertial regime (i.e. $L > L_{\sigma}$) we can say that the expression appearing at the right hand side is sub-leading and the magnetic flux lines evolve glued to the plasma element.

The other quantity which is conserved in the superconducting limit is the magnetic helicity

$$\mathcal{H}_M = \int_V d^3x \vec{A} \cdot \vec{B}, \quad (3.17)$$

where \vec{A} is the vector potential [¶]. In Eq. (3.17) the vector potential appears and, therefore it might seem that the expression is not gauge invariant. This is not the case. In fact $\vec{A} \cdot \vec{B}$ is not gauge invariant but, none the less, \mathcal{H}_M is gauge-invariant since the integration volume is defined in such a way that the magnetic field \vec{B} is parallel to the surface which bounds V and which we will call ∂V . If \vec{n} is the unit vector normal to ∂V then $\vec{B} \cdot \vec{n} = 0$ on ∂V [125].

The magnetic gyrotropy

$$\vec{B} \cdot \vec{\nabla} \times \vec{B} \quad (3.18)$$

it is a gauge invariant measure of the diffusion rate of \mathcal{H}_M at finite conductivity. In fact [55]

$$\frac{d}{d\eta} \mathcal{H}_M = -\frac{1}{\sigma} \int_V d^3x \vec{B} \cdot \vec{\nabla} \times \vec{B}. \quad (3.19)$$

The magnetic gyrotropy is a useful quantity in order to distinguish different mechanisms for the magnetic field generation. Some mechanisms are able to produce magnetic fields whose flux lines have a topologically non-trivial structure (i.e. $\langle \vec{B} \cdot \vec{\nabla} \times \vec{B} \rangle \neq 0$).

Usually the picture for the formation of galactic magnetic fields is related to the possibility of implementing the dynamo mechanism. By comparing the rotation period with the age of the galaxy (for a Universe with $\Omega_\Lambda \sim 0.7$, $h \sim 0.65$ and $\Omega_m \sim 0.3$) the number of rotations performed by the galaxy since its origin is approximately 30. During these 30 rotations the dynamo term of Eq. (3.12) dominates against the magnetic diffusivity. As a consequence an instability develops. This instability can be used in order to drive the magnetic field from some small initial condition up to its observed value. Eq. (3.12) is linear in the mean magnetic field. Hence, initial conditions for the mean magnetic field should be postulated at a given time and over a given scale. This initial mean field, postulated as initial condition of (3.12) is usually called seed.

Most of the work in the context of the dynamo theory focuses on reproducing the correct features of the magnetic field of our galaxy. The achievable amplification produced by the dynamo instability can be at most of 10^{13} , i.e. e^{30} . Thus, if the present value of the galactic magnetic field is 10^{-6} Gauss, its value right after the gravitational collapse of the protogalaxy might have been as small as 10^{-19} Gauss over a typical scale of 30–100 kpc.

There is a simple way to relate the value of the magnetic fields right after gravitational collapse to the value of the magnetic field right before gravitational collapse. Since the gravitational collapse occurs at high conductivity the magnetic flux and the magnetic

[¶]Notice that in conformally flat FRW spaces the radiation gauge is conformally invariant. This property is not shared by the Lorentz gauge condition [56].

helicity are both conserved. Right before the formation of the galaxy a patch of matter of roughly 1 Mpc collapses by gravitational instability. Right before the collapse the mean energy density of the patch, stored in matter, is of the order of the critical density of the Universe. Right after collapse the mean matter density of the protogalaxy is, approximately, six orders of magnitude larger than the critical density.

Since the physical size of the patch decreases from 1 Mpc to 30 kpc the magnetic field increases, because of flux conservation, of a factor $(\rho_a/\rho_b)^{2/3} \sim 10^4$ where ρ_a and ρ_b are, respectively the energy densities right after and right before gravitational collapse. The correct initial condition in order to turn on the dynamo instability is $B \sim 10^{-23}$ Gauss over a scale of 1 Mpc, right before gravitational collapse.

Since the flux is conserved the ratio between the magnetic energy density, $\rho_B(L, \eta)$ and the energy density sitting in radiation, $\rho_\gamma(\eta)$ is almost constant and therefore, in terms of this quantity (which is only scale dependent but not time dependent), the dynamo requirement can be rephrased as

$$r_B(L) = \frac{\rho_B(L, \eta)}{\rho_\gamma(\eta)} \geq 10^{-34}, \quad L \sim 1 \text{ Mpc}, \quad (3.20)$$

to be compared with the value $r_B \sim 10^{-8}$ which would lead to the galactic magnetic field only thanks to the collapse and without the need of dynamo action. This is the case when the magnetic field is fully primordial.

The estimate of Eq. (3.20) is, to say the least, rather generous and has been presented just in order to make contact with several papers (concerned with the origin of large scale magnetic fields) using such an estimate. Eq. (3.20) is based on the (highly questionable) assumption that the amplification occurs over thirty e-folds while the magnetic flux is completely frozen in. In the real situation, the achievable amplification is much smaller. Typically a good seed would not be 10^{-19} G after collapse (as we assumed for the simplicity of the discussion) but of the order of (or larger than) 10^{-12} – 10^{-13} G [37].

The possible applications of dynamo mechanism to clusters is still under debate and it seems more problematic [19, 20, 57]. The typical scale of the gravitational collapse of a cluster is larger (roughly by one order of magnitude) than the scale of gravitational collapse of the protogalaxy. Furthermore, the mean mass density within the Abell radius ($\simeq 1.5h^{-1}$ Mpc) is roughly 10^3 larger than the critical density. Consequently, clusters rotate less than galaxies since their origin and the value of $r_B(L)$ has to be larger than in the case of galaxies. Since the details of the dynamo mechanism applied to clusters are not clear, at present, it will be required that $r_B(L_{\text{Mpc}}) \gg 10^{-34}$ [for instance $r_B(L_{\text{Mpc}}) \simeq 10^{-12}$].

4 A twofold path to the origin

Back in the late sixties Harrison [58] suggested that the initial conditions of the magnetic diffusivity equation might have something to do with cosmology in the same way as

he suggested that the primordial spectrum of gravitational potential fluctuations (i.e. the Harrison-Zeldovich spectrum) might be produced in some primordial phase of the evolution of the Universe. Since then, several mechanisms have been invoked in order to explain the origin of the magnetic seeds [62, 64] and few of them are compatible with inflationary evolution. It is not my purpose to review here all the different mechanisms which have been proposed and good reviews exist already [59, 60, 61]. A very incomplete selection of references is, however, reported [62, 63, 64]. Furthermore, more details on this topic can be found in the contribution of D. Boyanovsky [65].

In spite of the richness of the theoretical models, the mechanisms for magnetic field generation can be divided, broadly speaking, into two categories: astrophysical [3, 37] and cosmological. The cosmological mechanisms can be divided, in their turn, into *causal* mechanisms (where the magnetic seeds are produced at a given time inside the horizon) and *inflationary* mechanisms where correlations in the magnetic field are produced outside the horizon. Astrophysical mechanisms have always to explain the initial conditions of Eq. (3.12). This is because the MHD are linear in the magnetic fields. It is questionable if purely astrophysical considerations can set a natural initial condition for the dynamo amplification.

4.1 Turbulence?

Causal mechanisms usually fail in reproducing the correct correlation scale of the field whereas inflationary mechanisms have problems in reproducing the correct amplitude required in order to turn on successfully the dynamo action. In the context of causal mechanisms there are interesting proposals in order to enlarge the correlation scale. These proposals have to do with the possible occurrence of turbulence in the early Universe. The ratio of the magnetic Reynolds number to the kinetic Reynolds number is the Prandtl number [43]

$$\text{Pr}_M = \frac{\text{Re}_M}{\text{Re}} = \nu\sigma, \quad (4.1)$$

where ν is the thermal diffusivity coefficient and σ , is, as usual, the conductivity. Consider, for instance, the case of the electroweak epoch [66, 67, 68, 70, 71, 72]. At this epoch taking $H_{\text{ew}}^{-1} \sim 3\text{cm}$ we get that $\text{Pr}_M \sim \nu\sigma \sim 10^6$ where the bulk velocity of the plasma is of the order of the bubble wall velocity at the epoch of the phase transition.

This means that the early universe is both kinetically and magnetically turbulent. The features of magnetic and kinetic turbulence are different. This aspect reflects in a spectrum of fluctuations is different from the usual Kolmogorov spectrum [43]. If the Universe is both magnetically and kinetically turbulent it has been speculated that an inverse cascade mechanism can occur [66, 67, 68, 72]. This idea was originally put forward in the context of MHD simulations [43]. The inverse cascade would imply a growth in the correlation scale of the magnetic inhomogeneities and it has been shown to occur numerically in the approximation of unitary Prandtl number [43]. Specific cascade models have been

also studied [66, 67, 68, 69]. A particularly important rôle is played, in this context, by the initial spectrum of magnetic fields (the so-called injection spectrum [68]) and by the topological properties of the magnetic flux lines. If the system has non vanishing magnetic helicity and magnetic gyrotropy it was suggested that the inverse cascade can occur more efficiently [73, 74]. Recently simulations have discussed the possibility of inverse cascade in realistic MHD models [74]. More analytic discussions based on renormalization group approach applied to turbulent MHD seem to be not totally consistent with the occurrence of inverse cascade at large scales [75].

4.2 Magnetic fields from dynamical gauge couplings

Large scale magnetic fluctuations can be generated during the early history of the Universe and can go outside the horizon with a mechanism similar to the one required in order to produce fluctuations in the gravitational (Bardeen) potential. In this case the correlation scale of the magnetic inhomogeneities can be large. However, the typical amplitudes obtainable in this class of models may be too small.

The key property allowing the amplification of the fluctuations of the scalar and tensor modes of the geometry is the fact that the corresponding equations of motion are not invariant under Weyl rescaling of a (conformally flat) metric of FRW type. In this sense the evolution equations of relic gravitons and of the scalar modes of the geometry are said to be *not conformally invariant*. If gauge couplings are dynamical, the evolution equations of the gauge field are also not conformally invariant. Interesting examples in this direction are models containing extra-dimensions and scalar-tensor theories of gravity where the gauge coupling is, effectively, a scalar degree of freedom evolving in a given geometry.

The remarkable similarity of the abundances of light elements in different galaxies leads to postulate that the Universe had to be dominated by radiation at the moment when the light elements were formed, namely for temperatures of approximately 0.1 MeV [76, 77]. Prior to the moment of nucleosynthesis even indirect informations concerning the thermodynamical state of our Universe are lacking even if our knowledge of particle physics could give us important hints concerning the dynamics of the electroweak phase transition [78].

The success of big-bang nucleosynthesis (BBN) sets limits on alternative cosmological scenarios. Departures from homogeneity [79] and isotropy [80] of the background geometry can be successfully constrained. In the same spirit, BBN can also set limits on the dynamical evolution of internal dimensions [81, 82]. Internal dimensions are an essential ingredient of theories attempting the unification of gravitational and gauge interactions in a higher dimensional background like Kaluza-Klein theories [83] and superstring theories [84].

Defining, respectively, b_{BBN} and b_0 as the size of the internal dimensions at the BBN time and at the present epoch, the maximal variation allowed to the internal scale factor from the BBN time can be expressed as $b_{BBN}/b_0 \sim 1 + \epsilon$ where $|\epsilon| < 10^{-2}$ [81, 82]. The

bounds on the variation of the internal dimensions during the matter dominated epoch are even stronger. Denoting with an over-dot the derivation with respect to the cosmic time coordinate, we have that $|\dot{b}/b| < 10^{-9}H_0$ where H_0 is the present value of the Hubble parameter [81]. The fact that the time evolution of internal dimensions is so tightly constrained for temperatures lower of 1 MeV does not forbid that they could have been dynamical prior to that epoch. Moreover, recent observational evidence [85, 86, 87] seem to imply that the fine structure constant can be changing even today.

Suppose that prior to BBN internal dimensions were evolving in time and assume, for sake of simplicity, that after BBN the internal dimensions have been frozen to their present (constant) value. Consider a homogeneous and anisotropic manifold whose line element can be written as

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = a^2(\eta)[d\eta^2 - \gamma_{ij}dx^i dx^j] - b^2(\eta)\gamma_{ab}dy^a dy^b, \\ \mu, \nu = 0, \dots, D-1 = d+n, \quad i, j = 1, \dots, d, \quad a, b = d+1, \dots, d+n. \quad (4.2)$$

[η is the conformal time coordinate related, as usual to the cosmic time $t = \int a(\eta)d\eta$; $\gamma_{ij}(x)$, $\gamma_{ab}(y)$ are the metric tensors of two maximally symmetric Euclidean manifolds parameterized, respectively, by the ‘‘internal’’ and the ‘‘external’’ coordinates $\{x^i\}$ and $\{y^a\}$]. The metric of Eq. (4.2) describes the situation in which the d external dimensions (evolving with scale factor $a(\eta)$) and the n internal ones (evolving with scale factor $b(\eta)$) are dynamically decoupled from each other [88]. The results of the present investigation, however, can be easily generalized to the case of n different scale factors in the internal manifold.

Consider now a pure electromagnetic fluctuation decoupled from the sources, representing an electromagnetic wave propagating in the d -dimensional external space such that $A_\mu \equiv A_\mu(\vec{x}, \eta)$, $A_a = 0$. In the metric given in Eq. (4.2) the evolution equation of the gauge field fluctuations can be written as

$$\frac{1}{\sqrt{-G}}\partial_\mu\left(\sqrt{-G}G^{\alpha\mu}G^{\beta\nu}F_{\alpha\beta}\right) = 0, \quad (4.3)$$

where $F_{\alpha\beta} = \nabla_{[\alpha}A_{\beta]}$ is the gauge field strength and G is the determinant of the D dimensional metric. Notice that if $n = 0$ the space-time is isotropic and, therefore, the Maxwell’s equations can be reduced (by trivial rescaling) to the flat space equations. If $n \neq 0$ we have that the evolution equation of the electromagnetic fluctuations propagating in the external d -dimensional manifold will receive a contribution from the internal dimensions which cannot be rescaled away.

In the radiation gauge ($A_0 = 0$ and $\nabla_i A^i = 0$) the evolution the vector potentials can be written as

$$A_i'' + n\mathcal{F}A_i' - \vec{\nabla}^2 A_i = 0, \quad \mathcal{F} = \frac{b'}{b}. \quad (4.4)$$

The vector potentials A_i are already rescaled with respect to the (conformally flat) $d+1$ dimensional metric. In terms of the canonical normal modes of oscillations $\mathcal{A}_i = b^{n/2}A_i$

the previous equation can be written in a simpler form, namely

$$\mathcal{A}_i'' - V(\eta)\mathcal{A}_i - \vec{\nabla}^2 \mathcal{A}_i = 0, \quad V(\eta) = \frac{n^2}{4}\mathcal{F}^2 + \frac{n}{2}\mathcal{F}'. \quad (4.5)$$

From this set of equations the induced large scale magnetic fields can be computed in various models for the evolution of the internal manifold [89]. It should be noticed that large magnetic seeds are produced in this context only if internal dimensions are rather large if compared to the Planck length. This requires a careful discussion of the localization properties of gauge fields in the presence of large extra-dimensions [90] which is beyond the scope of the present discussion.

The same effect of magnetic field generation can be also obtained in the case of time-evolving gauge coupling already in four dimensions. In order to emphasize this phenomenon it will now be shown how squeezed states of relic photons can be produced [91]. It will be imagined that quantum-mechanical fluctuations of the gauge field will be present at some initial stage in the evolution of the Universe.

The squeezed states formalism has been successfully applied to the analysis of tensor, scalar [92] and rotational [93] fluctuations of the metric by Grishchuk and collaborators. In the case of relic gravitons and relic phonons the analogy with quantum optics is certainly very inspiring. In the case of relic photons the analogy is even closer since the time variation of the dilaton coupling plays directly the rôle of the laser “pump” which is employed in order to produce experimentally observable squeezed states [94].

The effective action of a generic Abelian gauge field in four space-time dimensions reads

$$S = -\frac{1}{4} \int d^4x \sqrt{-G} \frac{1}{g^2} F_{\alpha\beta} F^{\alpha\beta}, \quad (4.6)$$

where $F_{\alpha\beta} = \nabla_{[\alpha} A_{\beta]}$ is the Maxwell field strength and ∇_α is the covariant derivative with respect to the string frame metric $G_{\mu\nu}$. In Eq. (4.6) g is the (four dimensional) gauge coupling which is related to the expectation value of a scalar degree of freedom.

From Eq. (4.6) it is possible to derive the Hamiltonian and the Hamiltonian density of the gauge field fluctuations

$$H(\eta) = \int d^3k \sum_\alpha \left[k (\hat{a}_{k,\alpha}^\dagger \hat{a}_{k,\alpha} + \hat{a}_{-k,\alpha}^\dagger \hat{a}_{-k,\alpha} + 1) + \epsilon(g) \hat{a}_{-k,\alpha} \hat{a}_{k,\alpha} + \epsilon^*(g) \hat{a}_{k,\alpha}^\dagger \hat{a}_{-k,\alpha}^\dagger \right], \quad \epsilon(g) = i \frac{g'}{g}. \quad (4.7)$$

where The (two-modes) Hamiltonian contains a free part and the effect of the variation of the coupling constant is encoded in the (Hermitian) interaction term which is quadratic in the creation and annihilation operators whose evolution equations, read, in the Heisenberg picture

$$\frac{d\hat{a}_{k,\alpha}}{d\eta} = -ik\hat{a}_{k,\alpha} - \frac{g'}{g}\hat{a}_{-k,\alpha}^\dagger,$$

$$\frac{d\hat{a}_{k,\alpha}^\dagger}{d\eta} = ik\hat{a}_{k,\alpha}^\dagger - \frac{g'}{g}\hat{a}_{-k,\alpha}. \quad (4.8)$$

The general solution of the previous system of equations can be written in terms of a Bogoliubov-Valatin transformation

$$\begin{aligned} \hat{a}_{k,\alpha}(\eta) &= \mu_{k,\alpha}(\eta)\hat{b}_{k,\alpha} + \nu_{k,\alpha}(\eta)\hat{b}_{-k,\alpha}^\dagger \\ \hat{a}_{k,\alpha}^\dagger(\eta) &= \mu_{k,\alpha}^*(\eta)\hat{b}_{k,\alpha}^\dagger + \nu_{k,\alpha}^*(\eta)\hat{b}_{-k,\alpha} \end{aligned} \quad (4.9)$$

where $\hat{a}_{k,\alpha}(0) = \hat{b}_{k,\alpha}$ and $\hat{a}_{-k,\alpha}(0) = \hat{b}_{-k,\alpha}$. Unitarity requires that the two complex functions $\mu_k(\eta)$ and $\nu_k(\eta)$ are subjected to the condition $|\mu_k(\eta)|^2 - |\nu_k(\eta)|^2 = 1$ which also implies that $\mu_k(\eta)$ and $\nu_k(\eta)$ can be parameterized in terms of one real amplitude and two real phases

$$\mu_k = e^{i\theta_k} \cosh r_k, \quad \nu_k = e^{i(2\phi_k - \theta_k)} \sinh r_k, \quad (4.10)$$

(r is sometimes called squeezing parameter and ϕ_k is the squeezing phase; from now on we will drop the subscript labeling each polarization if not strictly necessary). The total number of produced photons

$$\langle 0_{-k}0_k | \hat{a}_k^\dagger(\eta)\hat{a}_k(\eta) + \hat{a}_{-k}^\dagger\hat{a}_{-k} | 0_k0_{-k} \rangle = 2 \bar{n}_k. \quad (4.11)$$

is expressed in terms of $\bar{n}_k = \sinh^2 r_k$, i.e. the mean number of produced photon pairs in the mode k . Inserting Eqs. (4.9),(4.10) and (4.11) into Eqs. (4.8) we can derive a closed system involving only the \bar{n}_k and the related phases:

$$\frac{d\bar{n}_k}{d\eta} = -2f(\bar{n}_k)\frac{g'}{g} \cos 2\phi_k, \quad (4.12)$$

$$\frac{d\theta_k}{d\eta} = -k + \frac{g'}{g} \frac{\bar{n}_k}{f(\bar{n}_k)} \sin 2\phi_k, \quad (4.13)$$

$$\frac{d\phi_k}{d\eta} = -k + \frac{g'}{g} \frac{df(\bar{n}_k)}{d\bar{n}_k} \sin 2\phi_k, \quad (4.14)$$

where $f(\bar{n}_k) = \sqrt{\bar{n}_k(\bar{n}_k + 1)}$.

The two-point function of the magnetic fields

$$\mathcal{G}_{ij}(\vec{r}, \eta) = \langle 0_{-k}0_k | \hat{B}_i(\vec{x}, \eta)\hat{B}_j(\vec{x} + \vec{r}, \eta) | 0_k0_{-k} \rangle \quad (4.15)$$

can be expressed, using Eqs. (4.9) and (4.10)

$$\mathcal{G}_{ij}(\vec{r}) = \int d^3k \mathcal{G}_{ij}(k) e^{i\vec{k}\cdot\vec{r}} \quad (4.16)$$

where

$$\begin{aligned} \mathcal{G}_{ij}(k, \eta) &= \frac{g^2(\eta)\mathcal{K}_{ij}}{2(2\pi)^3 a^4(\eta)} k [2 \sinh^2 r_k + \sinh 2r_k \cos 2\phi_k] \\ \mathcal{K}_{ij} &= \sum_{\alpha} e_i^{\alpha}(k) e_j^{\alpha}(k) = \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right), \end{aligned} \quad (4.17)$$

(the vacuum contribution, occurring for $r_k = 0$, has been consistently subtracted). The intercept for $\vec{r} = 0$ of the two-point function traced with respect to the two polarizations is related to the magnetic energy density

$$\frac{d\rho_B}{d\ln\omega} \simeq \frac{g^2(\eta)\omega^4}{2\pi^2} [2\sinh^2 r_k + \sinh 2r_k \cos 2\phi_k] \quad (4.18)$$

(where $\omega = k/a$ is the physical frequency). The two-point function and its trace only depend upon \bar{n}_k and upon ϕ_k . Since Eqs. (4.12) and (4.14) do not contain any dependence upon θ_k we can attempt to solve the time evolution by solving them simultaneously. In terms of the new variable $x = k\eta$ Eqs. (4.12) and (4.14) can be written as

$$\frac{d\phi_k}{dx} = -1 + \frac{d\ln g}{dx} \frac{df(\bar{n}_k)}{d\bar{n}_k} \sin 2\phi_k, \quad (4.19)$$

$$\frac{d\bar{n}_k}{d\ln g} = -2f(\bar{n}_k) \cos 2\phi_k, \quad (4.20)$$

If $|(d\ln g/dx)(df(\bar{n}_k)/d\bar{n}_k) \sin 2\phi_k| > 1$, then Eqs. (4.19) and (4.20) can be written as

$$\frac{du_k}{d\ln g} = 2\frac{df(\bar{n}_k)}{d\bar{n}_k}u_k, \quad \frac{d\bar{n}_k}{d\ln g} = -2f(\bar{n}_k)\frac{1-u_k^2}{1+u_k^2} \quad (4.21)$$

where $\phi_k = \arctan u_k$. By trivial algebra we can get a differential relation between u_k and \bar{n}_k which can be exactly integrated with the result that $u_k^2 - f(\bar{n}_k)u_k + 1 = 0$. By inverting this last relation we obtain two different solutions with equivalent physical properties, namely

$$u_k(\bar{n}_k) = \left[\frac{1}{2}(\sqrt{\bar{n}_k(\bar{n}_k+1)} \pm \sqrt{\bar{n}_k(\bar{n}_k+1)-4}) \right]. \quad (4.22)$$

If we choose the minus sign in Eq. (4.22) we obtain that $\phi_k \sim (m+1)\pi/2$, $m = 0, 1, 2, \dots$ with corrections of order $1/\bar{n}_k$. In the opposite case $\phi_k \sim \arctan(\bar{n}_k/2)$ within the same accuracy of the previous case (i.e. $1/\bar{n}_k$). By using the relation between u_k and \bar{n}_k the condition $|(d\ln g/dx)(df(\bar{n}_k)/d\bar{n}_k) \sin 2\phi_k| > 1$ is equivalent to $x \lesssim 1$, if, as we are assuming, $|g'/g|$ vanishes as η^{-2} for $\eta \rightarrow \pm\infty$ and it is, piece-wise, continuous. By inserting Eq. (4.22) into Eq. (4.19) a consistent solution can be obtained, in this case, if we integrate the system between η_f and η_i defined as the conformal times where $|(d\ln g/dx)(df(\bar{n}_k)/d\bar{n}_k) \sin 2\phi_k| = 1$:

$$\begin{aligned} \bar{n}_k(\eta_f) &\sim \frac{1}{4} \left(\frac{g(\eta_f)}{g(\eta_i)} - \frac{g(\eta_i)}{g(\eta_f)} \right)^2, \\ \phi_k(\eta) &\sim (m+1)\frac{\pi}{2} + \mathcal{O}\left(\frac{1}{\bar{n}_k(\eta)}\right), \quad m = 0, 1, 2, \dots \end{aligned} \quad (4.23)$$

If $|(d \ln g/dx)(df(\bar{n}_k)/d\bar{n}_k) \sin 2\phi_k| < 1$ (i. e. $x > 1$) the consistent solution of our system is given by

$$\begin{aligned} \bar{n}_k(\eta_f) &= \sinh^2 \left(2 \int^{k\eta} \ln g(x') \sin 2x' dx' \right) \\ \phi_k &\sim -k\eta + \varphi_k, \quad \varphi_k \simeq \text{constant}. \end{aligned} \tag{4.24}$$

If the coupling constant evolves continuously between $-\infty$ and $+\infty$ with a (global) maximum located at some time η_r then, for $x > 1$, $\bar{n}_k \sim \text{const.}$. Indeed by taking trial functions with bell-like shape for $|g'/g|$ we can show that \bar{n}_k oscillates around zero for large ϕ_k .

From Eq. (4.23) the magnetic energy density of Eq. (4.18) can be computed in different scenarios and related to the ratio discussed in Eq. (3.20). Recalling that the present frequency corresponding to 1 Mpc is roughly $\omega_G \sim 10^{-14}$ Hz, the ratio $r(\omega_G)$ can be estimated. Time evolution of gauge coupling during an inflationary phase produces rather large seeds $r(\omega_G) \sim 10^{-12}$ [97]. Furthermore, pre-big bang models [98] also lead to large seeds which can be even $r(\omega_G) \sim 10^{-8}$ [99, 100]. There is a difference in the models discussed in [97] and in [99, 100]. In the case of [97] the gauge coupling is related to the expectation value of a scalar field which evolves during an inflationary phase of de Sitter (or quasi de Sitter type). This scalar degree of freedom is not the inflaton and it is not a source of the background geometry. On the contrary, in [99, 100] the gauge coupling is a source of the evolution of the background geometry since it is connected to the expectation value of the dilaton field whose specific evolution dictates the nature of the pre-big bang solutions used in order to describe the dynamics of the Universe in its early stages.

5 EWPT and BAU

In the previous Section it has been stressed that causal mechanisms have, in general problems with the correlation scale of the obtained field, while inflationary mechanisms may have problems with the seed amplitude. In spite of this, it should be borne in mind that magnetic fields are generated over all physical scales compatible with the plasma dynamics at a given epoch. Hence, even if the magnetic fields at large scales may be very minute, magnetic fields at smaller scales may have a very interesting impact on different moments of the life of the Universe. The physical picture we have in mind is then the following. Suppose that conformal invariance is broken at some stage in the evolution of the Universe, for instance thanks to the (effective) time variation of gauge couplings. Then, vacuum fluctuations will go outside the horizon and will be amplified. The amplified magnetic inhomogeneities will re-enter (crossing the horizon a second time) during different moments of the life of the Universe and, in particular, even before the BBN epoch.

In the following various effects of these magnetic fields will be considered starting with the EW epoch. The electroweak epoch occurs when the temperature of the plasma was

roughly $T \sim T_c \sim 100$ GeV. The physical size of the horizon was, at that time, $H_{\text{ew}}^{-1} \sim 3$ cm. The electroweak epoch occurs, approximately, when the Universe was 10^{-11} sec old.

At small temperatures and small densities of different fermionic charges the $SU_L(2) \otimes U_Y(1)$ is broken down to the $U_{\text{em}}(1)$ and the long range fields which can survive in the plasma are the ordinary magnetic fields. However, for sufficiently high temperatures the $SU_L(2) \otimes U_Y(1)$ is restored and non-screened vector modes correspond to hypermagnetic fields. At the electroweak epoch the typical size of the horizon is of the order of 3 cm. The typical diffusion scale is of the order of 10^{-9} cm. Therefore, over roughly eight orders of magnitude hypermagnetic fields can be present in the plasma without being dissipated [70]. The evolution of hypermagnetic fields can be obtained from the anomalous magnetohydrodynamical (AMHD) equations. The AMHD equations generalize the treatment of plasma effects involving hypermagnetic fields to the case of finite fermionic density [71].

Depending on their topology, hypermagnetic fields can have various consequences [70, 71]. If the hypermagnetic flux lines have a trivial topology they can have an impact on the phase diagram of the electroweak phase transition [111, 112]. If the topology of hypermagnetic fields is non trivial, hypermagnetic knots can be formed [104] and, under specific conditions, the BAU can be generated [105]. The gauge field fluctuations produced as a result of the parametric amplification of vacuum fluctuations always lead to hypermagnetic fields with topologically trivial structure (i.e. with zero magnetic helicity and gyrotropy). However, thanks to pseudo-scalar couplings, a topologically trivial background of hypermagnetic flux lines may lead to a non-zero magnetic gyrotropy and, hence, to some kind of hypermagnetic knot with topologically non-trivial structure.

A classical hypermagnetic background in the symmetric phase of the EW theory can produce interesting amounts of gravitational radiation in a frequency range between 10^{-4} Hz and the kHz. The lower tail falls into the LISA window while the higher tail falls in the VIRGO/LIGO window. For the hypermagnetic background required in order to seed the BAU the amplitude of the obtained GW can be even six orders of magnitude larger than the inflationary predictions. In this context, the mechanism of baryon asymmetry generation is connected with GW production [104, 105].

5.1 Hypermagnetic knots

It is possible to construct hypermagnetic knot configurations with finite energy and helicity which are localized in space and within typical distance scale L_s . Let us consider in fact the following configuration in spherical coordinates [105]

$$\begin{aligned} \mathcal{Y}_r(\mathcal{R}, \theta) &= -\frac{2B_0}{\pi L_s} \frac{\cos \theta}{[\mathcal{R}^2 + 1]^2}, \\ \mathcal{Y}_\theta(\mathcal{R}, \theta) &= \frac{2B_0}{\pi L_s} \frac{\sin \theta}{[\mathcal{R}^2 + 1]^2}, \end{aligned}$$

$$\mathcal{Y}_\phi(\mathcal{R}, \theta) = -\frac{2B_0}{\pi L_s} \frac{n\mathcal{R} \sin \theta}{[\mathcal{R}^2 + 1]^2}, \quad (5.1)$$

where $\mathcal{R} = r/L_s$ is the rescaled radius and B_0 is some dimensionless amplitude and n is just an integer number whose physical interpretation will become clear in a moment. The hypermagnetic field can be easily computed from the previous expression and it is

$$\begin{aligned} \mathcal{H}_r(\mathcal{R}, \theta) &= -\frac{4B_0}{\pi L_s^2} \frac{n \cos \theta}{[\mathcal{R}^2 + 1]^2}, \\ \mathcal{H}_\theta(\mathcal{R}, \theta) &= -\frac{4B_0}{\pi L_s^2} \frac{\mathcal{R}^2 - 1}{[\mathcal{R}^2 + 1]^3} n \sin \theta, \\ \mathcal{H}_\phi(\mathcal{R}, \theta) &= -\frac{8B_0}{\pi L_s^2} \frac{\mathcal{R} \sin \theta}{[\mathcal{R}^2 + 1]^3}. \end{aligned} \quad (5.2)$$

The poloidal and toroidal components of $\vec{\mathcal{H}}$ can be usefully expressed as $\vec{\mathcal{H}}_p = \mathcal{H}_r \vec{e}_r + \mathcal{H}_\theta \vec{e}_\theta$ and $\vec{\mathcal{H}}_t = \mathcal{H}_\phi \vec{e}_\phi$. The Chern-Simons number is finite and it is given by

$$N_{CS} = \frac{g'^2}{32\pi^2} \int_V \vec{\mathcal{Y}} \cdot \vec{\mathcal{H}}_Y d^3x = \frac{g'^2}{32\pi^2} \int_0^\infty \frac{8nB_0^2}{\pi^2} \frac{\mathcal{R}^2 d\mathcal{R}}{[\mathcal{R}^2 + 1]^4} = \frac{g'^2 n B_0^2}{32\pi^2}. \quad (5.3)$$

We can also compute the total helicity of the configuration namely

$$\int_V \vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y d^3x = \frac{256 B_0^2 n}{\pi L^2} \int_0^\infty \frac{\mathcal{R}^2 d\mathcal{R}}{(1 + \mathcal{R}^2)^5} = \frac{5B_0^2 n}{L_s^2}. \quad (5.4)$$

We can compute also the total energy of the field

$$E = \frac{1}{2} \int_V d^3x |\vec{\mathcal{H}}_Y|^2 = \frac{B_0^2}{2 L_s} (n^2 + 1). \quad (5.5)$$

and we discover that it is proportional to n^2 . This means that one way of increasing the total energy of the field is to increase the number of knots and twists in the flux lines. We can also have some real space pictures of the core of the knot (i.e. $\mathcal{R} = r/L_s < 1$). This type of configurations can be also obtained by projecting a non-Abelian SU(2) (vacuum) gauge field on a fixed electromagnetic direction [106] ^{||} These configurations have been also studied in [108, 109]. In particular, in [109], the relaxation of HK has been investigated with a technique different from the one employed in [104, 105] but with similar results. The problem of scattering of fermions in the background of hypermagnetic fields has been also studied in [110].

^{||}In order to interpret these solutions it is very interesting to make use of the Clebsh decomposition. The implications of this decomposition (beyond the hydrodynamical context, where it was originally discovered) have been recently discussed (see [107] and references therein). I thank R. Jackiw for interesting discussions about this point.

Topologically non-trivial configurations of the hypermagnetic flux lines lead to the formation of hypermagnetic knots (HK) whose decay might seed the Baryon Asymmetry of the Universe (BAU). HK can be dynamically generated provided a topologically trivial (i.e. stochastic) distribution of flux lines is already present in the symmetric phase of the electroweak (EW) theory [104, 105]. In spite of the mechanism generating the HK, their typical size must exceed the diffusivity length scale. In the minimal standard model (MSM) (but not necessarily in its supersymmetric extension) HK are washed out.

The importance of the topological properties of long range (Abelian) hypercharge magnetic fields has been stressed in the past [113, 114, 115, 116]. In [117] it was argued that if the spectrum of hypermagnetic fields is dominated by parity non-invariant Chern-Simons (CS) condensates, the BAU could be the result of their decay. Most of the mechanisms often invoked for the origin of large scale magnetic fields in the early Universe seem to imply the production of topologically trivial (i.e. stochastic) configurations of magnetic fields [62, 63, 64].

5.2 Hypermagnetic knots and BAU

Suppose that the EW plasma is filled, for $T > T_c$ with topologically trivial hypermagnetic fields $\vec{\mathcal{H}}_Y$, which can be physically pictured as a collection of flux tubes (closed because of the transversality of the field lines) evolving independently without breaking or intersecting with each other. If the field distribution is topologically trivial (i.e. $\langle \vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y \rangle = 0$) parity is a good symmetry of the plasma and the field can be completely homogeneous. We name hypermagnetic knots those CS condensates carrying a non vanishing (averaged) hypermagnetic helicity (i.e. $\langle \vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y \rangle \neq 0$). If $\langle \vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y \rangle \neq 0$ parity is broken for scales comparable with the size of the HK, the flux lines are knotted and the field $\vec{\mathcal{H}}_Y$ cannot be completely homogeneous.

In order to seed the BAU a network of HK should be present at high temperatures [70, 71, 117]. In fact for temperatures larger than T_c the fermionic number is stored both in HK and in real fermions. For $T < T_c$, the HK should release real fermions since the ordinary magnetic fields (present *after* EW symmetry breaking) do not carry fermionic number. If the EWPT is strongly first order the decay of the HK can offer some seeds for the BAU generation [117]. This last condition can be met in the minimal supersymmetric standard model (MSSM) [118, 119, 120, 121].

Under these hypotheses the integration of the $U(1)_Y$ anomaly equation [117] gives the CS number density carried by the HK which is in turn related to the density of baryonic number n_B for the case of n_f fermionic generations.

$$\frac{n_B}{s}(t_c) = \frac{\alpha'}{2\pi\sigma_c} \frac{n_f \langle \vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y \rangle}{s} \frac{M_0 \Gamma}{\Gamma + \Gamma_{\mathcal{H}}} \frac{1}{T_c^2}, \quad \alpha' = \frac{g'^2}{4\pi} \quad (5.6)$$

(g' is the $U(1)_Y$ coupling and $s = (2/45)\pi^2 N_{eff} T^3$ is the entropy density; N_{eff} , at T_c , is 106.75 in the MSM; $M_0 = M_P/1.66\sqrt{N_{eff}} \simeq 7.1 \times 10^{17} \text{GeV}$). In Eq. (5.6) Γ is

the perturbative rate of the right electron chirality flip processes (i.e. scattering of right electrons with the Higgs and gauge bosons and with the top quarks because of their large Yukawa coupling) which are the slowest reactions in the plasma and

$$\Gamma_{\mathcal{H}} = \frac{783}{22} \frac{\alpha'^2}{\sigma_c \pi^2} \frac{|\vec{\mathcal{H}}_Y|^2}{T_c^2} \quad (5.7)$$

is the rate of right electron dilution induced by the presence of a hypermagnetic field. In the MSM we have that $\Gamma < \Gamma_{\mathcal{H}}$ [122] whereas in the MSSM Γ can naturally be larger than $\Gamma_{\mathcal{H}}$. Unfortunately, in the MSM a hypermagnetic field can modify the phase diagram of the phase transition but cannot make the phase transition strongly first order for large masses of the Higgs boson [111]. Therefore, we will concentrate on the case $\Gamma > \Gamma_{\mathcal{H}}$ and we will show that in the opposite limit the BAU will be anyway small even if some (presently unknown) mechanism would make the EWPT strongly first order in the MSM.

HK can be dynamically generated [104, 105] (see also [123]). Gauge-invariance and transversality of the magnetic fields suggest that perhaps the only way of producing $\langle \vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y \rangle \neq 0$ is to postulate, a time-dependent interaction between the two (physical) polarizations of the hypercharge field Y_α . Having defined the Abelian field strength $Y_{\alpha\beta} = \nabla_{[\alpha} Y_{\beta]}$ and its dual $\tilde{Y}_{\alpha\beta}$ such an interaction can be described, in curved space, by the Lagrangian density

$$L_{eff} = \sqrt{-g} \left[-\frac{1}{4} Y_{\alpha\beta} Y^{\alpha\beta} + c \frac{\psi}{4M} Y_{\alpha\beta} \tilde{Y}^{\alpha\beta} \right]. \quad (5.8)$$

where $g_{\mu\nu}$ is the metric tensor and g its determinant, c is the coupling constant and M is a typical scale. This interaction is plausible if the $U(1)_Y$ anomaly is coupled, (in the symmetric phase of the EW theory) to dynamical pseudoscalar particles ψ (like the axial Higgs of the MSSM). Thanks to the presence of pseudoscalar particles, the two polarizations of $\vec{\mathcal{H}}_Y$ evolve in a slightly different way producing, ultimately, inhomogeneous HK.

Suppose that an inflationary phase with $a(\tau) \sim \tau^{-1}$ is continuously matched, at the transition time τ_1 , to a radiation dominated phase where $a(\tau) \sim \tau$. Consider then a massive pseudoscalar field ψ which oscillates during the last stages of the inflationary evolution with typical amplitude $\psi_0 \sim M$. As a result of the inflationary evolution $|\vec{\nabla} \psi| \ll \psi'$. Consequently, the phase of ψ can get frozen. Provided the pseudoscalar mass m is larger than the inflationary curvature scale $H_i \sim \text{const.}$, the ψ oscillations are converted, at the end of the quasi-de Sitter stage, in a net helicity arising as a result of the different evolution of the two (circularly polarized) vector potentials

$$Y_{\pm}'' + \sigma Y_{\pm}' + \omega_{\pm}^2 Y_{\pm} = 0, \quad \vec{H}_Y = \vec{\nabla} \times \vec{Y} \quad (5.9)$$

$$\omega_{\pm}^2 = k^2 \mp k \frac{c}{M} a \dot{\psi} \quad (5.10)$$

(where we denoted with $\vec{H}_Y = a^2 \vec{\mathcal{H}}_Y$ the curved space fields and with $\sigma = \sigma_c a$ the rescaled hyperconductivity; the prime denotes derivation with respect to conformal time τ whereas the over-dot denotes differentiation with respect to cosmic time t).

Since $\omega_+ \neq \omega_-$ the helicity gets amplified according to Eq. (5.9). There are two important points to stress in this context. First of all the plasma effects as well as the finite density effects are important. This means, in practical terms, that the dissipation scales of the problem should be borne in mind. This has not always been done. The second point is related to the first. By treating, consistently, the plasma and finite density effects in the context of AMHD [70, 104, 105], one realizes that the pseudoscalar coupling of ψ together with the coupling of the chemical potential are not sufficient in order to seed the BAU *unless* some hypermagnetic background is already present. In other words the scenario which leads to the generation of the BAU is the following. Correlation in the hypermagnetic fields are generated outside the horizon during inflation by direct breaking of conformal invariance. An example in this direction has been given in the previous Section. Then hypermagnetic fields re-entering at the electroweak epoch will participate in the dynamics and, in particular, they will feel the effect of the anomalous coupling either to ψ or to the chemical potential. The effect of the anomalous coupling will not be to amplify the hypermagnetic background. The anomalous coupling will only make the topology of the hypermagnetic flux lines non-trivial. So the statement is that *if* conformal invariance is broken and, *if* hypermagnetic have anomalous couplings, *then* a BAU $\gtrsim 10^{-10}$ can be achieved without spoiling the standard cosmological evolution [104, 105]. It is worth mentioning that this type of scenario may be motivated by the low energy string effective action where, by supersymmetry, Kalb-Ramond axions and dilatons are coupled, respectively, to $Y_{\mu\nu} \tilde{Y}^{\mu\nu}$ and to the gauge kinetic term [124]. It is interesting to notice that, in this scenario, the value of the BAU is determined by various particle physics parameters but also by the ratio of the hypermagnetic energy density over the energy density sitting in radiation during the electroweak epoch, namely, using the language of the previous Sections [104, 105],

$$\frac{n_B}{s} \propto r. \quad (5.11)$$

In order to get a sizable BAU r should be at least 10^{-3} if the anomalous coupling operates during a radiation phase. The value of r could be smaller in models where the anomalous coupling is relevant during a low scale inflationary phase [105].

6 BBN and matter–antimatter fluctuations

Large scale magnetic fields possibly present at the BBN epoch can have an impact on the light nuclei formation. By reversing the argument, the success of BBN can be used in order to bound the magnetic energy density possibly present at the time of formation of light nuclei.

These bounds are qualitatively different from the ones previously quoted and coming, alternatively, from homogeneity [79] and isotropy [80] of the background geometry at the BBN time. As elaborated in slightly different frameworks through the years [126, 127, 128, 129, 130], magnetic fields possibly present at the BBN epoch could have a twofold effect. On one hand they could enhance the rate of reactions (with an effect proportional to $\alpha\rho_B$) and, on the other hand they could artificially increase the expansion rate (with an effect proportional to ρ_B). It turns out that the latter effect is probably the most relevant [129]. In order to prevent the Universe from expanding too fast at the BBN epoch $\rho_B < 0.27\rho_\nu$ where ρ_ν is the energy density contributed by the standard three neutrinos for $T < 1$ MeV.

In the previous Section the case of a *topologically non-trivial* hypermagnetic background has been considered (i.e. $\langle \vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y \rangle \neq 0$). In this Section we will instead assume that the hypermagnetic background is *topologically trivial* (i.e. $\langle \vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y \rangle = 0$). In this case, fluctuations in the baryon to entropy ratio will be induced since $\langle (\vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y)^2 \rangle \neq 0$. These fluctuations are of *isocurvature* type and can be related to the spectrum of hypermagnetic fields at the EWPT. Defining as

$$\Delta(r, t_c) = \sqrt{\langle \delta\left(\frac{n_B}{s}\right)(\vec{x}, t_c) \delta\left(\frac{n_B}{s}\right)(\vec{x} + \vec{r}, t_c) \rangle}, \quad (6.1)$$

the fluctuations in the baryon to entropy ratio at $t = t_c$ [71], the value of $\Delta(r, t_c)$ can be related to the hypermagnetic spectrum which is determined in terms of its amplitude ξ and its slope ϵ . A physically realistic situation corresponds to the case in which the Green's functions of the magnetic hypercharge fields decay at large distance (i. e. $\epsilon > 0$) and this would imply either “blue” ($\epsilon \geq 0$) or “violet” ($\epsilon \gg 1$) energy spectra. The case of “red” spectra ($\epsilon < 0$) will then be left out of our discussion. The flat spectrum corresponds to $\epsilon \ll 1$ and may appear quite naturally in string cosmological models [99, 100].

Since the fluctuations in the baryon to entropy ratio are not positive definite, they will induce fluctuations in the baryon to photon ratio, η at the BBN epoch. The possible effect of matter–antimatter fluctuations on BBN depends on the typical scale of the baryon to entropy ratio at the electroweak epoch. Recalling that for $T \sim T_c \sim 100$ GeV the size of the electroweak horizon is approximately 3 cm, fluctuations whose scale is well inside the EW horizon at T_c have dissipated by the BBN time through (anti)neutron diffusion. The neutron diffusion scale at T_c is

$$r_n(T_c) = 0.3 \text{ cm}. \quad (6.2)$$

The neutron diffusion scale at $T = 1$ keV is 10^5 m, while, today it is 10^{-5} pc, i.e. of the order of the astronomical unit. Matter–antimatter fluctuations smaller than 10^5 m annihilate before neutrino decoupling and have no effect on BBN. Two possibilities can then be envisaged. We could require that the matter–antimatter fluctuations (for scales $r \geq r_n$) are small. This will then imply a bound, in the (ξ, ϵ) plane on the strength of

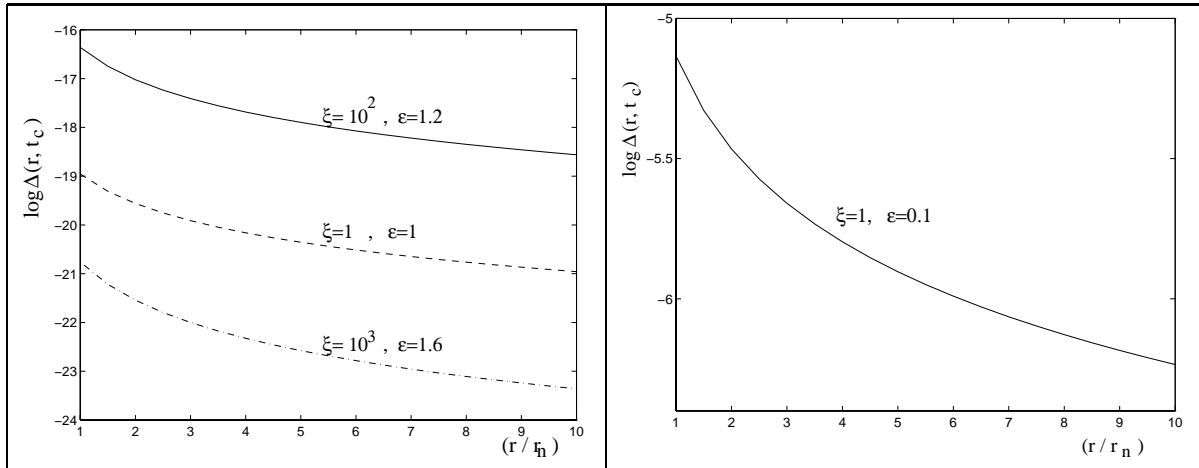


Figure 2: We report the value of the baryon number fluctuations for different parameters of the hypermagnetic background ξ and ϵ .

the hypermagnetic background. In Fig. 3 such an exclusion plot is reported with the full line. With the dashed line the bound implied by the increase in the expansion rate is reported. Finally with the dot-dashed line the critical density bound is illustrated for the same hypermagnetic background. The second possibility is to study the effects of large matter–matter domains. These studies led to a slightly different scenario of BBN [131], namely BBN with matter–antimatter regions [132, 133, 134, 135, 136, 137]. The idea is to discuss BBN in the presence of spherically symmetric regions of anti–matter characterized by their radius r_A and by the parameter R , i.e. the matter/antimatter ratio. Furthermore, in this scenario the net baryon-to-photon ratio, η , is positive definite and non zero. Antimatter domains larger than 10^5 m at 1 keV may survive until BBN and their dissipation has been analyzed in detail in [132, 133, 134, 135, 136, 137]. Antimatter domains in the range

$$10^5 \text{ m} \lesssim r_A \lesssim 10^7 \text{ m} \quad (6.3)$$

at 1 keV annihilate before BBN for temperatures between 70 keV and 1 MeV. Since the antineutrons annihilate on neutrons, the neutron to proton ratio gets smaller. As a consequence, the ^4He abundance gets reduced if compared to the standard BBN scenario. The maximal scale of matter–antimatter fluctuations is determined by the constraints following from possible distortions of the CMB spectrum. The largest scale is of the order of 100 pc (today), corresponding to 10^{12} m at 1 keV. Suppose that matter–antimatter regions are present in the range of Eq. (6.3). Then the abundance of ^4He get reduced. The yield of ^4He are reported as a function of R , the matter–antimatter ratio and r_A . Now, we do know that by adding extra-relativistic species the ^4He can be increased since the Universe expansion gets larger. Then the conclusion is that BBN with matter–antimatter domains allows for a larger number of extra-relativistic species if compared to the standard

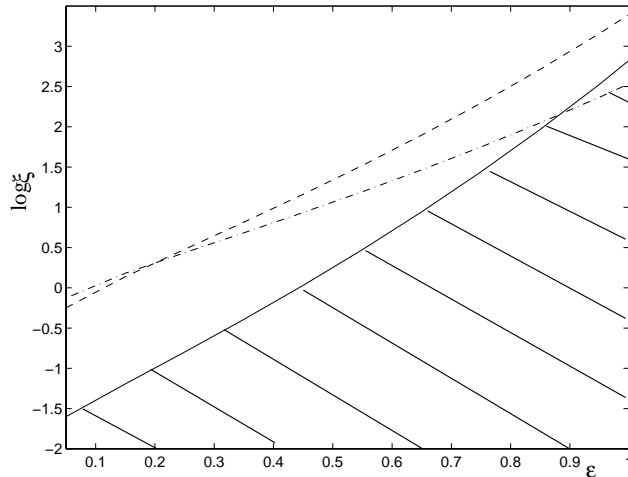


Figure 3: The parameter space of the hypermagnetic background in the case $\Delta(r, t_c) < n_B/s$ for $r > r_n$ (full line).

BBN scenario. This observation may have implications for the upper bounds on the stochastic GW backgrounds of cosmological origin [138] since the extra-relativistic species present at the BBN epoch can indeed be interpreted as relic gravitons.

7 GW backgrounds

If a hypermagnetic background is present for $T > T_c$, then, as also discussed in [125] in the context of ordinary MHD, the energy momentum tensor will acquire a small anisotropic component which will source the evolution equation of the tensor fluctuations $h_{\mu\nu}$ of the metric $g_{\mu\nu}$:

$$h_{ij}'' + 2\mathcal{H}h_{ij}' - \nabla^2 h_{ij} = -16\pi G\tau_{ij}^{(T)}. \quad (7.1)$$

where $\tau_{ij}^{(T)}$ is the *tensor* component of the *energy-momentum tensor* [125] of the hypermagnetic fields. Suppose now, as assumed in [111] that $|\vec{\mathcal{H}}|$ has constant amplitude and that it is also homogeneous. Then as argued in [141] we can easily deduce the critical fraction of energy density present today in relic gravitons of EW origin

$$\Omega_{\text{gw}}(t_0) = \frac{\rho_{\text{gw}}}{\rho_c} \simeq z_{\text{eq}}^{-1} r^2, \quad \rho_c(T_c) \simeq N_{\text{eff}} T_c^4 \quad (7.2)$$

($z_{\text{eq}} = 6000$ is the redshift from the time of matter-radiation, equality to the present time t_0). Because of the structure of the AMHD equations, stable hypermagnetic fields will be present not only for $\omega_{\text{ew}} \sim k_{\text{ew}}/a$ but for all the range $\omega_{\text{ew}} < \omega < \omega_\sigma$ where ω_σ is the diffusivity frequency. Let us assume, for instance, that $T_c \sim 100$ GeV and $N_{\text{eff}} = 106.75$.

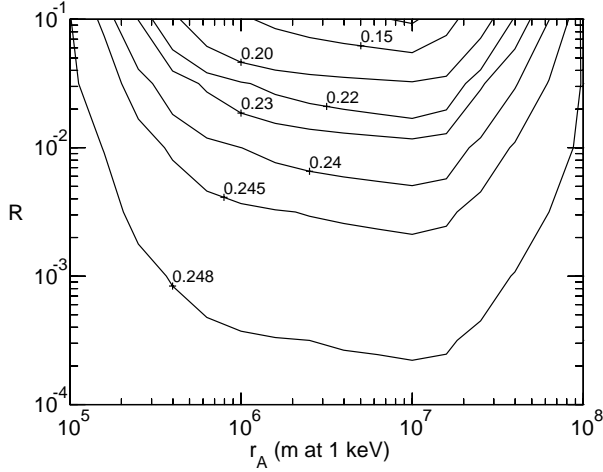


Figure 4: From [138] the ${}^4\text{He}$ yield is illustrated in the (R, r_A) plane for $\eta = 6 \times 10^{-10}$. As the matter/antimatter ratio decreases, we recover the standard ${}^4\text{He}$ yield.

Then, the (present) values of ω_{ew} is

$$\omega_{\text{ew}}(t_0) \simeq 2.01 \times 10^{-7} \left(\frac{T_c}{1\text{GeV}} \right) \left(\frac{N_{\text{eff}}}{100} \right)^{1/6} \text{ Hz.} \quad (7.3)$$

Thus, $\omega_\sigma(t_0) \sim 10^8 \omega_{\text{ew}}$. Suppose now that $T_c \sim 100$ GeV; than we will have that $\omega_{\text{ew}}(t_0) \sim 10^{-5}$ Hz. Suppose now, as assumed in [111], that

$$|\vec{\mathcal{H}}|/T_c^2 \gtrsim 0.3. \quad (7.4)$$

This requirement imposes $r \simeq 0.1\text{--}0.001$ and, consequently,

$$h_0^2 \Omega_{\text{GW}} \simeq 10^{-7} - 10^{-8}. \quad (7.5)$$

Notice that this signal would occur in a (present) frequency range between 10^{-5} and 10^3 Hz. This signal satisfies the presently available phenomenological bounds on the graviton backgrounds of primordial origin. The pulsar timing bound (which applies for present frequencies $\omega_P \sim 10^{-8}$ Hz and implies $h_0^2 \Omega_{\text{GW}} \leq 10^{-8}$) is automatically satisfied since our hypermagnetic background is defined for $10^{-5}\text{Hz} \leq \omega \leq 10^3\text{Hz}$. The large scale bounds would imply $h_0^2 \Omega_{\text{GW}} < 7 \times 10^{-11}$ but a at much lower frequency (i.e. 10^{-18} Hz). The signal discussed here is completely absent for frequencies $\omega < \omega_{\text{ew}}$. Notice that this signal is clearly distinguishable from other stochastic backgrounds occurring at much higher frequencies (GHz region) like the ones predicted by quintessential inflation [142]. It is equally distinguishable from signals due to pre-big-bang cosmology (mainly in the window of ground based interferometers [143]). The frequency of operation of the interferometric devices (VIRGO/LIGO) is located between few Hz and 10 kHz [143]. The

frequency of operation of LISA is well below the Hz (i.e. 10^{-3} Hz, approximately). In this model the signal can be located both in the LISA window and in the VIRGO/LIGO window due to the hierarchy between the hypermagnetic diffusivity scale and the horizon scale at the phase transition [104, 105].

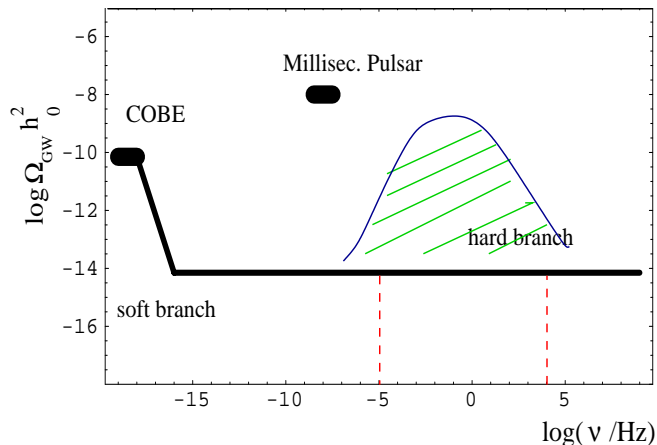


Figure 5: The stochastic background of GW produced by inflationary models with flat logarithmic energy spectrum, illustrated together with the GW background of hypermagnetic origin. The frequencies marked with dashed lines correspond to the electroweak frequency and to the hypermagnetic diffusivity frequency.

8 Faraday rotation of CMB ?

Large scale magnetic fields present at the decoupling epoch can have various consequences. For instance they can induce fluctuations in the CMB [99, 100], they can distort the Planckian spectrum of CMB [139], they can distort the acoustic peaks of CMB anisotropies [140] and they can also depolarize CMB [144].

The polarization of the CMB represents a very interesting observable which has been extensively investigated in the past both from the theoretical [145] and experimental points of view [146]. Forthcoming satellite missions like PLANCK [147] seem to be able to achieve a level of sensitivity which will enrich decisively our experimental knowledge of the CMB polarization with new direct measurements.

If the background geometry of the universe is homogeneous but not isotropic the CMB is naturally polarized [145]. This phenomenon occurs, for example, in Bianchi-type I models [148]. On the other hand if the background geometry is homogeneous and isotropic (like in the Friedmann-Robertson-Walker case) it seems very reasonable that the CMB acquires a small degree of linear polarization provided the radiation field has a non-vanishing quadrupole component at the moment of last scattering [149].

Before decoupling photons, baryons and electrons form a unique fluid which possesses only monopole and dipole moments, but not quadrupole. Needless to say, in a homogeneous and isotropic model of FRW type a possible source of linear polarization for the CMB becomes efficient only at the decoupling and therefore a small degree of linear polarization seems a firmly established theoretical option which will be (hopefully) subjected to direct tests in the near future. The linear polarization of the CMB is a very promising laboratory in order to directly probe the speculated existence of a large scale magnetic field (coherent over the horizon size at the decoupling) which might actually rotate (through the Faraday effect [1, 2, 3]) the polarization plane of the CMB.

Consider, for instance, a linearly polarized electromagnetic wave of physical frequency ω traveling along the \hat{x} direction in a cold plasma of ions and electrons together with a magnetic field (\overline{B}) oriented along an arbitrary direction (which might coincide with \hat{x} in the simplest case). If we let the polarization vector at the origin ($x = y = z = 0, t = 0$) be directed along the \hat{y} axis, after the wave has traveled a length Δx , the corresponding angular shift ($\Delta\alpha$) in the polarization plane will be :

$$\Delta\alpha = f_e \frac{e}{2m} \left(\frac{\omega_{pl}}{\omega} \right)^2 (\overline{B} \cdot \hat{x}) \Delta x \quad (8.1)$$

(conventions: $\omega_B = eB/m$ is the Larmor frequency; $\omega_{pl} = \sqrt{4\pi n_e e^2/m}$ is the plasma frequency n_e is the electron density and f_e is the ionization fraction ; we use everywhere natural units $\hbar = c = k_B = 1$). It is worth mentioning that the previous estimate of the Faraday rotation angle $\Delta\alpha$ holds provided $\omega \gg \omega_B$ and $\omega \gg \omega_{pl}$. From Eq. (8.1) by stochastically averaging over all the possible orientations of \overline{B} and by assuming that the last scattering surface is infinitely thin (i.e. that $\Delta x f_e n_e \simeq \sigma_T^{-1}$ where σ_T is the Thompson cross section) we get an expression connecting the RMS of the rotation angle to the magnitude of \overline{B} at $t \simeq t_{dec}$

$$\langle (\Delta\alpha)^2 \rangle^{1/2} \simeq 1.6^0 \left(\frac{B(t_{dec})}{B_c} \right) \left(\frac{\omega_M}{\omega} \right)^2, \quad B_c = 10^{-3} \text{ Gauss}, \quad \omega_M \simeq 3 \times 10^{10} \text{ Hz} \quad (8.2)$$

(in the previous equation we implicitly assumed that the frequency of the incident electromagnetic radiation is centered around the maximum of the CMB). We can easily argue from Eq. (8.2) that if $B(t_{dec}) \gtrsim B_c$ the expected rotation in the polarization plane of the CMB is non negligible. Even if we are not interested, at this level, in a precise estimate of $\Delta\alpha$, we point out that more refined determinations of the expected Faraday rotation signal (for an incident frequency $\omega_M \sim 30$ GHz) were recently carried out [150, 151] leading to a result fairly consistent with (8.1).

Then, the statement is the following. *If* the CMB is linearly polarized and *if* a large scale magnetic field is present at the decoupling epoch, *then* the polarization plane of the CMB can be rotated [144]. The predictions of different models can then be confronted with the requirements coming from a possible detection of depolarization of the CMB [144].

9 Concluding Remarks

The large scale magnetic fields observed today in the Universe may or may not be primordial and there could indeed be different possibilities. It could be that in the past history of the Universe very strong magnetic fields have been created. These fields could be strong enough to affect phase transitions and other phenomena in the life of the Universe but, at the same time, too weak to be responsible for the origin of large scale magnetic fields. It could also be that magnetic field were indeed strong enough to act as seeds of presently observed magnetic fields and, in this case we should be able to find evidence that this was indeed the case. In light of this perspective various “observables” , possibly affected by the existence of primordial magnetic fields, could be proposed. They include the stochastic GW backgrounds, the Faraday rotation of CMB and the baryon asymmetry of the Universe. From a more theoretical perspective, primordial magnetic fields can be connected to the existence of (small and large) extra-dimensions and to the possible dynamics of gauge couplings in the early stages of the evolution of the Universe.

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