Comparison of SUSY mass spectrum calculations

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Abstract

We provide a comparison of the results of four SUSY mass spectrum calculations in mSUGRA: Isajet, SuSpect, SoftSusy, and SPheno. In particular, we focus on the high $\tan\beta$ and focus point regions, where the differences in the results are known to be large.

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1 Introduction

Many SUSY studies rely on computer codes that calculate the mass spectrum of the minimal supersymmetric standard model (MSSM), the couplings, branching ratios, *etc.*, from given sets of model parameters. For the LHC, for instance, many simulations are done for particular benchmark scenarios or by mapping the $(m_0, m_{1/2})$ parameter plane. For such studies it is certainly important wheather a particular decay channel is open or not and what branching ratio it has. Also, theoretically or experimentally excluded regions depend on the details of the spectrum. Studies for an e^+e^- Linear Collider deal, in addition, with high precision measurements of (s)particle properties, with the determination of the underlying SUSY breaking parameters, their extrapolation to the GUT scale, model destinction, *etc.* Experimental accuracies of the per-cent or even per-mille level are expected. It is thus clear that we need theoretical predictions of a precision comparable to the experimental accuracy. However, it has been noticed [1, 2] that different programs can give quite different results for the same set of input parameters.

In this article, we compare the mass spectrum calculations of four public codes: Isajet 7.63 [3], SuSpect 2.005 [4], SoftSusy 1.4 [5], and SPheno 1.0 [6], in the minimal supergravity (mSUGRA) framework. We discuss the renormalization group (RG) running and the implementation of radiative corrections, concentrating on the parameter regions where the largest differences are encountered: large $\tan\beta$ and large m_0. An overview of which corrections are implemented in each of the four programs is given in Table 1.

2 Large $\tan\beta$

Large $\tan\beta$ has always been recognized as a difficult case since it requires a thorough treatement of the bottom Yukawa coupling h_b. It is well known [12] that h_b gets large $\tan\beta$ enhanced corrections from SUSY loops, the dominant contributions coming from $\tilde{b}\tilde{g}$ and $\tilde{t}\tilde{\chi}^+$ exchanges. These generate a $H_2^0 b\bar{b}$ coupling, which is forbidden at tree-level, $\mathcal{L} \sim h_b\, H_1^0 b\bar{b} + \Delta h_b\, H_2^0 b\bar{b}$. This modifies the tree-level relation between the bottom mass

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[1] Speaker

	Isajet 7.63	**SuSpect 2.005**	**SoftSusy 1.4**	**SPheno 1.0**
RGEs				
gauge + Yuk.	2–loop	2–loop	2–loop	2–loop
gaugino par.	2–loop	2–loop	2–loop	2–loop
scalar par.	2–loop	1–loop	1–loop	2–loop
SUSY masses				
$\tilde{\chi}^{\pm}, \tilde{\chi}^0$	some corr. for $\tilde{\chi}_1^{\pm}$	1–loop approx. for ΔM_1, ΔM_2, $\Delta\mu$		full 1–loop
$\tilde{t}$	—	$\tilde{t}g + t\tilde{g}$ + Yuk.	full 1–loop	full 1–loop
$\tilde{b}$	—	$\tilde{b}g + b\tilde{g}$	full 1–loop	full 1–loop
$\tilde{g}$	$g\tilde{g} + q\tilde{q}$ loops resummed			
Yukawa cpl.				
h_t	full 1–loop resum.	$tg + \tilde{t}\tilde{g}$	full 1–loop	full 1–loop
h_b	full 1–loop resum.	$bg + \tilde{b}\tilde{g} + \tilde{t}\tilde{\chi}^{\pm}$ corr. resummed		full 1–loop resum.
Higgs sector				
tadpoles	3rd gen. (s)fermions	complete 1–loop corrections [7]		
h^0, H^0	1–loop [8]	1–loop [9]	2–loop [10]	2–loop [11]

Table 1: RGEs and radiative corrections implemented in Isajet, SuSpect, SoftSusy, and SPheno.

and Yukawa coupling, $m_b = h_b v_1 \to m_b = h_b v_1(1+\Delta_b)$ with $\Delta_b = (\Delta h_b/h_b)\tan\beta$. In the programs under discussion this is taken into account as

$$h_b(M_Z) = \hat{m}_b^{\rm MSSM}(M_Z)/v_1(M_Z)\,, \quad \hat{m}_b^{\rm MSSM}(M_Z) = \frac{\hat{m}_b^{\rm SM}(M_Z)}{1+\Delta m_b/m_b}\,. \tag{1}$$

Here $\hat{m}_b^{\rm SM}$ is the $\overline{\rm DR}$ bottom mass in the Standard Model and $\Delta m_b = (\Delta m_b)^{\tilde{b}\tilde{g}+\tilde{t}\tilde{\chi}^+ + \ldots}$ contains the SUSY-loop corrections. The complete 1-loop expression for Δm_b is given in [7].[2] Compared to the naive 1-loop expansion $\hat{m}_b^{\rm MSSM} = \hat{m}_b^{\rm SM}(1-\Delta m_b/m_b)$, eq. (1) makes a numerical difference of about 10% in h_b and about 10–30% in m_A for large $\tan\beta$. The resummation of SUSY threshold corrections [13] will be discussed elsewhere [14]. Although all four programs now apply eq. (1), some numerical differences in h_b remain. These are partly due to differences in α_s: Suspect, SoftSusy and SPheno calculate α_s in the $\overline{\rm DR}$ scheme, Isajet uses the $\overline{\rm MS}$ value. Another reason is that Isajet uses $m_b = m_b(M_{SUSY})$ for the expression $\Delta m_b/m_b$ in eq. (1), while the other programs use $m_b(M_Z)$ or the bottom pole mass; also the gluino masses differ by about 5%. Moreover, the vacuum expectation values $v_{1,2}$ are not running in Isajet.

The bottom Yukawa coupling has its largest effect in the Higgs sector. Figure 1 shows the running of $m^2_{H_{1,2}}$ for $m_0 = 400$ GeV, $m_{1/2} = 300$ GeV, $A_0 = 0$, $\mu > 0$, and the two cases $\tan\beta = 10$ and $\tan\beta = 50$. As one can see, there is good agreement for not too large $\tan\beta$. However, for $\tan\beta = 50$, quite different results are obtained for $m^2_{H_1}$, whose evolution is driven by h_b:

$$\frac{dm^2_{H_1}}{dt} \sim \frac{3}{8\pi^2}\, h_b\, X_b + \ldots\,, \qquad X_b = (m^2_{\tilde{Q}} + m^2_{\tilde{D}} + m^2_{H_1} + A_b^2)\,. \tag{2}$$

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[2] Here note that [3, 4, 5, 6] and [7] partly have different conventions, *e.g.*, for the ordering of the squark mass eigenstates and the sign of μ.

Note in particular the dotted line which shows the result obtained with Isajet 7.58. In this version, the SUSY corrections to h_b were not yet resummed.

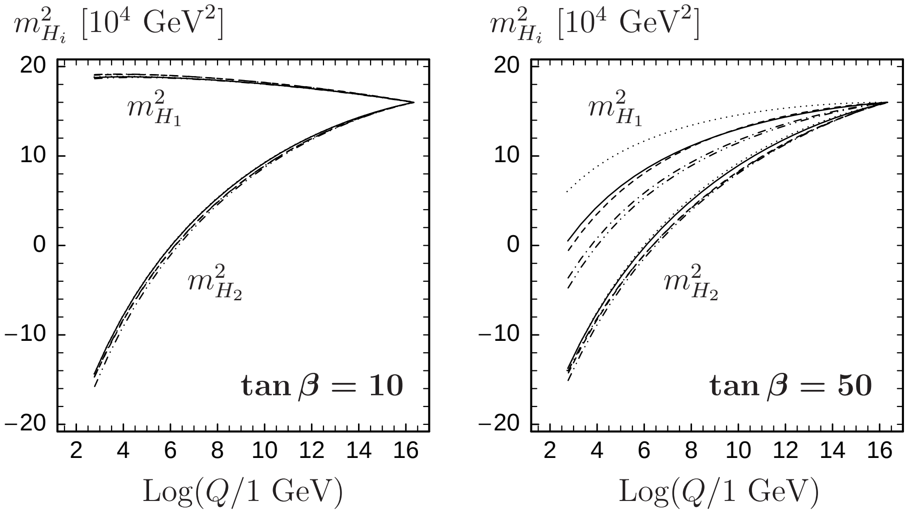

Figure 1: Running of $m^2_{H_{1,2}}$ as a function of the scale Q, for $m_0 = 400$ GeV, $m_{1/2} = 300$ GeV, $A_0 = 0$, $\mu > 0$, $\tan\beta = \{10, 50\}$, $M_t = 175$ GeV. The full (dotted) lines are for Isajet 7.63 (7.58), the dashed lines are for SuSpect 2.005, the dash-dotted ones for SoftSusy 1.4, and the dash-dot-dotted ones for SPheno 1.0.

The differences in $m^2_{H_1}$ directly translate into m^2_A and thus into the physical Higgs boson masses, since

$$m^2_A = \frac{1}{c_{2\beta}}\left(\overline{m}^2_{H_2} - \overline{m}^2_{H_1}\right) + \frac{s^2_\beta\, t_1}{v_1} + \frac{c^2_\beta\, t_2}{v_2} - M^2_Z\,. \tag{3}$$

Here $\overline{m}^2_{H_i} = m^2_{H_i} - t_i/v_i$, $i = 1, 2$, and $t_{1,2}$ are the tadpole contributions. The self energies of Z and A have been neglected in eq. (3). We note that including only the tadpoles from the third generation is in general a good approximation. The remaining 1-loop contributions account for a $\mathcal{O}(1\%)$ correction.

Figure 2 shows the Higgs boson masses obtained by the four programs as a function of $\tan\beta$. The new Isajet version 7.63 has led to a majer improvement compared to the situation discussed in [2, 15] (the results obtained by Isajet 7.58 are again shown as dotted lines in Fig. 2). For m_A and $m_{H^\pm}$ there is now agreement within $\sim 10\%$ up to $\tan\beta \sim 45$. Sources for the remaining differences are pointed out above. Moreover, it makes a difference whether one uses running couplings and/or masses for the tadpoles $t_{1,2}$. Here each program has a different approach. For the neutral scalars, however, the situation is not so good. Especially for m_{h^0}, a discrepancy of ~ 4 GeV is too large compared to the expected experimental accuracy. This discrepancy is mainly due to the different radiative corrections taken into account for the (h^0, H^0) system. They vary between 1- and 2-loop, effective potential and diagramatic calculations, see Table 1. Given the expected experimental accuracy for m_{h^0} it is clear that the best available calculation should be used.

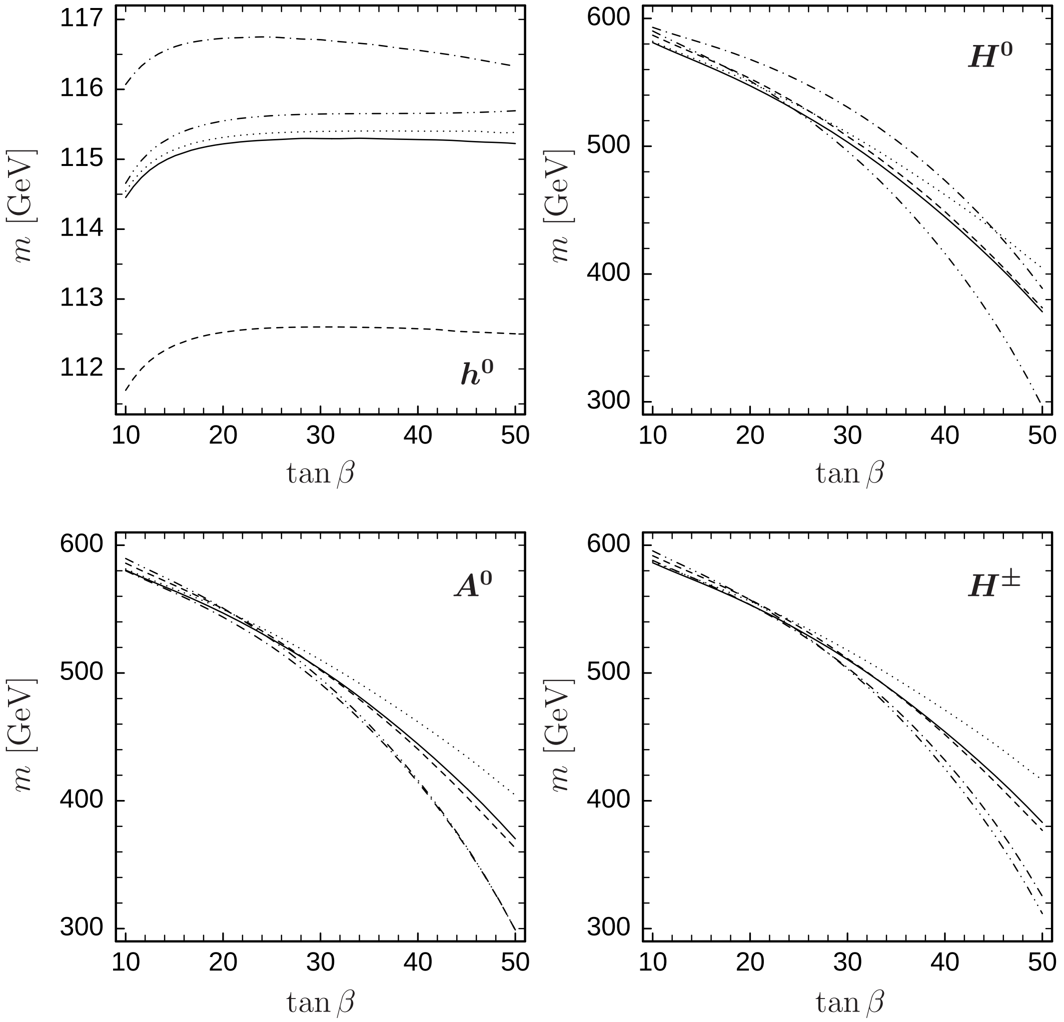

Figure 2: Higgs boson masses as a function of $\tan\beta$, for $m_0 = 400$ Gev, $m_{1/2} = 300$ GeV, $A_0 = 0$, $\mu > 0$, $M_t = 175$ GeV; full (dotted) lines: Isajet 7.63 (7.58), dashed: SuSpect 2.005, dash-dotted: SoftSusy 1.4, dash-dot-dotted: SPheno 1.0.

3 Large m_0

For large m_0, the running of $m^2_{H_2}$ becomes very steep and very sensitive to the top Yukawa coupling $h_t = \hat{m}_t/v_2$:

$$\frac{dm^2_{H_2}}{dt} \sim \frac{3}{8\pi^2}\, h_t\, X_t + \ldots\,, \qquad X_t = (m^2_{\tilde{Q}} + m^2_{\tilde{U}} + m^2_{H_2} + A_t^2)\,. \tag{4}$$

As a result, the μ parameter given by

$$\mu^2 = \frac{\overline{m}_{H_1} - \overline{m}^2_{H_2}\tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}\, M_Z^2 \tag{5}$$

becomes extremely sensitive to h_t. This is visualized in Fig. 3 where we show in (a) the running of $m^2_{H_{1,2}}$ for $m_0 = 1450$ GeV, and in (b) μ as a function of m_0. The other

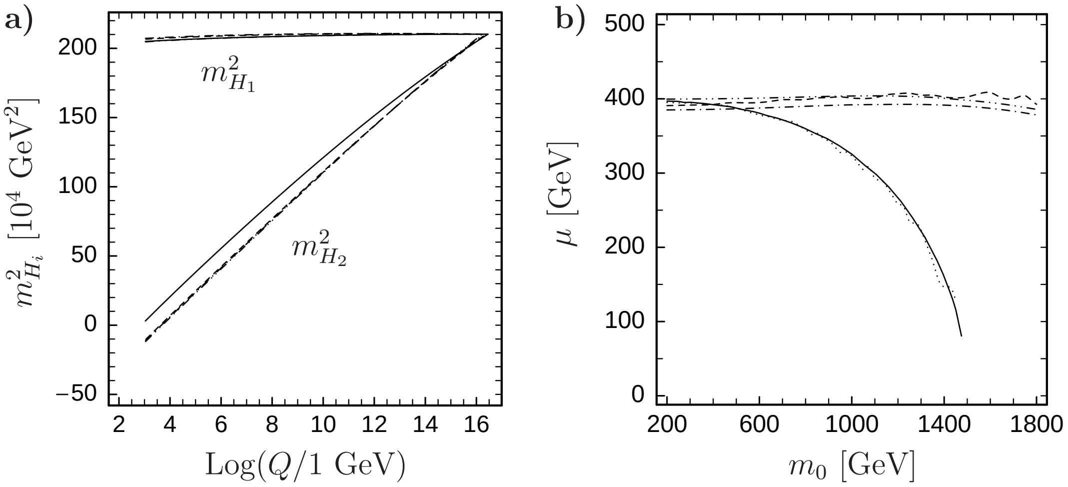

Figure 3: **a)** Running of $m^2_{H_{1,2}}$ for $m_0 = 1450$ GeV; **b)** μ as a function of m_0; for $m_{1/2} = 300$ GeV, $A_0 = 0$, $\mu > 0$, $\tan\beta = 10$, and $M_t = 175$ GeV; full (dotted) lines: Isajet 7.63 (7.58), dashed: SuSpect 2.005, dash-dotted: SoftSusy 1.4, dash-dot-dotted: SPheno 1.0.

parameters are $m_{1/2} = 300$ GeV, $A_0 = 0$, $\mu > 0$, and $\tan\beta = 10$. The large discrepancy in μ for $m_0 \gtrsim 800$ GeV lead to completely different chargino/neutralino properties and likewise to very different excluded regions in Isajet compared to the other programs. For instance, radiative EWSB breaks down in Isajet for $m_0 \gtrsim 1.5$ TeV. In SuSpect, SoftSusy, and SPheno, this happens only for $m_0 \gtrsim 2.5$–2.8 TeV.[3]

In order to understand the behaviour in Fig. 3b it is useful to write eq. (5) in the form

$$\mu^2 \simeq c_1\, m_0^2 + c_2\, m_{1/2}^2 - 0.5 M_Z^2\,. \tag{6}$$

Approximate analytical expressions for c_1 and c_2 can be found *e.g.*, in [16, 17]. For $A_0 = 0$ and $\tan\beta = 10$ we get [17]

$$c_1 \sim \left(\frac{\hat{m}_t}{156.5\ \text{GeV}}\right)^2 - 1\,, \qquad c_2 \sim \left(\frac{\hat{m}_t}{102.5\ \text{GeV}}\right)^2 - 0.52\,. \tag{7}$$

Since the Higgs potential is minimized at $M_{SUSY} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, we take $\hat{m}_t$ in eq. (7) as $\hat{m}_t = \hat{m}_t(M_{SUSY})$. The m_0 dependence seen in Isajet is reproduced for $\hat{m}_t \sim 151$ GeV. The one of SuSpect, SoftSusy and SPheno is reproduced for $\hat{m}_t \sim 155$ GeV. Figure 4 shows a contour plot of μ in the $(m_0, \hat{m}_t)$ plane. Notice the fast increasing dependence on $\hat{m}_t$ for increasing m_0. Notice also that for $\hat{m}_t \sim 156$–157 GeV, μ becomes almost independent of m_0, which is the actual focus point condition.

There are some obvious differences in the calculations. For instance, M_{SUSY}, the scale where the SUSY parameters are frozen out and the Higgs potential is minimized, varies by about 100 GeV due to different radiative corrections to the stop masses, *c.f.* Table 1. In the loop corrections to m_t, analogous differences occur as discussed above for $\Delta m_b/m_b$.

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[3] After the conference, a sign error was corrected in SPheno. As a consequence, its results for large m_0 now nicely agree with those of SoftSusy and SuSpect (contrary to what was presented in the talk).

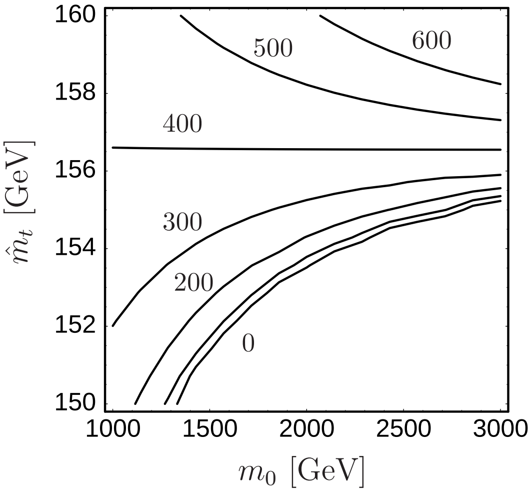

Figure 4: The parameter μ as given by eq. (6) in the $(m_0, \hat{m}_t)$ plane, for $m_{1/2} = 300$ GeV, $A_0 = 0$, and $\tan\beta = 10$.

Also the evolution of h_t between M_Z and M_t and the inclusion of threshold effects are delicate points. However, this is not yet sufficient to explain the observed discrepancies. More work is needed to clarify the situation.

4 Conclusions

For the calculation of the SUSY mass spectrum from GUT scale boundary conditions, there are two particular difficult parameter regions where large numerical differences have been noticed: large $\tan\beta$ and large m_0. These regions are very sensitive to the bottom and top Yukawa couplings, respectively.

The inclusion of the SUSY 1-loop corrections to h_b has led to a considerable improvement in the large $\tan\beta$ case. In particular, the four programs now agree on m_A within $\lesssim 10\%$ for $\tan\beta \lesssim 45$. Further improvements are of course desirable.

For large m_0, there are still very large numerical discrepancies due to the corrections to h_t. As a matter of fact, h_t is much smaller in Isajet than in the other programs. Some differences in the calculation of h_t have been pointed out, but these do not satisfingly explain the observed discrepancies. Work is in progress to clarify the situation [14].

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