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A Solution to the Doublet-Triplet Splitting Problem in the Type IIB Supergravity

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Abstract

The doublet–triplet mass splitting problem is one of the most serious problems in supersymmetric grand unified theories (GUTs). A class of models based on a product gauge group, such as the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(3)_{\mathrm{H}}$ or the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(2)_{\mathrm{H}}$, realize naturally the desired mass splitting that is protected by an unbroken R symmetry. It has been pointed out that various features in the models suggest that these product-group unification models are embedded in a supersymmetric brane world. We show an explicit construction of those models in the supersymmetric brane world based on the Type IIB supergravity in ten dimensions. We consider $\mathbf{T}^6/(\mathbf{Z}_{12}\times\mathbf{Z}_2)$ orientifold for the compactified six extra dimensions. We find that all of the particles needed for the GUT-symmetry-breaking sector are obtained from the D-brane fluctuations. The three families of quarks and leptons are introduced at an orbifold singularity, although their origin remains unexplained. This paper includes extensive discussion on anomaly cancellation in a given orbifold geometry. Relation to the Type IIB string theory, realization of R symmetry as a rotation of extra-dimensional space, and effective superpotential at low energies are also discussed.

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1 Introduction

Supersymmetric (SUSY) Grand Unified Theories (GUTs) have attracted many people for a long time because of a number of theoretical beauties [1]. They have become even more attractive recently since the precise measurements of the standard-model gauge coupling constants support their SUSY SU(5) unification [2, 3].

However, there are serious problems in the SUSY SU(5) models. The most severe problem is to provide coloured-triplet Higgs multiplets with masses of the order of the GUT scale, $\sim 10^{16}$ GeV, keeping two Higgs doublets almost massless. Another problem is the absence of the dimension-5 proton decay [4, 5]. This problem is closely related to the previous problem, because both indicate a particular structure of the mass matrix of the coloured Higgs multiplets and a symmetry behind it.

A class of models of the SUSY GUTs in Refs. [6, 7, 8, 9] is one of the solutiions to these problems. A discrete R symmetry plays a crucial role there [9]. This R symmetry forbids both large masses of the Higgs doublets and dimension-5 operators for proton decay, simultaneously. Coloured Higgs multiplets are provided with mass terms through a kind of missing-partner mechanism [10]. Their mass partners (triplets without doublets) emerge as composite states, at the price of introducing a new gauge group above the GUT scale. Thus, we call it the product-group unification[2].

This class of models has to give up the "unification" by a simple group. The gauge coupling constants are not universal either; the SU(5) gauge group has weak coupling, while the newly introduced group(s) has(have) to have relatively large coupling(s). However, we do not consider these features as ugly, because they are quite naturally explained along with a number of other features of the models, if the models are embedded in a SUSY brane world [14, 15][3]. Here, the extra dimensions are assumed to be smaller than the inverse of the GUT scale. Qualitative arguments in Ref. [14] show that the SUSY brane-world structure behind the models is quite a natural possibility. The present authors briefly show in the previous letter [15] an explicit construction of the brane-world model by adopting the D3-D7 system of the ten-dimensional supergravity. This article provides an extensive and more detailed construction. Theoretical consistency and relation to string theories are also discussed.

[2]Various unification models based on the SU(5)×SU(5) gauge group have been proposed to explain naturally the doublet–triplet mass splitting [11, 12, 13]. However, in this paper, we mean by the "product-group unification" the class of models developed in Refs. [6, 7, 8, 9], which was referred to as "semi-simple unification" in [14, 15, 16, 17].

[3]A similar model has been proposed in [18].

This article is organized as follows. The original product-group unification in the four-dimensional space-time is briefly reviewed in section 2. Motivation for extending the original models to the SUSY brane world are summarized in section 3. Section 4 explains our principle of the brane-world model construction in the Type IIB supergravity. The geometry of the particular orbifold compactification we adopt is explained in section 5. Sections 6 and 7 are devoted to explicit constructions of two different models of the product-group unification. We describe the D-brane configurations on the orbifold and orbifold projection conditions. The whole sector relevant to the SU(5)-symmetry breaking is perfectly obtained from massless modes on D-branes, although we cannot find the origin of quarks and leptons. Anomaly cancellation on the orbifold is discussed for both models. Relation to the Type IIB string theory is also briefly mentioned in subsubsection 7.2.2. Particles that have a definite origin in extra-dimensional space, i.e. particles obtained from massless modes on D-branes and from the supergravity multiplet in the bulk, cannot have arbitrary R charges. We show that the R-charge assignment in the extra-dimensional construction of the models can coincide with the one required for successful phenomenology. Effective superpotentials below the Kaluza–Klein scale are discussed at the ends of both sections. Section 8 provides a summary of this paper and a brief discussion of phenomenological consequences of the present models.

2 Product-Group Unification Models in Four Dimensions

The symmetry that governs the product-group unification models is a discrete R symmetry: (mod 4)-R symmetry [9]. The R charges of all the fields in the minimal SUSY standard model (MSSM) are given in Table 1. Higgsino does not have a mass of the order of the fundamental scale because of this symmetry, and the Giudice–Masiero mechanism [19] provides the SUSY-invariant mass term (μ-term) for Higgs multiplets of the order of the electroweak (TeV) scale after the (mod 4)-R symmetry is broken by the non-vanishing vacuum value of the superpotential. The (mod 4)-R is actually the unique symmetry in the MSSM compatible with the $\mathrm{SU}(5)_{\mathrm{GUT}}$ that satisfies the above two properties and that may have [20] a vanishing mixed anomaly [21, 22] with the $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge group.

An immediate consequence of this symmetry is the absence of the dimension-5 proton decay. At the same time, this symmetry implies that additional $\mathrm{SU}(3)_C$-triplets (without $\mathrm{SU}(2)_L$-doublets) should be introduced as mass partners of the coloured Higgs multiplets.

They should have R charge 2 rather than 0. The other possibility[4] is that the $SU(5)_{\rm GUT}$ covariant fields, $H^i(\mathbf{5})$ and $\bar{H}_i(\mathbf{5}^*)$, contain only doublets as one-particle degrees of freedom without triplets in the $SU(5)_{\rm GUT}$-breaking phase. The product-group unification we discuss in this paper is a framework [6] that provides explicit models for the above two possibilities.

Let us first explain a model based on a product gauge group $SU(5)_{\rm GUT}\times U(3)_{\rm H}$ [9]. Quarks and leptons are singlets of the $U(3)_{\rm H}$ gauge group and form three families of $\mathbf{5}^*$+$\mathbf{10}$ of the $SU(5)_{\rm GUT}$. Higgs multiplets that contain two Higgs doublets are $H(\mathbf{5})^i$ and $\bar{H}(\mathbf{5}^*)_i$, which are also singlets of the $U(3)_{\rm H}$. Fields introduced for the $SU(5)_{\rm GUT}$ breaking are given as follows: $X^\alpha{}_\beta(\alpha,\beta=1,2,3)$ transforming as ($\mathbf{1}$,$\mathbf{adj.}$=$\mathbf{8}+\mathbf{1}$) under the $SU(5)_{\rm GUT}\times U(3)_{\rm H}$ gauge group, and $Q^\alpha_i(i=1,...,5)+Q^\alpha_6$ and $\bar{Q}^i{}_\alpha(i=1,...,5)+\bar{Q}^6{}_\alpha$ transforming as ($\mathbf{5}^*$+$\mathbf{1}$,$\mathbf{3}$) and ($\mathbf{5}$+$\mathbf{1}$,$\mathbf{3}^*$). Indices i are for the $SU(5)_{\rm GUT}$ and α or β for the $U(3)_{\rm H}$. The chiral superfield $X^\alpha{}_\beta$ is also written as $X^c(t_c)^\alpha{}_\beta(c=0,1,...,8)$, where $t_a(a=1,...,8)$ are Gell-Mann matrices of the $SU(3)_{\rm H}$ gauge group[5] and $t_0\equiv\mathbf{1}_{3\times3}/\sqrt{6}$, where $U(3)_{\rm H}\simeq SU(3)_{\rm H}\times U(1)_{\rm H}$. The R charges (mod 4) of these fields are given in Table 2. The mixed anomaly (R mod 4)$[SU(3)_{\rm H}]^2$ happens to vanish [20]. The most general superpotential is given [9] by

$$\begin{aligned} W \;=\; & \sqrt{2}\lambda_{3\rm H}\bar{Q}^i{}_\alpha X^a(t_a)^\alpha_\beta Q^\beta_i+\sqrt{2}\lambda'_{3\rm H}\bar{Q}^6{}_\alpha X^a(t_a)^\alpha_\beta Q^\beta_6 \\ & +\sqrt{2}\lambda_{1\rm H}\bar{Q}^i{}_\alpha X^0(t_0)^\alpha_\beta Q^\beta_i+\sqrt{2}\lambda'_{1\rm H}\bar{Q}^6{}_\alpha X^0(t_0)^\alpha_\beta Q^\beta_6 \\ & -\sqrt{2}\lambda_{1\rm H}v^2X^\alpha_\alpha \\ & +h'\bar{H}_i\bar{Q}^i{}_\alpha Q^\alpha_6+h\bar{Q}^6{}_\alpha Q^\alpha_i H^i \\ & +y_{\mathbf{10}}\mathbf{10}\cdot\mathbf{10}\cdot H+y_{\mathbf{5}^*}\mathbf{5}^*\cdot\mathbf{10}\cdot\bar{H}+\cdots, \end{aligned} \tag{1}$$

where the parameter v is taken to be of the order of the GUT scale, $y_{\mathbf{10}}$ and $y_{\mathbf{5}^*}$ are Yukawa coupling constants of the quarks and leptons, and $\lambda_{3\rm H},\lambda'_{3\rm H},\lambda_{1\rm H},\lambda'_{1\rm H},h'$ and h are dimensionless coupling constants. Ellipses stand for neutrino mass terms and non-renormalizable terms. The fields Q^α_i and $\bar{Q}^i{}_\alpha$ in the bifundamental representations acquire vacuum expectation values (VEV's), $\langle Q^\alpha{}_i\rangle=v\delta^\alpha{}_i$ and $\langle\bar{Q}^i{}_\alpha\rangle=v\delta^i{}_\alpha$, because of the second and third lines in (1). Thus, the gauge group $SU(5)_{\rm GUT}\times U(3)_{\rm H}$ is broken down to that of the standard model. The mass terms of the coloured Higgs multiplets arise from the fourth line in (1) in the GUT-breaking vacuum. No unwanted particle remains massless after the gauge group is broken

[4]The only way to avoid providing only triplets or only doublets in SU(5) unified theories is to introduce two sets of infinite number of the SU(5)-($\mathbf{5}$+$\mathbf{5}^*$), where one set has R charge 0 and the other has R charge 2. This is what is done in [23, 24].

[5]The normalization condition $\mathrm{tr}(t_at_b)=\delta_{ab}/2$ is understood. Note that the normalization of the following t_0 is determined such a way that it also satisfies $\mathrm{tr}(t_0t_0)=1/2$.

down to that of the standard model. In other words, the above model is constructed so that the $\mathrm{U}(3)_{\mathrm{H}}$ gauge interactions leave only two composite massless fields (moduli), $(\bar{Q}^i{}_\alpha Q^\alpha{}_6)$ and $(\bar{Q}^6{}_\alpha Q^\alpha{}_i)$, after they are integrated out. These two composite fields have R charge 2 (see Table 2), and contain only $\mathrm{SU}(3)_C$-triplets (without $\mathrm{SU}(2)_L$-doublets) as one-particle degrees of freedom. Thus, they can be the mass partners of the coloured Higgs multiplets. Therefore, the doublet–triplet mass splitting problem is naturally solved.

Fine structure constants of the $\mathrm{SU}(3)_{\mathrm{H}} \times \mathrm{U}(1)_{\mathrm{H}}$ must be larger than that of the $\mathrm{SU}(5)_{\mathrm{GUT}}$. This is because the gauge coupling constants α_C, α_L and α_Y of the MSSM are given by

$$\frac{1}{\alpha_C} = \frac{1}{\alpha_{\mathrm{GUT}}} + \frac{1}{\alpha_{3\mathrm{H}}}, \tag{2}$$

$$\frac{1}{\alpha_L} = \frac{1}{\alpha_{\mathrm{GUT}}}, \tag{3}$$

and

$$\frac{3/5}{\alpha_Y} = \frac{1}{\alpha_{\mathrm{GUT}}} + \frac{2/5}{\alpha_{1\mathrm{H}}}, \tag{4}$$

at tree level, where $\alpha_{\mathrm{GUT}}, \alpha_{3\mathrm{H}}$ and $\alpha_{1\mathrm{H}}$ are fine structure constants of $\mathrm{SU}(5)_{\mathrm{GUT}}$, $\mathrm{SU}(3)_{\mathrm{H}}$ and $\mathrm{U}(1)_{\mathrm{H}}$, respectively. The values of $1/\alpha_{3\mathrm{H}}$ and $1/\alpha_{1\mathrm{H}}$ must be within a few percent of the $1/\alpha_{\mathrm{GUT}}$ at the GUT scale to reproduce the approximate unification of α_C, α_L and $5\alpha_Y/3$.

The large coupling constant of the $\mathrm{SU}(3)_{\mathrm{H}}$ required above, however, is not stable in the renormalization group running [16]. Although its beta function is zero at 1-loop, renormalization at higher-loop levels is not negligible; n-th loop effects arise with $(3 \times \alpha_{3\mathrm{H}}/(4\pi))^n \sim 1$. At 2-loop level, it is easy to see that the $\mathrm{SU}(3)_{\mathrm{H}}$ coupling becomes infinity immediately above the GUT scale. In other words, the $\mathrm{SU}(3)_{\mathrm{H}}$ gauge interactions immediately become weak below the cut-off scale. Thus, the approximate $\mathrm{SU}(5)_{\mathrm{GUT}}$ relation of the MSSM gauge couplings is not a natural consequence unless the large coupling constant of the $\mathrm{SU}(3)_{\mathrm{H}}$ is stable against the radiative corrections.

An interesting way to solve this problem is to impose a specific relation,

$$\frac{(\lambda_{3\mathrm{H}})^2}{4\pi} \simeq \frac{(\lambda'_{3\mathrm{H}})^2}{4\pi} \simeq \alpha_{3\mathrm{H}}. \tag{5}$$

This relation is stable under the renormalization group, because there is a symmetry in the limit of $g_{\mathrm{GUT}}, h, h', y_{\mathbf{10}}, y_{\mathbf{5}^*} \to 0$: an $\mathcal{N}=2$ SUSY. The matter contents of the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector have a multiplet structure of the $\mathcal{N}=2$ SUSY [8]: the $\mathrm{U}(3)_{\mathrm{H}}$ vector multiplet and the $\mathrm{U}(3)_{\mathrm{H}}$-**adj.** chiral multiplet, $X^\alpha{}_\beta$, form an $\mathcal{N}=2$ vector multiplet, and the vector-like pairs $(Q^\alpha_k, \bar{Q}^k{}_\alpha)$ $(k = 1, ..., 6)$ in this sector form $\mathcal{N}=2$ hypermultiplets. The superpotential

(1) from the first to the third line exhibits the form of interactions of the $\mathcal{N}=2$ SUSY gauge theories [14]. The approximate $\mathcal{N}=2$ SUSY exists when the $\mathcal{N}=2$ relation in Eq. (5) is satisfied. This relation, in turn, is stable because of the symmetry. Then, the perturbative renormalization to the gauge coupling is 1-loop-exact in this $\mathcal{N}=2$ SUSY limit. Higher-loop renormalization to the $\mathrm{SU}(3)_{3\mathrm{H}}$ gauge coupling only appears by involving weak couplings, $g_{\mathrm{GUT}}, h, h', y_{\mathbf{10}}$ and y_{5^*}, and hence the large gauge coupling can be preserved under the renormalization group. This is the main reason why we impose the approximate $\mathcal{N}=2$ SUSY in the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector. We also impose $(\lambda_{1\mathrm{H}})^2/(4\pi) \simeq (\lambda'_{1\mathrm{H}})^2/(4\pi) \simeq \alpha_{1\mathrm{H}}$ so that the approximate $\mathcal{N}=2$ SUSY is maintained in the full $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector.

The gauge coupling constant of the $\mathrm{U}(1)_{\mathrm{H}}$ is asymptoticcally non-free. The coupling, which is already strong at the GUT scale, becomes infinity below the Planck scale, $M_{\mathrm{Pl}} \simeq 2.4 \times 10^{18}$ GeV. Even the $\mathcal{N}=2$ SUSY does not solve this problem. Thus, the cut-off scale (in other words, the fundamental scale) M_* of this model should lie below the Planck scale. On the other hand, the fundamental scale should be higher than the GUT scale by at least one order of magnitude, so that the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking corrections to the gauge coupling constants, through non-renormalizable interactions such as

$$W = \mathrm{tr}\left(\left(\frac{1}{g^2} + \frac{\langle \bar{Q}Q \rangle}{M_*^2}\right) \mathcal{W}^{\alpha, SU(5)} \mathcal{W}_\alpha^{SU(5)}\right), \tag{6}$$

are suppressed below 10^{-2}.

The other model of the product-group unification is based on an $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(2)_{\mathrm{H}}$ gauge group, where $\mathrm{U}(2)_{\mathrm{H}} \simeq \mathrm{SU}(2)_{\mathrm{H}} \times \mathrm{U}(1)_{\mathrm{H}}$. This model provides two Higgs doublets without triplets as one-particle degrees of freedom in massless composite fields, after the U(2) gauge group is integrated out. This model realizes the other possibility discussed at the beginning of this section.

Matter contents of this model are $X^\alpha{}_\beta$ ($\mathbf{1}$,$\mathbf{adj.}$=$\mathbf{3}$+$\mathbf{1}$), $Q^\alpha{}_i + Q^\alpha{}_6(\mathbf{5}^*$+$\mathbf{1}$,$\mathbf{2})$ and $\bar{Q}^i_\alpha + \bar{Q}^6_\alpha(\mathbf{5}$+$\mathbf{1}$,$\mathbf{2}^*)$ $(\alpha, \beta = 4, 5; i = 1, ..., 5)$ in addition to the three families of quarks and leptons, $(\mathbf{5}^*$+$\mathbf{10}$,$\mathbf{1})$. The ordinary Higgs fields $H^i(\mathbf{5})$ and $\bar{H}_i(\mathbf{5}^*)$ are not introduced. The R charges of those fields are given in Table 3. Mixed anomaly (R mod 4)$[\mathrm{SU}(2)_{\mathrm{H}}]^2$ happens to vanish again. The superpotential is given by

$$\begin{aligned} W \;=\; & \sqrt{2}\lambda_{2\mathrm{H}} \bar{Q}^i{}_\alpha X^a (t_a)^\alpha_\beta Q^\beta_i + \sqrt{2}\lambda'_{2\mathrm{H}} \bar{Q}^6{}_\alpha X^a (t_a)^\alpha_\beta Q^\beta_6 \\ & + \sqrt{2}\lambda_{1\mathrm{H}} \bar{Q}^i{}_\alpha X^0 (t_0)^\alpha_\beta Q^\beta_i + \sqrt{2}\lambda'_{1\mathrm{H}} \bar{Q}^6{}_\alpha X^0 (t_0)^\alpha_\beta Q^\beta_6 \\ & - \sqrt{2}\lambda_{1\mathrm{H}} v^2 X^\alpha_\alpha \\ & + c_{\mathbf{10}} \mathbf{10}^{i_1 i_2} \mathbf{10}^{i_3 i_4} (\bar{Q}Q)^{i_5}{}_6 + c_{\mathbf{5}^*} (\bar{Q}Q)^6{}_i \cdot \mathbf{10}^{ij} \cdot \mathbf{5}^*_j + \cdots, \end{aligned} \tag{7}$$

where $t_a (a = 1, 2, 3)$ is now one half of the Pauli matrices. The $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(2)_{\mathrm{H}}$ symmetry is broken down to that of the standard model through the expectation values $\langle Q^{\alpha}{}_{i} \rangle = v \delta^{\alpha}{}_{i}$ and $\langle \bar{Q}^{i}{}_{\alpha} \rangle = v \delta^{i}{}_{\alpha}$. When the $\mathrm{U}(2)_{\mathrm{H}}$ gauge interactions are integrated out, two moduli remain massless in addition to the chiral quarks and leptons, which are $(\bar{Q}^{i}{}_{\alpha} Q^{\alpha}{}_{6})$ and $(\bar{Q}^{6}{}_{\alpha} Q^{\alpha}{}_{i})$. These two composite fields contain only $\mathrm{SU}(2)_L$-doublets as one-particle degrees of freedom in the $\mathrm{SU}(5)_{\mathrm{GUT}}$ breaking phase. Thus, they play the role of the two Higgs doublets in the MSSM. Their R charges are 0 (see Table 3) as required. There is no unwanted massless particle in this model either.

The gauge coupling constants of the $\mathrm{SU}(2)_{\mathrm{H}} \times \mathrm{U}(1)_{\mathrm{H}}$ should also be relatively large, for the same reason as in the $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(3)_{\mathrm{H}}$ model. An $\mathcal{N} = 2$ SUSY relation:

$$\frac{(\lambda_{2\mathrm{H}})^2}{4\pi} \simeq \frac{(\lambda'_{2\mathrm{H}})^2}{4\pi} \simeq \alpha_{2\mathrm{H}}, \qquad \frac{(\lambda_{1\mathrm{H}})^2}{4\pi} \simeq \frac{(\lambda'_{1\mathrm{H}})^2}{4\pi} \simeq \alpha_{1\mathrm{H}}, \tag{8}$$

also stabilizes the large coupling constant of the $\mathrm{SU}(2)_{\mathrm{H}}$ gauge group. The cut-off scale should lie below the Planck scale, as explained in the previous model.

Here, we summarize five remarkable features that are common to the two models described above. First of all, the gauge groups of these "unification theories" have a product-group structure, and secondly, the $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge coupling constant is small while the rest of the gauge couplings are relatively large. Third, there is an approximate $\mathcal{N} = 2$ SUSY in the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector. The $\mathcal{N} = 2$ SUSY is crucial in maintaining the approximate $\mathrm{SU}(5)_{\mathrm{GUT}}$ unification of the MSSM gauge couplings at the GUT scale. It is quite remarkable that the matter contents and interactions support the approximate $\mathcal{N} = 2$ SUSY. Fourth, the cut-off scale should lie below the Planck scale because of the asymptotically non-free running of the $\mathrm{U}(1)_{\mathrm{H}}$ gauge coupling constant. Finally, the discrete R symmetry that governs these models should be preserved in an accuracy better than the 10^{-14} level to keep the two Higgs doublets almost massless.

3 Motivations of Product-Group Unification in Type IIB Supergravity

3.1 Product-Group Unification in Supersymmetric Brane World

The five features listed at the end of the previous section are understood quite naturally when we embed the models in a SUSY brane world [14, 15], as we briefly explain in this subsection.

This is the reason why we develop explicit construction of models in a SUSY brane world in this paper.

It is quite reasonable to think of supersymmetric higher dimensions when one considers the approximate $\mathcal{N}=2$ SUSY in the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector as an indication of an extended SUSY at short distances (rather than as an accident); any extended SUSY can be easily broken down to the $\mathcal{N}=1$ SUSY through extra-dimensional geometry, while it is difficult to obtain successful models using the partial SUSY-breaking mechanisms in the four-dimensional space-time [25]. The $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector should be localized in an extra-dimensional manifold, or otherwise the low energy matter contents would be chiral and have the multiplet structure of only the $\mathcal{N}=1$ SUSY. This is the primary reason why we embed the original models into a SUSY brane world.

There is a possibility that a localized sector has $\mathcal{N}=2$ SUSY (eight SUSY charges), when the short-distance physics possesses $\mathcal{N}=4$ SUSY (sixteen SUSY charges)[6]; sixteen SUSY charges are necessary to realize the $\mathcal{N}=2$ SUSY on a localized sector, because the localized sector itself breaks translational symmetry in the extra dimensions and hence SUSY is broken by at least half. Partial breaking of the translational symmetry on the localized sector *can* leave half of the original SUSY charges unbroken when the localized sector satisfies BPS conditions (e.g. see appendix of [14]). Then, the $\mathcal{N}=2$ SUSY is preserved if we impose sixteen SUSY charges in extra-dimensional space-time and if the local geometry around the localized sector are not the extra source of further breakings of SUSY charges. Then, the approximate $\mathcal{N}=2$ SUSY (the third feature) is no longer an accident.

The whole geometry of the compactified manifold, on the other hand, is chosen so that it preserves only the $\mathcal{N}=1$ SUSY of the four-dimensional space-time. It is only the local geometry around the localized $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector that is required to preserve the $\mathcal{N}=2$ SUSY.

On the other hand, the $\mathrm{SU}(5)_{\mathrm{GUT}}$ vector multiplet should propagate in the extra space dimensions. Indeed, gauge fields of the $\mathrm{SU}(5)_{\mathrm{GUT}}$ should propagate around the localized sector, since the hypermultiplets in the bifundamental representation $(\bar{Q}^i{}_\alpha, Q^\alpha{}_i)$ are charged under the $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge group, while the $\mathrm{SU}(5)_{\mathrm{GUT}}$ vector multiplet should not be confined in the localized sector since we should not have chiral multiplets of the $\mathrm{SU}(5)_{\mathrm{GUT}}$-**adj.** representation (i.e. $\mathcal{N}=2$ SUSY partner) in the models explained in the previous section.

When the compactified manifold has a moderately large volume in the M_* units, the

[6]The $\mathcal{N}=4$ SUSY is understood as (1,1) SUSY in six dimensions, and is understood as $\mathcal{N}=1$ SUSY in more than six dimensions.

effective four-dimensional Planck scale $M_{\rm Pl}$ is higher than the fundamental scale, which is given by

$$M_{\rm pl}^2 \simeq M_*^2 (M_*^\delta \times \text{volume}), \tag{9}$$

where δ is the number of sligtly large dimensions of the compactified manifold. The cut-off scale lies below the effective Planck scale of the four-dimensional gravity. Therefore, the fourth feature is translated into the moderately large volume of the extra dimensions. Now the gauge coupling constant of the $\mathrm{SU}(5)_{\rm GUT}$ Kaluza–Klein zero mode becomes weak with respect to that of the $\mathrm{U}(3)_{\rm H}$ ($\mathrm{U}(2)_{\rm H}$); this is because only the $\mathrm{SU}(5)_{\rm GUT}$ gauge field propagates in the extra dimensions and its gauge coupling is suppressed as

$$\frac{1}{\alpha_{\rm GUT}} \simeq \frac{1}{\alpha_*}(M_*^\delta \times \text{volume}), \qquad \frac{1}{\alpha_{\rm 3H,2H,1H}} \simeq \frac{1}{\alpha_*}. \tag{10}$$

Thus, the disparity in the gauge coupling constants (the second feature) follows naturally. We impose[7]

$$(M_*^\delta \times \text{volume}) \sim 10^2 \tag{11}$$

to maintain the approximate $\mathrm{SU}(5)_{\rm GUT}$ unification. Then, in turn, $M_* \simeq 10^{-1} M_{\rm pl}$ follows[8] from Eq. (9), which is also a desirable value for the cut-off scale.

Another benefit of higher dimensions is that R symmetries can be realized as discrete gauged symmetries below the compactification scale [26]. A discrete rotational symmetry of the compactified manifold is in general recognized as an R symmetry below the compactification scale. The rotational symmetry is a gauge symmetry, since it is a subgroup of the extra-dimensional Lorentz group. The R symmetry is thus exact, unless broken spontaneously. The fifth feature finds its natural explanation when the (mod 4)-R symmetry is identified with a suitable rotational symmetry of the compactified manifold.

As we have seen so far, an effective field theory with a localized sector and with an higher-dimensional SUSY is able to explain basic structures of the product-group unification models. Both the extended SUSY and the localization of gauge fields are natural ingredients of higher-dimensional supergravities. In fact, a number of indications have been obtained, which suggest that SUSY gauge theories are localized on solitonic solutions of the higher-dimensional supergravities [27] called D-branes[9]. Once we adopt this picture, then

[7] The volume for the gravity in Eq. (9) and the volume for the $\mathrm{SU}(5)_{\rm GUT}$ gauge field in Eq. (10) are not necessarily the same, in general. However, we have no motivations to consider such a situation.

[8] This relation is independent of the number of extra dimensions δ.

[9] Those solitonic solutions were formerly called "black p-branes" [27]. We make no distinction between "D-branes" in supergravities and "black p-branes".

the product-group structure of the "unified gauge group" (the first feature) is quite a natural consequence since each stack of D-branes provides each factor of the product group.

3.2 In Type IIB Supergravity

Therefore, it is quite interesting to consider that the product-group unification models are realized on D-branes in higher-dimensional supergravities. References [14, 15] identify the D3–D7 system (bound states of D3- and D7-branes) of the Type IIB supergravity, in ten-dimensional space-time, with the origin of the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(3)_{\mathrm{H}}$ ($\mathrm{U}(2)_{\mathrm{H}}$) gauge group.

The Type IIB supergravity (in ten-dimensional space-time) has the maximally extended SUSY and highly restricted multiplet structure. There are thirty-two SUSY charges (eight times those of the four-dimensional $\mathcal{N}=1$ SUSY), which are combined into two SUSY generators $\mathcal{Q}$ and $\mathcal{Q}'$, irreducible under the SO(9,1). Both the $\mathcal{Q}$ and $\mathcal{Q}'$ are Weyl and Majorana spinors of the SO(9,1), and hence each one contains sixteen SUSY charges. The Type IIB supergravity allows only one SUSY multiplet, the supergravity multiplet. No other multiplet is allowed as a massless representation of the SUSY generators and the Lorentz symmetry SO(9,1). The supergravity multiplet consists of one hundred and twenty-eight bosonic states and one hundred and twenty-eight fermionic states. The one hundred and twenty-eight bosonic states are described by ten-dimensional metric, which contains thirty-five on-shell states, two real scalar fields ϕ called dilaton and $C_{(0)}$, two 2-form fields $B_{\mu\nu}$ and $C_{(2)}$, both containing twenty-eight states, and a self-dual 4-form field $C^{+}_{(4)}$ containing thirty-five states. The one hundred and twenty-eight fermionic states consist of two Weyl and Majorana gravitinos (fifty-six states each) and two Weyl and Majorana spinors (eight states each).

D-branes are soliton solutions made of those fields. D7-branes are soliton solutions that extend in seven spatial dimensions. There are two codimensions in the ten-dimensional space-time. D7-branes are made[10] of the metric, the dilaton ϕ and the $C_{(0)}$, whose discretized 7-brane charges are measured by

$$\oint dC_{(0)}, \tag{12}$$

where the closed path is taken so that it winds around the D7-branes. D3-branes are solitons

[10] The D7-brane is a multi-valued solution of $C_{(0)}$, as is evident from the formula of 7-brane charges for D7-branes, Eq. (12). The multi-valued soliton solution is a natural possibility because the D7-branes have only two codimensions. The $\tau \equiv (C_{(0)} + ie^{-\phi})$ of D7-brane solutions is considered as single-valued up to $\mathrm{SL}_2(\mathbf{Z})$ monodromy around them, where this $\mathrm{SL}_2(\mathbf{Z})$ is a subgroup of the $\mathrm{SL}_2(\mathbf{R})$ symmetry of the Type IIB supergravity that acts on τ. (See, e.g. [28].)

that extend in three spatial dimensions made of the metric and the $C^+_{(4)}$. The 3-brane charges of the D3-branes are measured by

$$\int_{S_5} dC^+_{(4)}, \tag{13}$$

where the 5-form is integrated over a 5-sphere that wraps the D3-branes. The 5-brane charges of the D5-branes are measured by

$$\int_{S_3} dC_{(2)} \tag{14}$$

on a 3-sphere surrounding the D5-branes, although this solution is not relevant to our construction. What is called the D3–D7 system is a bound state of the D3- and D7-branes.

The D7-branes are BPS solutions of the Type IIB supergravity, on which half of SUSY charges (sixteen SUSY charges, i.e. the $\mathcal{N} = 1$ SUSY in eight dimensions) are realized linearly [27, 29, 30]. The SUSY charges preserved in the presence of D7-branes are[11]

$$\mathcal{Q} - \Gamma^{98}\mathcal{Q}'. \tag{15}$$

The SUSY charges preserved in the presence of D3-branes are

$$\mathcal{Q} + \Gamma^{987654}\mathcal{Q}'. \tag{16}$$

The SUSY charges on the D3–D7 system, i.e. the SUSY charges that belong both to (15) and to (16), are equivalent to (15) that satisfy an additional constraint

$$-\Gamma^{7654}(\mathcal{Q} - \Gamma^{98}\mathcal{Q}') = (\mathcal{Q} - \Gamma^{98}\mathcal{Q}'). \tag{17}$$

Since the Γ^{7654} has two eigenvalues[12], 1 and -1, both with the same multiplicity, the SUSY charges are further broken by half on the D3–D7 system [29, 30]. Eight SUSY charges (SUSY charges in the eigenspace $(-\Gamma^{7654}) = 1$) are left unbroken among the sixteen SUSY charges in (15).

Now we see that the D3–D7 system preserves the $\mathcal{N} = 2$ SUSY, which is necessary to the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector. Therefore, we consider the D3–D7 system as the origin of the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector[13]. The $\mathrm{U}(3)_{\mathrm{H}}$ or the $\mathrm{U}(2)_{\mathrm{H}}$ gauge group is expected to arise on D3-branes and the $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge group on D7-branes.

[11] $\Gamma^{98} \equiv \Gamma^9\Gamma^8$. A similar notation is used throughout this paper.

[12] Note that $(-\Gamma^{7654})^2 = 1$. This is why four extra dimensions, transverse to the localized $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector, are necessary.

[13] There is another system that possesses the $\mathcal{N} = 2$ SUSY: i.e. NS5–D4–D6 system [30]. We do not discuss that system in this paper. Degrees of freedom on the NS5-brane are also relevant because the extra dimensions are compactified, and moreover the weak coupling limit is not applicable in the case that interests us.

Discrete rotational symmetry of the plane transverse to the D7-branes is identified [14, 15] with an origin of the (mod 4)-R symmetry, which is crucial in the product-group unification models. This is the primary reason why we construct models in *ten*-dimensional space-time.

3.3 Purpose of This Paper

The discussion given above does not go beyond qualitative arguments. Explicit models should be constructed to examine theoretical consistencies in the Type IIB supergravity, which is the purpose of this paper.

It is quite difficult to handle a six-dimensional compactified manifold, unless it has a simple geometry. We only adopt an orbifold of a six-dimensional torus as the compactified manifold[14]. The purpose of the following sections is to see whether the basic idea is consistently realized in the orbifold compactification of the Type IIB supergravity. Namely, we want to see if the $SU(5)_{GUT}$-breaking sector of the product-group unification is naturally realized on the D3–D7 system.

Once the orbifold geometry is fixed, we can calculate the low-energy spectrum on a set of D-brane configuration and orbifold projection conditions. We show that the whole $SU(5)_{GUT}$-breaking sector is obtained from fluctuations localized on D-branes. Although not all the particles of the whole theory are obtained as D-brane fluctuations in our construction (quarks and leptons are missing), we consider missing particles that do not arise as D-brane fluctuations arise as fields at fixed points (see also discussion in section 4). Once particles are obtained from the D-brane fluctuations, then their R charges are far from arbitrary. Now that the (mod 4)-R symmetry is identified with a rotational symmetry of the compactified manifold, we can determine how each particle transforms under the rotation and equivalently under the R symmetry. Therefore, the model building in such a higher-dimensional space-time is subject to a stringent consistency check. Anomaly cancellation at orbifold fixed points is also discussed. This also serves as a non-trivial consistency check.

4 Our Principle of Model Construction in Type IIB Supergravity

Our discussion is based on supergravity, and we do not assume the Type IIB string theory. It is true that the Type IIB string theory is one of the candidates of quantum gravity that

[14]We use the word "manifold" even if it has singularities. It is an abuse of terminology, though.

effectively provides the Type IIB supergravity below the fundamental scale (i.e. the string scale), but there is no proof that it is the only one. The Type IIB string theory has a definitely fixed spectrum that extends up to infinity above the fundamental scale. We have no strong motivation that directly suggests to us to impose such a stringent restriction[15]. Our study keeps genericity and is independent of the ultraviolet (UV) spectrum above the cut-off scale of the supergravity. Thus, we have more freedom to construct realistic models, since we do not specify the UV physics. In particular, we do not ask the question of whether the UV physics required for our model is contained in the vast variety of vacua of what is called the M-theory [31]. This is not a question to be solved at present, since we do not have a precise definition of the M-theory.

Massless matter contents and interactions on D-branes are known very well if one assumes the Type IIB string theory. On the other hand, localized massless sectors on D-branes are not well understood when one assumes only the Type IIB supergravity[16]. However, various studies on the AdS/CFT correspondence [32] seem to suggest that those sectors in supergravity are the same as those predicted by string theories in particular cases[17]; there are a number of evidences [33], when the 't Hooft coupling $g^2_{\mathrm{Yang-Mills}}N$ is large, that $\mathrm{U}(N)$ gauge theories with sixteen SUSY charges are localized on coincident N D3-branes of the supergravity, which is the same as the predictions of the Type IIB string theory. Although such studies have not yet given *proof* that the gauge theories localized on *any* D-brane configuration in supergravities with *arbitrary* 't Hooft coupling are the same as those in the Type IIB string theory, we assume this to be the case [34].

Since string theories provide massless matter contents and their interactions that are known to obey all consistency conditions of the higher-dimensional supergravity, it is convenient to adopt those string predictions as the starting point of our model construction. This is another reason why we assume the same massless matter contents and interactions on D-branes as in the string theories.

We consider that the $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge group comes from five D7-branes and the $\mathrm{U}(3)_{\mathrm{H}}$ or $\mathrm{U}(2)_{\mathrm{H}}$ gauge group from three or two D3-branes, respectively [14, 15]. The gauge group

[15] Even if one does not assume the exact spectrum of the Type IIB string theory, the $\mathrm{SL}_2(\mathbf{Z})$ symmetry is necessary in the UV spectrum, since this symmetry is crucial for the existence of the D7-brane solution.

[16] T.W. thanks M. Nishimura for useful discussion.

[17] Most of the studies on the AdS/CFT does not aim at showing a correspondence between supergravities and gauge theories, but rather a correspondence between string theories and gauge theories [32]. However, most of the evidences of this correspondence have been obtained for large 't Hooft couplings, where string corrections are not important (i.e. only the supergravity is relevant). Such results, which are rather independent of the UV spectrum, are just what we are interested in here.

would be U(5) on D7-branes rather than $SU(5)_{GUT}$, but it is not a problem as will be shown in subsections 6.4 and 7.4. We show that all the matter contents of the $SU(5)_{GUT}$-breaking sector are obtained from the fluctuations of the D3-D7 system. We try to understand as many particles of the models as possible as the D-brane fluctuations (i.e. massless fields on D-branes), but quarks and leptons are not obtained.

The Type IIB string theory also has definite predictions on the massless matter contents and interactions at orbifold singularities. Those massless matter contents are called the twisted sector. However, we do not restrict ourselves to the matter contents of the twisted sector determined by the Type IIB string theory.

The twisted sectors of the Type IIB string theory play important roles in the following two aspects: first, in restoring the modular invariance of the string world-sheet and, second, in keeping the unitarity of the theory. The first aspect, the modular invariance of world-sheets, is crucial to make the string theories UV-finite [29]. Theories with modular invariance are constructed so that the 1-loop amplitude with UV momentum, where an infinite number of massive particles are in the loop, is equivalent to the 1-loop amplitude with infrared (IR) momentum, where only massless particles are in the loop. This equivalence between the UV and IR amplitudes enables one to cut out the UV part from theories, rendering the theories UV-finite. This clearly shows that modular invariance is a constraint between UV physics and IR physics, not a constraint on purely IR physics. In particular, this means that the necessary massless contents at the orbifold singularities are different when the UV spectrum in the bulk is different. Moreover, we are not sure whether the modular invariance in string theories is the only way to make a theory UV-finite. Therefore, we have no reason to adopt the massless matter contents at orbifold singularities predicted by the Type IIB string theory; our study is based on a generic Type IIB supergravity and we do not specify the theory above the cut-off scale. Thus, our framework is not restrictive enough to determine the massless contents at orbifold singularities in a top-down way.

The second aspect, the unitarity, is a well-defined notion within field theories. Anomalies appear at orbifold singularities, even if the theory is consistent in the *ten*-dimensional spcetime. Some of the massless fields in the twisted sector in the Type IIB string theory provide massless degrees of freedom necessary to cancel the pure gravitational anomalies, and some of them realize the (generalized) Green–Schwarz mechanism [35, 36] at orbifold singularities, cancelling the anomalies. Therefore, we also require in our model that the massless matter contents at singularities are such that anomalies are all cancelled.

We expect that suitable particles can be supplied at singularities by a certain mechanism of a fundamental theory if they are required for theoretical consistency there. We do not specify the mechanism in this paper. General interactions localized at singularities are also considered as long as they satisfy the symmetries around there. Since we do not know the origin of particles that arise at singularities, there is no way of determining their interactions other than theoretical consistencies and symmetries.

There are consistency conditions called Ramond–Ramond tadpole cancellation [37] in (string-based) Type IIB orientifolds [38]. These conditions are closely related to the matter contents and interactions of the twisted sector. These conditions are *generically* equivalent to conditions for vanishing non-Abelian anomalies at fixed-point singularities [39, 40], and at the same time they ensure automatically that the mixed anomalies arising at fixed points are cancelled through the fixed-point interactions (generalized Green–Schwarz mechanism) predicted by the Type IIB string theory [38, 41, 40].

However, it is also known that the Ramond–Ramond tadpole cancellation is sometimes more stringent than the triangle anomaly cancellation [40]. We clarify the relation between the Ramond–Ramond tadpole cancellation and the anomaly cancellation in subsubsection 7.2.2, in addition to the arguments on the anomaly cancellation in subsections 6.2 and 7.2. We conclude, there, that various anomalies vanish or can be cancelled through the (generalized) Green–Schwarz mechanism, while one cannot argue some of the Ramond–Ramond tadpole cancellation conditions in our models, since we do not specify the UV spectrum and the matter contents at singularities.

5 Geometry of the $\mathbf{T}^6/(\mathbf{Z}_{12}\langle\sigma\rangle \times \mathbf{Z}_2\langle\Omega R_{89}\rangle)$ Orientifold

The extra dimensions should be compactified to obtain realistic low-energy physics. Then, the fluxes of gauge fields $C_{(0)}$ and $C^{+}_{(4)}$ have nowhere to escape in the compactified extra dimensions. This implies that the totality of 7-brane charges and that of 3-brane charges scattered within the compact manifold should be zero. However, it is impossible for D7-branes alone (D3-branes alone) to have vanishing total 7-brane charges (3-brane charges) without breaking SUSY, because the unbroken half-SUSY on each BPS D-brane is determined by the sign of its 7- (3-)brane charge [42]. This is a well-known phenomenon for the BPS monopoles and instantons [42]. Therefore, in order for the 7- (3-)brane charges to cancel out within the compactified manifold, there should be new objects with 7- (3-)brane charge whose sign is opposite to the charge of D7- (D3-)branes and which also preserve the same SUSY as D7- (D3-)branes do. Such candidates are known in string theories: orientifold 7-planes (O7-

planes) and orientifold 3-planes (O3-planes) [43]. Orientifold planes emerge in string theories when the world-sheet parity (Ω) (flipping of the orientation of strings) is gauged. One can gauge an arbitrary combination of the world-sheet parity and order-2 space transformation ($g^2 = 1; g \in$SO(6)) rather than a simple world-sheet parity, i.e. one can gauge Ωg. Orientifold planes are (loci of) fixed points of such order-2 space transformations.

Although the orientifold planes are well-formulated in string theories [29, 30], their origin is not clearly understood in general field theories[18]. However, we assume that there exist "orientifold planes" even in the Type IIB supergravity, since there is no clear obstruction against this assumption. Furthermore, we assume that the "orientifold planes" have almost the same properties as those in the Type IIB string theories. Namely, Op-planes have p-brane charges opposite to that of the Dp-branes, and they are always (loci of) $\mathbf{Z}_2$-fixed points as explained above. Here, an orientifold p-plane carries -2^{p-4} times the p-brane charge of a single Dp-brane, as in string theories [29, 30][19].

The D7-branes, O7-planes, D3-branes and O3-planes are put in a six-dimensional torus $\mathbf{T}^6$. We will obtain most of matter contents of the product-group unification models by gauging the orientifold projection, and by gauging the discrete rotational symmetry of the $\mathbf{T}^6$. The geometry we adopt in this paper is $\mathbf{T}^6/(\mathbf{Z}_{12}\langle\sigma\rangle \times \mathbf{Z}_2\langle\Omega R_{89}\rangle)$. Let us first explain the geometry of this manifold. The geometry is important when we discuss the D-brane configuration, anomaly cancellation, discrete R symmetry and effective superpotential. (However, the reader can skip the rest of this section for now, and come back to it when necessary.)

$\mathbf{T}^6$ denotes a six-dimensional torus $(\mathbf{C}^3 = \{(z_1, z_2, z_3)|z_1, z_2, z_3 \in \mathbf{C}\})/\Gamma_0$ in which the lattice Γ_0 is spanned by six base vectors $\mathbf{e}_4$, $\mathbf{e}_5$, $\mathbf{e}_6$, $\mathbf{e}_7$, $\mathbf{e}_8$, and $\mathbf{e}_9$. In other words, two points $\mathbf{y}, \tilde{\mathbf{y}} \in \mathbf{C}^3$ are identified with each other if and only if

$$\tilde{\mathbf{y}} = \mathbf{y} + n_m \mathbf{e}_m \qquad (n_m \in \mathbf{Z}, \quad m = 4, \cdots, 9). \tag{18}$$

The base vectors $\mathbf{e}_{4,5,6,7}$ will be chosen so that they span the first two complex planes $\mathbf{C}^2 = \{(z_1, z_2)|z_1, z_2 \in \mathbf{C}\}$, and $\mathbf{e}_{8,9}$ for the last complex plane, whose coordinate is z_3, in the orbifold we adopt in this paper.

[18] An O6-plane is realized as a solitonic solution of eleven-dimensional supergravity (in the context of M-theory) in [44]. This O6-plane solution is also associated to a $\mathbf{Z}_2$ projection and is found to have the same 6-brane charge as that predicted by string theories.

[19] It is not clear at all whether we have to impose this p-brane charge of the orientifold planes for the consistency of models in a field theory. Our construction of models, nevertheless, does not change so much even if the p-brane charges of the orientifold planes are different from the predictions of the string theories. There is a related discussion at the beginning of subsection 6.1.

We assume D7-branes stretched in the first two complex planes, and hence we need O7-planes parallel to the D7-branes to cancel those 7-brane charges. We gauge the combination ΩR_{89}, where R_{89} denotes the angle-π rotation in the last complex plane, having loci of fixed points that extend parallel to the D7-branes. The loci of the R_{89}-fixed points are the O7-planes. Although this orientifold projection breaks half of the thirty-two SUSY charges of the Type IIB supergravity, the unbroken sixteen SUSY charges are the same as those preserved on the D7-branes (i.e. (15)), since the O7-planes are parallel to the D7-branes. There are four O7-planes within the six-dimensional torus, whose z_3-coordinates are given by

$$z_3 = \frac{1}{2} n_{m''} \mathbf{e}_{m''}, \qquad (\text{mod } n_{m''}\mathbf{e}_{m''}|_{m''=8,9} \quad \text{where } n_{m''} \in \mathbf{Z}). \tag{19}$$

The total 7-brane charge of the O7-planes is -32 because each O7-plane carries[20] the 7-brane charge $= -8$. Thus, there should be thirty-two D7-branes within the six-dimensional torus $\mathbf{T}^6$.

We also need O3-planes parallel to the D3-branes so that the totality of the 3-brane charges vanishes in the six-dimensional torus. This implies that ΩR_{456789} should also be gauged, where R_{456789} reverses all six extra dimensions. In other words, R_{4567} should also be gauged, where R_{4567} reverses all $\mathbf{e}_{4,5,6,7}$. Indeed, gauging $\mathbf{Z}_2 \langle R_{4567} \rangle$ is equivalent to gauging $\mathbf{Z}_2 \langle \Omega R_{456789} \rangle$, under the condition that ΩR_{89} is already gauged because of the isomorphism $\mathbf{Z}_2 \langle R_{4567} \rangle \times \mathbf{Z}_2 \langle \Omega R_{89} \rangle \simeq \mathbf{Z}_2 \langle \Omega R_{456789} \rangle \times \mathbf{Z}_2 \langle \Omega R_{89} \rangle$. Thus, the orbifold group should contain $\mathbf{Z}_2 \langle R_{4567} \rangle$ as a subgroup. The SUSY charges broken by this orbifold projection $\mathbf{Z}_2 \langle R_{4567} \rangle$ are the same as those broken in the presence of the D3-branes. Indeed, the SUSY charges on which $-\Gamma^{7654}$ acts trivially are not twisted by the R_{4567} since the $-\Gamma^{7654}$ is the same as the spinor representation of $R_{4567} = \exp((\pi\Gamma^{45} - \pi\Gamma^{67})/2)$. Hence the eight SUSY charges ($\mathcal{N} = 2$ SUSY in four-dimensional spce-time) are preserved in the D3–D7 system along with the O7- and O3-planes. There are sixty-four R_{456789}-fixed points, whose coordinates are

$$\mathbf{y}' = \frac{1}{2} n_{m'} \mathbf{e}_{m'}|_{m'=4,5,6,7} \quad \text{and} \quad z_3 = \frac{1}{2} n_{m''} \mathbf{e}_{m''}|_{m''=8,9} \tag{20}$$

mod $n_m \mathbf{e}_m|_{m=4,\ldots,9}$, where $n_m \in \mathbf{Z}$. Thus, the total 3-brane charges from those O3-planes are -32 because each O3-plane carries the 3-brane charge $= -1/2$. Therefore, thirty-two D3-branes are required in the six-dimensional torus. The orbifold group should be much larger,

[20] Some works state that each O7-plane carries the 7-brane charge $= -4$, and that the total 7-brane charge of O7-planes is cancelled by sixteen D7-branes. This discrepancy comes from adopting two different descriptions: counting 7-brane charges either in projected space $\mathbf{T}^6/\mathbf{Z}_2 \langle \Omega R_{89} \rangle$, or in the covering space $\mathbf{T}^6$. We adopt the latter counting throughout this paper.

so that the SUSY of the whole geometry preserves only $\mathcal{N}=1$ SUSY of the four-dimensional spce-time, i.e. only four SUSY charges. Thirteen $\mathbf{Z}_n$-type orbifold groups[21] are listed in [45] that can be imposed on the six-dimensional torus, keeping the $\mathcal{N}=1$ SUSY. Four of them preserve even $\mathcal{N}=2$ SUSY, and two others do not contain $\mathbf{Z}_2\,\langle R_{4567}\rangle$ as their subgroup. We adopt the $\mathbf{Z}_{12}\,\langle\sigma\rangle \equiv \{\sigma^k | k=0,...,11\}$ orbifold[22] among the seven remaining candidates. There are a couple of reasons why we choose this group, each of which is explained in the course of the following discussion.

The generator σ of the present orbifold group, $\mathbf{Z}_{12}\,\langle\sigma\rangle$, acts on the $\mathbf{C}^3$ as

$$\sigma : \mathbf{y} \equiv (z_b)|_{b=1,2,3} \in \mathbf{C}^3 \longmapsto \sigma\cdot\mathbf{y} \equiv (e^{2\pi i v_b} z_b)|_{b=1,2,3} \in \mathbf{C}^3 \tag{21}$$

with $(v_b)|_{b=1,2,3} = (1/12, -5/12, 4/12)$. Note that $\sigma^6 = R_{4567}$, and hence the $\mathbf{Z}_{12}\,\langle\sigma\rangle$ contains $\mathbf{Z}_2\,\langle\sigma^6 = R_{4567}\rangle$ as required, and hence there surely exist O3-planes (and D3-branes) in the orbifold. On the other hand, since the σ^6 is the only order-2 element in the $\mathbf{Z}_{12}\,\langle\sigma\rangle$, only the ΩR_{89} and $\Omega R_{89}\sigma^6$ in $\mathbf{Z}_{12}\,\langle\sigma\rangle \times \mathbf{Z}_2\,\langle\Omega R_{89}\rangle$ are the elements that lead to the existence of orientifold planes. Therefore, the O7- (O3-)planes transverse to the $\mathbf{e}_{8,9}$ ($\mathbf{e}_{4,5,6,7,8,9}$) directions are the only orientifold planes, and no other types of orientifold planes exist in this manifold.

The six-dimensional torus $\mathbf{T}^6$ should be chosen so that it has the $\mathbf{Z}_{12}\,\langle\sigma\rangle$ symmetry. Thus, the Γ_0, which determines the six-dimensional torus, is chosen as

$$\mathbf{e}_4 = (1,1,0)L_4, \qquad \mathbf{e}_5 = (\zeta, \zeta^{-5}, 0)L_4, \tag{22}$$

$$\mathbf{e}_6 = (\zeta^2, \zeta^2, 0)L_4, \qquad \mathbf{e}_7 = (\zeta^3, \zeta^{-3}, 0)L_4, \tag{23}$$

$$\mathbf{e}_8 = (0,0,1)L_2, \qquad \mathbf{e}_9 = (0,0,\omega)L_2, \tag{24}$$

where $\zeta = e^{2\pi i/12}$, $\omega = e^{2\pi i/3}$ and L_4, L_2 are two independent length scales of the six-dimensional torus; L_4 corresponds to the size of the first four-dimensional torus and L_2 to that of the remaining two-dimensional torus.

The D7-branes are stretched in the $\mathbf{e}_{4,5,6,7}$ directions, on which the $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge fields are expected to propagate. Therefore, we require that the L_4 be slightly larger than the fundamental-scale inverse, because the volume of these extra four dimensions should be large enough to account for the disparity between gauge couplings of the $\mathrm{SU}(5)_{\mathrm{GUT}}$ and $\mathrm{U}(3)_{\mathrm{H}}$ or

[21] We restrict our attention only to orbifolds of the form $\mathbf{T}^6/\mathbf{Z}_n$. One can, in principle, consider orbifolds such as $\mathbf{T}^6/(\mathbf{Z}_n \times \mathbf{Z}_m)$ or more complicated ones. However, the variety of higher-dimensional constructions does not become very much richer by considering such possibilities.

[22] We follow the notation of [38].

$\mathrm{U}(2)_{\mathrm{H}}$. Equations (10) and (11) imply that

$$\frac{1}{4}(M_* L_4)^4 \sim 10^2, \tag{25}$$

where $(1/12) \times 3(L_4)^4$ is the volume of extra dimensions where the $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge field propagates. Here, the D7-branes that provide the $\mathrm{SU}(5)_{\mathrm{GUT}}$ are assumed to reside at a $\mathbf{Z}_{12}\,\langle\sigma\rangle$-fixed[23] locus. On the other hand, the L_2 is considered to be of the order of the inverse of the fundamental scale.

The generator σ in (21) rotates three complex planes separately, and satisfies $\sum_{b=1}^{3} v_b = 0$. Thus, it belongs to an SU(3) subgroup of the six-dimensional rotational symmetry SO(6) $\simeq$ SU(4). The σ is regarded as an SU(4) element, which is written as[24]

$$\sigma = e^{-2\pi i\,\mathrm{diag}(\tilde{v}_a)_{a=0,1,2,3}} = e^{-2\pi i\,\mathrm{diag}(0,\frac{1}{12},\frac{-5}{12},\frac{4}{12})} \in \mathrm{SU}(4), \tag{26}$$

where $\tilde{v}_0 \equiv (v_1 + v_2 + v_3)/2$ and $\tilde{v}_b \equiv v_b - \tilde{v}_0$ for b=1,2,3, or equivalently,

$$\mathrm{diag}(\tilde{v}_a)|_{a=0,1,2,3} \equiv \mathrm{diag}(\frac{v_1+v_2+v_3}{2}, \frac{v_1-v_2-v_3}{2}, \frac{-v_1+v_2-v_3}{2}, \frac{-v_1-v_2+v_3}{2}). \tag{27}$$

Note that $\tilde{v}_0 = 0$ and $\tilde{v}_b = v_b$ (for $b = 1, 2, 3$) when $\sum_{b=1}^{3} v_b = 0$. The σ belongs to an SU(3) subgroup at the lower-right corner of the SU(4). This $\tilde{v}_0 = 0$, or equivalently an eigenvalue $e^{-2\pi i\tilde{v}_0} = 1$, implies that SUSY charges are partially preserved in the orbifold geometry [46]. The unbroken SUSY charges, which correspond to the first entry of the **fund.** representation of the SU(4) (i.e. $\sigma = e^{-2\pi i\tilde{v}_0}$ eigenspace), are half of the $\mathcal{N} = 2$ SUSY of the D3–D7 system, because $((-\Gamma^{7654}) = \sigma^6 = e^{-2\pi i\,\mathrm{diag}(\tilde{v}_a)\times 6}) = 1$ eigenspaces are the first and fourth entries of the **fund.** representation of SU(4) ($\simeq$ **spinor** representation of SO(6)). That is, the $\mathcal{N} = 1$ SUSY, which is one half of the $\mathcal{N} = 2$ SUSY preserved in the D3–D7 system, is also preserved in the entire orbifold geometry.

There are several types of singularities on this geometry. Loci of points fixed under the σ^6 or σ^3 form two-dimensional singularities in the $\mathbf{T}^6$ that extend in the directions spanned by $\mathbf{e}_8$ and $\mathbf{e}_9$ (i.e. singularities with (five+one) spce-time dimensions). This is because $v_3 \times 3 \in \mathbf{Z}$ or, in other words, the σ^3 does not rotate the last complex plane. The coordinates of these

[23] The orientifold projection $\mathbf{Z}_2\,\langle\Omega R_{89}\rangle$ does not reduce the volume on which $\mathrm{SU}(5)_{\mathrm{GUT}}$ propagates because it acts only in the transverse directions to the D7-branes.

[24] Section(s) 3 (and 5) and appendices of Ref. [14] might be useful to understand this paragraph and the following sections. SO(6) $\simeq$ SU(4) transformation properties of gauge fields, scalars, fermions and SUSY charges are explicitly written there.

singularities in the first two complex planes, $\mathbf{y}' = (z_1, z_2) \in \mathbf{C}^2$, are given by

$$\sigma^6\text{-fixed} \quad \mathbf{y}' = n_{m'} \frac{\mathbf{e}_{m'}}{2}|_{m'=4,\cdots,7}, \qquad ({}^\forall z_3), \tag{28}$$

$$\sigma^3\text{-fixed} \quad \mathbf{y}' = n_4 \frac{\mathbf{e}_4 + \mathbf{e}_5 + \mathbf{e}_6}{2} + n_5 \frac{\mathbf{e}_4 + \mathbf{e}_7}{2} + n_6 \mathbf{e}_6 + n_7 \mathbf{e}_7, \qquad ({}^\forall z_3), \tag{29}$$

where $n_{m'}|_{m'=4,5,6,7}$ are integers. There are sixteen loci of σ^6-fixed points of within the covering space $\mathbf{T}^6$, four of which are loci of points fixed under the σ^3. One locus ($\mathbf{y}' = \mathbf{0}$) among the latter four is fixed under the σ^1 as a locus. Three remaining loci among the four in $\mathbf{T}^6$ form an orbit of $\mathbf{Z}_{12} \langle \sigma \rangle / \mathbf{Z}_4 \langle \sigma^3 \rangle$, and become a single singularity in $\mathbf{T}^6 / \mathbf{Z}_{12} \langle \sigma \rangle$. Twelve remaining loci of σ^6-fixed points in $\mathbf{T}^6$ form two distinct orbits of $\mathbf{Z}_{12} \langle \sigma \rangle / \mathbf{Z}_2 \langle \sigma^6 \rangle$ and become two distinct singularities in $\mathbf{T}^6 / \mathbf{Z}_{12} \langle \sigma \rangle$. Thus, there are four distinct two-dimensional singularities in $\mathbf{T}^6 / \mathbf{Z}_{12} \langle \sigma \rangle$. The $\mathbf{Z}_2 \langle \Omega R_{89} \rangle$ projection acts only within each two-dimensional singularity, and hence there are four two-dimensional singularities in $\mathbf{T}^6 / (\mathbf{Z}_{12} \langle \sigma \rangle \times \mathbf{Z}_2 \langle \Omega R_{89} \rangle)$. The isotropy group is $\mathbf{Z}_4 \langle \sigma^3 \rangle$ at a generic point of the first two singularities and is $\mathbf{Z}_2 \langle \sigma^6 \rangle$ at a generic point of the last two singularities.

Other singularities are points in the $\mathbf{T}^6$ (i.e. singularities with only (three+one) spce-time dimensions), whose coordinates $\mathbf{y} = (\mathbf{y}', z_3) \in \mathbf{C}^3$ are

$$\sigma^4\text{-fixed} \qquad \mathbf{y}' = n_4 \frac{\mathbf{e}_4 + \mathbf{e}_6}{3} + n_5 \frac{\mathbf{e}_5 + \mathbf{e}_7}{3} + n_6 \mathbf{e}_6 + n_7 \mathbf{e}_7, \tag{30}$$

$$\sigma^{2,1}\text{-fixed} \qquad \mathbf{y}' = n_{m'} \mathbf{e}_{m'}|_{m'=4,\ldots,7} \tag{31}$$

in the first two complex planes, where $n_{m'}|_{m'=4,\ldots,7}$ are integers, and

$$\sigma^{4,2,1}\text{-fixed} \quad z_3 = \mathbf{0}, \pm \frac{\mathbf{e}_8 + 2\mathbf{e}_9}{3} \qquad (\text{mod } \mathbf{e}_{8,9} \quad \text{where } n_{8,9} \in \mathbf{Z}) \tag{32}$$

in the last complex plane. Note that all fixed points of σ^2 are fixed also under the σ^1 in this geometry.

Both four-dimensional loci in the $\mathbf{T}^6$ determined by $z_3 = (\mathbf{e}_8 + 2\mathbf{e}_9)/3$ and $z_3 = -(\mathbf{e}_8 + 2\mathbf{e}_9)/3)$ are fixed under the σ^1 as a four-dimensional locus, respectively. Although each point on those loci are moved by σ^1 except $\mathbf{y}' = 0$, they move only within each four-dimensional locus. These two four-dimensional $\mathbf{Z}_{12} \langle \sigma \rangle$-fixed loci, which are (seven+one)-dimensional sub-spce-time, are distant from the loci of $\mathbf{Z}_2 \langle \Omega R_{89} \rangle$ fixed points, where O7-planes reside. The existence of this (seven+one)-dimensional fixed loci away from O7-planes is one of the reasons why we chose the $\mathbf{Z}_{12} \langle \sigma \rangle$ orbifold. We put D7-branes on these (seven+one)-dimensional fixed loci in the next section. D7-branes should be put on fixed loci; otherwise orbifold projection conditions would not be imposed. Incidentally, unwanted massless matter contents would remain in the spectrum when a D3–D7 system coincides with orientifold planes. Therefore, we need a fixed locus away from O7-planes. Only the $\mathbf{Z}_6$, $\mathbf{Z}'_6$ and $\mathbf{Z}_{12}$ orbifolds[25] have such a fixed locus among the seven candidates that reduce the higher-dimensional SUSY down to four-dimensional $\mathcal{N} = 1$ SUSY accommodating the D3–D7 system.

6 SU(5)$_{\rm GUT}\times$ U(2)$_{\rm H}$ Model

In the following two sections we explicitly construct the product-group unification models in the Type IIB supergravity. We begin with the construction of the SU(5)$_{\rm GUT}\times$U(2)$_{\rm H}$ model, because its structure is simpler in some aspects. The SU(5)$_{\rm GUT}\times$U(3)$_{\rm H}$ model is discussed in section 7.

6.1 D-brane Configuration and Orbifold Projection

Matter contents below the Kaluza–Klein scale depend on the D-brane configuration (locations of D-branes on the orbifold geometry) and the orbifold projection conditions. Let us first describe how the SU(5)$_{\rm GUT}\times$U(2)$_{\rm H}$ model is obtained.

We assume that the gauge theory on N coincident D7-branes (distant from any O7-planes) consists of a U(N) vector multiplet ($\Sigma^k{}_l$) ($k, l = 1, ..., N$) of the $\mathcal{N} = 1$ SUSY of eight-dimensional space-time. M coincident D3-branes are also expected to have a U(M) vector multiplet ($X^\alpha{}_\beta$) ($\alpha, \beta = 1, ..., M$) of the $\mathcal{N} = 4$ SUSY of four-dimensional space-time. There would be a hypermultiplet ($\bar{Q}^k{}_\alpha$,$Q^\alpha{}_k$) of the four-dimensional $\mathcal{N} = 2$ SUSY when the M D3-branes are on the N D7-branes. Here, the $\mathcal{N} = 1$ chiral multiplets $\bar{Q}^k{}_\alpha$ and $Q^\alpha{}_k$ are in the bifundamental representation under the U(N)$\times$U(M) gauge group, transforming ($\mathbf{N}, \mathbf{M}^*$) and ($\mathbf{N}^*$,$\mathbf{M}$), respectively. These are the particles we assume at the starting point of our model construction in addition to the Type IIB supergravity multiplet in the ten-dimensional bulk. These are exactly the same massless modes as in the D3–D7 system of the Type IIB string theory. D7-branes are labelled by indices k, l, and D3-branes by α, β.

We are concerned only with those massless modes in the following, except for massless modes that are required at orbifold singularities. It is clear that those matter contents on D-branes satisfy all consistency conditions of the supergravity. Anomalies would arise only at orbifold singularities. These issues are discussed in subsection 6.2.

[25] Notations are based on [38].

We do not specify the spectrum of massive particles above the cut-off scale. Those particles are not relevant to the physics at the GUT scale, or at low energies. It is true that the theoretical consistency conditions can be modified if there are infinite numbers of massive particles. However, we do not consider such theoretical conditions involving massive sectors, since they are highly dependent on the UV spectrum above the cut-off scale.

In particular, string theories predict so-called winding modes. These are massive excitations of a "string" in which a "string" winds around the circle of a torus. We do not require theoretical consistency conditions that involve the contributions from winding modes[26]. This is because those conditions depend on the UV spectrum above the cut-off scale, which we do not specify, and are not conditions of the physics below the cut-off scale.

We use twelve out of thirty-two D7-branes to provide the $\mathrm{SU}(5)_{\mathrm{GUT}}$ vector multiplet. They should be put on (seven+one)-dimensional loci fixed under the $\mathbf{Z}_{12}\,\langle\sigma\rangle$ orbifold group, so that the vector multiplet with sixteen SUSY charges is projected out except for a four-dimensional $\mathcal{N}=1$ $\mathrm{SU}(5)_{\mathrm{GUT}}$ vector multiplet (without chiral multiplets in the $\mathrm{SU}(5)_{\mathrm{GUT}}$-**adj.** representation). Six of them are put at a fixed locus $z_3 = \frac{\mathbf{e}_8+2\mathbf{e}_9}{3}$ and six remaining D7-branes are at the $\mathbf{Z}_2\,\langle\Omega R_{89}\rangle$-image of the fixed locus, i.e. at $z_3 = -\frac{\mathbf{e}_8+2\mathbf{e}_9}{3}$. The reason why we put six D7-branes at each fixed locus rather than five will become clear later in this subsection 6.1 (it is because we require that $\bar{Q}^6{}_\alpha$ and $Q^\alpha{}_6$ are obtained from the D3–D7 system).

The twenty D7-branes that have not been used are placed at the other fixed locus $z_a = \mathbf{0}$ or are floating in the bulk. Their existence is irrelevant to the dynamics of the $\mathrm{SU}(5)_{\mathrm{GUT}}$-symmetry breaking, while they provide a room for constructing the SUSY-breaking sector, inflation sector and some other gauge theories we do not know yet.

Once the configuration of 7-branes is fixed, we can calculate the behaviour of the dilaton VEV through equations of motion. In particular, the F-theory [48] implies that $\tau \equiv (C_{(0)} + ie^{-\phi})$ goes to $i\infty$ at the fixed loci where we put six D7-branes [49]. However, equations of

[26]Winding modes are equivalent to Kaluza–Klein modes through the T-duality in string theories. Thus, the gauge theories on D7-branes or on D3-branes are essentially ten-dimensional gauge theories, in some aspects, in the presence of the winding modes. In particular, the hexagonal anomalies of ten-dimensional gauge theories are required to vanish, which is the case when both D7-branes and D3-branes have "ten-dimensional" SO(32) gauge theory with sixteen SUSY charges (or its spontaneous breakdown) [47, 37]. This provides an independent reason for the total number of D7-branes and D3-branes to be thirty-two within the flat six-dimensional torus $\mathbf{T}^6$. We do not respect this reason, since it is heavily dependent on the presence of the winding modes, but we still respect the total number of D-branes since we borrow "orientifold planes" from string theories, which might not be fully justified within pure supergravity, as we discussed in section 5. However, the total number of D-branes is not an important constraint for our model construction.

motion derived from the Type IIB supergravity is not reliable in the vicinities of D7-branes within $1/M_*$ (since we do not specify short-distance physics above the cut-off scale M_*), and hence it does not make sense to discuss the short-distance behaviour of the dilaton VEV. Moreover, a precise relation is not known in supergravities between the dilaton VEV around the D7-branes and the effective coupling constant of the gauge theories on D7-branes[27]. Therefore, we cannot discuss what the natural values are for the effective coupling constants $1/\alpha_*$ and $(M_*^\delta \text{volume})/\alpha_*$ in (10). The relative ratio, however, is reasonable, since gauge theories on D7-branes become non-dynamical in the large-volume limit.

Now, the fields on the D7-branes at $z_3 = \frac{\mathbf{e}_8+2\mathbf{e}_9}{3}$ are identified with their image at $z_3 = -\frac{\mathbf{e}_8+2\mathbf{e}_9}{3}$ under the projection condition associated with the ΩR_{89}. Thus, we only need discuss the fields on one of these images. Those fields are a U(6) vector multiplet $\Sigma^k{}_l(x, \mathbf{y}')$ ($x \in \mathbf{R}^{3,1}$, $\mathbf{y}' = (z_1, z_2) \in \mathbf{C}^2$) of the $\mathcal{N} = 1$ SUSY in eight-dimensional space-time. Fields contained in this multiplet are also described in terms of four-dimensional $\mathcal{N} = 1$ SUSY: one vector multiplet $(\Sigma_0)^k{}_l(x, \mathbf{y}', \theta) \equiv \mathcal{W}_\alpha^{\mathrm{U}(6)}(x, \mathbf{y}', \theta)$ and three chiral multiplets $(\Sigma_b)^k{}_l(x, \mathbf{y}', \theta)$ ($b = 1, 2, 3$).

Let us see how those fields transform[28] under the rotational symmetry of $\mathbf{C}^3$ before we discuss the orbifold projection conditions. Let us consider three independent rotations $z_b \mapsto e^{i\alpha_b} z_b$ for ($b = 1, 2, 3$). The four-dimensional gauge-field strength in Σ_0 is a singlet under the rotation of extra-dimensional space. The complex scalars of $\Sigma_{b=1,2}$ receive the phase factors $e^{i\alpha_b}$ since they originate from polarizations of gauge fields in higher-dimensional space pointing at extra dimensions. Four fermions (Weyl in four-dimensional space-time) in Σ_a's ($a = 0, ..., 3$) receive phase factors $e^{i\tilde{\alpha}_a}$ due to the Lorentz transformation of the higher-dimensional space-time in the spinor representation, where $\tilde{\alpha}_0 \equiv (\alpha_1 + \alpha_2 + \alpha_3)/2$ and $\tilde{\alpha}_b \equiv \alpha_b - \tilde{\alpha}_0$ for ($b = 1, 2, 3$). Thus, in terms of four-dimensional $\mathcal{N} = 1$ superfields, they transform as

$$\Sigma_0(x, \mathbf{y}', \theta) \quad \mapsto \quad e^{i\tilde{\alpha}_0} \Sigma_0(x, \tilde{\mathbf{y}'}, e^{-i\tilde{\alpha}_0}\theta), \tag{33}$$

$$\Sigma_b(x, \mathbf{y}', \theta) \quad \mapsto \quad e^{i\alpha_b} \Sigma_b(x, \tilde{\mathbf{y}'}, e^{-i\tilde{\alpha}_0}\theta) \quad \text{for } b = 1, 2, \tag{34}$$

where $\tilde{\mathbf{y}'}$ is a point that is moved to the point $\mathbf{y}'$ under the rotation, i.e. $\tilde{\mathbf{y}'} = (e^{-i\alpha_1} z_1, e^{-i\alpha_2} z_2)$ and $\mathbf{y}' = (z_1, z_2)$. The complex scalar component of Σ_3 receives the phase factor $e^{i\alpha_3}$, so that

$$\Sigma_3(x, \mathbf{y}', \theta) \mapsto e^{i\alpha_3} \Sigma_3(x, \tilde{\mathbf{y}'}, e^{-i\tilde{\alpha}_0}\theta), \tag{35}$$

[27]See also the question raised in the introduction of [50].

[28]Ref. [14] might be useful again.

similarly to $\Sigma_{b=1,2}$. This choice of the phase factor is to ensure that the sixteen-SUSY-charge symmetric interaction in the superpotential,

$$d^2\theta\left(W = \sqrt{2}g_{U(6)}2\,\mathrm{tr}(\Sigma_2[\Sigma_3, \Sigma_1])\right), \tag{36}$$

is invariant under the three independent rotations; indeed, we see that

$$\Sigma_1\Sigma_2\Sigma_3(\theta) \mapsto e^{i(\alpha_1+\alpha_2+\alpha_3)}\Sigma_1\Sigma_2\Sigma_3(e^{-i(\alpha_1+\alpha_2+\alpha_3)/2}\theta). \tag{37}$$

Let us now turn to the orbifold projection. The orbifold projection associated with $\mathbf{Z}_{12}\,\langle\sigma\rangle$ extracts only singlets of a $\mathbf{Z}_{12}$ symmetry generated by a σ-transformation of fields, and all other states are projected out of the theory. Namely, the following condition is imposed:

$$\Sigma_a(x,\mathbf{y}',\theta)^k{}_l = \widetilde{\Sigma_a}(x,\mathbf{y}',\theta)^k{}_l, \quad \text{where} \quad \sigma : \Sigma_a(x,\mathbf{y}',\theta)^k{}_l \mapsto \widetilde{\Sigma_a}(x,\mathbf{y}',\theta)^k{}_l. \tag{38}$$

The whole sector comprised of all singlets of a symmetry consistently becomes a self-closed theory. (Singular points of the geometry should be treated carefully, since the field theories are not well defined there. This is the subject of the next subsection 6.2.) Here, the σ-transformation of fields is primarily determined by the geometric rotation (21). However, we also have one degree of freedom in determining the σ-transformation of fields on the geometry — the geometric rotation of fields can be accompanied by a non-trivial twist through a rigid gauge transformation by $\tilde{\gamma}_{\sigma;7}\in$U(6). Notice that the σ-transformation is still an exact symmetry of the unorbifolded theory. Now, the σ-transformation is given as follows:

$$\sigma: \quad (\Sigma_0)^k{}_l(x,\mathbf{y}',\theta) \mapsto (\widetilde{\Sigma_0})^k{}_l(x,\mathbf{y}',\theta) \equiv (\tilde{\gamma}_{\sigma;7})^k{}_{k'}(\Sigma_0)^{k'}{}_{l'}(x,\sigma^{-1}\cdot\mathbf{y}',\theta)(\tilde{\gamma}^{-1}_{\sigma;7})^{l'}{}_l, \tag{39}$$

$$\sigma: \quad (\Sigma_b)^k{}_l(x,\mathbf{y}',\theta) \mapsto (\widetilde{\Sigma_b})^k{}_l(x,\mathbf{y}',\theta) \equiv e^{2\pi i v_b}(\tilde{\gamma}_{\sigma;7})^k{}_{k'}(\Sigma_b)^{k'}{}_{l'}(x,\sigma^{-1}\cdot\mathbf{y}',\theta)(\tilde{\gamma}^{-1}_{\sigma;7})^{l'}{}_l, \tag{40}$$

where $\tilde{\alpha}_a = 2\pi\tilde{v}_a$ are substituted into Eqs. (33)–(35) and we take the 6 by 6 unitary matrix[29] $\tilde{\gamma}_{\sigma;7}$ as

$$(\tilde{\gamma}_{\sigma;7})^k{}_l = \mathrm{diag}\left(\overbrace{e^{-\frac{1}{12}\pi i},\ldots,e^{-\frac{1}{12}\pi i}}^{5}, -e^{-\frac{1}{12}\pi i}\right). \tag{43}$$

[29]This 6 by 6 unitary matrix $\tilde{\gamma}_{\sigma;7}$ is related to 32 by 32 unitary matrices $\gamma_{\sigma;D7}$ found in references such as [37, 51, 38] through

$$\gamma_{\sigma;D7} = \tilde{\gamma}_{\sigma;7}\oplus\tilde{\gamma}^{-1}_{\sigma;7}\oplus(\text{20 by 20 matrix}). \tag{41}$$

The unitary matrix $\gamma_{\Omega R_{89};D7}$ associated to the projection condition of ΩR_{89} is expressed in this basis as

$$\gamma_{\Omega R_{89};D7} = \mathbf{1}_{6\times6}\otimes\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\oplus\mathbf{1}_{10\times10}\otimes\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}. \tag{42}$$

First of all, in the above equations, Eqs. (39) and (40), the Grassmann variable θ of the four-dimensional $\mathcal{N}=1$ SUSY does not receive a phase rotation because $\sigma \in \mathrm{SU}(3) \subset \mathrm{SU}(4) \simeq \mathrm{SO}(6)$ (i.e. $\tilde{\alpha}_0 = 2\pi\tilde{v}_0 = 0$). This ensures [46] that the four-dimensional $\mathcal{N}=1$ SUSY is preserved in the spectrum obtained from the orbifold projection conditions (38). Secondly, we add a twist in the orbifold projection through the U(6) rigid transformation by $\tilde{\gamma}_{\sigma;7}$, which causes U(6)-symmetry breaking.

The specific choice of $\tilde{\gamma}_{\sigma;7}$ in Eq. (43) is based on the following reasons. First, the twelve times repeated application of $\tilde{\gamma}_{\sigma;7}$ leads to an adjoint action by $\tilde{\gamma}_{\sigma^{12};7} \equiv (\tilde{\gamma}_{\sigma;7})^{12} = -\mathbf{1} \propto \mathbf{1}$, whose effect is trivial. This is a required property[30], since σ^{12} acts trivially on the space $\mathbf{T}^6$. Secondly, the sixth diagonal entry of the $\tilde{\gamma}_{\sigma;7}$ is chosen differently from the first five entries in order to break the U(6) gauge symmetry down to U(5)×U(1). Then, it follows from the requirement $(\tilde{\gamma}_{\sigma;7})^{12} \propto \mathbf{1}$ that only the allowed difference is the twelfth root of unity between the sixth and other entries. Thirdly, we avoid the phase difference $\{e^{2\pi i n/12} | n = 0, 1, 4, 5, 7, 8, 11\}$, since we do not want SU(5)-charged matter particles to appear on the D7-branes. This is first because the elementary Higgs particles $H^i(\mathbf{5})$ and $\bar{H}_i(\mathbf{5}^*)$ are not necessary in the $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(2)_{\mathrm{H}}$ model, and second because we give up trying to obtain whole matter contents of the quarks and leptons from D-branes[31]. Fourthly, we do not use the phase difference $\{e^{2\pi i n/12} | n = 3, 9\}$, since we want the U(6) symmetry to be restored, at least, at the σ^6-projection, as we explain later in this subsection 6.1. Finally, among the remaining candidates[32] $\{e^{2\pi i n/12} | n = 2, 6, 10\}$, we use $n = 6$, since two other possibilities are excluded by anomaly arguments in subsection 6.2. The overall phase $e^{-\pi i/12}$ in $\tilde{\gamma}_{\sigma;7}$ is not

[30] The $\gamma_{\sigma;D7}$ given in (41) satisfies an algebraic constraint

$$(\gamma_{\sigma;D7})(\gamma_{\Omega R_{89};D7})(\gamma_{\sigma;D7})^T = (\gamma_{\Omega R_{89};D7}), \tag{44}$$

which is also required in [51].

[31] It is quite a difficult subject to obtain at the same time (i) *three families of* quarks and leptons, (ii) a SUSY SU(5) unified gauge group, and finally (iii) a sector that breaks the the unified symmetry, from open strings on D-branes. There are some trials as follows, yet they are not satisfactory. The non-SUSY standard model is obtained in [52], the SUSY standard model is obtained in [53] with exotic chiral multiplets, and the flipped SU(5)-unified model is obtained in [54] with some necessary particles missing. Note also that all these models listed above are constructed using the intersecting D6–D6 system [55] in the Type IIA string theory. This framework is not in a simple T-dual to the D3–D7 system in the Type IIB string theory.

[32] The $\mathbf{Z}_6(1/6, 1/6, -2/6)$ and $\mathbf{Z}'_6(1/6, -3/6, 2/6)$, which have (seven+one)-dimensional fixed loci away from O7-planes, do not have such candidates. For the $\mathbf{Z}_6$, $\{e^{2\pi i n/6} | n = 0, 1, 2, 4, 5\}$ are excluded from the third criterion in the text, but the fourth criterion requires $\{e^{2\pi i n/6} | n = 0, 2, 4\}$. For the $\mathbf{Z}'_6$ orbifold, all possibilities of choosing a phase difference in $\tilde{\gamma}$ are excluded by the third criterion. In short, both orbifolds have too simple structure to fulfil our requirements. The $\mathbf{Z}'_6$ orbifold is disfavoured also by anomaly arguments, as mentioned briefly in subsection 6.2.

important since the $\tilde{\gamma}_{\sigma;7}$ acts on Σ_a's through **adj.** representation of the U(6). This choice is just to make the expression similar to the conventions [37, 51, 38] among string theorists.

Only the U(5)×U(1)$_6$ vector multiplet of the four-dimensional $\mathcal{N}=1$ SUSY survive these orbifold projection in the low-energy spectrum. We identify the SU(5) part of the U(5) $\simeq$ SU(5)×U(1)$_5$ gauge group with the SU(5)$_{\rm GUT}$ gauge group. Some linear combination of the U(1)$_5$×U(1)$_6$ is identified with the fiveness[33] gauge symmetry in subsection 6.4. There is no massless chiral multiplet arising from D7-branes. Although we do not need Higgs multiplets in the SU(5)$_{\rm GUT}$×U(2)$_{\rm H}$ model, quarks and leptons (and right-handed neutrinos) are missing. It is not easy to accommodate all the three families of quarks and leptons along with the model of SU(5)$_{\rm GUT}$ breaking we discuss. We consider that the three families of quarks and leptons reside at a fixed point of the orbifold geometry, although we cannot specify their origin.

Let us now derive from the D3–D7 bound state the matter contents of the SU(5)$_{\rm GUT}$-breaking sector. D3-branes are put on a fixed point that preserves $\mathcal{N}=2$ SUSY [15]. We refer such fixed points to the $\mathcal{N}=2$ fixed points. There would be unwanted SU(2)$_{\rm H}$-**adj.** chiral multiplets if the D3-branes were not put at a fixed point; however, the multiplet structure of the $\mathcal{N}=2$ SUSY would be lost if they were put on a fixed point the $\mathcal{N}=2$ SUSY is not preserved.

SUSY of local geometry at a given fixed point is determined by its isotropy group, that is a subgroup of the orbifold group that fixes the point. The isotropy group determines the local geometry around the fixed point, and hence the SUSY. Matter contents from D-branes located at that fixed point are also determined by imposing orbifold projection conditions associated to the isotropy group.

The $\mathcal{N}=2$ fixed points should have isotropy group contained in a particular SU(2) subgroup of the SU(4) $\simeq$ SO(6) rotation. This is because the local geometry, which is determined by the isotropy group, should preserve the $\mathcal{N}=2$ SUSY of the D3–D7 system. Since the SUSY charges of this system are in the $-\Gamma^{7654}=1$ eigenspace, the isotropy group should act only on the $-\Gamma^{7654}\neq 1$ eigenspace. It is the SU(2) rotation of the two complex planes $\mathbf{C}^2=\{(z_1,z_2)|z_1,z_2\in\mathbf{C}\}$ that acts on this eigenspace; indeed this SU(2) subgroup (one of SU(2)×SU(2) $\simeq$ (SO(4) of $\mathbf{C}^2$)) is given by

$$\exp\left(\phi_a \bar{\eta}^a_{m'n'}\Gamma^{m'n'}\right), \tag{45}$$

where $\bar{\eta}^a_{m'n'}$ is the 't Hooft η-symbol [56] with $a=1,2,3; m',n'=4,5,6,7$, whose generators

[33] The fiveness is equivalent to B−L in the standard model.

$\bar{\eta}^a_{m'n'}\Gamma^{m'n'}$ are trivial on the $-\Gamma^{7654}=1$ eigenspace:

$$\bar{\eta}^a_{m'n'}\Gamma^{m'n'} = \bar{\eta}^a_{m'n'}\Gamma^{m'n'}(-\Gamma^{7654}) = -\bar{\eta}^a_{m'n'}\Gamma^{m'n'}, \tag{46}$$

since $\bar{\eta}^a_{m'n'}\epsilon_{m'n'k'l'} = -2\bar{\eta}^a_{k'l'}$.

There are essentially two different candidates of the $\mathcal{N}=2$ fixed points on the D7-branes: two points at which the isotropy group is $\mathbf{Z}_2\langle\sigma^6\rangle$ and a point at which the isotropy group is $\mathbf{Z}_4\langle\sigma^3\rangle$. All of them[34] are at the intersections of the two-dimensional singularities in the $\mathbf{T}^6$ with the D7-branes. Note that σ^3,σ^6 and σ^9 belong to the above-mentioned SU(2) subgroup, since it does not rotate the third complex plane spanned by $\mathbf{e}_8$ and $\mathbf{e}_9$ because of $v_3\times 3\in\mathbf{Z}$.

We put the D3–D7 bound state at an $\mathcal{N}=2$ fixed point where the original U(6) symmetry is enhanced on the D7-branes. This is because the $\mathcal{N}=2$ hypermultiplet $(\bar{Q}^6{}_\alpha, Q^\alpha{}_6)$ is necessary in the $\mathrm{SU(5)_{GUT}}$-breaking sector in addition to the hypermultiplet $(\bar{Q}^i{}_\alpha, Q^\alpha{}_i)$ $(i=1,...,5)$. The U(6) symmetry would be enhanced only at fixed points with isotropy group $\mathbf{Z}_3\langle\sigma^4\rangle$, where there is only $\mathcal{N}=1$ SUSY, if we were to adopt the phase difference $\{e^{2\pi in/12}|n=3,9\}$ in (43). Thus, there would be no fixed point where the $\mathcal{N}=2$ SUSY and the U(6) symmetry are simultaneously obtained. This is the reason why we reject the phase difference $\{e^{2\pi in/12}|n=3,9\}$ in the $\tilde{\gamma}_{\sigma;7}$. On the contrary, when we take the phase difference $\{e^{2\pi in/12}|n=2,6,10\}$, the symmetry is enhanced up to U(6) at fixed points with isotropy group $\mathbf{Z}_2\langle\sigma^6\rangle$, where the $\mathcal{N}=2$ SUSY is also restored.

Therefore, we put two D3-branes on one of the fixed points where the isotropy group is $\mathbf{Z}_2\langle\sigma^6\rangle$. There are two such fixed points on the $\mathbf{T}^6/(\mathbf{Z}_{12}\langle\sigma\rangle\times\mathbf{Z}_2\langle\Omega R_{89}\rangle)$, and the resulting phenomenology is different. However, we postpone choosing one from these two candidates until subsection 6.4, since they make no difference in the theoretical construction of the models. Such a fixed point, whichever one chooses, consists of twelve points in the covering space $\mathbf{T}^6$, which are identified under the $(\mathbf{Z}_{12}\langle\sigma\rangle\times\mathbf{Z}_2\langle\Omega R_{89}\rangle)/\mathbf{Z}_2\langle\sigma^6\rangle$. Thus, two D3-branes have to be introduced at each of these twelve points. Twenty-four D3-branes are necessary as a whole in the covering space $\mathbf{T}^6$. The rest of the D3-branes can be used for other sectors, which are not directly observed today.

We concentrate on a set of fields at one of these twelve images on which the orbifold projection associated with the isotropy group $\mathbf{Z}_2\langle\sigma^6\rangle$ is imposed: those fields are a U(2) vector multiplet $(X)^\alpha{}_\beta$ $(\alpha,\beta=4,5)$ of the four-dimensional $\mathcal{N}=4$ SUSY and a hypermultiplet of the four-dimensional $\mathcal{N}=2$ SUSY in the $(\mathbf{6},\mathbf{2}^*)$ representation of the U(6)×U(2) gauge

[34] Intersection of the two-dimensional singularity at $\mathbf{y}'=\mathbf{0}$ with the D7-branes, however, has only $\mathcal{N}=1$ SUSY, because the isotropy group is $\mathbf{Z}_{12}\langle\sigma\rangle$ there.

group. They are decomposed into irreducible multiplets of the $\mathcal{N}=1$ SUSY: a U(2) vector multiplet $(X_0)^\alpha{}_\beta(x,\theta) \equiv (\mathcal{W}^{\mathrm{U}(2)})(x,\theta)$ and three chiral multiplets $(X_b)^\alpha{}_\beta(x,\theta)$ $(b=1,2,3)$ in the U(2)-**adj.** representation, chiral superfields $Q^\alpha{}_k(x,\theta)$ in the $(\mathbf{6}^*,\mathbf{2})$ representation and $\bar{Q}^k{}_\alpha(x,\theta)$ in the $(\mathbf{6},\mathbf{2}^*)$ representation.

Let us see how these fields transform under the three independent rotational symmetries on the $\mathbf{C}^3$: $z_b \mapsto e^{i\alpha_b} z_b$ for $b=1,2,3$. The four chiral multiplets of four-dimensional $\mathcal{N}=1$ SUSY X_a's, which are under the control of sixteen SUSY charges, transform in the same way as Σ_a's on the D7-branes (except for $\mathbf{y}'$-dependence):

$$
\begin{aligned}
X_0(x,\theta) &\mapsto e^{i\tilde{\alpha}_0} X_0(x, e^{-i\tilde{\alpha}_0}\theta), &(47)\\
X_b(x,\theta) &\mapsto e^{i\alpha_b} X_b(x, e^{-i\tilde{\alpha}_0}\theta) \quad \text{for } b=1,2,3. &(48)
\end{aligned}
$$

The AdS/CFT correspondence on the D3-branes in the large 't Hooft coupling limit provides sufficient evidence for this determination [32]. Transformation properties of $(\bar{Q},Q)$ are determined through field theoretical arguments. The four-dimensional $\mathcal{N}=2$ SUSY interaction,

$$
W = \sqrt{2} g_{\mathrm{U}(2)} \bar{Q}^k{}_\alpha (X_3)^\alpha{}_\beta Q^\alpha{}_k, \tag{49}
$$

requires that the product $\bar{Q}Q$ transforms as

$$
\bar{Q}Q(x,\theta) \mapsto e^{i(\alpha_1+\alpha_2)} \bar{Q}Q(x, e^{-i\tilde{\alpha}_0}\theta). \tag{50}
$$

On the other hand, vanishing U(2)[rotation]^2 anomalies require that $\bar{Q}$ and Q have the same rotational charge. Thus

$$
\begin{aligned}
\bar{Q}(x,\theta) &\mapsto e^{i(\alpha_1+\alpha_2)/2} \bar{Q}(x, e^{-i\tilde{\alpha}_0}\theta), &(51)\\
Q(x,\theta) &\mapsto e^{i(\alpha_1+\alpha_2)/2} Q(x, e^{-i\tilde{\alpha}_0}\theta). &(52)
\end{aligned}
$$

These fields transform under the σ^6 as

$$
\begin{aligned}
\sigma^6: \quad & (X_0)^\alpha{}_\beta(x,\theta) \mapsto (\widetilde{X_0})^\alpha{}_\beta(x,\theta) \equiv (\tilde{\gamma}_{\sigma^6;3})^\alpha{}_{\alpha'} (X_0)^{\alpha'}{}_{\beta'}(x,\theta) (\tilde{\gamma}^{-1}_{\sigma^6;3})^{\beta'}{}_\beta, &(53)\\
\sigma^6: \quad & (X_b)^\alpha{}_\beta(x,\theta) \mapsto (\widetilde{X_b})^\alpha{}_\beta(x,\theta) \equiv e^{2\pi i v_b 6} (\tilde{\gamma}_{\sigma^6;3})^\alpha{}_{\alpha'} (X_b)^{\alpha'}{}_{\beta'}(x,\theta) (\tilde{\gamma}^{-1}_{\sigma^6;3})^{\beta'}{}_\beta, &(54)
\end{aligned}
$$

$$
\begin{aligned}
\sigma^6: \quad & Q^\alpha{}_k(x,\theta) \mapsto \widetilde{Q}^\alpha{}_k(x,\theta) \equiv e^{\pi i(v_1+v_2)6} (\tilde{\gamma}_{\sigma^6;3})^\alpha{}_{\alpha'} Q^{\alpha'}{}_{k'}(x,\theta) (\tilde{\gamma}^{-6}_{\sigma;7})^{k'}{}_k, &(55)\\
\sigma^6: \quad & \bar{Q}^k{}_\alpha(x,\theta) \mapsto \widetilde{\bar{Q}}^k{}_\alpha(x,\theta) \equiv e^{\pi i(v_1+v_2)6} (\tilde{\gamma}^{6}_{\sigma;7})^k{}_{k'} \bar{Q}^{k'}{}_{\alpha'}(x,\theta) (\tilde{\gamma}^{-1}_{\sigma^6;3})^{\alpha'}{}_\alpha, &(56)
\end{aligned}
$$

where we take the 2 by 2 unitary matrix for the rigid U(2) transformation[35] $\tilde{\gamma}_{\sigma^6;3}$ as

$$\tilde{\gamma}_{\sigma^6;3} = \mathrm{diag}(e^{-\frac{1}{2}\pi i}, e^{-\frac{1}{2}\pi i}). \tag{58}$$

Here, $(\tilde{\gamma}_{\sigma^6;3})^2 = -\mathbf{1} \propto \mathbf{1}$ is satisfied. Phase $e^{-\pi i/2}$ is chosen so that the $(\bar{Q}, Q)$ survive the orbifold projection conditions

$$X_a = \widetilde{X_a}, \quad Q = \widetilde{Q}, \quad \bar{Q} = \widetilde{\bar{Q}}, \tag{59}$$

with $a = 0, 1, 2, 3$.

Massless modes that survive the orbifold projection are a U(2) vector multiplet (X_0,X_3) of the four-dimensional $\mathcal{N} = 2$ SUSY, and a hypermultiplet (${Q^\alpha}_k$,${\bar{Q}^k}_\alpha$) ($k = 1, ..., 6$). This is exactly the matter content of the $\mathrm{SU(5)_{GUT}}$-breaking sector in the product-group unification model based on the $\mathrm{SU(5)_{GUT}}\times\mathrm{U(2)_H}$ gauge group. An unwanted $\mathcal{N} = 2$ hypermultiplet in the U(2)-**adj.** representation (X_1,X_2) has been projected out.

6.2 Anomaly Cancellation

There is no anomaly in the ten-dimensional bulk. However, anomalies generally arise at orbifold singularities. This subsection is devoted to the analysis of such anomalies. We mainly consider the anomalies due to the $\mathbf{Z}_{12}$ $\langle\sigma\rangle$ projection. Anomalies due to the $\mathbf{Z}_2$ $\langle\Omega R_{89}\rangle$ projection are briefly touched upon in the course of the discussion.

6.2.1 Triangle Anomalies

The matter contents we obtained below the Kaluza–Klein scale is free from triangle anomalies. However, this only implies that the total sum of triangle anomalies localized at all the fixed points cancel one another. Thus, anomalies at each fixed point can be non-zero. If it is the case, then there will be violation of unitarity at an energy scale higher than the Kaluza–Klein

[35] The 32 by 32 matrix $\gamma_{\sigma;D3}$ in Refs. [37, 51, 38] is expressed in terms of the $\tilde{\gamma}_{\sigma^6;3}$ as

$$\gamma_{\sigma;D3} = \begin{pmatrix} & & & & & 1 \\ 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \end{pmatrix} \otimes (\tilde{\gamma}_{\sigma^6;3})^{\frac{1}{6}} \oplus \begin{pmatrix} & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \\ 1 & & & & & \end{pmatrix} \otimes (\tilde{\gamma}_{\sigma^6;3})^{-\frac{1}{6}} \oplus (\text{8 by 8 matrix}). \tag{57}$$

.

scale, which means that the description using higher-dimensional field theories is no longer valid.

Kaluza–Klein particles of higher-dimensional massless fields play a crucial role in the determination of the triangle anomaly distribution at fixed points, although the total sum of the anomaly is determined only from the four-dimensional massless particles. This is intuitively obvious from the fact that the anomaly distribution only from Kaluza–Klein zero modes is homogeneous over the orbifold, because of their homogeneous wave functions. It is the Kaluza–Klein towers that collect and redistribute to fixed points the anomalies carried by zero modes (see below for a more concrete explanation). The resulting distribution depends on the Kaluza–Klein spectrum.

Higher-dimensional massive excitations above the fundamental scale, if they exist, also change the anomaly distribution. Their existence can results in replacement of the anomaly that is once localized at a fixed point by the Kaluza–Klein towers to another fixed point. This effect will be described by the Chern–Simons terms on the D7-branes after those massive excitations are integrated out [57]. Thus, in general, one can expect that triangle anomalies vanish at all fixed points whenever the total sum of these anomalies vanish; the anomaly distribution determined by the Kaluza–Klein towers of higher-dimensional massless fields can be gathered at a single fixed point to vanish in the presence of unknown massive excitations.

However, the general possibility described above does not work straightforwardly in the presence of highly extended SUSY in the extra-dimensional space (on D7-branes). Let us discuss this issue by taking the effective theory description (i.e. massive particles are integrated out and only massless fields of the Type IIB supergravity (including D-brane fluctuations) are used). It is true that the effective Chern–Simons term on the D7-branes,

$$\int_{D7} (dC_{(2)}) \wedge \mathrm{tr}(AFF + \cdots), \tag{60}$$

which is allowed by the $\mathcal{N} = 1$ SUSY of eight-dimensional space [58], replaces [59, 60] triangle anomalies[36] from a fixed point to another, provided the $C_{(2)}$ field has a background configuration such that

$$d(dC_{(2)}) = \sum_{\mathbf{y}'_* \in \{\text{fixed points}\}} n_{\mathbf{y}'_*} \delta^4(\mathbf{y}' - \mathbf{y}'_*), \tag{61}$$

where $\mathbf{y}'$ is the coordinate on the D7-branes $\mathbf{y}' = (z_1, z_2)$ and $\delta^4(\mathbf{y}' - \mathbf{y}'_*)$ denotes, here, a

[36]Discussed in Ref. [59] is the replacement of box anomalies, which can be done without breaking SUSY.

delta-function-supported 4-form. However, this condition on the background means that

$$\int_{S_3} dC_{(2)} = \int_{B_4} d(dC_{(2)}) = n_{\mathbf{y}'_*}, \tag{62}$$

where the S_3 is a 3-sphere in D7-branes surrounding a fixed point $\mathbf{y}'_*$ and $\partial B_4 = S_3$. In other words, there are $n_{\mathbf{y}'_*}$ 5-branes intersecting the D7-branes at $\mathbf{y}'_*$. Since the SUSY charges preserved in the presence of 5-branes alone are

$$\mathcal{Q} - \Gamma^{7654}\mathcal{Q}', \tag{63}$$

these SUSY charges contain no common subset with the SUSY charges on the D7-branes (15). Thus, it is in SUSY-violating vacua that the triangle anomalies on the D3–D7 system can be replaced through the interaction (60).

Therefore, we require that the triangle anomalies vanish at all of the fixed points on the D7-branes, in the distribution determined from the Kaluza–Klein spectrum. The simplest way to calculate the triangle anomaly distribution is given in [61].

In the case of the $S^1/\mathbf{Z}_2$ orbifold [61], zero modes have distribution function 1/2, the n-th excited Kaluza–Klein modes have distribution functions $\cos^2(ny/L)$ or $\sin^2(ny/L)$, with anomaly coefficient opposite in sign. The total summation of all these contributions from all the Kaluza–Klein towers,

$$\frac{1}{2} + \sum_{n>0} \left(\cos^2\left(\frac{ny}{L}\right) - \sin^2\left(\frac{ny}{L}\right) \right), \tag{64}$$

leads to a delta-function distribution of the anomaly supported on the $S^1/\mathbf{Z}_2$ fixed points.

Now the generalization to higher-dimensional orbifold is straightforward. The anomaly distribution is calculated as the total summation of absolute square of each Kaluza–Klein wave function weighted by its anomaly coefficients:

$$\sum_{I,a} A_I \frac{1}{12} \sum_{\mathbf{p}' \in \Lambda'_0} |\psi_{I,a,\mathbf{p}'}(\mathbf{y}')|^2. \tag{65}$$

Kaluza–Klein towers are labelled by I and a, where $a = 0, 1, 2, 3$ runs over all components of the SO(3,1)-irreducible decomposition of eight-dimensional Weyl fermions (the fermion contents of the U(6)-**adj.** $\mathcal{N} = 1$ vector multiplet of the eight-dimensional space-time), while the I runs over irreducible representations $I \in \{\mathbf{24}^{(0,0)}, \mathbf{1}^{(0,0)}, \mathbf{5}^{(1,-1)}, \mathbf{5}^{*(-1,1)}\}$ (which form

U(6)-**adj.**) of the unbroken gauge group $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(1)_5\times\mathrm{U}(1)_6$. The anomaly coefficient A_I of a given type (such as $[\mathrm{SU}(5)_{\mathrm{GUT}}]^3$, $\mathrm{U}(1)_5[\mathrm{SU}(5)_{\mathrm{GUT}}]^2$, etc.) is determined as usual:

$$\mathrm{tr}(\{t_I^b, t_I^c\}t_I^a) = A_I\,\mathrm{tr}(\{t_I^b, t_I^c\}t_I^a))|_{I=\textbf{fund.}\text{ of }\mathrm{SU}(5)_{\mathrm{GUT}}}. \tag{66}$$

Each Kaluza–Klein particle in a Kaluza–Klein tower is labelled by its Kaluza–Klein momentum $\mathbf{p}'$. In (65) the Λ'_0 denotes the dual lattice of the four-dimensional space lattice Γ'_0 spanned by $\mathbf{e}_4$, $\mathbf{e}_5$, $\mathbf{e}_6$, $\mathbf{e}_7$, over which the D7-branes are stretched. The Kaluza–Klein wave function on the $\mathbf{Z}_{12}\langle\sigma\rangle$ orbifold is given by

$$\psi_{I,a,\mathbf{p}'} = \frac{1}{\sqrt{12\mathrm{vol}(\mathbf{T}^4)}}\sum_{g\in\mathbf{Z}_{12}\langle\sigma\rangle}\rho_a(g)\rho_I(g)e^{-i\mathbf{p}'\cdot(g\cdot\mathbf{y}')} \tag{67}$$

with phase factors $\rho_a(g)$ and $\rho_I(g)$ being fixed by the local Lorentz rotation and by the U(6) rigid transformation associated with the orbifold transformation $g\in\mathbf{Z}_{12}$, respectively. These two phase factors are explicitly given by

$$\rho_a(\sigma^k) = (e^{i\pi\tilde{v}_a})|_{a=0,1,2,3} \tag{68}$$

for four Weyl fermions of SO(3,1); $\rho_I(\sigma^k) = (-1)^k$ for $I\in\{\mathbf{5}^{(1,-1)},\mathbf{5}^{*(-1,1)}\}$ and $\rho_I(\sigma^k)=1$ for $I\in\{\mathbf{24}^{(0,0)},\mathbf{1}^{(0,0)}\}$.

The distribution function of triangle anomalies (65) is rewritten as

$$\frac{1}{12}\sum_{g\in\mathbf{Z}_{12}\langle\sigma\rangle}\left(\frac{1}{\mathrm{vol}(\mathbf{T}^4)}\sum_{\mathbf{p}'\in\Lambda'_0}e^{-i\mathbf{p}'\cdot(g\cdot\mathbf{y}'-\mathbf{y}')}\right)\sum_I A_I\rho_I(g)\sum_{a=0}^{3}\rho_a(g), \tag{69}$$

where algebraic relations $\rho_a(g')^*\rho_a(g) = \rho_a(g'^{-1}\cdot g)$ and $\rho_I(g')^*\rho_I(g) = \rho_I(g'^{-1}\cdot g)$ are used. The Poisson formula tells us that

$$\frac{1}{\mathrm{vol}(\mathbf{T}^4)}\sum_{\mathbf{p}'\in\Lambda'_0}e^{-i(g^{-1}\cdot\mathbf{p}'-\mathbf{p}')\cdot\mathbf{y}'} = \frac{1}{|\Gamma'_g:\Gamma'_0|}\sum_{\mathbf{y}'_*\in\Gamma'_g}\delta^4(\mathbf{y}'-\mathbf{y}'_*). \tag{70}$$

Here, a momentum-space lattice $\Lambda'_g\equiv\{g^{-1}\cdot\mathbf{p}'-\mathbf{p}'|\mathbf{p}'\in\Lambda'_0\}$ is a superlattice of Λ'_0, and Γ'_g in the above expression denotes the sublattice of Γ'_0 dual to Λ'_g. This Γ'_g is also characterized as a set of g-fixed points on the D7-branes. $|\Gamma'_g:\Gamma'_0|$ is the number of the g-fixed points within the covering space $\mathbf{T}^4$, which is the world volume of the D7-branes before the orbifold projection. Thus, the integration of the (70) over a single unit cell of $\mathbf{T}^4$ yields unity. Therefore, the triangle-anomaly distribution over the orbifold geometry is decomposed into parts, each of which corresponds to an element σ^k of the orbifold group and is supported on the σ^k-fixed points. The total amount of anomaly carried by this component is

$$\frac{1}{12} i \sum_{a=0}^{3} \rho_a(\sigma^k) \sum_I (-iA_I \rho_I(\sigma^k)) = \frac{1}{12} 4 \left(\prod_{b=1}^{3} \sin(\pi v_b k) + i \prod_{b=1}^{3} \cos(\pi v_b k) \right) \sum_I (-iA_I \rho_I(\sigma^k)). \tag{71}$$

$1/|\Gamma'_{\sigma^k} : \Gamma'_0|$ of (71) is distributed at each σ^k-fixed point through the σ^k-component. The anomaly localized at a fixed point is given by the sum of all such g-component contribution, where the fixed point is g-fixed The cosine part in this expression cancels with that of the σ^{12-k}-component, and hence only the sine part is of importance.

Now it is easy to see that all triangle anomalies vanish at all fixed points on the D7-branes in the $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(2)_{\mathrm{H}}$ model described in the previous subsection 6.1. It is sufficient, though not necessary, to see that all $(-i \sum_I A_I \rho_I(\sigma^k))$ vanish for $k = 0, ..., 11$. It is indeed the case, since $\rho_{I=\mathbf{5}^{(1,-1)}}(\sigma^k) = \rho_{I=\mathbf{5}^{*(-1,1)}}(\sigma^k) = (-1)^k$, and $A_{I=\mathbf{5}^{(1,-1)}} = -A_{I=\mathbf{5}^{*(-1,1)}}$ for all types of triangle anomalies between $\mathrm{SU}(5)_{\mathrm{GUT}}$, $\mathrm{U}(1)_5$, $\mathrm{U}(1)_6$ and gravity. It is extremely encouraging that there is a set of geometry and U(6)-twisting matrix $\tilde{\gamma}_{\sigma;7}$ where the triangle anomalies vanish at all fixed points.

If we were to take $e^{2\pi i(n=\pm 2)/12}$ as the phase difference in $\tilde{\gamma}_{\sigma;7}$, rather than $e^{2\pi i(n=6)/12}$ as in the text, it turns out that $[\mathrm{SU}(5)_{\mathrm{GUT}}]^3$ triangle anomalies are distributed at all fixed points contained in $\Gamma'_{\sigma^{\pm 4}}$ and $\Gamma'_{\sigma^{\pm 1}} = \Gamma'_{\sigma^{\pm 2}} = \Gamma'_{\sigma^{\pm 5}}$ by a fractional amount that cannot be cancelled by introducing new particles at fixed points. This is the reason why we do not adopt $e^{2\pi i(n=\pm 2)/12}$ as the phase difference in (43).

6.2.2 More Anomalies

Now that the triangle anomalies vanish at all of the fixed points, the next question is whether they are the only ones that we have to care about or not. We point out in the rest of this subsection 6.2 that there are two other issues we have to discuss — higher-dimensional anomalies and "anomalies in internal dimensions". In the end, however, it is concluded that two consistency conditions that come from those two issues depend on UV physics, and hence there is a chance that these conditions would be satisfied by choosing suitable UV physics. Therefore, they are not necessary conditions for physics below the cut-off scale.

Let us first discuss higher-dimensional anomalies. Although the ten-dimensional space-time is directly compactified to the four-dimensional space-time, singular loci, on which anomalies may be localized, are not necessarily (three+one)-dimensional sub-space-time.

The $\mathbf{T}^6/\mathbf{Z}_{12}\langle\sigma\rangle$-compactified ten-dimensional space-time possesses a couple of (five+one)-dimensional singularities, on which box anomalies may appear.

First, we show that the box anomalies *do* appear at those singularities. Indeed, six-dimensional (box) anomalies arise on the same footing as the triangle anomalies in the Fujikawa method [62] extended to orbifold-compactified geometry [63]. Massless particles in the bulk and massless particles on D-branes are already specified, and hence it is a well-defined question how the box anomalies arise from these particles. We first show that the distribution of the triangle anomalies are re-obtained in the Fujikawa method and then discuss how the box anomalies arise. We use non-gravitational (pure gauge) anomalies as examples, just for illustration. Non-gravitational anomalies only arise from gauge-charged Weyl fermions, and hence it is easy to calculate them. Anomalies involving gravity can also be treated in a similar way by regarding the gravity as local Lorentz gauge theory. Actual calculation of gravitational anomaly is presented later.

Gauge fermions on the D7-branes are the only source of pure gauge anomalies, since fields on the D3–D7 bound state propagate only in four-dimensional space-time and since they keep a vector-like structure even after the orbifold projection. We calculate the anomalies at each fixed point that come from the fermions of the U(6) vector multiplet, which propagates in the eight-dimensional space-time.

Anomalies are understood as anomalous variations of the functional measure [62]. The daAnomalous variation of an action due to a chiral transformation is given by

$$\delta S = \mathrm{Tr}\left(\delta^4(\tilde{x} - x)\gamma_5\right) \tag{72}$$

from the measure of a four-dimensional Weyl fermion, where the trace is taken over space-time coordinates $\tilde{x} = x$, spinor indices and gauge indices. One can think of anomaly to a more general symmetry transformation by replacing γ_5 to the generator t^a of that symmetry transformation. A straightforward extension to the higher-dimensional space-time compactified over orbifold geometry is simply given [63] by

$$\delta S = \mathrm{Tr}_{co,sp,\omega,I}\left(\left(\frac{1}{12}\sum_{g\in\mathbf{Z}_{12}\langle\sigma\rangle}\delta^4(\tilde{\mathbf{y}'} - g\cdot\mathbf{y}')\delta^4(\tilde{x} - x)\rho_{sp}(g)\rho_I(g)\right)t_I^a\right), \tag{73}$$

where t_I^a is a suitable representation of the generator t^a of the transformation of which we consider the anomaly; $(\delta^4(\tilde{x} - x))$ in four dimensions is replaced by its orbifold analogue; the delta function on the orbifold geometry is given by

$$\frac{1}{12}\sum_{g\in\mathbf{Z}_{12}\langle\sigma\rangle}\delta^4(\tilde{\mathbf{y}'}-g\cdot\mathbf{y}')\delta^4(\tilde{x}-x)\rho_{sp}(g)\rho_I(g). \tag{74}$$

Note that the gauge fermions of interest propagate in eight-dimensional space-time. Coordinates of four extra dimensions are denoted by $\mathbf{y}', \tilde{\mathbf{y}'} \in \mathbf{T}^4 \equiv \mathbf{C}^2/\Gamma'_0$. Here, the Γ'_0 denotes the lattice spanned by $\mathbf{e}_4$, $\mathbf{e}_5$, $\mathbf{e}_6$ and $\mathbf{e}_7$; $\mathbf{y}'$ and $g\cdot\mathbf{y}'(g\in\mathbf{Z}_{12}\langle\sigma\rangle)$ are the same point on the orbifold $\mathbf{T}^4/\mathbf{Z}_{12}\langle\sigma\rangle$. Fields on these two points are identified up to two internal transformations, $\rho_{sp}(g)$, which denotes the effect of local Lorentz rotation under $g\in\mathbf{Z}_{12}\langle\sigma\rangle$, and $\rho_I(g)$, which denotes the effect of the rigid U(6) transformation given by $\tilde{\gamma}_{\sigma;7}$, as before. Here, $\rho_{sp}(g)$ plays the same role as $\rho_a(g)$ in (68), which is now given by

$$\rho_{sp}(\sigma^k)\equiv\prod_{b=1}^{3}\left(\cos(\pi v_b k)+\sin(\pi v_b k)\Gamma^{(2b+2)(2b+3)}\right). \tag{75}$$

The trace is taken over the space-time coordinates (i.e. summation over x and $\mathbf{y}'$ with $\tilde{x}=x$ and $\tilde{\mathbf{y}'}=\mathbf{y}'$ imposed), over eight spinor indices (since we consider a Weyl fermion of eight-dimensional space-time), over weights (ω) in a single irreducible representation of the unbroken gauge groups, and finally over different irreducible representations $I \in \{\mathbf{24}^{(0,0)}, \mathbf{1}^{(0,0)}, \mathbf{5}^{(1,-1)},$
$\mathbf{5}^{*(-1,1)}\}$.

The CPT conjugate of an eight-dimensional Weyl fermion gives a hermitian conjugate of (73). The (73) and its hermitian conjugate as a whole are expressed just through taking the trace over the whole sixteen spinor components of the eight dimensions. This is because the CPT-conjugate of a Weyl fermion of eight dimensions has the chirality opposite to the original one, and also because the whole relevant fermions form the vector-like (**adj.**) representation of the U(6). The Γ^{89} in (75) is understood as 16 by 16 part (which acts on the U(6) gauge fermion) of the 32 by 32 matrix of Γ^{89}.

The trace over the coordinate in the $(x,\mathbf{y}')$-base can be rewritten in the momentum-space base as

$$\begin{aligned}\delta S \;=\; & \int\left(\frac{dp}{2\pi}\right)^4\frac{1}{\mathrm{vol}(\mathbf{T}^4)}\sum_{\mathbf{p}'\in\Lambda'_0}\int d^4\tilde{x}d^4\tilde{\mathbf{y}'}\int d^4x d^4\mathbf{y}'\delta^4(\tilde{x}-x) \\ & \frac{1}{12}\sum_{g\in\mathbf{Z}_{12}\langle\sigma\rangle}\mathrm{tr}_{\,sp,\omega,I}\left(e^{-i(p\cdot\tilde{x}+\mathbf{p}'\cdot\tilde{\mathbf{y}'})}\delta^4(\tilde{\mathbf{y}'}-g\cdot\mathbf{y}')\rho_{sp}(g)\rho_I(g)e^{i(p\cdot x+\mathbf{p}'\cdot\mathbf{y}')}t^a_I\right),\end{aligned} \tag{76}$$

$$= \int d^4x d^4\mathbf{y}' \int\!\left(\frac{dp}{2\pi}\right)^4 \frac{1}{\mathrm{vol}(\mathbf{T}^4)} \sum_{\mathbf{p}'\in\Lambda'_0} \frac{1}{12} \sum_{g\in\mathbf{Z}_{12}\langle\sigma\rangle} \mathrm{tr}_{\,sp,\omega,I} \left(e^{-i\mathbf{p}'\cdot(g\cdot\mathbf{y}'-\mathbf{y}')}\rho_{sp}(g)\rho_I(g)t_I^a\right), \quad (77)$$

where now the trace runs only over the spinor and gauge indices.

This anomalous variation of the functional measure is regularized in a gauge-invariant way by inserting $e^{\not{D}\cdot\not{D}/2M^2}$, where M is an energy scale of the regulator and $\not{D}$ the Dirac operator of eight-dimensional space-time. We adopt, here, the SO(7,1)-symmetric regularization. Then,

$$\delta S = \frac{1}{12}\sum_{g\in\mathbf{Z}_{12}\langle\sigma\rangle}\int d^4x d^4\mathbf{y}' \ \frac{1}{\mathrm{vol}(\mathbf{T}^4)}\sum_{\mathbf{p}'\in\Lambda'_0} e^{-i(g^{-1}\cdot\mathbf{p}'-\mathbf{p}')\cdot\mathbf{y}'} \quad (78)$$
$$\lim_{M\to\infty} i\left(\frac{M^2}{2\pi}\right)^2 \mathrm{tr}_{\,sp,\omega,I}\left(e^{-\frac{\mathbf{p}'\cdot\mathbf{p}'+\frac{i}{2}F_{AB}\Gamma^{AB}}{2M^2}}\rho_{sp}(g)\rho_I(g)t_I^a\right).$$

The space-time indices A and B run from 0 to 7. Here, we can see a structure similar to the anomaly distribution obtained in Eq. (69). The anomalies are decomposed into parts, each of which corresponds to each element g of the orbifold group $\mathbf{Z}_{12}\langle\sigma\rangle$. Each component has its own distribution function determined by g. The major difference from Eq. (69) is that the expression for the anomaly of each component

$$\frac{1}{12}\times\lim_{M\to\infty} i\left(\frac{M^2}{2\pi}\right)^2 e^{-\frac{\mathbf{p}'\cdot\mathbf{p}'}{2M^2}}\,\mathrm{tr}_{\,sp,\omega,I}\left(e^{-i\frac{\Gamma\cdot F}{4M^2}}\rho_{sp}(g)\rho_I(g)t_I^a\right), \quad (79)$$

contains not only the triangle anomalies but also more information, including box anomalies as we see below.

The triangle anomalies are obtained from (79) in the following way. The $\rho_{sp}(\sigma^k)$ in the above expression contains a term $\prod_{b=1}^3\sin(\pi v_b k)\Gamma^{456789}$. Then, the trace over spinor indices becomes non-vanishing when the regulator $e^{-i(\Gamma\cdot F)/(4M^2)}$ provides $-(\Gamma\cdot F)^2/(2(4M^2)^2)$. The decoupling limit of the regulator $M\to\infty$ leaves convergent and (generally) non-vanishing quantities. The σ^k-component includes triangle anomalies as

$$\frac{1}{12}i\left(\frac{M^2}{2\pi}\right)^2\left(\prod_{b=1}^3\sin(\pi v_b k)\right)\sum_I\rho_I(g)\,\mathrm{tr}_{\,sp,\omega}\left(t_I^a\frac{-1}{2(4M^2)}(\Gamma^{MN}F_{MN})^2\Gamma^{456789}\right) \quad (80)$$
$$= \frac{\mathrm{tr}_{\,\omega}(t^a\{t^b,t^c\})|_{I=\mathbf{fund.}}}{32\pi^2}(F^b\cdot\tilde{F}^c)|_{4D}\frac{4}{12}\left(\prod_{b=1}^3\sin(\pi v_b k)\right)\sum_I(-i\rho_I(g)A_I)\frac{\mathrm{tr}_{\,sp}\left(\Gamma^{0123456789}_{16\text{ by }16}\right)}{16},$$

which coincides with (71).

Let us now turn our attention to the box anomalies. The six-dimensional singularities are associated with elements of the orbifold group that do not rotate one of three complex planes. All the six-dimensional singularities on the $\mathbf{T}^6/\mathbf{Z}_{12}\langle\sigma\rangle$ orbifold extend in the third complex plane labeled by z_3. The isotropy groups at these singularites are generically $\mathbf{Z}_4\langle\sigma^3\rangle$ or $\mathbf{Z}_2\langle\sigma^6\rangle$, as explained in section 5. The $\sigma^{3k}(k=0,1,2,3)$ do not rotate this third complex plane. In other words, the loci of σ^{3k}-fixed points are no longer points in $\mathbf{T}^6$, but rather fixed two-dimensional planes in $\mathbf{T}^6$, and hence they are (five+one)-dimensional singularities.

Since the gauge fields do not propagate in two dimensions among the six dimensions of those singularities, we do not have to care about the pure gauge box anomalies on these six-dimensional singularities. This is one of the benefits of the $\mathbf{Z}_{12}\langle\sigma\rangle$ orbifold, and one of the reasons why we adopt the $\mathbf{Z}_{12}$ orbifold. When the $\mathbf{Z}'_6$ orbifold is adopted, where $(v_b)|_{b=1,2,3} = (1/6,-3/6,2/6)$, six-dimensional singularities develop in the directions to which the D7-branes are stretched. In this case, one has to take care of the pure gauge box anomalies.

However, since the gravitational field propagates in all ten dimensions, pure gravitational box anomalies may be localized on those six-dimensional singularities. We require that all of them be cancelled out on each singularity. The distribution function in the first line of Eq. (78) is now given by

$$\frac{1}{\mathrm{vol}(\mathbf{T}^6)}\sum_{\mathbf{p}\in\Lambda_0} e^{-i(g^{-1}\cdot\mathbf{p}-\mathbf{p})\cdot\mathbf{y}}, \tag{81}$$

where Λ_0 is the dual lattice of the Γ_0. This distribution function is independent of z_3, when $g=\sigma^{3k}$ does not rotate the third complex plane. Thus, the σ^{3k}-component of anomalies are localized on these six-dimensional singularities. At the same time, (79) yields box anomalies for σ^{3k}. Indeed, the $\rho_{sp}(\sigma^{3k})$ do not contain the Γ^{456789} term, since $\sin(\pi v_3(3k))=0$, but rather another term

$$\prod_{b=1}^{2}(\sin(\pi v_b(3k)))\cos(\pi v_3(3k))\Gamma^{4567}. \tag{82}$$

The trace over the spinor indices becomes non-trivial when the regulator provides $i(\Gamma\cdot F)^3/(3!(4M^2)^3)$; the F is understood as the field strength R of the local Lorentz symmetry. One can see that the decoupling limit of the regulator $M\to\infty$ leaves convergent and (generally) non-vanishing results $\propto \mathrm{tr}_{sp}((R\wedge R\wedge R)|_{6D}t^a)|_I$.

The box anomalies consist of only σ^{3k}-components $(0,1,2,3)$. All other transformations in the orbifold group are irrelevant. This implies that the box anomalies can be calculated by assuming only the $\mathbf{Z}_4\langle\sigma^3\rangle$-orbifold projection. This is also intuitively reasonable. Since anomalies are, in some sense, a UV phenomenon, they can be determined by looking at only local geometry. Then, since the local geometry around the six-dimensional singularities of $\mathbf{T}^6/\mathbf{Z}_{12}\langle\sigma\rangle$ is the same as that in $\mathbf{C}\times(\mathbf{T}^4/\mathbf{Z}_4\langle\sigma^3\rangle)$, box anomalies calculated in the latter geometry should be the same locally as those calculated in the former one. Now, the ten-dimensional space-time is compactified on the $\mathbf{T}^4/\mathbf{Z}_4\langle\sigma^3\rangle$ orbifold, and we have Kaluza–Klein towers of six-dimensional particles. Thus, we can calculate box anomalies by summing up the absolute square of Kaluza–Klein wave functions, just as we have done for the triangle anomalies at the beginning of this subsection 6.2. The actual calculation is much easier in this way than having to calculate the anomalies with various representations of SO(9,1) in the Fujikawa method extended for orbifold geometry. The anomaly on the $\Gamma'_{\sigma^{3k}}$ is again given by

$$\frac{1}{\#\mathbf{Z}_4\langle\sigma^3\rangle}\sum_I A_I^{\rm box}\rho_I(\sigma^{3k}). \tag{83}$$

The supergravity multiplet of the Type IIB supergravity has the following fields that contribute to the pure gravitational anomalies: two Weyl and Majorana gravitinos (two times fifty-six on-shell states), two Weyl and Majorana fermions (two times eight on-shell states) and real self-dual 4-form fields (thirty-five on-shell states). These fields are irreducible representations of ten-dimensional space-time, but they are decomposed into various fields in the six-dimensional space-time after the Kaluza–Klein reduction, which we label I. The term $\rho_I(g)$ denotes the phase factor due to the local Lorentz transformation associated with $g\in\mathbf{Z}_4\langle\sigma^3\rangle$.

A pair of the Weyl and Majorana gravitinos and Weyl and Majorana fermions is an $\mathbf{8}_v\otimes\mathbf{8}_s$ representation of the little group SO(8) in ten dimensions, where $\mathbf{8}_v$ is the vector representaion of the SO(8) and $\mathbf{8}_s$ the spinor representation. This is decomposed into Kaluza–Klein towers of six-dimensional fields $(\mathbf{4}_v+\mathbf{2}_{scl+}+\mathbf{2}_{scl-})\otimes(\mathbf{4}_-+\mathbf{4}_+)$, where $\mathbf{4}_v$ is vector representation, $\mathbf{2}_{scl\pm}$ are complex scalars, and $\mathbf{4}_\pm$ are Dirac spinors with opposite Γ^7-chiralities. All these towers contribute to the pure gravitational box anomalies. The transformation $(z_1,z_2)\mapsto(e^{i\alpha}z_1,e^{-i\alpha}z_2)$, to which the $\mathbf{Z}_4\langle\sigma^3\rangle$ belong, acts on each Kaluza–Klein tower through $\mathbf{4}_v,\mathbf{4}_-\mapsto\mathbf{4}_v,\mathbf{4}_-$, $\mathbf{2}_{scl\pm}\mapsto e^{\pm i\alpha}\mathbf{2}_{scl\pm}$ and $\mathbf{4}_+\mapsto e^{i\alpha}\mathbf{4}_+$. The phase factor ρ_I is calculated from these transformations for each Kaluza–Klein tower I.

The self-dual 4-form is decomposed into Kaluza–Klein towers of two scalars, four vector fields, one 2-form field, one complex rank-2 tensor contained in $\mathbf{4}_+\otimes\mathbf{4}_+$ and one complex rank-2 tensor in $\mathbf{4}_-\otimes\mathbf{4}_-$. Only the last two of them contribute to the box anomalies. The tensor in $\mathbf{4}_+\otimes\mathbf{4}_+$ is multiplied by $e^{2i\alpha}$, while the tensor in $\mathbf{4}_-\otimes\mathbf{4}_-$ is multiplied by 1 under the transformation described in the previous paragraph.

Let us calculate the irreducible part of the box anomalies. The coefficient $A_I^{\text{box,irr}}$ of the irreducible part of the pure gravitational box anomalies is given as follows: a gravitino in $\mathbf{4}_v \otimes \mathbf{4}_-$ has $A_I^{\text{box,irr}} = 245/360$, $A_I^{\text{box,irr}} = 1/360$ for a Dirac fermion $\mathbf{4}_-$, and $A_I^{\text{box,irr}} = -56/360$ for a complex rank-2 tensor in $\mathbf{4}_+ \otimes \mathbf{4}_+$. Fields of opposite chirality have $A_I^{\text{box,irr}}$ with opposite sign.

The coefficient on the $\Gamma'_{\sigma^{3k}}|_{k=0,1,2,3}$ lattice is calculated as

$$\frac{1}{4}\frac{1}{360}(2 \times (245(1-i^k) - 1(1-i^k) + 2(i^k - i^{2k} + i^{-k} - 1)) - 56(i^{2k} - 1)), \tag{84}$$

where $e^{i\alpha} = i$ for σ^3 has been used. The coefficients do not vanish for $k = 1, 2, 3$. Thus, we examine how much anomaly is distributed to each six-dimensional singularity. Sixteen points in Γ'_{σ^6} receive 1/16 of 240/360 from the σ^6-component, and four of them, which are also in $\Gamma'_{\sigma^3} = \Gamma'_{\sigma^9}$, also receive 1/4 of $(1/360) \times ((122(1-i)+28)+(122(1+i)+28)) = 300/360$ from the σ^3- and σ^9-components of the anomaly. Thus, the twelve loci of σ^6-fixed points in the covering space $\mathbf{T}^6$ have 15/360 pure gravitational irreducible anomaly, and the remaining four loci of σ^3-fixed points in the $\mathbf{T}^6$ have 90/360 anomaly. Those sixteen singularities in the covering space $\mathbf{T}^6$ correspond to four distinct two-dimensional singularities of the $\mathbf{T}^6/\mathbf{Z}_{12}\langle\sigma\rangle$. 90/360, 270/360, 90/360 and 90/360 of anomalies are localized on the four singularities, respectively.

One can, in general, cancel these irreducible pure gravitational anomalies, by introducing gauge singlet fields at these singularities. However, the situation is not so simple in our case of interest. It is because the matter contents that can be introduced on these singularities are quite limited, since there is extended SUSY there. Since SUSY is broken at these singularities only by $\mathbf{Z}_2\langle\sigma^6\rangle$- or $\mathbf{Z}_4\langle\sigma^3\rangle$-orbifold, there are sixteen SUSY charges, which form (0,2) SUSY of the six-dimensional space-time (e.g. [59]). The minimal SUSY multiplet of the (0,2) SUSY theories is a tensor multiplet. Other SUSY multiplets cannot be introduced because they always include fields of spin more than 1. The tensor multiplet consists of five real scalars, one real rank-2 tensor in $\mathbf{4}_+ \otimes \mathbf{4}_+$, and two Dirac fermions in $\mathbf{4}_+$. Therefore, a single tensor multiplet of the (0,2) SUSY contributes to the irreducible part of the pure gravitational box anomaly by $(-28 - 2 \times 1)/360 = -30/360$ (e.g. [59]).

The irreducible part of the pure gravitational box anomaly can be cancelled by introducing the tensor multiplets of the (0,2) SUSY at each of four six-dimensional singularities. This is because the amount of anomaly happens to be an integral multiple of $-30/360$, with opposite sign at each singularity. All the box anomalies are cancelled out if the number of tensor multiplets at those singularities is 3, 9, 3 and 3, respectively. This is also a miraculous result.

There will be non-vanishing reducible part of pure gravitational anomalies, but they can be cancelled by the Green–Schwarz mechanism [59].

Finally, the $\mathbf{Z}_2$ $\langle \Omega R_{89} \rangle$-projection does not give rise to anomalies since the theory obtained by gauging only ΩR_{89} is nothing but the Type I$'$ theory known to be consistent.

We have discussed the box anomaly cancellation on six-dimensional singularities, and the pentagonal anomalies on eight dimensions. In particular, we have shown that the irreducible part of the pure gravitational anomalies can be cancelled by introducing suitable massless fields in the six-dimensional singularities. However, the introduction of new massless fields might not be necessary in a situation such as the following. When there are an infinite number of massive excitations on D3-branes, D7-branes or four-dimensional fixed points, those particles sometimes give rise to higher-dimensional anomalies. Winding modes on D-branes in the Type IIB string theory are good examples. Since those particles can contribute to the pure gravitational box anomalies, it does not make sense, in principle, to discuss higher-dimensional anomalies without specifying a spectrum of infinite massive particles on four-dimensional space-time[37,38]. We show above that it is possible to cancel the anomaly in a genuinely field-theoretical manner (cancelling box anomalies through six-dimensional massless fields). We do not consider that this is the only way, but rather we claim that there is at least one way of cancelling the anomaly.

We have examined so far anomalies over untwisted space-time. We have discussed triangle anomalies at (three+one)-dimensional singularities and box anomalies at (five+one)-dimensional singularities. They are anomalies over space-time that extend in untwisted directions. However, there is another class of "anomalies" that arise from Eq. (79), and these are over space in the twisted directions. This is the issue discussed in the rest of this subsection 6.2. We only treat such anomalies on D7-branes, but the following discussion can easily be extended to include the gravity in the bulk.

Equation (79) implies that there are other "anomalies". Terms in $\rho_{sp}(g)$ proportional to $\mathbf{1}$, Γ^{45}, Γ^{67} and Γ^{4567} give rise to non-vanishing values after the trace over the spinor index is taken by extracting terms proportional to themselves from the regulator $e^{-(\mathbf{p}'\cdot\mathbf{p}'+\frac{i}{2}\Gamma_{AB}\cdot F^{AB})/2M^2}$ we adopt. We call them "anomalies in internal space dimensions". The σ^k-component of these anomalies is given by

$$\frac{4M^4}{\pi^2}\left(\prod_{b=1}^{3}\cos(\pi v_b k)\right)\left(\sum_I i\rho_I(\sigma^k)\,\mathrm{tr}_{\,\omega}(t_I^a)\right) \tag{85}$$

[37] For a related discussion, see subsubsection 7.2.2.

[38] We expect that things are much the same for global anomalies, which we do not discuss in this paper.

$$+ \quad \frac{2M^2}{\pi^2}\sin(\pi v_1 k)\left(\prod_{b=2}^{3}\cos(\pi v_b k)\right)\left(\sum_I -\rho_I(\sigma^k)\,\mathrm{tr}_{\,\omega}(t_I^a t_I^b)F_{45}^b\right) \tag{86}$$

$$+ \quad \frac{2M^2}{\pi^2}\sin(\pi v_2 k)\left(\prod_{b=3}^{1}\cos(\pi v_b k)\right)\left(\sum_I -\rho_I(\sigma^k)\,\mathrm{tr}_{\,\omega}(t_I^a t_I^b)F_{67}^b\right) \tag{87}$$

$$+ \quad \frac{\mathrm{tr}_{\,\omega}(t^a\{t^b,t^c\})|_{I=\mathbf{fund.}}}{32\pi^2}(F^b\tilde{F}^c)|_{\text{on 4567-th plane}} \tag{88}$$

$$4\left(\prod_{b=1}^{2}\sin(\pi v_b k)\right)\cos(\pi v_3 k)\left(\sum_I -i\rho_I(\sigma^k)A_I\right).$$

Now, these expressions explicitly depend on the regulator mass M. This is in sharp contrast with the ordinary anomalies discussed before. This suggests that these "anomalies in internal dimensions" are regularization-dependent, and their values can be changed by UV physics ($\simeq$ regularization). Thus, the "anomalies" (85) – (87) do not lead to reliable constraints on low-energy physics.

The "anomaly" (88), which is proportional to $(F\wedge F)|_{\text{on 4567-th plane}}$, seems to be regularization-independent, but it is not the case either. We have only discussed so far the consequences of adopting the regulator $e^{\not{D}\cdot\not{D}/2M^2}$. This regulator fully respects the whole SO(7,1) Lorentz symmetry. However, it does not have to do so, since the geometry of the orbifold already breaks this symmetry. It is true that there exists the SO(7,1) local Lorentz symmetry on the D7-brane world volume away from singularities. It is also true that there exists the SO(5,1) local Lorentz symmetry around the six-dimensional singularities and that there exists the SO(3,1) local Lorentz symmetry at any point in the orbifold, even around the four-dimensional singularities. However, the regulator does not have to respect broken SO(7,1)/SO(3,1) symmetry (SO(7,1)/SO(5,1) symmetry) at four-dimensional (six-dimensional) singularities, respectively. Then, regulators can include $(F\wedge F)|_{\text{on 4567-th plane}}/M^4$ explicitly at singularities, without being accompanied by Γ^{4567}, although it is not easy to write down explicitly such regularization in the momentum-diagonal base. Notice that the gauge transformation around the singularities cannot be topologically non-trivial[39] in the twisted $\mathbf{e}_{4,5,6,7}$ directions; hence the explicit $(F\wedge F)|_{\text{on 4567-th plane}}/M^4$ (at singularities) in the regulator is not forbidden by the remaining gauge symmetry. The $\left(\prod_{b=1}^{3}\cos(\pi v_b k)\right)$ term in (75), thus, can also give rise to the "anomaly in internal dimensions" proportional to $(F\wedge F)|_{\text{on 4567-th plane}}$. This contribution depends on the regulariza-

[39]One can understand this statement by considering the gauge symmetry on S_1-compactified and $S_1/\mathbf{Z}_2$-compactified five dimensions.

tion. Therefore, the "anomaly" (88) also depends on regularization (more specifically, on UV physics at singularities), and does not lead to a constraint only on low-energy physics.

It is clear from the above argument that the anomaly in the untwisted directions is not susceptible to these variation in the regulators. Regulators do not contain F_{01}, F_{23} or $(F\wedge F)|_{\text{on 0123-th plane}}$ because of the unbroken local SO(3,1) symmetry and topologically non-trivial gauge-symmetry transformation. Thus, terms in $\rho_{sp}(g)$ that are not proportional to Γ^{456789} do not give rise to the anomalies proportional to $(F\wedge F)|_{\text{on 0123-th plane}}$. The situation is the same for the box anomalies $(R\wedge R\wedge R)|_{\text{on 012389-th plane}}$ in the untwisted directions.

We conclude that all the "anomalies in internal dimensions" are not constraints on low-energy physics. Although they might be constraints on UV physics, especially on UV physics at singularities, we do not discuss this issue further.

6.3 Discrete R Symmetry

Orbifold geometry preserves a discrete rotational symmetry, which is a subgroup of the SO(6) rotational symmetry of the $\mathbf{C}^3$. This subsection is devoted to consequences of this symmetry.

Now that all the particles in the $SU(5)_{\rm GUT}$-breaking sector are obtained from D-branes, we know how those fields transform under the discrete rotational symmetry of the orbifold geometry. On the other hand, such a rotational symmetry is regarded as an internal symmetry at energies below the Kaluza–Klein scale. This symmetry, in general, rotates SUSY charges since these are in the spinor representation of the space rotational symmetry. Thus, it becomes an R symmetry below the Kaluza–Klein scale. Therefore, we can figure out how those fields transform under the discrete R symmetry.

The R symmetry obtained in this way is a gauged symmetry. Indeed the rotation is nothing but the combined action of a general coordinate transformation and a local Lorentz symmetry, both of which are gauged. Thus, the discrete gauge R symmetry (obtained in this way) is exact unless it is spontaneously broken. This is quite important because the (mod 4)-R symmetry of the product-group unification models should be preserved at the 10^{-14} level to keep the two Higgs doublets almost massless.

The (mod 4)-R symmetry is expected to be spontaneously broken by vacuum condensation of the superpotential, which is related to the spontaneous breaking of $\mathcal{N}=1$ SUSY. Spontaneous symmetry breaking of geometry is related to the spontaneous breaking of SUSY also in string theories [64]; the SUSY breaking causes tadpoles of NS–NS fields, leading to instabilities of the geometry. Deformations of geometry due to this instability (due to SUSY breaking) will lead to the spontaneous breaking of the discrete R symmetry. One might expect that a similar thing could happen in a model based on supergravity.

The transformation properties of various fields under the rotation of extra dimensions have been already given in subsection 6.1. One can easily see that the R charge is properly assigned for all the particles in the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector when the R symmetry is identified[40] with the rotational symmetry of the third complex plane $z_3 \mapsto e^{i\alpha} z_3$.

The toroidal compactification breaks the SO(6) rotational symmetry of the $\mathbf{C}^3$ down to a discrete subgroup of the SO(6). Discrete rotation that corresponds to the mod 4 part of the R symmetry should be preserved by the geometry; otherwise the R symmetry, although gauged, is spontaneously broken at the Kaluza–Klein scale and does not play any role in phenomenology. Notice that the mod 4 discrete subgroup *is* naturally preserved by geometry since that the subgroup corresponds to rotation of the third complex plane $z_3 \in (\mathbf{C}/(\mathbf{Z}_{12}\langle\sigma\rangle \times \mathbf{Z}_2\langle\Omega R_{89}\rangle)$ by an angle π.

The supergravity multiplet of the Type IIB supergravity also provides Kaluza–Klein zero modes that survive the orbifold projection associated to $\mathbf{Z}_{12}\langle\sigma\rangle \times \mathbf{Z}_2\langle\Omega R_{89}\rangle$. Those matter contents (all are gauge singlets) consist [38] of one supergravity multiplet of four-dimensional $\mathcal{N}=1$ supergravity and four chiral multiplets. Three of the four chiral multiplets correspond to the metric and the 2-form in three different complex planes, and the other one corresponds to the dilaton and an axion. R charges of all these chiral multiplets are 0. Therefore, these moduli are massless without (mod 4)-R symmetry breaking unless particles appear at singularities whose R charges are 2.

Since we do not specify the origin of the quarks and leptons, it is impossible to determine the R charges of those particles. We just expect that their R charges are determined as those given in Table 3.

It is discovered in [20] that the (mod 4)-R symmetry has a vanishing anomaly with the $\mathrm{SU}(2)_{\mathrm{H}}$ gauge group, and can be anomaly-free with $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge group, if there is an extra pair of $(\mathbf{5},\mathbf{1})$ and $(\mathbf{5}^*,\mathbf{1})$ chiral multiplets of the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(2)_{\mathrm{H}}$ gauge group. The higher-dimensional construction motivates the existence of this $\mathrm{SU}(5)_{\mathrm{GUT}}$-charged vector-like pair at the TeV scale. The vector-like pair cannot have mass unless (mod 4)-R symmetry is broken. Notice, however, that the (mod 4 R)$[\mathrm{SU}(5)_{\mathrm{GUT}}]^2$ can be cancelled also by the generalized Green–Schwarz mechanism.

Although we examined whether triangle anomalies are cancelled at all fixed points, we do

[40] The required R-charge assignment is properly obtained as long as $\alpha_1 = -\alpha_2$. We put $\alpha_1 = \alpha_2 = 0$ in the text just because of its simplicity. The unbroken subgroup discussed in the next paragraph is the mod 4 subgroup whenever $(\alpha_1 = -\alpha_2) \in \alpha_3\mathbf{Z}$.

not have to check that the (mod 4)-R symmetry has vanishing anomalies at each fixed point. This is because the angle-π rotation that we are interested in is a rigid rotation of the whole orbifold. We are not interested in a space rotation with the angle changing point by point.

6.4 Toward a Realistic Model

The most successful feature of this higher-dimensional construction is that the superpotential of $\mathcal{N} = 2$ SUSY (the first and the second lines of (7)) is automatically obtained from (49). The approximate $\mathcal{N} = 2$ SUSY relation is naturally expected as a result of the extended SUSY in the UV physics. The third line of (7) is also allowed by the $\mathcal{N} = 2$ SUSY and the (mod 4)-R symmetry (the Fayet–Iliopoulos F-term).

Let us now discuss what is needed to make the model realistic beyond the orbifold construction obtained so far. The first issue to be discussed is the necessary particles that we could not obtain from the orbifold construction.

Quarks and leptons, $\mathrm{SU}(5)_{\mathrm{GUT}}$-$\mathbf{10}$ and -$\mathbf{5}^*$, are not obtained in our orbifold construction. There is no model using D-brane construction based on string theories that has succeeded in obtaining all of (i) the three families of quarks and leptons, (ii) a unified gauge group and (iii) a sector to break that symmetry; this is not a difficulty limited to our construction. We consider that the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector with $\mathcal{N} = 2$ SUSY is a strong indication of the structure of higher-dimensional space, and we, therefore, construct the model so that this structure is manifestly realized. The orbifold geometry thus obtained provides a good description of the structure of the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector, but not of the quarks and leptons; these are described as particles put by hand at a fixed point. The fixed point where they reside should preserve neither the U(6) symmetry nor $\mathcal{N} = 2$ SUSY. There is only one candidate for such a fixed point: $\mathbf{y}' = \mathbf{0}$ on the D7-branes.

As we have discussed in subsection 6.2, the pure gravitational anomalies localized on six-dimensional singularities requires that some particles be newly introduced. The tensor multiplet of (0,2) SUSY of the six dimensions with the number specified in subsection 6.2 is one of the possibilities. Towers of infinite massive particles on D3-branes or D7-branes, if they exist, may also contribute to the anomaly cancellation.

The gauge symmetry of our model is now $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(1)_5\times\mathrm{U}(1)_6\times\mathrm{U}(2)_{\mathrm{H}}$. However, when quarks and leptons are introduced at the $\mathbf{y}' = \mathbf{0}$ fixed point, it is probable that only one linear combination of $\mathrm{U}(1)_5$ and $\mathrm{U}(1)_6$ remains free from mixed anomaly with the $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge group; the other combination will be anomalous, but its anomaly will be cancelled by the generalized Green–Schwarz mechanism [36]. The mixed-anomaly free combination must be what is called the "fiveness" U(1) symmetry[41], since this is the unique assignment that leads to a vanishing mixed anomaly. However, there are still non-vanishing $\mathrm{U}(1)_{\mathrm{fiveness}}[\mathrm{gravity}]^2$ and $[\mathrm{U}(1)_{\mathrm{fiveness}}]^3$ triangle anomalies. These can be cancelled (i) by the generalized Green–Schwarz mechanism or (ii) by introducing another particle. Three families of right-handed neutrinos are sufficient to cancel these two anomalies simultaneously. In case (ii), the $\mathrm{U}(1)_{\mathrm{fiveness}}$ symmetry is not broken at the Kaluza–Klein scale, and it should be spontaneously broken at the intermediate scale to explain the small neutrino masses via seesaw mechanism [65].

The $\mathbf{T}^6/(\mathbf{Z}_{12}\left<\sigma\right> \times \mathbf{Z}_2 \left<\Omega R_{89}\right>)$ geometry would have more discrete symmetries in addition to the (mod 4)-R symmetry discussed above. We consider that they may be broken by condensations of gauge-singlet fields introduced at singularities, although their geometrical interpretations are not clear. Therefore, we do not take such discrete symmetries seriously as a symmetry that determines the low-energy physics.

Now finally, at the end of this section 6, we discuss the low-energy superpotential. Yukawa interactions of quarks and leptons are

$$W = c\mathbf{10}\ \mathbf{10}\ \frac{(\bar{Q}^i{}_\alpha Q^\alpha{}_6)}{M_*} + c' \frac{(\bar{Q}^6{}_\alpha Q^\alpha{}_i)}{M_*} \cdot \mathbf{10}^{ij} \cdot \mathbf{5}^*_j \tag{89}$$

in the $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(2)_{\mathrm{H}}$ model. These interactions can be induced (i) by massive particle exchange and (ii) by unknown non-perturbative effects.

The wave-function renormalization of composite states $(\bar{Q}^i{}_\alpha Q^\alpha{}_6)$ and $(\bar{Q}^6{}_\alpha Q^\alpha{}_i)$ in the Kähler potential may be[42] protected from the strong $\mathrm{U}(2)_{\mathrm{H}}$ couplings by the approximate $\mathcal{N}$ = 2 SUSY [66]. The $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge interactions, which do not preserve $\mathcal{N} = 2$ SUSY, do not lead to a sizeable wave-function renormalization. Thus, the effective Yukawa coupling may include a $\left<Q\right>/M_* \simeq v/M_* \simeq 10^{-1}$ suppression factor in both cases (i) and (ii).

The effective coefficients c and c' include an exponential suppression factor if they are generated by massive particle exchanges (in case (i)). This is because these interactions can be generated only by particles whose masses are of the order of the fundamental scale M_* and because the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector, to which the $Q, \bar{Q}$ belong, and quarks and leptons reside at different fixed points. The Kaluza–Klein modes of the U(6) vector multiplet cannot induce the Yukawa couplings, since those particles that have suitable gauge charges

[41]The "fiveness" U(1) symmetry is given by a linear combination of the U(1) B−L symmetry and $\mathrm{U}(1)_Y$ of the standard model that commutes with all generators in the $\mathrm{SU}(5)_{\mathrm{GUT}}$.

[42]The hyper-Kähler metric is not renormalized in general $\mathcal{N} = 2$ SUSY gauge theories [66]. However, our case of interest is asymptotic non-free, and hence it is not obvious that it is indeed the case in the present model.

do not have non-vanishing wave functions at the fixed where quarks and leptons reside. The exponential suppression factor is $e^{-M_* L_4/2}$ or $e^{-M_* L_4}$, depending on which σ^6-fixed point the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector resides. Note that we have not yet chosen one from two candidates of the σ^6-fixed points in subsection 6.1. The former choice is preferred because of its moderate suppression factor. The effective Yukawa coupling includes $e^{-M_* L_4/2} \times (v/M_*) \sim 10^{-2}$ suppression factor as a whole. However, there is no way to estimate the effective coupling between the massive particle in the extra dimensions and the quarks and leptons, since their origins are not known. Thus, we see that the suppression of the order of 10^{-2} is marginal[43] to obtain the top Yukawa coupling of order 1.

The disunification between strange quarks and muons is obtained through an operator

$$W = \frac{c''}{M_*^3} (\bar{Q}Q)^6{}_i \mathbf{10}^{ij} (\bar{Q}Q)^k{}_j \mathbf{5}^*_k. \tag{90}$$

Thus, the effective coefficient would involve extra $(v/M_*)^2 \sim 10^{-2}$ suppression with respect to the Yukawa couplings of bottom quarks and tau leptons.

When there are non-perturbative contributions to the Yukawa couplings (in case (ii)), it is impossible to determine how the effective couplings c and c' are suppressed. Yukawa couplings may be obtained without an extra suppression factor if they are generated by non-perturbative effects. But the study of such effects is beyond the scope of this paper.

7 $\mathbf{SU(5)_{\mathrm{GUT}} \times U(3)_{\mathrm{H}}}$ Model

7.1 D-brane Configuration and Orbifold Projection

Let us now describe how the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector of the $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(3)_{\mathrm{H}}$ model is derived. We adopt the same geometry, $\mathbf{T}^6/(\mathbf{Z}_{12}\langle\sigma\rangle \times \mathbf{Z}_2\langle\Omega R_{89}\rangle)$, as in the previous model. We basically assume the same massless field contents on the D-branes at the beginning, i.e. $\mathrm{U}(N)$ vector multiplet on N coincident D-branes, etc. What is different between the construction of the two models is the D-brane configuration, and also the choice of unitary matrices $\tilde{\gamma}_{\sigma^k;7}$ and $\tilde{\gamma}_{\sigma^k;3}$ that appear in the orbifold projection conditions.

We put seven D7-branes at the two $\mathbf{Z}_{12}\langle\sigma\rangle$-fixed loci $z_3 = \frac{\mathbf{e}_8+2\mathbf{e}_9}{3}$ and $z_3 = -\frac{\mathbf{e}_8+2\mathbf{e}_9}{3}$. The $\mathbf{Z}_2\langle\Omega R_{89}\rangle$ only identifies the fields on both fixed loci, and the identified fields are subject

[43] Although figures larger that 1 are certainly required other than the exponential suppression factor in c and c', the effective coefficients c and c' themselves do not exceed the bound from Born unitarity below the cut-off scale M_* as long as $c, c' \lesssim 4\pi$.

only to the orbifold projection conditions of the $\mathbf{Z}_{12}\,\langle\sigma\rangle$. Projection conditions are written in the same way as in Eqs. (39), (40) and (38). The only difference is that the vector multiplet $(\Sigma)^k{}_l$ $(k,l=1,...,7)$ is of the U(7) gauge group rather than of the U(6). The 7 by 7 unitary matrix $\tilde{\gamma}_{\sigma;7}$ is now chosen as

$$(\tilde{\gamma}_{\sigma;7})^k{}_l = \mathrm{diag}\left(\overbrace{e^{-\frac{1}{12}\pi i},...,e^{-\frac{1}{12}\pi i}}^{5}, e^{-\frac{9}{12}\pi i}, e^{-\frac{11}{12}\pi i}\right), \tag{91}$$

instead of (43).

The sixth diagonal entry and seventh diagonal entry are chosen differently from the first five entries, so that the U(7) gauge symmetry is broken down to U(5)$\times$U(1)$_6\times$U(1)$_7$. The phase difference between the first five entries and the sixth is chosen as $e^{-2\pi i4/12}$ for the following reasons. Since we require that the U(6) gauge symmetry be restored at the σ^3-projection, for a reason that is explained later, the phase difference should be the third root of unity. The phase difference can be $e^{+2\pi i4/12}$, but the model is not essentially different from the model with the phase difference $e^{-2\pi i4/12}$.

No matter which phase difference we adopt, there exists a U(5)-**(anti-)fund.** chiral multiplet that survives the orbifold projection. On the other hand, we observe in the next subsection 7.2 that it is hard to cancel the triangle anomalies at each fixed point unless we take a construction such that the U(5)-conjugate particle also survives the orbifold projection. This is the reason why we start from U(7) vector multiplet. As a result, the SU(5)$_{\rm GUT}\times$U(3)$_{\rm H}$ model inevitably includes SU(5)-($\mathbf{5}+\mathbf{5}^*$) pair in the Kaluza–Klein zero modes from D7-branes. The phase difference between the first five entries and the seventh entry is chosen so that U(5)-conjugate matter appears in the low-energy spectrum. There are only two possibilities — $e^{-2\pi i5/12}$ and $e^{2\pi i/12}$. Both possibilities essentially lead to the same physics. Under the choice of the phase difference in (91), i.e. $e^{-2\pi i5/12}$, the U(7) symmetry is not enhanced at the σ^3-projection. If the U(7) symmetry were to remain at the σ^3-fixed point on the D7-branes, then the unwanted hypermultiplet $(\bar{Q}^7{}_\alpha, Q^\alpha{}_7)$ would appear in the SU(5)$_{\rm GUT}$-breaking sector.

The Kaluza–Klein zero modes that survive the orbifold projection in Eq. (38) are as follows: $\mathcal{N}=1$ vector multiplets of U(5)$\times$U(1)$_6\times$U(1)$_7$, where the SU(5) subgroup of the U(5) $\simeq$ SU(5)$\times$U(1)$_5$ is identified with the SU(5)$_{\rm GUT}$ gauge group, and chiral multiplets, $(\Sigma_3)^6{}_i$, $(\Sigma_2)^i{}_7$ and $(\Sigma_1)^7{}_6$, which transform $(\mathbf{5}^*)^{(-1,1,0)}$, $(\mathbf{5})^{(1,0,-1)}$ and $(\mathbf{1})^{(0,-1,1)}$ under the gauge group SU(5)$_{\rm GUT}\times$U(1)$_5\times$U(1)$_6\times$U(1)$_7$. The index "i" now runs from 1 to 5, not to 7. Anomalies of the above gauge group are discussed in the next subsection 7.2. We identify $(\Sigma_3)^6{}_i$ and $(\Sigma_2)^i{}_7$ with Higgs multiplets $\bar{H}_i(\mathbf{5}^*)$ and $H^i(\mathbf{5})$ in subsections 7.4 and 7.3, respectively.

Three D3-branes are put on a fixed point on the D7-branes where the isotropy group[44] is $\mathbf{Z}_4\,\langle\sigma^3\rangle$. There is only one such a candidate in the $\mathbf{T}^6/(\mathbf{Z}_{12}\,\langle\sigma\rangle\times\mathbf{Z}_2\,\langle\Omega R_{89}\rangle)$ geometry. These three D3-branes provide the $\mathrm{U}(3)_\mathrm{H}$ gauge group in the $\mathrm{SU}(5)_\mathrm{GUT}$-breaking sector. Eighteen D3-branes (six images of three D3-branes) are necessary as a whole within the covering space $\mathbf{T}^6$, yet fourteen remaining D3-branes can be used for other sectors.

Fields are restricted only under the orbifold projection by the isotropy group $\mathbf{Z}_4\,\langle\sigma^3\rangle$. Thus, the U(6) symmetry, which is required to be restored in the $\mathrm{SU}(5)_\mathrm{GUT}$-breaking sector, is required to be restored on D7-branes under the σ^3-projection. This is the major reason why we take the phase difference between the first five diagonal entries and the sixth diagonal entry as the third root of unity in (91). Fields on the D3-branes transform under the $\mathbf{Z}_4\,\langle\sigma^3\rangle$ as

$$\sigma^3: \quad (X_0)^\alpha{}_\beta \mapsto (\widetilde{X_0})^\alpha{}_\beta \equiv (\tilde{\gamma}_{\sigma^3;3})^\alpha{}_{\alpha'}(X_0)^{\alpha'}{}_{\beta'}(\tilde{\gamma}^{-1}_{\sigma^3;3})^{\beta'}{}_\beta, \tag{92}$$

$$\sigma^3: \quad (X_b)^\alpha{}_\beta \mapsto (\widetilde{X_b})^\alpha{}_\beta \equiv e^{2\pi i\tilde{v}_b 3}(\tilde{\gamma}_{\sigma^3;3})^\alpha{}_{\alpha'}(X_b)^{\alpha'}{}_{\beta'}(\tilde{\gamma}^{-1}_{\sigma^3;3})^{\beta'}{}_\beta, \tag{93}$$

$$\sigma^3: \quad Q^\alpha{}_k \mapsto \widetilde{Q}^\alpha{}_k \equiv e^{\pi i(v_1+v_2)3}(\tilde{\gamma}_{\sigma^3;3})^\alpha{}_{\alpha'}Q^{\alpha'}{}_{k'}(\tilde{\gamma}^{-3}_{\sigma;7})^{k'}{}_k, \tag{94}$$

$$\sigma^3: \quad \bar{Q}^k{}_\alpha \mapsto \widetilde{\bar{Q}}^k{}_\alpha \equiv e^{\pi i(v_1+v_2)3}(\tilde{\gamma}^{3}_{\sigma;7})^k{}_{k'}\bar{Q}^{k'}{}_{\alpha'}(\tilde{\gamma}^{-1}_{\sigma^3;3})^{\alpha'}{}_\alpha, \tag{95}$$

where the (X_a)'s form a U(3) vector multiplet of the four-dimensional $\mathcal{N}=4$ SUSY and the $(\bar{Q},Q)$ is a hypermultiplet of the four-dimensional $\mathcal{N}=2$ SUSY in the $(\mathbf{7},\mathbf{3}^*)$ representation of the U(7)×U(3) gauge group. The 3 by 3 unitary matrix $\tilde{\gamma}_{\sigma^3;3}$ is chosen as

$$\tilde{\gamma}_{\sigma^3;3} = \mathrm{diag}(e^{\frac{3}{4}\pi i}, e^{\frac{3}{4}\pi i}, e^{\frac{3}{4}\pi i}), \tag{96}$$

so that the hypermultiplets $(\bar{Q}^k{}_\alpha, Q^\alpha{}_k)$ $(k=1,...,6;\ \alpha=1,2,3)$ survive the following orbifold projection conditions. The orbifold projection imposes Eq. (59). Projected out by these conditions are the $\mathcal{N}=2$ hypermultiplet (X_1,X_2) in the U(2)-**adj.** representation and $(\bar{Q}^7{}_\alpha, Q^\alpha{}_7)$. What is left is exactly the matter contents of the $\mathrm{SU}(5)_\mathrm{GUT}$-breaking sector. It is very encouraging that the full multiplets for the $\mathrm{SU}(5)_\mathrm{GUT}$-breaking sector are obtained with the $\mathcal{N}=2$ SUSY structure.

[44] The $\mathrm{SU}(5)_\mathrm{GUT}\times\mathrm{U}(3)_\mathrm{H}$ model cannot be constructed by putting the D3-branes at a fixed point where the isotropy group is $\mathbf{Z}_2\,\langle\sigma^6\rangle$. This is because thirty-six D3-branes (twelve images of three D3-branes) are necessary within the covering space $\mathbf{T}^6$ in this case. Thirty-six D3-branes are too many to cancel their 3-brane charges by negative charges of O3-planes.

7.2 Anomaly Cancellation and Tadpole Cancellation

7.2.1 Anomaly Cancellation

The $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(3)_{\mathrm{H}}$ model discussed in this section differs from the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(2)_{\mathrm{H}}$ model in the previous section only in the D-brane configuration and the unitary matrices $\tilde{\gamma}_{\sigma^k;7}$ and $\tilde{\gamma}_{\sigma^k;3}$. Therefore, the discussion in subsubsection 6.2.2 exactly holds also in this model. In particular, the pure gravitational box anomalies are cancelled because only the supergravity multiplet in the bulk is relevant, whose Kaluza–Klein spectrum is exactly the same as in the previous model. Anomalies in internal dimensions are not the conditions on low energy physics either in this model, just because of the ambiguity of regularization at singularities. However, triangle anomalies, which are related to gauge fields on D7-branes, should be examined in this model again.

Let us discuss triangle anomalies between the unbroken gauge groups $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(1)_5\times\mathrm{U}(1)_6\times\mathrm{U}(1)_7$ and gravity. We first discuss the $[\mathrm{SU}(5)_{\mathrm{GUT}}]^3$ anomaly cancellation at each fixed point because this type of anomaly cannot be cancelled out by incorporating the generalized Green–Schwarz mechanism.

Formulae (69) with (71) (or equivalently, (78) with (80)) are applicable also to the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(3)_{\mathrm{H}}$ model discussed in the previous subsection 7.1. What is obvious from the expression (80) is that no anomaly arises on a fixed-point lattice Γ'_{σ^k} in which ${}^{\exists}v_b k \in \mathbf{Z}$. In particular, the fixed points with $\mathcal{N}=2$ SUSY, which are only on one of the $\Gamma'_{\sigma^{3k}}$'s, do not have any kind of triangle anomalies. This is because a vector-like structure is still kept at those fixed points. Triangle anomalies are carried by σ^k-components with $k=1,2,4,5,7,8,10$ and 11. These eight components have only two independent distribution functions. One is the σ^4-fixed lattice, Γ'_{σ^4} $(=\Gamma'_{\sigma^8})$, which contains nine points within a torus $\mathbf{T}^4\equiv\mathbf{C}^2/\Gamma'_0$. The other is the $\Gamma'_{\sigma^{1,2,5,7,10,11}}$, all of which consist of only one and the same fixed point, $\mathbf{y}'=\mathbf{0}$. Thus, the anomaly in the $\sigma^{\pm4}$-components and the total anomaly that comes from the $\sigma^{\pm1}$, $\sigma^{\pm2}$ and $\sigma^{\pm5}$-components should separately vanish, so that the triangle anomalies vanish at all the fixed points.

The $[\mathrm{SU}(5)_{\mathrm{GUT}}]^3$ anomaly of the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(3)_{\mathrm{H}}$ model on the Γ'_{σ^4} vanishes because

$$\sigma^4+\sigma^8 : \frac{8}{12}\times\left(-\frac{3\sqrt{3}}{8}\right)\times 2\mathrm{Im}(e^{2\pi i\frac{4}{3}}+e^{2\pi i\frac{5}{3}})=0. \qquad (97)$$

Now we do not have to make more calculations to arrive at the conclusion that there is no $[\mathrm{SU}(5)_{\mathrm{GUT}}]^3$ anomaly on the $\Gamma'_{\sigma^{1,2,5,7,10,11}}$ lattice (i.e. fixed point $\mathbf{y}'=\mathbf{0}$) either, and hence at any fixed points. This is because the total amount of the $[\mathrm{SU}(5)_{\mathrm{GUT}}]^3$ anomaly integrated over the whole compact space vanishes (due to the anomaly-free spectrum of the zero modes), and also because this total anomaly consists of anomalies only on the $\Gamma'_{\sigma^{\pm 4}}$ and on $\mathbf{y}' = \mathbf{0}$.

We also show, however, the explicit calculation of the anomaly at the fixed point $\mathbf{y}' = \mathbf{0}$ just as a preparation for discussion in the next subsubsection 7.2.2. The $[\mathrm{SU}(5)_{\mathrm{GUT}}]^3$ anomalies from $\sigma^{\pm 1}$, $\sigma^{\pm 2}$ and $\sigma^{\pm 5}$-components are

$$\sigma^1 + \sigma^{11}: \qquad \frac{8}{12} \times \left(-\frac{\sqrt{3}}{8} \right) \times 2\mathrm{Im} \left(e^{2\pi i \frac{4}{12}} + e^{2\pi i \frac{5}{12}} \right), \tag{98}$$

$$\sigma^5 + \sigma^7: \qquad \frac{8}{12} \times \left(\frac{\sqrt{3}}{8} \right) \times 2\mathrm{Im} \left(e^{2\pi i 5 \frac{4}{12}} + e^{2\pi i 5 \frac{5}{12}} \right), \tag{99}$$

$$\sigma^2 + \sigma^{10}: \qquad \frac{8}{12} \times \left(-\frac{\sqrt{3}}{8} \right) \times 2\mathrm{Im} \left(e^{2\pi i 2 \frac{4}{12}} + e^{2\pi i 2 \frac{5}{12}} \right), \tag{100}$$

and the sum of all these three contributions vanishes, though each component does not.

If we were to start from the U(6) vector multiplet rather than the U(7), then the $\sigma^{\pm 4}$-components and the sum of $\sigma^{\pm 1}$, $\sigma^{\pm 2}$ and $\sigma^{\pm 5}$-components are separately non-vanishing. The former has $-3/4$ and the latter has $-1/4$ times the anomaly of the $\mathrm{SU}(5)_{\mathrm{GUT}}$-**anti-fund.** representation. These anomalies are distributed into four different $\mathcal{N} = 1$ SUSY fixed points, and the amount of anomaly at each fixed point is a fractional number. These anomalies cannot be cancelled even by introducing $\mathrm{SU}(5)_{\mathrm{GUT}}$ charged particles[45].

Let us now discuss the anomalies of the U(1) gauge groups. First, the $\mathrm{U}(1)_{5+6+7}$ gauge field decouples from all the matter contents on the D7-branes and hence there is no anomaly associated to this gauge group. Second, it is easy to see, from the same calculation as in the case of the $[\mathrm{SU}(5)_{\mathrm{GUT}}]^3$ anomaly, that the $\mathrm{U}(1)_5$ gauge group does not have $\mathrm{U}(1)_5[\mathrm{SU}(5)_{\mathrm{GUT}}]^2$, $\mathrm{U}(1)_5[\mathrm{gravity}]^2$ and $[\mathrm{U}(1)_5]^3$ anomalies. Finally, the $\mathrm{U}(1)_{6-7}$ gauge group has a number of anomalies at various fixed points, and hence these anomalies should be cancelled by the generalized Green–Schwarz mechanism at all these fixed points. Thus, the $\mathrm{U}(1)_{6-7}$ symmetry is spontaneously broken.

7.2.2 Relation to the Ramond–Ramond Tadpole Cancellation

Here, we clarify the relation between the Ramond–Ramond tadpole cancellation in string theories and the anomaly cancellation discussed in subsection 6.2 and in subsubsection 7.2.1.

[45] In the previous paper [15], there is a mistake in this calculation. The model described there is also valid if it is possible to replace triangle anomalies from one fixed point to another, though.

This part is not necessary for the rest of this paper. The following discussion is basically along the line of Ref. [40]. It is argued there that both conditions are *generically* equivalent, while sometimes different.

The Ramond–Ramond tadpole cancellation conditions for the $\mathbf{Z}_{12}$ $\langle\sigma\rangle$-orbifold is given by [38, 67]

$$\mathrm{tr}(\tilde{\gamma}_{\sigma^k;7} \oplus \tilde{\gamma}^{-1}_{\sigma^k;7}) - \mathrm{tr}_{\,\mathbf{y}'=0}(\tilde{\gamma}_{\sigma^k;3} \oplus \tilde{\gamma}^{-1}_{\sigma^k;3}) = 0 \quad \text{for} \quad k = 1, 2, 5, 7, 10, 11, \tag{101}$$

$$\mathrm{tr}(\tilde{\gamma}_{\sigma^k;7} \oplus \tilde{\gamma}^{-1}_{\sigma^k;7}) + 3\,\mathrm{tr}_{\,\mathbf{y}'\in\mathrm{Eq.}(30)}(\tilde{\gamma}_{\sigma^k;3} \oplus \tilde{\gamma}^{-1}_{\sigma^k;3}) = 0 \quad \text{for} \quad k = 4, 8, \tag{102}$$

and

$$\mathrm{tr}(\gamma_{\sigma^k;7}) + 2\,\mathrm{tr}_{\,\mathbf{y}'\in\mathrm{Eq.}(29)}(\gamma_{\sigma^k;3}) = 0 \quad \text{for} \quad k = 3, 9, \tag{103}$$

$$\mathrm{tr}(\gamma_{\sigma^k;7}) + 4\,\mathrm{tr}_{\,\mathbf{y}'\in\mathrm{Eq.}(28)}(\gamma_{\sigma^k;3}) = 0 \quad \text{for} \quad k = 6, \tag{104}$$

with $\mathrm{tr}(\gamma_{\sigma^0;7}) = 32$ and $\mathrm{tr}(\gamma_{\sigma^0;3}) = 32$. Here, the $\gamma_{\sigma^k;7}$ denotes a 32 by 32 unitary matrix $(\gamma_{\sigma;D7})^k$ obtained from (41) and the $\gamma_{\sigma^k;3}|_{\mathbf{y}'\in\mathrm{Eq.}(29)}$ or $\gamma_{\sigma^k;3}|_{\mathbf{y}'\in\mathrm{Eq.}(28)}$ a diagonal block of $(\gamma_{\sigma;D3})^k$ obtained from (57) that corresponds to D3-branes at six-dimensional singularities whose $\mathbf{y}'$ coordinates are one of Eq. (29) or Eq. (28). These matrices are replaced by corresponding ones when the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(3)_{\mathrm{H}}$ model is considered. Equations (102), (103) and (104) are imposed at each of their fixed points separately, e.g. Eq. (102) for $k = 4$ contains nine equations for nine fixed points given in Eq. (30). There are more Ramond–Ramond tadpole cancellation conditions, which, however, do not restrict the projection on the D7-branes and D3-branes in our model. Thus, we do not list them here.

Let us first show that Eqs. (101) and (102) are derived *generically* from the cancellation of non-Abelian triangle anomalies on the D7-branes. Let us take the $\tilde{\gamma}_{\sigma;7}$ as

$$\tilde{\gamma}_{\sigma;7} = \mathrm{diag}(e^{2\pi i\frac{1}{24}}\mathbf{1}_{n_1\times n_1}, \ldots, e^{2\pi i\frac{2j-1}{24}}\mathbf{1}_{n_j\times n_j}, \ldots, e^{2\pi i\frac{23}{24}}\mathbf{1}_{n_{12}\times n_{12}}). \tag{105}$$

This is a generalization of (43) and (91). Then, the gauge group from these D7-branes is $\prod_{j=1}^{12} U(n_j)$. Now the σ^k-component of the $[\mathrm{SU}(n_j)]^3$ anomaly is proportional to

$$\begin{aligned}
& \frac{4}{12}\left(\prod_{b=1}^{3}\sin(\pi v_b k)\right)(-i\sum_I A_I \rho_I(\sigma^k)) \\
= \;& \frac{4}{12}\left(\prod_{b=1}^{3}\sin(\pi v_b k)\right)(-i)\left(e^{2\pi i\frac{k}{24}(2j-1)}\,\mathrm{tr}(\tilde{\gamma}^*_{\sigma^k;7}) - e^{-2\pi i\frac{k}{24}(2j-1)}\,\mathrm{tr}(\tilde{\gamma}_{\sigma^k;7})\right) \\
= \;& \frac{4}{12}\left(\prod_{b=1}^{3}\sin(\pi v_b k)\right)(-2)\mathrm{Im}\left(e^{-2\pi i\frac{k}{24}(2j-1)}\,\mathrm{tr}(\tilde{\gamma}_{\sigma^k;7})\right).
\end{aligned} \tag{106}$$

When it is required that all the σ^k-components separately[46] vanish for all $[\mathrm{SU}(n_j)]^3$ anomalies ($j = 1, ..., 12$), then

$$\mathrm{tr}(\tilde{\gamma}_{\sigma^k;7}) = 0 \qquad \text{for } k = 1, 2, 5, 7, 10, 11, 4, 8 \tag{107}$$

follows. No condition follows from the triangle anomaly cancellation for $k = 0, 3, 6, 9$, because $\sin(\pi v_3 k) = 0$. Equations (107) are the same conditions as Eqs. (101) and (102) in the absence of the D3-branes at those fixed points. Notice that it is the case in our construction, since D3-branes are put only at Γ'_{σ^6} in the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(2)_{\mathrm{H}}$ model or at $\Gamma'_{\sigma^{\pm 3}}$ in the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(3)_{\mathrm{H}}$ model, not at $\Gamma'_{\sigma^{\pm 1,2,5}}$ or $\Gamma'_{\sigma^{\pm 4}}$.

The triangle anomaly cancellation (for the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(3)_{\mathrm{H}}$ model discussed in this subsection) does not require that each σ^k-component (106) separately vanishes, but rather, it is sufficient to require vanishing sum of components that have the same distribution function. In particular, the sum of (98), (99) and (100) vanishes, but not separately. This is one of the differences between the triangle anomaly cancellation and the Ramond–Ramond tadpole cancellation. The other difference is that we do not have to impose a non-Abelian triangle anomaly cancellation when $n_j < 3$. In particular, the $[\mathrm{SU}(n_{12} = 5)]^3$ anomaly cancellation is imposed, while no other "$[\mathrm{SU}(n_{j\neq 12})]^3$ anomaly" does not have to be cancelled, since $n_{7,8} = 1$ and all other n_j's are 0.

The origin of quarks and leptons is not identified, but they reside on the $\mathbf{y}' = \mathbf{0}$ fixed point, which is exactly the $\Gamma'_{\sigma^1} = \Gamma'_{\sigma^5} = \Gamma'_{\sigma^2}$. Therefore, they can also give certain contributions[47] to each of (98), (99) and (100), although we cannot calculate each contribution in terms of $\mathrm{tr}_{\,\mathbf{y}'=0}(\tilde{\gamma}_{\sigma^k;3})$. In particular, there is a possibility that each σ^k-component vanishes separately owing to contributions from particles whose origins are not well-specified yet.

The Ramond–Ramond tadpole cancellation conditions for $k = 0, 3, 6, 9$, Eqs. (103) and (104) are not obtained from the triangle anomaly cancellation [40]. However, Eq. (103)[48] is a condition for pure gauge box anomaly cancellation, assuming the massive spectrum in the Type IIB string theory. It will be easily guessed from discussion in subsection 6.2. Since the infinite towers of string excitations winding in the z_3-direction on D7-branes and D3-branes behave as Kaluza–Klein towers through T-duality, gauge theories on those branes effectively extend in ten-dimensional and six-dimensional space-time, respectively. Thus, one has to consider the cancellation of (the irreducible part of) pure gauge box anomalies. Equation

[46] The Ref. [40] does not require that each component of triangle anomalies separately vanish. Requiring only vanishing total triangle anomalies integrated over the orbifold is sufficient to derive the Eq. (107) as far as the triangle anomalies are concerned.

[47] The authors thank M. Cvetic for useful discussion.

[48] Equation (104) is trivially satisfied as long as one takes the $\gamma_{\sigma;D7}$ and $\gamma_{\sigma;D3}$ as in (41) and (57).

(103) ensures that we can cancel that anomaly using the anomaly inflow to the singularities. In the above situation, where the winding modes in the z_3-direction play an important role, all the fields on D7-branes and D3-branes, whatever the z_3-coordinates are, contribute to the same box anomalies on the six-dimensional singularities; z_3-coordinates of initial points and end points of strings are no longer important in the sense of local field theories when they can wind around the torus of the z_3-direction. Therefore, all thirty-two D7-branes and all the D3-branes in a given six-dimensional singularity contribute to Eq. (103).

However, this condition depends highly on the spectrum above the cut-off scale (including the existence of the winding modes), and on the configuration of D7- and D3-branes that are away from the fixed loci at $z_3 = \pm(\mathbf{e}_8 + 2\mathbf{e}_9)/3$. This is the reason why we do not take this condition, Eq. (103), seriously in our generic study based on supergravity. The same thing is expressed in another way also in subsection 6.2.2.

7.3 Discrete R Symmetry

The matter contents obtained from D-branes are the whole $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector and three chiral multiplets $(\Sigma_1)^7{}_6$, $(\Sigma_2)^i{}_7$ and $(\Sigma_3)^6{}_i$. We regard the rotational symmetry $z_b \mapsto e^{i\alpha_b} z_b$ of $\mathbf{C}^3$ with $\alpha_1 = -\alpha_2 = \alpha_3$ as the principal origin of the (mod 4)-R symmetry. Note that the (mod 4)-subgroup is preserved by the geometry, since it is generated by the rotation of three complex planes by an angle π. The zero modes have the following R charges under this rotation: 2 for $(X_3)^\alpha{}_\beta$, 0 for $\bar{Q}^k{}_\alpha$ and $Q^\alpha{}_k$, 2 for $(\Sigma_1)^7{}_6$, -2 for $(\Sigma_2)^i{}_7$ and 2 for $(\Sigma_3)^6{}_i$.

As shown in the next subsection 7.4, it is reasonable to identify the chiral multiplet $(\Sigma_3)^6{}_i$ with one of the Higgs multiplets $\bar{H}_i(\mathbf{5}^*)$. Then, the R charge of the $(\Sigma_3)^6{}_i$ should be 0 mod 4, while the charge obtained from the rotation is 2. Thus, we consider that the (mod 4)-R symmetry is a suitable linear combination of the rotational symmetry and anomaly-free $\mathrm{U}(1)_{6+7}$ symmetry[49], so that the $(\Sigma_3)^6{}_i$ has R charge 0. Then, it follows that $(\Sigma_2)^i{}_7$ also has R charge 0. The $(\Sigma_2)^i{}_7$ has the same $\mathrm{SU}(5)_{\mathrm{GUT}}$ charge and the same R charge as the Higgs $H^i(\mathbf{5})$. Therefore, we identify the $(\Sigma_2)^i{}_7$ with the Higgs multiplet $H^i(\mathbf{5})$. The R charges (mod 4) of all the zero modes are now obtained exactly as in Table 2, including those of the $\bar{Q}^6{}_\alpha$ and $Q^\alpha{}_6$. We also note here that the $\mathrm{SU}(5)_{\mathrm{GUT}}$-singlet $(\Sigma_1)^7{}_6$ has R charge 2.

[49] The (mod 4)-R symmetry can be a linear combination of the $\mathrm{U}(1)_{\mathrm{H}}$ and the $\mathrm{U}(1)_5$ symmetry in addition to the geometric rotation and the $\mathrm{U}(1)_{6+7}$. We do not exclude this possibility. The choice made in the text is just to simplify the description.

7.4 Toward a Realistic Model

We have obtained the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(1)_5\times\mathrm{U}(1)_{6+7}$ vector multiplet and three chiral multiplets $S \equiv (\Sigma_1)^7{}_6$, $\bar{H}_i(\mathbf{5}^*) \equiv (\Sigma_3)^6{}_i$ and $H^i(\mathbf{5}) \equiv (\Sigma_2)^i{}_7$ from the D7-branes. The $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector is exactly obtained on the D3-branes. Interactions determined by extended SUSY provide tree-level interactions of these (Kaluza–Klein zero mode) fields. Some of them are written in the superpotential as:

$$W = \sqrt{2}g_{\mathrm{H}}\bar{Q}^i{}_\alpha(X_3)^\alpha{}_\beta Q^\alpha{}_i + \sqrt{2}g_{\mathrm{H}}\bar{Q}^6{}_\alpha(X_3)^\alpha{}_\beta Q^\alpha{}_6 \tag{108}$$

$$+\sqrt{2}g_{\mathrm{GUT}}Q^\alpha{}_6(\Sigma_3)^6{}_i\bar{Q}^i{}_\alpha + \sqrt{2}g_{\mathrm{GUT}}(\Sigma_1)^7{}_6(\Sigma_3)^6{}_i(\Sigma_2)^i{}_7. \tag{109}$$

The first line is the $\mathcal{N}=2$ SUSY interaction in (1), whose natural explanation is one of the main purposes of our higher-dimensional construction. We identify the $(\Sigma_3)^6{}_i$ as one of the Higgs multiplets $\bar{H}_i(\mathbf{5}^*)$, because the first term in the second line gives the first term of the fourth line of the superpotential (1). The last term implies that there exists a trilinear term

$$W = \sqrt{2}g_{\mathrm{GUT}}S\bar{H}_iH^i. \tag{110}$$

All particle contents have been obtained, except for three families of quarks and leptons, $\mathbf{5}^*$+$\mathbf{10}$. They are introduced at the fixed point $\mathbf{y}'=\mathbf{0}$, just as in the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(2)_{\mathrm{H}}$ model. Only one linear combination of the $\mathrm{U}(1)_5$ and the $\mathrm{U}(1)_{6+7}$ gauge groups is expected to be free from mixed anomaly with the $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge group in the presence of quarks and leptons. It should be the $\mathrm{U}(1)_{\text{fiveness}}$. The other candidate of the anomaly-free gauge symmetry is the (mod 4)-R symmetry discussed in the previous subsection 7.3. This symmetry is a linear combination of the rotational symmetry of the extra dimensions and $\mathrm{U}(1)_{6+7}$. It was discovered in Ref. [20] that this symmetry has vanishing mixed anomalies, not only with the $\mathrm{SU}(3)_{\mathrm{H}}$ gauge group but also with the $\mathrm{SU}(5)_{\mathrm{GUT}}$ gauge group, provided there is an extra pair of $\mathrm{SU}(5)_{\mathrm{GUT}}$-($\mathbf{5}$+$\mathbf{5}^*$) chiral multiplets. In the presence of these extra particles, this anomaly-free discrete gauge R symmetry can be kept unbroken at low energies, until the vacuum condensation of the superpotential breaks it. We consider that other linear combinations are anomalous, and that their anomalies will be cancelled by the generalized Green–Schwrz mechanism or rather simply spontaneously broken. Thus, those symmetries are not preserved at low energies. In the absence of the extra pair of $\mathrm{SU}(5)_{\mathrm{GUT}}$-($\mathbf{5}$+$\mathbf{5}^*$), the (mod 4)-R symmetry is also anomalous, whose anomaly is also cancelled by the generalized Green–Schwarz mechanism.

Yukawa couplings of quarks and leptons and a coloured Higgs mass term $W = h\bar{Q}^6{}_\alpha Q^\alpha{}_i H^i$ are expected to be generated through non-perturbative effects. We cannot estimate the

Yukawa couplings since we do not know the dynamics that generates these couplings. We expect all terms allowed by symmetries, namely the $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(3)_{\mathrm{H}}$ gauge symmetry, the (mod 4)-R symmetry, and $\mathrm{U}(1)_{\text{fiveness}}$ (which is assumed to be spontaneously broken at some intermediate scale), are generated dynamically.

The symmetries listed in the previous paragraph allows a superpotential

$$W = \lambda S^3 + m^2 S. \tag{111}$$

However, the order of magnitude of m does not allow any expectation since it highly depends on UV the cut-off.

Although both Higgs multiplets $\bar{H}_i(\mathbf{5}^*)$ and $H^i(\mathbf{5})$ originate from higher-dimensional polarizations of the U(7) gauge fields, the U(7) gauge transformation, which causes inhomogeneous shifts also to the higher-dimensional polarizations, does not prevent the Yukawa couplings from being generated. Both Higgs multiplets do not transform inhomogeneously under the U(7) gauge transformation, since they are zero modes, although Kaluza–Klein excitations do. Therefore, the Yukawa couplings can be generated and can be finite in the effective action below the compactification scale, where the spontaneous breaking of higher-dimensional Lorentz symmetry is already taken into account.

8 Summary and Phenomenological Consequences

The product-group unification constructed in four-dimensional space-time has been proposed to solve the doublet-triplet mass splitting problem in SUSY GUT's, which has a number of interesting features. Models use product group as a "unified gauge group" with strong gauge coupling constants for extra gauge groups. The $\mathcal{N} = 2$ SUSY is necessary to maintain the strong coupling, and the structures of the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sectors of these models accommodate the $\mathcal{N} = 2$ SUSY. The cut-off scale of the models should lie somewhat lower than the Planck scale. Finally, the symmetry principle of these models, the (mod 4) R symmetry, can be a discrete gauge symmetry, shedding some light on the required 10^{-14} precision to keep light Higgs doublets at the TeV scale.

All these features can be naturally explained when these models are embedded into an extra-dimensional space with extended SUSY, where the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector is expected to be localized on a point in the extra dimensions.

We have considered in this paper that the above localization mechanism of the SUSY gauge theories is realized on solitonic solutions of the ten-dimensional Type IIB supergravity.

Although the localization of a particular SUSY gauge theory predicted by the Type IIB string theory is not perfectly proved within the Type IIB supergravity, we assume that the same massless contents are realized on the D3–D7 system as in the Type IIB string theory. The D3–D7 system preserves $\mathcal{N} = 2$ SUSY before the orbifold projection condition is imposed, which is necessary in the models.

We have pursued the basic idea that the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector with $\mathcal{N} = 2$ SUSY is realized on the D3–D7 system. We have shown that the whole of the sector is obtained from D-brane fluctuations together with the $\mathcal{N} = 2$ SUSY, while the whole system has only $\mathcal{N} = 1$ SUSY; $\mathbf{T}^6/(\mathbf{Z}_{12}\langle\sigma\rangle \times \mathbf{Z}_2\langle\Omega R_{89}\rangle)$ is adopted as the compactified manifold. Quarks and leptons are assumed to reside on one of the fixed points, since they are not obtained from D-brane fluctuations. Anomalies are suitably cancelled within the framework of field theories. The R charges are suitably obtained for particles that are identified with the D-brane fluctuations.

We finally summarize a couple of phenomenological consequences of these models. The first issue is the proton decay. The analysis of Refs. [16, 17] has made two assumptions. One is the approximate $\mathcal{N} = 2$ SUSY relations in Eqs. (5) and (8), and the other is the absence of a tree-level contribution that involves $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking VEV such as the second term in (6). Both assumptions are justified in our construction, because the $\mathcal{N} = 2$ SUSY is a symmetry at short distances, and the second term in (6) has an extra suppression factor of 10^{-2} relative to the first term in (6) because the first term comes from the whole D7-branes, while the second term is only on D3-branes.

Threshold corrections to the MSSM gauge coupling constants arising from particles in the $\mathrm{SU}(5)_{\mathrm{GUT}}$-breaking sector almost cancel each other in both the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(2)_{\mathrm{H}}$ and the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(3)_{\mathrm{H}}$ models. This is because of the approximate $\mathcal{N} = 2$ SUSY relation. Cancellation of the threshold corrections enables one to estimate the GUT gauge boson mass, leading to a prediction of the lifetime of the proton. Typically $\tau(p \to e^+\pi^0) \simeq (3 - 10) \times 10^{34}$ yrs is the prediction common to both models [16, 17][50], which is a fairly short lifetime compared with the typical prediction for ordinary grand unified theories, $\tau \simeq 10^{36}$yrs. Although there is an SU(5) unification model [24, 69] that also predicts a short lifetime of the proton (typically $\tau \sim 10^{34}$ yrs), their model and the present models can be distinguished experimentally because all the decay modes $p \to e^+\pi^0, \mu^+\pi^0, e^+K^0, \mu^+K^0, \pi^+\bar{\nu}, K^+\bar{\nu}$ can have sizeable branching ratios in [69], while the standard decay mode $p \to e^+\pi^0$ is the dominant

[50]The analysis in [16, 17] is based on models in four-dimensional space-time. Although the higher-dimensional effects would not change the prediction very much, a detailed analysis of their effects will be given elsewhere [68].

one in the models we discuss in this paper.

The second issue is the gaugino mass. This mass does not necessarily satisfy the SU(5) GUT relation [70], since there are contributions from masses of $\mathrm{SU}(2)_{\mathrm{H}}\times\mathrm{U}(1)_{\mathrm{H}}$ (or $\mathrm{SU}(3)_{\mathrm{H}}\times\mathrm{U}(1)_{\mathrm{H}}$) gauginos. We cannot determine the gaugino masses without fixing how the SUSY breaking is mediated, however. Contact interaction between the $\mathrm{U}(2)_{\mathrm{H}}$ ($\mathrm{U}(3)_{\mathrm{H}}$) vector multiplet and chiral multiplets carrying the SUSY-breaking F-term VEV is, in general, forbidden by local $\mathcal{N}=2$ SUSY, and then gaugino masses only come from the $\mathrm{SU}(5)_{\mathrm{GUT}}$, and the SU(5) GUT relation is almost satisfied. However, such an $\mathcal{N}=2$ SUSY-violating interaction can be generated in an effective action below the Kaluza–Klein scale, and hence there is no definite prediction.

The third issue is the discrete gauge R symmetry. Now the (mod 4)-R symmetry is a gauged symmetry. Although it has vanishing mixed anomaly with $\mathrm{SU}(2)_{\mathrm{H}}$ or $\mathrm{SU}(3)_{\mathrm{H}}$ gauge group, the mixed anomaly (mod 4 R)$[\mathrm{SU}(5)_{\mathrm{GUT}}]^2$ does not vanish. This anomaly might be cancelled through the generalized Green–Schwarz mechanism, or otherwise, new $\mathrm{SU}(5)_{\mathrm{GUT}}$ charged particles are required. Those particles do not have masses without SUSY breaking, which breaks the (mod 4)-R symmetry down to R parity, and hence they are expected, if they exist, around the TeV scale [20].

Finally, there are possibilities that gauge-singlet particles exist (moduli) with masses of the order of the TeV scale. The Kaluza–Klein zero modes that survive the orbifold projection become moduli fields, unless they have mass partners whose R charges are 2. Those particles may have interesting implications in the thermal history of the Universe. Another possibility is the gauge singlet $S \equiv (\Sigma_1)^7{}_6$ in the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(3)_{\mathrm{H}}$ model, which is characterized by its trilinear coupling with the two Higgs doublets in (110). This particle remains in the low-energy spectrum as long as there is no mass partner having R charge 0. If the tadpole term is not generated for the gauge singlet field S, then the model becomes the so-called next-to-minimal SSM [71].

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References

[1] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974) 438.

[2] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D **24** (1981) 1681.

[3] U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B **260** (1991) 447;
P. Langacker and M. x. Luo, Phys. Rev. D **44** (1991) 817;
J. R. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B **260** (1991) 131.

[4] N. Sakai and T. Yanagida, Nucl. Phys. B **197** (1982) 533 ;
S. Weinberg, Phys. Rev. D **26** (1982) 287.

[5] J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B **402** (1993) 46 [arXiv:hep-ph/9207279];
T. Goto and T. Nihei, Phys. Rev. D **59** (1999) 115009 [arXiv:hep-ph/9808255];
H. Murayama and A. Pierce, Phys. Rev. D **65** (2002) 055009 [arXiv:hep-ph/0108104].

[6] T. Yanagida, Phys. Lett. B **344** (1995) 211 [arXiv:hep-ph/9409329].

[7] T. Hotta, K. I. Izawa and T. Yanagida, Phys. Rev. D **53** (1996) 3913 [arXiv:hep-ph/9509201], and D **54** (1996) 6970 [arXiv:hep-ph/9602439].

[8] J. Hisano and T. Yanagida, Mod. Phys. Lett. A **10** (1995) 3097 [arXiv:hep-ph/9510277].

[9] K. I. Izawa and T. Yanagida, Prog. Theor. Phys. **97** (1997) 913 [hep-ph/9703350].

[10] A. Masiero, D. V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. B **115** (1982) 380.

[11] R. Barbieri, G. R. Dvali and A. Strumia, Phys. Lett. B **333** (1994) 79 [arXiv:hep-ph/9404278].

[12] K. I. Izawa and T. Yanagida, Prog. Theor. Phys. **99** (1998) 423 [arXiv:hep-ph/9710218].

[13] E. Witten, arXiv:hep-ph/0201018.

[14] Y. Imamura, T. Watari and T. Yanagida, Phys. Rev. D **64** (2001) 065023 [arXiv:hep-ph/0103251].

[15] T. Watari and T. Yanagida, Phys. Lett. B **520** (2001) 322 [arXiv:hep-ph/0108057].

[16] M. Fujii and T. Watari, Phys. Lett. B **527** (2002) 106 [arXiv:hep-ph/0112152].

[17] M. Fujii and T. Watari, *in Proc.* Workshop on Neutrino Oscillation and its Origin (NOON)2001, to be published by World Scientific.

[18] C. Csaki, G. D. Kribs and J. Terning, Phys. Rev. D **65** (2002) 015004 [arXiv:hep-ph/0107266].

[19] G. F. Giudice and A. Masiero, Phys. Lett. B **206** (1988) 480.

[20] K. Kurosawa, N. Maru and T. Yanagida, Phys. Lett. B **512** (2001) 203 [hep-ph/0105136].

[21] L. E. Ibanez and G. G. Ross, Phys. Lett. B **260** (1991) 291, and Nucl. Phys. B **368** (1992) 3. L. E. Ibanez, Nucl. Phys. B **398** (1993) 301 [arXiv:hep-ph/9210211].

[22] T. Banks and M. Dine, Phys. Rev. D **45** (1992) 1424 [arXiv:hep-th/9109045].

[23] E. Witten, Nucl. Phys. B **258** (1985) 75.

[24] L. J. Hall and Y. Nomura, Phys. Rev. D **64** (2001) 055003 [arXiv:hep-ph/0103125].

[25] I. Antoniadis, H. Partouche and T. R. Taylor, Phys. Lett. B **372** (1996) 83 [arXiv:hep-th/9512006].

[26] M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory* (Cambridge University Press, UK, 1987).

[27] G. T. Horowitz and A. Strominger, Nucl. Phys. B **360** (1991) 197.

[28] J. H. Schwarz, Phys. Lett. B **360** (1995) 13 [Erratum, ibid. B **364** (1995) 252] [arXiv:hep-th/9508143].

[29] J. Polchinski, *String Theory* (Cambridge University Press, UK, 1998).

[30] For a review article, see A. Giveon and D. Kutasov, Rev. Mod. Phys. **71** (1999) 983 [arXiv:hep-th/9802067].

[31] C. M. Hull and P. K. Townsend, Nucl. Phys. B **438** (1995) 109 [arXiv:hep-th/9410167]; E. Witten, Nucl. Phys. B **443** (1995) 85 [arXiv:hep-th/9503124]; Ref. [28], J. H. Schwarz, Nucl. Phys. Proc. Suppl. **55B** (1997) 1 [arXiv:hep-th/9607201]; P. K. Townsend, arXiv:hep-th/9612121, and arXiv:hep-th/9712004; C. Vafa, arXiv:hep-th/9702201;

[32] J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231 [Int. J. Theor. Phys. **38** (1999) 1113] [arXiv:hep-th/9711200].

[33] For a review areticle, see O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Phys. Rep. **323** (2000) 183 [arXiv:hep-th/9905111].

[34] J. P. Gauntlett, R. Portugues, D. Tong and P. K. Townsend, Phys. Rev. D **63** (2001) 085002 [arXiv:hep-th/0008221].

[35] M. B. Green and J. H. Schwarz, Phys. Lett. B **149** (1984) 117.

[36] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B **289** (1987) 589.

[37] E. G. Gimon and J. Polchinski, Phys. Rev. D **54** (1996) 1667 [arXiv:hep-th/9601038].

[38] G. Aldazabal, A. Font, L. E. Ibanez and G. Violero, Nucl. Phys. B **536** (1998) 29 [arXiv:hep-th/9804026].

[39] Y. Cai and J. Polchinski, Nucl. Phys. B **269** (1988) 91;
J. Blum and K. Intrilligator, Nucl. Phys. B **506** (1997) 223 [arXiv:hep-th/9705030], Nucl. Phys. B **506** (1997) 199 [arXiv:hep-th/9705044];
R. G. Leigh and M. Rozali, Phys. Rev. D **59** (1999) 026004 [arXiv:hep-th/9807082];
J. Park and A. Uranga, Nucl. Phys. B **542** (1999) 139 [arXiv:hep-th/9808161].

[40] G. Aldazabal, D. Badagnani, L. E. Ibanez and A. M. Uranga, JHEP **9906** (1999) 031 [arXiv:hep-th/9904071].

[41] L. E. Ibanez, R. Rabadan and A. M. Uranga, Nucl. Phys. B **542** (1999) 112 [arXiv:hep-th/9808139].

[42] For example, C. G. Callan, J. A. Harvey and A. Strominger, arXiv:hep-th/9112030.

[43] For example, J. Polchinski, S. Chaudhuri and C. V. Johnson, arXiv:hep-th/9602052.

[44] A. Sen, JHEP **9709** (1997) 001 [arXiv:hep-th/9707123].

[45] L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B **274** (1986) 285.

[46] P. Candelas, G. T. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B **258** (1985) 46.

[47] Ref. [35]; M. B. Green and J. H. Schwarz, Phys. Lett. B **151** (1985) 21.

[48] C. Vafa, Nucl. Phys. B **469** (1996) 403 [arXiv:hep-th/9602022].

[49] A. Sen, Nucl. Phys. B **475** (1996) 562 [arXiv:hep-th/9605150];
T. Banks, M. R. Douglas and N. Seiberg, Phys. Lett. B **387** (1996) 278 [arXiv:hep-th/9605199].

[50] N. Seiberg, Phys. Lett. B **384** (1996) 81 [arXiv:hep-th/9606017].

[51] M. R. Douglas and G. W. Moore, arXiv:hep-th/9603167.

[52] L. E. Ibanez, F. Marchesano and R. Rabadan, JHEP **0111** (2001) 002 [arXiv:hep-th/0105155].

[53] M. Cvetic, G. Shiu and A. M. Uranga, Phys. Rev. Lett. **87** (2001) 201801 [arXiv:hep-th/0107143], Nucl. Phys. B **615** (2001) 3 [arXive:hep-th/0107166].

[54] J. R. Ellis, P. Kanti and D. V. Nanopoulos, arXiv:hep-th/0206087.

[55] M. Berkooz, M. R. Douglas and R. G. Leigh, Nucl. Phys. B **480** (1996) 265 [arXiv:hep-th/9606139].

[56] G. 't Hooft, Phys. Rev. D **14** (1976) 3432 [Erratum, ibid. D **18** (1978) 2199].

[57] C. G. Callan and J. A. Harvey, Nucl. Phys. B **250** (1985) 427.

[58] M. R. Douglas, arXiv:hep-th/9512077.

[59] E. Witten, Nucl. Phys. B **463** (1996) 383 [arXiv:hep-th/9512219].

[60] M. B. Green, J. A. Harvey and G. W. Moore, Class. Quant. Grav. **14** (1997) 47 [arXive:hep-th/9605033].

[61] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B **516** (2001) 395 [arXiv:hep-th/0103135].

[62] K. Fujikawa, Phys. Rev. Lett. **42** (1979) 1195.

[63] R. Barbieri, R. Contino, P. Creminelli, R. Rattazzi and C. A. Scrucca, arXiv:hep-th/0203039.

[64] E. Dudas and J. Mourad, Phys. Lett. B **486** (2000) 172 [arXive:hep-th/0004165];
R. Blumenhagen and A. Font, Nucl. Phys. B **599** (2001) 241 [arXive:hep-th/0011269].

[65] T. Yanagida, *in* Proc. Workshop on the Unified Theory and the Baryon Number in the Universe, Tsukuba, 1979, eds. O. Sawada and S. Sugamoto, Report KEK-79-18 (1979);
M. Gell-mann, P. Ramond and R. Slansky, *in* "Supergravity" North-Holland, Amsterdam, 1979, eds. D.Z. Freedman and P. van Nieuwenhuizen.

[66] R. Barbieri, S. Ferrara, L. Maiani, F. Palumbo and C. A. Savoy, Phys. Lett. B **115** (1982) 212;
B. de Wit, P. G. Lauwers and A. Van Proeyen, Nucl. Phys. B **255** (1985) 569. P. C. Argyres, M. Ronen Plesser and N. Seiberg, Nucl. Phys. B **471** (1996) 159 [arXiv:hep-th/9603042].

[67] M. Cvetic, A. M. Uranga and J. Wang, Nucl. Phys. B **595** (2001) 63 [arXiv:hep-th/0010091].

[68] M. Fujii, M. Ibe and T. Watari, in preparation.

[69] L. J. Hall and Y. Nomura, arXiv:hep-ph/0205067.

[70] N. Arkani-Hamed, H. C. Cheng and T. Moroi, Phys. Lett. B **387** (1996) 529 [arXiv:hep-ph/9607463].

[71] J. R. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D **39** (1989) 844, and references therein.

Fields	$Q, \bar{U}, \bar{E}$	$\bar{D}, L$	H_u	$\bar{H}_d$
R charges (mod 4)	$1 - n/5$	$1 + 3n/5$	$0 + 2n/5$	$0 - 2n/5$

Table 1: R charges (mod 4) of the fields in the MSSM are given here. n is an arbitrary integer.

Fields	$\mathbf{10}^{ij}$	$\mathbf{5}^*_i$	$H(\mathbf{5})^i$	$\bar{H}(\mathbf{5}^*)_i$	X	$Q_i, \bar{Q}^i$	Q_6	$\bar{Q}^6$
R charges	1	1	0	0	2	0	2	-2

Table 2: R charges (mod 4) of the fields in the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(3)_{\mathrm{H}}$ model are given here.

Fields	$\mathbf{10}^{ij}$	$\mathbf{5}^*_i$	X	$Q_i, \bar{Q}^i$	$Q_6, \bar{Q}^6$
R charges	1	1	2	0	0

Table 3: R charges (mod 4) of the fields in the $\mathrm{SU}(5)_{\mathrm{GUT}}\times\mathrm{U}(2)_{\mathrm{H}}$ model are given here.