Perturbative Inaccessibility of Conformal Fixed Points in Nonsupersymmetric Quiver Theories

Paul H. Frampton [1,2], and Peter Minkowski [1,3]
[1] *TH Division, CERN, CH1211 Geneva 23, Switzerland, and*
[2] *University of North Carolina,Chapel Hill, NC 27599, USA.*
[3] *Institute for Theoretical Physics, University of Bern, Bern, Switzerland.*

The possibility that non-supersymmetric quiver theories may have a renormalization-group fixed point at which there is conformal invariance requires non-perturbative information.

The AdS/CFT correspondence of Maladacena [1] provides a powerful tool to study non-gravitational gauge field theories in flat *e.g.* four-dimensional spacetime. In particular, it has been suggested [2] that nonsupersymmetric quiver gauge theories obtained from compactification of the Type IIB superstring on $AdS_5 \times S^5/\Gamma$ may, for finite number N of $D3-$branes, possess a conformal fixed point at the TeV scale and that one such theory may be the correct path to go beyond the standard model of particle phenomenology.

Three years ago, in a paper [3] an argument based on perturbation theory was used to criticize this whole approach to string phenomenology. Our purpose here is not just to comment on that paper but on what we perceive to be a widespread belief that the way to approach the issue is through perturbative analysis.

Here we shall show that the S-duality of the underlying IIB superstring constrains the relevant RG beta function $\beta(\alpha)$ to suggest a conformal fixed point but that to any order of perturbation theory such a fixed point must remain inaccessible. We shall show this by an illustration of the possible form of $\beta(\alpha)$ which may be analytic around $\alpha = 0$ and $\alpha = \infty$ as a function of α with arbitrary coefficients in the Taylor expansion for small α.

Let us assume that the quiver theory has one independent coupling constant which is asymptotically free for small α so that the perturbative expansion is (here $t = \ln\mu$)

$$\beta = \frac{d\alpha}{dt} = b_0\alpha^2 + b_1\alpha^3 + b_2\alpha^4 + ... \tag{1}$$

with $b_0 < 0$. As already mentioned in [4] the underlying S-duality under $\alpha \to 1/\alpha$ implies by

$$\beta(1/\alpha) = -\frac{1}{\alpha^2}\beta(\alpha) \tag{2}$$

that $\beta(1) = 0$.

Let us illustrate the solution of Eq.(2) in the form

$$\beta(\alpha) = -\alpha(\alpha - \frac{1}{\alpha})F(\alpha) \tag{3}$$

where $F(\alpha) = +F(1/\alpha)$. In particular consider

$$F(\alpha) = \Sigma_{n=0}^{n=\infty} C_n \alpha^{n+2}(1+\alpha^{2n+4})^{-1} \tag{4}$$

Then

$$\beta(\alpha) = (1-\alpha^2)[C_0\alpha^2(1+\alpha^4)^{-1} + C_1\alpha^3(1+\alpha^6)^{-1} + ...] \tag{5}$$

so that

$$\beta(\alpha) = C_0\alpha^2 + C_1\alpha^3 + (C_2 - C_0)\alpha^4 + ... \tag{6}$$

can reproduce any expansion of the perturbative type Eq.(1) by appropriate choice of the coefficients C_n. This form for $\beta(\alpha)$ can be analytic for both $|\alpha| < 1$ and $|\alpha| > 1$ but must be singular at least somewhere on the unit circle $|\alpha| = 1$. In particular, $\beta(\alpha)$ can be analytic and vanishing at the points $\alpha = 0$ and ∞. The specific illustration chosen has a simple zero on the real axis at $\beta(1) = 0$. It is clear that analysis based on perturbative calculation of the coefficients b_n in Eq.(1) could neither prove nor disprove the existence of this zero in β.

The point is that while calculation of the perturbative expansion for $\beta(\alpha)$ in Eq.(1) may well lead to supporting evidence (but not rigorous proof) of a fixed point for weak coupling, as in *e.g.* the well-known example [5] of QCD with 16 flavors, such a calculation can never by itself confirm the presence or absence of a conformal fixed point.

The work of PHF was supported in part by the Office of High Energy, US Department of Energy under Grant No. DE-FG02-97ER41036. We thank Luis Alvarez-Gaume for a discussion.

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2,** 231 (1998). hep-th/9711200.
[2] P.H. Frampton, Phys. Rev. **D60,** 041901 (1999). hep-th/9812117.

[3] C. Csaki, W. Skiba and J. Terning, Phys. Rev. **D61,** 025019 (2000). hep-th/9906057.
[4] P.H. Frampton and C. Vafa, hep-th/9903226.
[5] T. Banks and A. Zaks, Nucl. Phys. **B196,** 189 (1982).