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Exact anomalous dimensions for $\mathcal{N}=2$ ADE SCFTs

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Abstract: We consider four-dimensional $\mathcal{N}=2$ superconformal field theories based on *ADE* quiver diagrams. We use the procedure of [3] and compute the exact anomalous dimensions of operators with large $\mathrm{U}(1)_R$ charge to all orders in perturbation in the planar limit. The results are in agreement with the string computation in the dual pp-wave backgrounds.

Keywords: Penrose limit and pp-wave background, Extended Supersymmetry.

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1. Introduction

The pp-wave background has received much attention recently since it can be obtained by taking a Penrose limit of $\mathrm{AdS}_5 \times S^5$ [1]. Via the AdS/CFT correspondence (for a review, see [2]), it corresponds to a particular limit of $\mathcal{N} = 4$ SYM theory, where the number of colors N is taken to infinity, the Yang-Mills coupling g_{YM} is kept fixed and one considers operators with infinite R-charge J such that $J/\sqrt{N}$ is fixed. Since string theory on a pp-wave background is solvable, it was argued in [1] that the anomalous dimensions of a class of operators with large R-charge can be predicted to all orders in perturbation in the planar limit.

Obviously, it would be nice if the string theory predictions can be confirmed on the CFT side. For $\mathcal{N} = 4$ SYM theory, the authors of [3] performed an exact computation of the anomalous dimensions. The results were in agreement with the string theory analysis.

The aim of this letter is to perform a similar computation for $\mathcal{N} = 2$ superconformal field theories (SCFTs) based on ADE quiver diagrams. These theories are conjectured to be dual to IIB string on $\mathrm{AdS}_5 \times S^5/\Gamma$ [4] where Γ is one of the ADE finite subgroup of SU(2).

The letter is organized as follows. In section 2 we describe the construction of the $\mathcal{N} = 2$ quiver gauge theories. In section 3 we compute the exact spectrum of anomalous dimensions.

2. Quiver gauge theories

The quiver gauge theories can be constructed by considering $N|\Gamma(G)|$ D3-branes on the covering space of $\mathbb{C}^2/\Gamma(G)$ and performing a $\Gamma(G)$ projection on the worldvolume fields and the Chan-Paton factors [5, 6, 7]. We start with $\mathcal{N} = 4$ SYM theory with the gauge group $\mathrm{SU}(N|\Gamma(G)|)$ and the action given by

$$\begin{aligned} S = &\int d^4x\, d^4\theta\, \mathrm{tr}\left(e^{-g_{\mathrm{YM}}\boldsymbol{V}}\bar{\boldsymbol{\Phi}}_i\, e^{g_{\mathrm{YM}}\boldsymbol{V}}\boldsymbol{\Phi}^i\right) + \\ &+ \frac{1}{2g_{\mathrm{YM}}^2}\int d^4x\, d^2\theta\, \mathrm{tr}\left(\boldsymbol{W}^\alpha \boldsymbol{W}_\alpha\right) + \frac{ig_{\mathrm{YM}}}{3!}\int d^4x\, d^2\theta\, \epsilon_{ijk}\, \mathrm{tr}\left(\boldsymbol{\Phi}^i[\boldsymbol{\Phi}^j, \boldsymbol{\Phi}^k]\right) + \mathrm{c.c.} \end{aligned} \tag{2.1}$$

The $\mathcal{N}=1$ vector superfield $\boldsymbol{V}$ and chiral superfields $\boldsymbol{\Phi}^i$, $i=1,2,3$ belong to the adjoint representation of $\mathrm{SU}(N|\Gamma(G)|)$. $g_{\mathrm{YM}}^2 = 4\pi g_s$ is the gauge coupling of the $\mathcal{N}=4$ SYM. We follow the notations adopted in [3]. The manifest global symmetries of this action are

$$\begin{aligned} \mathrm{U}(1)_X: &\quad \boldsymbol{W}(\theta) \to e^{i\alpha}\boldsymbol{W}(e^{-i\alpha}\theta),\ \boldsymbol{\Phi}^i(\theta) \to e^{i\frac{2}{3}\alpha}\boldsymbol{\Phi}^i(e^{-i\alpha}\theta), \\ \mathrm{SU}(3): &\quad \boldsymbol{W}(\theta) \to \boldsymbol{W}(\theta), \quad \boldsymbol{\Phi}^i(\theta) \to U^i_j\, \boldsymbol{\Phi}^j(\theta)\,. \end{aligned} \tag{2.2}$$

We denote $\boldsymbol{\Phi} \equiv \boldsymbol{\Phi}^1$, $\boldsymbol{\Psi} \equiv \boldsymbol{\Phi}^2$, $\boldsymbol{Z} \equiv \boldsymbol{\Phi}^3$.

The orbifold group $\Gamma(G)$ is a subgroup

$$\Gamma(G) \subset \mathrm{SU}(2) \subset \mathrm{SU}(3)\,. \tag{2.3}$$

By this orbifolding, $\mathcal{N}=4$ SUSY is broken to $\mathcal{N}=2$ SUSY where the $\mathrm{U}(1)_R$ symmetry is an appropriate linear combination of $\mathrm{U}(1)_X$ and $\mathrm{U}(1)'$. $\mathrm{U}(1)'$ is the normal subgroup to $\mathrm{SU}(2) \subset \mathrm{SU}(3)$. There is also an $\mathrm{SU}(2)_R$ symmetry under which the scalar components of $\boldsymbol{\Phi}, \bar{\boldsymbol{\Psi}}$ transform as a doublet. The orbifolding for the worldvolume fields is implemented by the conditions

$$R\boldsymbol{W}R^{-1} = \boldsymbol{W}\,, \qquad R\boldsymbol{Z}R^{-1} = \boldsymbol{Z}\,, \qquad \begin{pmatrix} R\boldsymbol{\Phi}R^{-1} \\ R\boldsymbol{\Psi}R^{-1} \end{pmatrix} = Q \begin{pmatrix} \boldsymbol{\Phi} \\ \boldsymbol{\Psi} \end{pmatrix}. \tag{2.4}$$

R is the $N|\Gamma(G)|$-dimensional regular representation of $\Gamma(G)$ which can be decomposed as $\oplus_{i=1}^r (n_i r_i \otimes I_N)$ with r_i being the irreducible representations of $\Gamma(G)$. $r = \mathrm{rank}G$, and Q is a two-dimensional representation of $\Gamma(G)$.

2.1 A_{k-1} orbifolds

Consider $\Gamma(A_{k-1}) = \mathbb{Z}_k$. R can be taken to be $\mathrm{diag}(1, \omega^{-1}, \omega^{-2}, \ldots, \omega^{-k+1})$ where $\omega = e^{2\pi i/k}$ and each block is proportional to an $N \times N$ unit matrix. Q is the 2×2 matrix of the form $\mathrm{diag}(\omega, \omega^{-1})$. It is easy to verify that the following components survive the projections

$$\begin{aligned} \boldsymbol{W} &= \begin{pmatrix} W_0 & & & \\ & W_1 & & \\ & & \ddots & \\ & & & W_{k-1} \end{pmatrix} & \boldsymbol{Z} &= \begin{pmatrix} Z_0 & & & \\ & Z_1 & & \\ & & \ddots & \\ & & & Z_{k-1} \end{pmatrix} \\ \boldsymbol{\Phi} &= \begin{pmatrix} 0 & \Phi_{01} & & \\ & 0 & \Phi_{12} & \\ & & \ddots & \ddots \\ \Phi_{k-1,0} & & & \end{pmatrix} & \boldsymbol{\Psi} &= \begin{pmatrix} 0 & & & \Psi_{k-1,0} \\ \Psi_{01} & 0 & & \\ & \Psi_{12} & \ddots & \\ & & \ddots & 0 \end{pmatrix} \end{aligned} \tag{2.5}$$

The resulting theory is an $\prod_{i=0}^{k-1} \mathrm{SU}(N)_i$ gauge theory with the $\mathcal{N}=1$ chiral superfields $\Phi_{i,i+1}$ in the $(\mathbf{N}, \bar{\mathbf{N}})$ representation of $\mathrm{SU}(N)_i \times \mathrm{SU}(N)_{i+1}$, and $\Psi_{i,i+1}$ in the $(\bar{\mathbf{N}}, \mathbf{N})$ representation of $\mathrm{SU}(N)_i \times \mathrm{SU}(N)_{i+1}$.[1]

[1] The sub-index k is equivalent to 0.

2.2 D, E orbifolds

$\Gamma(D_k) = \mathbf{D}_{k-2}$ is the binary extension of the dihedral group of order $4(k-2)$. $\Gamma(E_6) = \mathcal{T}$, $\Gamma(E_7) = \mathcal{O}$ and $\Gamma(E_8) = \mathcal{I}$ are the binary tetrahedral, octahedral and icosahedral groups of order 24, 48 and 120, respectively. The gauge group and matter content of the D and E theories are worked out by the same procedure. The results are summarized in terms of quiver diagrams corresponding to the extended Dynkin diagrams of the group. The gauge group is associated with the nodes of the diagram. It is $\prod_{i=0}^{r} G_i$ with $G_i = \mathrm{SU}(n_i N)$ where n_i is the dimension of the irreducible representation of $\Gamma(G)$ corresponding to the node i. The matter content is associated with the links of the diagram. A link between the node i and j is a hypermultiplet $H_{\langle ij\rangle} = (\Phi_{\langle ij\rangle}, \bar{\Psi}_{\langle ij\rangle})$ which transforms in the bifundamental representation of $G_i \times G_j$. The $\mathcal{N} = 1$ chiral superfields in the adjoint representation of $\prod_i G_i$ which are part of the $\mathcal{N} = 2$ vector multiplets are denoted by Z_i. Recall that the $\mathrm{U}(1)_R$ charge of $\boldsymbol{Z}$ is 2 while $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ are neutral.

3. Anomalous dimensions

The action of the quiver gauge theories is given by (2.1) divided by $|\Gamma(G)|$ with only the fields that remain after the projections (2.4). The field equations read

$$\begin{aligned} \bar{D}^2\bar{\boldsymbol{\Phi}} + ig_{\mathrm{YM}}(\boldsymbol{\Psi}\boldsymbol{Z} - \boldsymbol{Z}\boldsymbol{\Psi}) &= 0\,, \\ \bar{D}^2\bar{\boldsymbol{\Psi}} + ig_{\mathrm{YM}}(\boldsymbol{Z}\boldsymbol{\Psi} - \boldsymbol{\Psi}\boldsymbol{Z}) &= 0\,, \\ \bar{D}^2\bar{\boldsymbol{Z}} + ig_{\mathrm{YM}}(\boldsymbol{\Phi}\boldsymbol{\Psi} - \boldsymbol{\Psi}\boldsymbol{\Phi}) &= 0\,. \end{aligned} \tag{3.1}$$

The propagators take the form

$$\begin{aligned} \langle Z_i(z)\bar{Z}_j(z')\rangle &= \delta_{ij}\,\frac{|\Gamma(G)|}{4\pi^2 n_i}\,\bar{D}^2\frac{\delta^4(\theta-\theta')}{|x-x'|^2}\overleftarrow{D}^2\,, \\ \langle \Phi_{\langle ij\rangle}(z)\bar{\Phi}_{\langle i'j'\rangle}(z')\rangle &= \langle \Psi_{\langle ij\rangle}(z)\bar{\Psi}_{\langle i'j'\rangle}(z')\rangle = c_{\langle ij\rangle}\delta_{ij}\delta_{i'j'}\,\frac{|\Gamma(G)|}{4\pi^2}\,\bar{D}^2\frac{\delta^4(\theta-\theta')}{|x-x'|^2}\overleftarrow{D}^2\,. \end{aligned} \tag{3.2}$$

$c_{\langle ij\rangle}$ are constants that are fixed by the action. For $G = A_{k-1}$ $c_{\langle ij\rangle} = 1$ for all $\langle ij\rangle$. For $G = D_4$ $c_{\langle ij\rangle} = 1/2$. We will not work out $c_{\langle ij\rangle}$ in general as it is not needed in the sequel.

Consider now the dictionary between CFT operators and string states on the pp-wave background [8, 9, 10].[2] We will work in the light-cone gauge. Let us discuss first the ground states $|0, 2p^+\rangle_q$, where $q = 0, 1, \ldots, r$ label the q-twisted sectors of the string theory. These ground states can be identified with the CFT operators $\mathrm{tr}(R_q\boldsymbol{Z}^J)$. R_q are representatives of the conjugacy classes of the $N|\Gamma(G)|$ dimensional regular representation of $\Gamma(G)$. R_0 corresponds to the identity. Recall that the operators depend only on the conjugacy classes by using an appropriate constant gauge transformation of $\mathrm{SU}(N|\Gamma(G)|)$. Now consider the

[2] For an earlier work on comparison of the spectrum between AdS orbifolds and the dual $\mathcal{N} = 2$ quiver gauge theories, see [11, 12].

CFT operators

$$
\begin{aligned}
\mathcal{O}_J &= \left(\frac{|\Gamma(G)|}{4\pi^2}\right)^{-(J+1)/2} N^{-J/2} \sum_l e^{i\varphi(q)l} R_q \boldsymbol{Z}^l \boldsymbol{\Phi} \boldsymbol{Z}^{J-l}\,,\\
\mathcal{U}_J &= \left(\frac{|\Gamma(G)|}{4\pi^2}\right)^{-(J+1)/2} N^{-J/2} \sum_l e^{i\varphi(q)l} R_q \boldsymbol{Z}^l \bar{\boldsymbol{\Psi}} \boldsymbol{Z}^{J-l}\,,
\end{aligned} \tag{3.3}
$$

where $\varphi(q) = 2\pi n(q)/J$. The overall normalization factors are chosen in order to get later a finite result for the two-point functions in the large N and J limit. The string state corresponding to $\mathcal{O}_J$ is obtained by acting with a creation operator in the $\mathbb{C}^2/\Gamma(G)$ part with the oscillation number $n(q)$. Note that we are working in the "dilute gas" approximation [1, 13] that is valid for $J \gg 1$. This implies that the diagrams relevant to the anomalous dimension of an operator with a large number of impurities involve only one impurity and $\boldsymbol{Z}$ fields next to the impurity field. The operators $\mathcal{O}_J$,$\mathcal{U}_J$ are the building blocks for it.

We are interested in the exact form of the anomalous dimension of $\mathcal{O}_J$, γ. To compute this, we first notice that (3.1) implies the relation

$$
\langle \bar{D}^2 \mathcal{U}_J(z) D^2 \bar{\mathcal{U}}_J(z')\rangle = -\frac{g_{\rm YM}^2 N|\Gamma(G)|}{4\pi^2}\,\alpha(q)\,\langle \mathcal{O}_{J+1}(z)\bar{\mathcal{O}}_{J+1}(z')\rangle\,, \tag{3.4}
$$

with $z = (x, \theta, \bar{\theta})$ being the $\mathcal{N} = 1$ superspace coordinates and $\alpha(q) = -(e^{i\varphi(q)}-1)(e^{-i\varphi(q)}-1)$. The computation of γ proceeds exactly the same way as in the case of $\mathcal{N} = 4$ SYM [3]. Let us first compute the two-point functions of $\mathcal{O}_{J+1}$ and $\mathcal{U}_J$. Notice that $\mathcal{O}_{J+1}$ and $\mathcal{U}_J$ consist of the blocks of the form

$$
\begin{aligned}
(\mathcal{O}_{J+1})_{\langle ij\rangle} &\equiv \left(\frac{|\Gamma(G)|}{4\pi^2}\right)^{-(J+2)/2} N^{-(J+1)/2} \sum_l e^{i\varphi(q)l} (Z_i)^l\, \Phi_{\langle ij\rangle}\,(Z_j)^{J-l+1},\\
(\mathcal{U}_J)_{\langle ij\rangle} &\equiv \left(\frac{|\Gamma(G)|}{4\pi^2}\right)^{-(J+1)/2} N^{-J/2} \sum_l e^{i\varphi(q)l} (Z_i)^l\, \bar{\Psi}_{\langle ij\rangle}\,(Z_j)^{J-l}\,.
\end{aligned} \tag{3.5}
$$

Using the propagators (3.2), the two-point functions at tree level read

$$
\begin{aligned}
\langle (\mathcal{O}_{J+1})_{\langle ij\rangle}(z)\,(\bar{\mathcal{O}}_{J+1})_{\langle ij\rangle}(z')\rangle &= c_{\langle ij\rangle}\Bigg(\bar{D}^2 D^2 - \frac{i}{4}\frac{\Delta_{\mathcal{O}}-\omega_{\mathcal{O}}}{\Delta_{\mathcal{O}}}\left[D^\alpha, \bar{D}^{\dot\alpha}\right]\partial_{\alpha\dot\alpha} -\\
&\qquad -\frac{1}{4}\frac{(\Delta_{\mathcal{O}}+\omega_{\mathcal{O}})(\Delta_{\mathcal{O}}-\omega_{\mathcal{O}})}{\Delta_{\mathcal{O}}(\Delta_{\mathcal{O}}-1)}\Box\Bigg)\frac{\delta^4(\theta-\theta')}{|x-x'|^{2\Delta_{\mathcal{O}}}}\,,\\
\langle \bar{D}^2(\mathcal{U}_J)_{\langle ij\rangle}(z)\,D^2(\bar{\mathcal{U}}_J)_{\langle ij\rangle}(z')\rangle &= (\Delta_{\mathcal{U}}-\omega_{\mathcal{U}})(\Delta_{\mathcal{U}}-\omega_{\mathcal{U}}-2)\,c_{\langle ij\rangle}\,\bar{D}^2 D^2\frac{\delta^4(\theta-\theta')}{|x-x'|^{2(\Delta_{\mathcal{U}}+1)}}\,.
\end{aligned} \tag{3.6}
$$

Here $\Delta_{\mathcal{O},\mathcal{U}}$ and $\omega_{\mathcal{O},\mathcal{U}}$ are the scaling dimensions and the chiral weights of $\mathcal{O}_{J+1}$,$\mathcal{U}_J$ and given by $\Delta = h + \bar{h}, \omega = h - \bar{h}$ with $h, \bar{h}$ being the number of chiral and anti-chiral superfields, respectively.

The two-point functions receive quantum corrections: the scaling dimensions shift by the anomalous dimensions $\Delta_{\mathcal{U},\mathcal{O}} \to \Delta_{\mathcal{U},\mathcal{O}} + \gamma_{\mathcal{U},\mathcal{O}}$, and the overall factor $c_{\langle ij\rangle}$ should be replaced by some functions of the coupling constant. Furthermore the chiral weights may change as $\omega_{\mathcal{U},\mathcal{O}} \to \omega_{\mathcal{U},\mathcal{O}} + \delta\omega_{\mathcal{U},\mathcal{O}}$. It then follows that the exact two-point functions in the planar and large J limit take the form

$$\begin{aligned}\langle \bar{D}^2(\mathcal{U}_J)_{\langle ij\rangle}(z)\, D^2(\bar{\mathcal{U}}_J)_{\langle ij\rangle}(z')\rangle &= (\gamma_{\mathcal{U}} - \delta\omega_{\mathcal{U}})(\gamma_{\mathcal{U}} - \delta\omega_{\mathcal{U}} - 2)\, f^{\mathcal{U}}_{\langle ij\rangle} \bar{D}^2 D^2 \frac{\delta^4(\theta-\theta')}{|x-x'|^{2(J+2+\gamma_{\mathcal{U}})}}, \\ \langle (\mathcal{O}_{J+1})_{\langle ij\rangle}(z)\, (\bar{\mathcal{O}}_{J+1})_{\langle ij\rangle}(z')\rangle &= f^{\mathcal{O}}_{\langle ij\rangle}\, \bar{D}^2 D^2 \frac{\delta^4(\theta-\theta')}{|x-x'|^{2(J+2+\gamma_{\mathcal{O}})}}\,, \end{aligned} \tag{3.7}$$

with $f^{\mathcal{U},\mathcal{O}}_{\langle ij\rangle} = c_{\langle ij\rangle} + O(g^2_{\mathrm{YM}}N/J^2)$. Here we assume that γ and $\delta\omega$ are of order one. We now observe that

$$\gamma_{\mathcal{U}} = \gamma_{\mathcal{O}} \equiv \gamma\,, \qquad \delta\omega_{\mathcal{U}} = \delta\omega_{\mathcal{O}} \equiv \delta\omega\,, \qquad f^{\mathcal{U}}_{\langle ij\rangle} = f^{\mathcal{O}}_{\langle ij\rangle} \equiv f_{\langle ij\rangle}\,. \tag{3.8}$$

To see this, recall that $\mathcal{O}_{J+1}$ differs from $\mathcal{O}_J$ only by the presence of an extra $\boldsymbol{Z}$-field. As pointed out in [3], this difference is irrelevant to the renormalization in the dilute gas approximation. Also since the scalar components of $\mathcal{O}_J$ and $\mathcal{U}_J$ belong to the same $\mathcal{N}=2$ multiplet, one can use $\mathrm{SU}(2)_R$ symmetry to see that $\langle \mathcal{U}_J(z)\bar{\mathcal{U}}_J(z')\rangle = \langle \mathcal{O}_J(z)\,\bar{\mathcal{O}}_J(z')\rangle$.

Plugging into (3.4), we obtain

$$(\gamma - \delta\omega)(\gamma - \delta\omega - 2) = -\frac{g^2_{\mathrm{YM}}N|\Gamma(G)|}{4\pi^2}\,\alpha(q)\,. \tag{3.9}$$

Under the assumption that $\delta\omega = 0$, the anomalous dimension in the large J limit can be solved to be

$$\gamma = -1 + \sqrt{1 + \frac{g^2_{\mathrm{YM}}N|\Gamma(G)|}{J^2}\, n(q)^2}\,. \tag{3.10}$$

When setting $g^2_{\mathrm{YM}} = 4\pi g_s$, this result coincides with the string computation by choosing $n(q)$ to be the oscillation number.

$G = A_{k-1}$. For $G = A_{k-1}$, one has $n(q) = n + q/k$ with $n \in \mathbb{Z}$ and $q = 0, 1, \ldots, k-1$.

$G = D_k$. The $4(k-2)$ elements of the two-dimensional defining representation of $\Gamma(D_k)$ consist of the matrices of the form[3]

$$F_l = \begin{pmatrix} e^{\frac{i\pi l}{k-2}} & 0 \\ 0 & e^{-\frac{i\pi l}{k-2}} \end{pmatrix}, \qquad G_l = \begin{pmatrix} 0 & ie^{-\frac{i\pi l}{k-2}} \\ ie^{\frac{i\pi l}{k-2}} & 0 \end{pmatrix}, \tag{3.11}$$

with $l = 0, 1, \ldots, 2k-5$. There are $k+1$ conjugacy classes:

- KE contains only the identity F_0.
- KZ contains the central extension $\mathcal{Z} \equiv F_{k-2} = -1$.
- KG_{e} contains the elements $G_{2\nu}$ with $\nu = 1, 2, \ldots, k-3$.
- KG_{o} contains the elements $G_{2\nu+1}$.
- There exist $k-3$ classes denoted by KF_μ, $\mu = 0, 1, \ldots, k-3$. Each of these contains the pair of the elements $\{F_\mu, F_{2(k-2)-\mu}\}$.

[3] See, for instance, [14].

The matrices in each conjugacy class set a boundary condition for the 2d fields of the orbifold CFT on $\mathbb{C}^2/\Gamma(D_k)$. One can verify that the resulting (un)twisted sectors contain string modes with the following oscillation numbers

$$n(KE) = n\,, \quad n(KZ) = n + \frac{1}{2}\,, \quad n(KG_\mathrm{e}) = n(KG_\mathrm{o}) = n + \frac{1}{4}\,, \quad n(KF_\mu) = n + \frac{\mu}{2(k-2)}\,. \tag{3.12}$$

One can work out in a similar way the explicit oscillation numbers for $G = E_k$.

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References

[1] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and PP waves from* $N=4$ *super Yang-Mills*, *J. High Energy Phys.* **04** (2002) 013 [hep-th/0202021].

[2] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

[3] A. Santambrogio and D. Zanon, *Exact anomalous dimensions of* $N=4$ *Yang-Mills operators with large R charge*, *Phys. Lett.* **B 545** (2002) 425 [hep-th/0206079].

[4] S. Kachru and E. Silverstein, *4d conformal theories and strings on orbifolds*, *Phys. Rev. Lett.* **80** (1998) 4855 [hep-th/9802183].

[5] M.R. Douglas and G.W. Moore, *D-branes, quivers and ALE instantons*, hep-th/9603167.

[6] C.V. Johnson and R.C. Myers, *Aspects of type-IIB theory on ALE spaces*, *Phys. Rev.* **D 55** (1997) 6382 [hep-th/9610140].

[7] A.E. Lawrence, N. Nekrasov and C. Vafa, *On conformal field theories in four dimensions*, *Nucl. Phys.* **B 533** (1998) 199 [hep-th/9803015].

[8] M. Alishahiha and M.M. Sheikh-Jabbari, *The PP-wave limits of orbifolded* $AdS_5 \times S^5$, *Phys. Lett.* **B 535** (2002) 328 [hep-th/0203018].

[9] N.-W. Kim, A. Pankiewicz, S.-J. Rey and S. Theisen, *Superstring on PP-wave orbifold from large-N quiver gauge theory*, *Eur. Phys. J.* **C 25** (2002) 327 [hep-th/0203080].

[10] T. Takayanagi and S. Terashima, *Strings on orbifolded PP-waves*, *J. High Energy Phys.* **06** (2002) 036 [hep-th/0203093].

[11] Y. Oz and J. Terning, *Orbifolds of* $AdS_5 \times S^5$ *and 4d conformal field theories*, *Nucl. Phys.* **B 532** (1998) 163 [hep-th/9803167].

[12] S. Gukov, *Comments on* $N=2$ *AdS orbifolds*, *Phys. Lett.* **B 439** (1998) 23 [hep-th/9806180].

[13] D.J. Gross, A. Mikhailov and R. Roiban, *Operators with large R charge in* $N=4$ *Yang-Mills theory*, *Ann. Phys. (NY)* **301** (2002) 31 [hep-th/0205066].

[14] D. Anselmi, M. Billo, P. Fre, L. Girardello and A. Zaffaroni, *ALE manifolds and conformal field theories*, *Int. J. Mod. Phys.* **A 9** (1994) 3007 [hep-th/9304135].