Universal Freeze-Out Criterion and Phase-Space Density in Ultrarelativistic Heavy-Ion Collisions

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We explore the consequences of a freeze-out criterion for ultrarelativistic heavy-ion collisions, based on pion escape probabilities from the hot and dense but rapidly expanding collision region. We study to what extent the recently observed increase in pion phase space density from SPS to RHIC can be understood by the smaller nucleon contribution to the pion scattering rate. We find that the dependence of the pion scattering rate on the momentum of the escaping particle can contribute significantly to the transverse momentum slope of transverse HBT radius parameters, and that its temperature dependence favours rather low freeze-out temperatures of 100 MeV at most.

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Hadrons at low transverse momentum form the bulk of the particles produced in ultrarelativistic heavy-ion collisions. They are produced at the end of the hot and dense partonic phase of the collision, but they subsequently undergo multiple scattering in the confined hadronic phase prior to decoupling from the collision system ("freeze-out"). Hadronic one- and two-particle spectra thus carry information about the collision region at freeze-out. In recent years, significant progress has been made in reconstructing from these spectra bulk properties of the collision at freeze-out, such as the spatial extent of the collision region, the freeze-out time, the freeze-out temperature, the average velocity and velocity gradients in the particle emitting source. A first step in gaining a dynamical understanding of the origin of these freeze-out parameters, is the study of the so-called "freeze-out criterion". This is the condition which has to be satisfied locally for decoupling to take place.

A realistic freeze-out criterion certainly depends strongly on the particle phase space density at freeze-out, since the hadronic scattering probability in a medium grows with particle density. However, the recently observed increase in average pion phase space density from SPS to RHIC [1, 2] indicates that—in contrast to an earlier suggestion [3]—this freeze-out criterion must take additional factors into account. The main purpose of the present letter is to discuss quantitatively how hadrochemical composition [4, 5], local temperature as well as collective velocity gradients [6] may influence the decoupling of hadrons from the collision region. Since hadrochemistry does, and freeze-out temperature or velocity gradients may change in going from SPS to RHIC energies, this study also provides the basis for discussing a possible dependence of hadronic freeze-out on bombarding energy.

The starting point of our study is the particle escape probability [7, 8]

$$\mathcal{P}(x,p,\tau) = \exp\left(- \int_\tau^\infty d\bar\tau\, \mathcal{R}(x + v\bar\tau, p) \right) . \qquad (1)$$

Here, v is the velocity of the escaping particle and $\mathcal{R}(x,p)$ denotes the scattering rate which is defined as the inverse of the mean time between collisions for a particle at position x with momentum p. The opacity integral in (1) integrates this scattering rate along the trajectory of the particle, starting from a time τ. As universal freeze-out criterion, we consider the condition that the escape probability $\mathcal{P}(x,p,\tau=\tau_{\rm fo})$ takes at freeze-out a characteristic *universal* value.

To illustrate how dynamical properties of the collision region enter this freeze-out criterion, we consider first a toy model for the scattering rate,

$$\mathcal{R}^{\rm toy}(x,p) = \mathcal{R}(\tau) = \sigma \rho(\tau) \bar{v}_{\rm rel} \, . \qquad (2)$$

Here, σ denotes the (averaged) cross-section for scattering within the medium, $\bar{v}_{\rm rel}$ the average velocity relative to other particles and $\rho(\tau)$ the density of scattering partners. For simplicity, we choose here a density which does not depend on the trajectory of the escaping particle but shows a power law decrease in proper time,

$$\rho(\tau) = \rho_{\rm fo} \left(\frac{\tau_{\rm fo}}{\tau} \right)^\alpha . \qquad (3)$$

This applies e.g. to particles of vanishing transverse momentum which do not propagate through layers of different density but—due to expansion—find themselves in a medium of rapidly decreasing density. To estimate the expansion parameter α, we take the four-velocity field of the collective flow at freeze-out to be a superposition of a longitudinally boost-invariant and a transverse expansion gradient,

$$\begin{aligned} u^\mu \; = \; & (\cosh\eta\, \cosh\eta_t\, , \cos\phi\, \sinh\eta_t\, , \\ & \sin\phi\, \sinh\eta_t\, , \sinh\eta\, \cosh\eta_t)\, . \end{aligned} \qquad (4)$$

Here, $\eta = \frac{1}{2} \ln\left[\frac{t+z}{t-z}\right]$ denotes the space-time rapidity. The strength of the transverse expansion gradient is controlled by the parameter ξ,

$$\eta_t = \xi r \, . \qquad (5)$$

How does the expansion parameter α in (3) depend on the flow strength ξ? The continuity equation $\partial_\mu(u^\mu \rho) = 0$

connects density and flow and leads to [9]

$$-\frac{1}{\rho}\frac{\partial\rho}{\partial\tau} = -\frac{1}{\rho}\, u^\mu \partial_\mu \rho = \partial_\mu u^\mu \,. \tag{6}$$

Inserting the model ansatz (4), (5) for u_μ, one finds

$$-\frac{1}{\rho}\, u^\mu \partial_\mu \rho = \frac{1}{\tau_{\rm fo}} + 2\xi \,, \tag{7}$$

where $\tau_{\rm fo} = \sqrt{t^2 - z^2}$ is the longitudinal proper time at freeze-out. Comparing to the ansatz (3) for the density, one finds,

$$\alpha = 1 + 2\xi\tau_{\rm fo} \,. \tag{8}$$

The opacity integral in (1) can now be determined in terms of the scattering rate $\mathcal{R}_{\rm fo} = \sigma\rho_{\rm fo}\bar{v}_{\rm rel}$ at freeze-out, and the strength ξ of the transverse flow gradient,

$$\int_{\tau_{\rm fo}}^{\infty} d\tau\, \mathcal{R}(\tau) = \frac{\mathcal{R}_{\rm fo}}{\alpha - 1}\tau_{\rm fo} = \frac{\mathcal{R}_{\rm fo}}{2\xi} \,. \tag{9}$$

We shall not try to extent this calculation to particles at finite $p_\perp$ which pass through regions of different density. We rather take the pocket formula (9) as an estimate for how the strength of the transverse expansion affects particle freeze-out in heavy-ion collisions. As seen from (9), an increase in the scattering rate $\mathcal{R}_{\rm fo}$ at freeze-out can be compensated by stronger transverse flow gradients. These lead to a faster density decrease in the collision region thus keeping the time-integrated opacity (9) constant. It is noteworthy that it is the flow *gradient* which enters here, and not the average transverse flow velocity which affects the slope of single-particle spectra. Indeed, depending on the transverse extension of the collision region, the same average transverse expansion velocity can correspond to rather different flow gradients.

After these preliminaries, we turn to calculating the scattering rate at freeze-out for the hadronic final state produced at SPS and RHIC. To be specific, we consider a pion with momentum p whose scattering rate due to interactions with particles of type i and momentum k can be written as [10]

$$\mathcal{R}(x,p;i,k) = \rho_i(x,k)\,\sigma_i(s)\,\frac{\sqrt{(s-s_a)(s-s_b)}}{2\sqrt{m_\pi^2+p^2}\sqrt{m_i^2+k^2}} \,. \tag{10}$$

Here, $\rho_i(x,k)$ is the density of scattering partners of type i and $\sigma_i(s)$ is the corresponding total cross-section for collinear collision. We use $s_a = (m_i + m_\pi)^2$, $s_b = (m_i + m_\pi)^2$, and the cms energy $s = s(k,p)$. The total scattering rate is obtained by integrating over momentum k and summing over all species i

$$\mathcal{R}(x,p) = \frac{1}{2\sqrt{m_\pi^2+p^2}} \times \sum_i \int d^3k\,\rho_i(x,k)\,\sigma_i(s)\,\frac{\sqrt{(s-s_a)(s-s_b)}}{\sqrt{m_i^2+k^2}} \,. \tag{11}$$

The momentum distribution of scattering partners can be described in a thermal model in which hadrochemical abundances are fixed in terms of chemical potentials μ_i,

$$\rho_i(k) = \frac{1}{(2\pi)^3}\,\left[\exp\left((E_k - \mu_i)/T\right) \pm 1\right]^{-1} \,. \tag{12}$$

This is known to provide a good description of the hadronic final state at SPS and RHIC. All pion scattering rates are computed in the rest frame of the hadron gas.

We include pions, (anti)nucleons, kaons, rhos, and (anti)deltas as scattering partners. The total cross section is parametrised as (here for meson-baryon scattering) [11]

$$\sigma(\sqrt{s}) = \sum_r \frac{\langle j_b,\, m_b,\, j_m,\, m_m \,||\, J_r,\, M_r\rangle\,(2S_r+1)}{(2S_b+1)(2S_m+1)} \times \frac{\pi}{p_{\rm cms}^2}\,\frac{\Gamma_{r\to mb}\Gamma_{\rm tot}}{(M_r - \sqrt{s})^2 + \Gamma_{\rm tot}^2/4} \,. \tag{13}$$

Summation in (13) runs over all relevant resonance states listed in [11]. Particle properties are taken from [12] except for higher excitations with large uncertainties which were taken from [11]. In addition to (13), a 5 mb momentum-independent contribution was added for meson-meson scattering to account for elastic processes. Furthermore, the cross-section with baryons was taken to saturate at 30 mb for $\sqrt{s}$ above 3 GeV/c. This, however, has a negligible effect on the calculation since high momentum particles are suppressed in the thermal distribution.

To fix the hadrochemical composition of scattering partners at SPS and RHIC, an estimate of the chemical potentials entering (12) is needed. We take the chemical potential for direct pions from data on pion phase-space densities, and those for all other particle species from ratios of dN/dy. For SPS at $\sqrt{s} = 17\,A$GeV/c and RHIC at $\sqrt{s} = 130\,A$GeV/c we determine the chemical potentials for three different temperatures: 90, 100 and 120 MeV.

In more detail: the pion chemical potential at the SPS is obtained by comparing the $m_\perp$-dependence of the average phase-space density measured by NA44 [13] and WA98 [14] to the result expected from a thermalized boost-invariant source with box transverse density profile and a transverse flow profile $\eta_t = \sqrt{2}\eta_f r/R_{\rm box}$ [15]. The transverse momentum spectra relate transverse flow and temperature, thus that $\eta_f \simeq 0.7$ corresponds to $T = 90$ - 100 MeV and $\eta_f \simeq 0.55$ for $T = 120$ MeV. From the ratios of dN/dy at midrapidity measured for pions, protons, antiprotons, positive and negative kaons [16, 17], particle densities and corresponding chemical potentials are extracted under the assumption that all particles originate from the same thermal source. Resonance decay contribution to pion production is accounted for. The density of neutrons is assumed to be the same as that of protons.

temperature	90 MeV	100 MeV	120 MeV
μ_p	550-571	518-540	456-489
$\mu_{\bar{p}}$	373-394	322-344	221-254
μ_π	50-65	38-53	10-30
μ_Δ	600-636	556-593	466-519
$\mu_{\bar{\Delta}}$	423-460	360-397	231-284
μ_ρ	100-130	76-106	20-60
μ_K	172-194	146-169	96-129
$\mu_{\bar{K}}$	121-143	90-112	28-61

TABLE I: Chemical potentials used in the calculation for SPS at $\sqrt{s} = 17\,A\mathrm{GeV}/c$. All values in MeV.

temperature	90 MeV	100 MeV	120 MeV
μ_p	499-549	454-517	361-449
$\mu_{\bar{p}}$	464-518	416-483	316-409
μ_π	78-100	70-95	50-85
μ_Δ	577-648	524-614	411-534
$\mu_{\bar{\Delta}}$	542-618	485-580	366-494
μ_ρ	156-200	140-194	100-170
μ_K	200-250	177-241	130-218
$\mu_{\bar{K}}$	185-239	160-228	109-203

TABLE II: Chemical potentials used in the calculation for RHIC at $\sqrt{s} = 130\,A\mathrm{GeV}$. All values in MeV.

Chemical potentials for ρ's, Δ's and $\bar{\Delta}$'s are deduced by requiring detailed balance [18]

$$\mu_\rho = 2\mu_\pi\,, \quad (14)$$

$$\mu_\Delta = \mu_p + \mu_\pi\,, \quad (15)$$

$$\mu_{\bar{\Delta}} = \mu_{\bar{p}} + \mu_\pi\,. \quad (16)$$

The chemical potentials extracted in this way are summarized in Table I. We associate to them rather conservative error estimates. This ensures insensitivity of our results against fine-tuning of parameters.

The same procedure is repeated for RHIC at 130 AGeV. Results are summarized in Table II. In contrast to SPS, the available data on pion phase-space density [2] are still preliminary, which is reflected in the much larger error estimates for the chemical potentials. The yields at midrapidity for pions, (anti)protons, and kaons were taken from [19]. Feed-down to (anti)proton multiplicity from weakly decaying particles was taken into account in order to be consistent with the data.

Fig. 1 shows scattering rates calculated for T = 100 MeV and the upper bounds of the chemical potentials listed in Tables I and II. This plot demonstrates that the smaller contribution of nucleons to the pion scattering rate at RHIC is largely compensated by antinucleons. Despite the increased pion phase space density at RHIC, the pionic contribution does not dominate the total scattering rate because of the small pion-pion cross section.

Fig. 2 summarizes our results for the various sets of freeze-out temperature and chemical potentials which limit the range of values consistent with data from SPS and RHIC [2]. Although particle densities are kept approximately the same for all temperatures, we conclude that the scattering rate is very sensitive to the temperature. This can be understood from the observation that most interactions with thermally moving scattering partners occur at cms energies below the lowest resonance. Increasing the thermal excitation in the collision region thus increases the contributions from resonance peaks and thus results in a larger scattering rate.

In principle, the temperature dependence of the scattering rate allows to constrain the range of allowed freeze-out temperatures. This, however, is subject to uncertainties: Our calculation considers the escaping particle in the rest frame of the locally thermalized medium. For an expanding fireball, higher $p_\perp$ particles come from transversely boosted source regions and the momentum with respect to the medium is smaller than $p_\perp$ and results in a higher scattering rate. On the other hand, if a pion with finite $p_\perp$ escapes the collision region at finite time τ', then the opacity integral entering (1) is cut off at τ'; this increases the escape probability. Finally, while freeze-out should occur when the escape probability (1) increases above a critical value of $\approx 50\%$, there is no a priori argument which fixes this value exactly. We are thus unable to determine the escape probability for finite $p_\perp$ particles without knowing the space-time evolution of the fireball, but we can set its *lower* bound by using the estimate (9) and the scattering rate for $p_\pi = 0$.

To establish this lower bound, we choose a realistic estimate for the transverse flow gradient $\xi = 0.08\,\mathrm{fm}^{-1}$ [20] which for a particle of vanishing transverse momentum and $\mathcal{R}_{\mathrm{fo}} = 0.7\,(\mathrm{fm}/c)^{-1}$ results in a very low escape probability of 1.2%. A reduction of the opacity integral

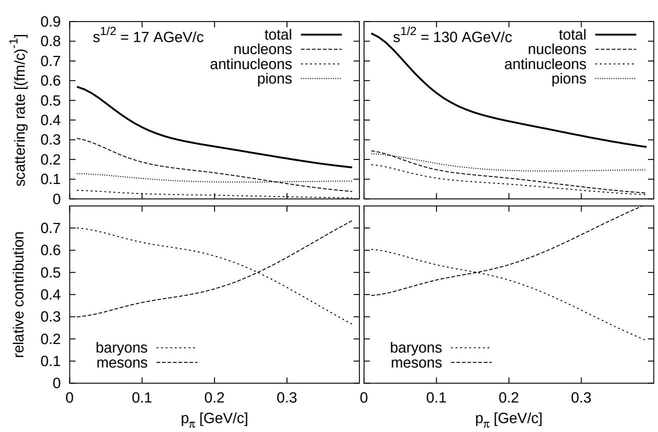

FIG. 1: Pion scattering rate as a function of pion momentum calculated at $T = 100\,\mathrm{MeV}$ and the highest possible values of chemical potentials for SPS (left column) and RHIC (right column). Contributions to the total scattering rate from scattering on nucleons, antinucleons and pions are indicated. The lower row shows the baryonic and mesonic relative contributions.

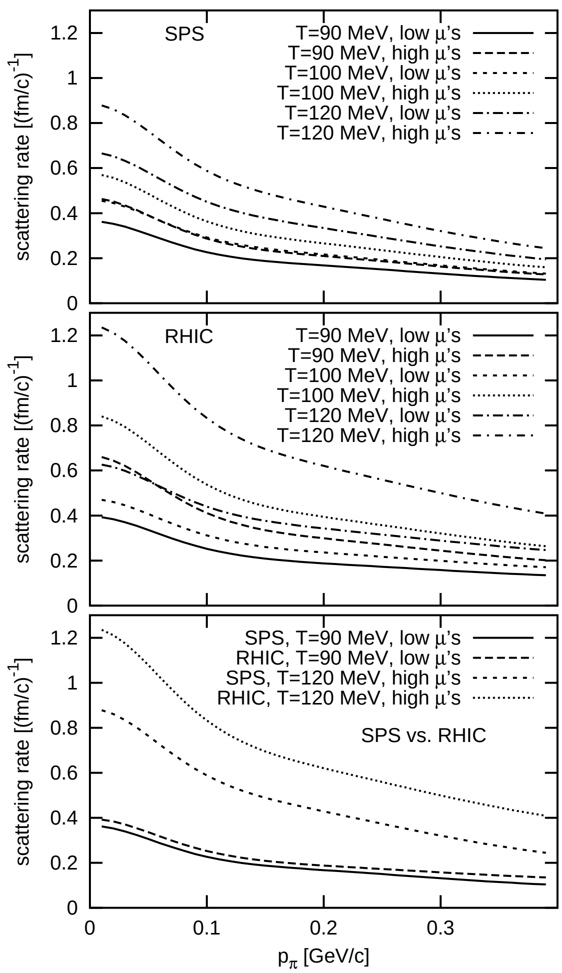

FIG. 2: Pion scattering rate as a function of pion momentum calculated for temperatures 90, 100, and 120 MeV and all corresponding sets of chemical potentials from Tables I (SPS at $\sqrt{s} = 17$ AGeV/c) and II (RHIC at $\sqrt{s} = 130$ AGeV/c). Lowest panel summaries the lowest and the highest curves from both SPS and RHIC.

in (1) by a factor 2 increases the escape probability to $\approx 11\%$, a reduction by a factor 4 results in $\approx 33\%$. Despite the above-mentioned caveats, at face value, a scattering rate $\mathcal{R}_{\rm fo} < 0.2\,({\rm fm}/c)^{-1}$ as found for $p_\pi > 200$ MeV and a rather low freeze-out temperature $T = 90$ MeV (see Fig. 2) is compatible with an escape probability of the order 1/3 to 1/2 while the scattering rates obtained for freeze-out temperatures of 120 MeV are not. This indicates that the freeze-out criterion investigated here favours rather low freeze-out temperatures of at or below 100 MeV.

We finally comment on the generic decrease of the scattering rate with the projectile momentum p_π. If plotted as a function of transverse momentum, this decrease will be flatter since a finite $p_\perp$ corresponds to a lower p_π measured in the rest frame of the comoving source. However, for a finite $p_\perp$ particle, the opacity entering (1) is cut off by finite size effects. Thus, we expect that the result of an escape probability increasing monotonously with $p_\perp$ will be robust against refinements of our calculation. This implies that high-$p_\perp$-particles can decouple earlier and may thus originate from a smaller source. It is an intriguing possibility that the rather steep transverse momentum slopes of the out and side HBT radius parameters, which are generically underpredicted in model calculations [20], receive a significant contribution from this effect.

In calculating the scattering rate, this work has singled out the dominant effects relevant for freeze-out from the many other details of a fully dynamical (event generator) calculation of ultrarelativistic heavy-ion collisions. We expect that this will prove useful for a better understanding of global features of the collision region, such as the observed freeze-out temperature, transverse velocity gradients, and absolute sizes and slopes of HBT radius parameters.

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