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# RLA design : RF optics

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**Abstract**

Opposite to the synchrotrons which work at fixed optics, the Recirculating Linear Accelerators work with a variable optics in the RF system. This is due to the increase of the particle energy at subsequent passages, which results in a decrease of the normalised quadrupole strengths. It is demonstrated in this note that this effect does not introduce any limitation in the total energy gain if FODO cells are used in the RF system. Consequently a single recirculator could be designed to accelerate from 2GeV to 50GeV if there is no need of a first recirculator for any other reasons like bunch shaping or matching.

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## 1 Introduction

In the present design of a neutrino factory at CERN [1], the muons provided by the cooling section are accelerated by a LINAC to 2GeV and then pass through two Recirculating Linear Accelerators (RLA) before being injected into a storage ring where they decay at constant energy.

The two RLA's system was proposed in the Fermilab study1 in order to match the bunches to the second recirculator which featured an RF frequency of 400MHz [2] . In the CERN project, the existence of two RLA comes from the assumption that "a momentum gain of more than a factor 10 probably requires more than one recirculator" [3]. The need of a bunch matching is not mentioned.

In subsequent studies on $\nu$-factory at CERN, it appeared that, under the assumption that LEP RF cavities are reused, no design could be found for the first RLA. The solution found eventually, after discussion at the Tsukuba workshop, was to use 220MHz cavities and the FNAL design [4] for this RLA.

This is why an approach from scratch is desirable in order to check where are the limits to the possible injection momentum and momentum gain in an RLA.

In this note we examine only limitations associated with the optics in the RF cells. As it has been shown that a triplet optics is not more attractive than a FODO optics [5], we concentrate on the latter. As a low frequency RF system is necessary in any case to make a sufficient beam acceptance, for acceleration up to about 10GeV, a single recirculator with 220 MHz cavities is proposed to accelerate from 2 to 50GeV.

## 2 Basic limitation associated with FODO cells

It is well known and shown in appendix 1 that the maximum value $\hat{\beta}$ of the $\beta$-function in FODO cells goes to infinity when the beam energy is increased with constant quadrupole gradients. However the adiabatic damping of the betatron oscillations make the emittance decrease with the inverse of the beam momentum. In order to describe the variation of the beam size during acceleration, equation 1 of appendix 1 has merely to be divided by the beam momentum. In other words, this equation has to be multiplied by the momentum ratio $r = Po/P$. Then the minimum possible beam size scales with momentum like :

$$\left(\frac{(1-gn)(1+r.gn)}{(1+gn)(1-r.gn)}\right)^{\frac{1}{4}}$$

This function has been normalised to have the value of 1 for $r = 1$, i.e. for the reference momentum. $gn$ is equal to the product of the integrated normalised quadrupole strength, which makes $\hat{\beta}$ minimum, with one quarter of the cell length. The variation of this function with $r$ is shown on figure 1, from which we can conclude that there is no fundamental reason associated with the RF optics which limits the amount of acceleration with an RLA. However this argument concerns a single cell and does not prove that any acceleration is possible for a given ensemble of cells. In order to proceed, we have to examine what happens for such an ensemble.

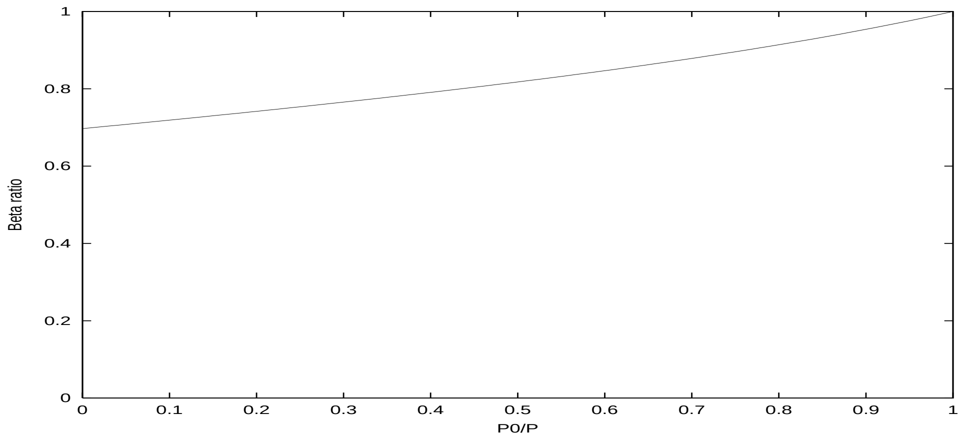


Figure 1: Relative variation of the size of a beam of a given normalised emittance passing through a matched FODO cell as a function of the inverse of the beam momentum.

## 3 Actual RF optics with many FODO cells

We start with a linac with constant focusing, i.e. with a perfectly periodic beam envelope for the first beam passage. This means that the quadrupoles have increasing strengths along the linac in order to match the energy increase.

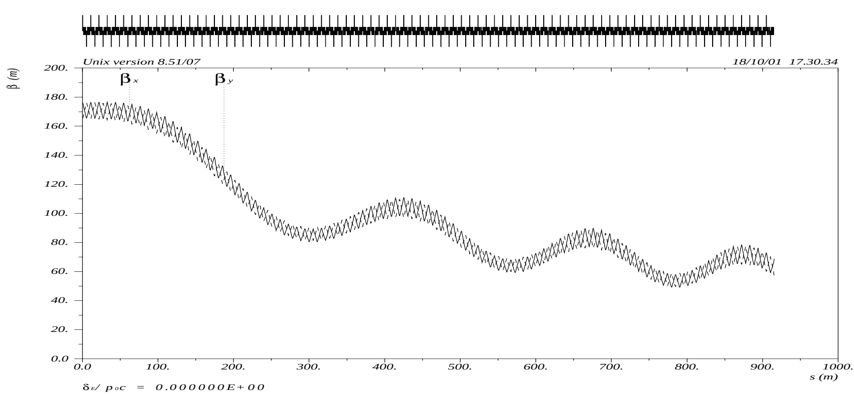


Figure 2: $\beta$-functions along the first linac (see next section) at the fourth beam passage. Initial energy : 48GeV, acceleration : 5.36GeV. The gradients are set for a constant focusing at the injection energy of 2GeV and the same acceleration.

For a higher beam energy, at a subsequent passage, we assume that the upstream arc is matched to the first cell. The maximum $\beta$-function in this first cell is larger than for the lowest energy as shown in the preceding section. In the subsequent cells, the focusing is stronger than in the first cell. Indeed the relative energy increase is smaller than for the lowest energy case, the beam energy being larger and the acceleration being the same. Consequently , the $\beta$-functions decrease on average along the LINAC. This is shown on figure 2 for the case of the linac described in the next section. This result was already given in [7].

So we can conclude that **the total energy gain in a recirculator with FODO cells in the LINAC's is not limited by the change of the LINAC's optics with energy if the first passage is done at constant focusing**. For the case of constant gradient this might not be true as the $\beta$-function increases from cell to cell at all passages.

## 4 RF cells with 220MHz cavities

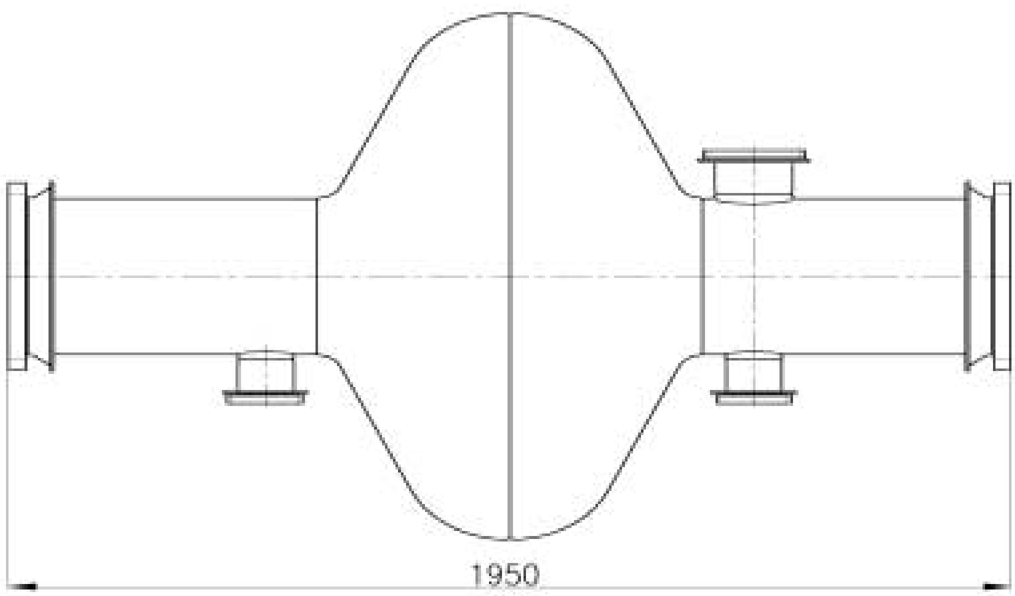


Figure 3: 200MHz super-conducting RF cavity under development at CERN.

In the CERN proposal of 2001 of a neutrino factory [1], the acceptance of the RLA's is limited by the size of the beam pipe associated with LEP cavities. As most of the LEP cavities are used in the proton driver linac, it is desirable to use more modern cavities in the RLA. We can think of 220MHz cavities. The value of 220MHz is five times the value of the frequency (44MHz) of the cavities used for bunch rotation before the cooling section. The dimensions of these cavities can be scaled from a 200MHz cavity built at CERN which will be tested soon in CESR. A sketch of this cavity is shown on fig. 3.

220MHz cavities of this sort provide a large acceptance as their associated beam pipe has a radius of 0.18m (scaling the cavity dimensions with the inverse of the frequency).

The 200MHz cavity is supposed to provide an acceleration field of 15MV/m. As this number might be optimistic, we consider the number of 13MV/m. This number is kept for 220MHz cavities which have a shorter length of 1.77m from flange to flange (scaled from the length in figure 3). Thus the accelerating voltage of 220MHz cavities (of length 0.68m) could be 8.8MV.

Possible RF cells are obtained as follows. The cavities could be separated by an integer number of wave lengths ($\lambda_{RF} = 1.3627$m) in order to preserve their high gradient, as it is not obvious that multicell cavities could achieve a similar gradient. A number $N_c$ of cavities will occupy a space of $(N_c - 1).\lambda_{RF}$. Then a space of length $2\lambda_{RF}$ is left to accommodate a short quadrupole. Consequently, a possible length of a half cell containing$N_c$ cavities and one quadrupole, is $(N_c + 1).\lambda_{RF}$.

The average accelerating field is then $8.8.N_c/[(N_c+1).\lambda_{RF}]$, i.e. $6.46.N_c/[(N_c+1)$MV/m. For $N_c = 1$, it is already larger than 2.62MV/m, the value in the present design of RLA2 [8].

The number of RF cavities per cell can be chosen to match a desired acceptance, as the acceptance associated with $N_c = 1$ is already extremely large. It could be estimated from the following considerations :

- The optimum integrated quadrupole strength is (see appendix 1) $2(\sqrt{5}-1)/l$, $l$ being the cell length. For the smallest value of 5.45m, which corresponds to $N_c = 1$, we obtain $K_Q L_Q = 0.454m^{-1}$. For a quadrupole length of 0.3m, this corresponds to a gradient of 40T/m at 8GeV (end of first linac, see next section) and about 4T on the pole. The length of 0.3m should be reasonable as a space of 1.3m has been left to accommodate the quadrupole which must be super-conducting.
- The maximum value of the $\hat{\beta}$ is $1.6651l$, i.e. 9.07m for the case of $N_c = 1$. The associated normalised emittance is 0.071rad.m (the present one is 0.015rad.m). This value is even larger than the maximum value of 6cm considered in the present cooling scheme [9]. The intensity of the muon beam could be easily increased by the use of 220MHz cavities. An order of magnitude of this increase can be inferred from fig. 4.

## 5 Possible design of the linacs of a 50GeV RLA

In order to maximise the average acceleration field a space as large as possible must be occupied by RF cavities. This implies that the two straight sections of the RLA are filled with cavities. Moreover the acceleration efficiency is increased if the density of cavities can be natched to the beam energy. This is examined below.

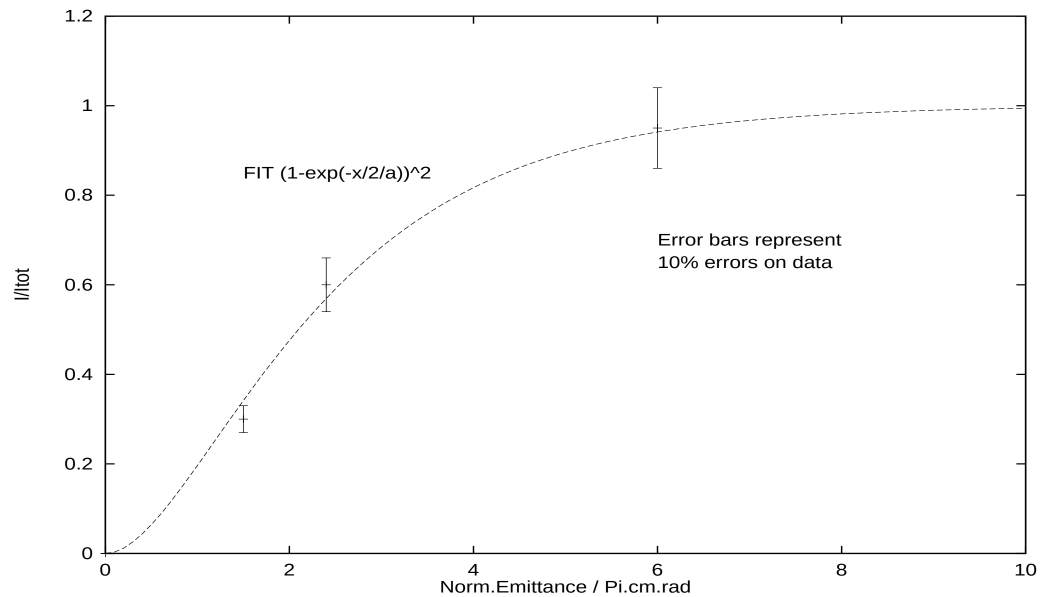


Figure 4: Muon intensity versus emittance. The emittance cut is assumed to be done in both plane in the same way, which leads to a squared Gaussian distribution. The data were provided by A. Lombardi.

### 5.1 First linac

If we want to accelerate with a single RLA to about 50GeV in four turns, the acceleration per turn must be 12GeV. We design the first linac for a normalised acceptance about twice as large as that of the present design, i.e. 0.03rad.m. This is achieved with a value of $\hat{\beta}$ of 21.5m. The cell length is 12.9m, which corresponds to $N_c \leq$3.73. For $N_c = 3$, the cell length is $l = 10.9$m and $\hat{\beta}$ is equal to 18.15m. The associated normalised acceptance, associated with a radius of the beam pipe of 0.18m, is 0.0355 $\pi$rad.m (for 2 GeV muons, $\beta\gamma$=19.9). The average accelerating field is 4.84MV/m.

The integrated quadrupole strength is 0.2268m$^{-1}$. Their integrated gradient at the end of the LINAC is 6.05T for an energy of about 8GeV. For a quadrupole length of 0.3m, the gradient is 20T/m and the field on the pole is about 2T, which means that super-conducting quadrupoles have to be used. There is little optimisation to be done but it has to be checked that the fringe field can be screened well enough to make the operation of the super-conducting RF cavities possible.

### 5.2 Second linac

As the energy at the first exit of the first linac is of the order of 8GeV, the adiabatic damping of the transverse emittance makes it possible to increase the cell length of the second linac by about four in order to fill the cavity aperture.

In order to be safe, we make the cell length equal to three times that of the first linac. Accordingly there will be 11 cavities per half cell : 9 cavities, as in three half cells of the first linac, plus two which are installed at the old location of quadrupoles. Thus the density of RF cells is increased by the factor 11/9. The average accelerating voltage becomes 11/9 times that of the first linac. So the accelerating length of the whole RLA is equivalent to 20/9 times the length of the first linac. In order to obtain a total of about 12GeV per turn, the length of the first linac must be about 1.12km. As the number of cell must be an integer multiple of 3, we take 102 cells, i. e. 1.1118km. This is a little more than one half that of the present RLA2 [1].

### 5.3 Discussion

The energy at the exit of the first linac at the first pass is 7.38GeV and the associated emittance reduction is 2/7.38. This smaller than the value of 1/3 that we assumed. The energy gain per turn is 11.95GeV if we forget the beam loading. The exit energy is 49.8GeV.

The integrated quadrupole strength in the second linac has the value 1/3 of that of the first linac, i.e. is 0.0756m$^{-1}$. The integrated gradient at the end of this linac (energy 14GeV) is 3.53T. For a length of 0.4m, the gradient takes the value of 8.8T/m and the pole field is 1.6T. This is a little too much for warm quadrupoles. It has to be examined whether a longer length is possible or whether super-conducting quadrupoles have to be used.

The design described above remains valid even if the cavity voltage is reduced by 20% and provides only 4.3GeV for the first linac, which corresponds to a reduction of the emittance by a factor of three after the first pass. For this case, the field in the cavities is 10.4MV/m and the exit energy is 40GeV.

The two linac have a length of 2.47km. This represent 2.34 times the LEP2 RF system [10], the cost of which was about 300MSF. However the 220MHz cavities are larger than the LEP cavities, so the cost per unit of length has to be scaled with a power of the inverse of the frequency comprised between 1 and 2. Taking 1.5, the RF system alone represents at least 1.4GSF.

Of course the design of the RF cells is only a part of the RLA design. It remains to be checked whether

- arcs and spreaders can be matched to these RF cells,
- longitudinal beam parameters are satisfactory

## 6 Possible design of a cheap RLA

In the case where the proton driver is a rapid cycling synchrotron, the LEP RF system becomes available for the RLA. As shown previously, this system can be used only at high energy because of the small aperture of the 352MHz cavities.

The total voltage available is 3.63GV for 288 cavities installed in 72 modules. Each module has a length of 12.5m (this includes pumping ports and valves). The length of cells made of two modules and two quadrupoles could be 26m. The associated value of $\hat{\beta}$ is 43.3m. The aperture of the beam pipe can be extended to 0.09925m provided the end tapers are replaced with cylinders [3]. The normalised acceptance at 2GeV is then 0.00453rad.m, which is much too small. Consequently the LEP cavities can only be used for the second linac.

A possible strategy consists of using The LEP system for the second linac and a first linac similar to that described in the preceding section, with the same length as the LEP RF system and with a frequency equal for instance to 235MHz (2/3 of 352MHz). This frequency choice imposes that the arc length be a multiple of $3\lambda_{RF,LEP}$.

### 6.1 Modification of the LEP system

As its length is around 1km, the first linac will provide an acceleration of about 5GeV. The energy at the entrance of the second linac will be around 7 GeV. In order to provide a normalised acceptance comparable with that of the first linac, the value of $\hat{\beta}$ has to be 18.6m. This results in a cell length of 11.16m.

This imposes that the modules are rearranged. The simplest procedure is probably to cut them into two pieces of 7m each. With a space of 1m to include short quadrupoles, the cell length is 16m and the normalised acceptance at 7GeV is 0.0248. If this is considered to be too small, each RF cell has to installed in its own cryostat and the cell length will be comparable to that of the first RLA described in [3], i.e. 8m (this would make a normalised acceptance of 0.05).

We consider a cut of the modules into two parts to limit the cost in what follows. The experience of the SPL study shows that the cost saving might be marginal is too much work is involved in the modification of the RF cavities. The length of the second linac will be 1.152km.

### 6.2 first linac

The first linac has a length of 1.152km. With the same design as above, this represents 105 cells and an accelerating voltage of 5.5GeV.

### 6.3 Performance

The total voltage per turn is 9.1GeV. With four turns and an injection energy of 2GeV, the final energy is 38.4GeV.

The acceptance is limited by the second linac to the normalised value of 0.026rad.m.

The cost of the RLA will be dominated by that of the first linac which can be estimated to about 0.6GSF.

## 7 Conclusion

It has been shown that no practical limitation to the acceleration arises from using FODO cells in the RF if the injection optics in both transverse planes is matched to the injection energy.

Using high performance 220MHz cavities, it is conceivable to design a single RLA to accelerate from 2GeV to 50GeV with an acceptance equal to twice the present one. However a problem lies probably in the design of the arcs as it has been already seen in the previous study [1]. In particular the longitudinal optics could introduce more limitations than the transverse optics [11].

A cheap RLA using the whole LEP RF system could provide muon beams with an energy of 38GeV and a maximum normalised emittance of 0.026rad.m.

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# Appendix 1

### FODO Cell at various beam energies

A FODO cells is made from a sequence of a drift space of length $l/2$, a focusing quadrupole of integrated normalised strength $-K_Q.L_Q = -4x/l$, a drift space of length $l/2$, and a defocusing quadrupole of integrated normalised strength $4x/l$. In the thin lens approximation, the transfer matrix of such a system is given by the product of the four transfer matrices associated with these elements :

$$\begin{pmatrix} 1 & 0 \\ x/l & 1 \end{pmatrix} \begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -x/l & 1 \end{pmatrix} \begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix}$$

l being the total length of the cell. The phase advance of the this system is obtain from the trace of the product matrix. We obtain the well known result [6]):

$$\sin \mu/2 = x$$

The maximum value $\hat{\beta}$ of the envelope function in the cell occurs at the location of the focusing quadrupole. Therefore it is obtained dividing the (1,2) matrix element by $\sin \mu$ :

$$\hat{\beta} = l(1+x)/\sin \mu = l(1+x)/2x\sqrt{1-x^2}$$

The derivative of the function $(1+x)/x\sqrt{1-x^2}$ is proportional to $x^2 - x + 1$. It goes to zero for $x = \frac{\sqrt{5}-1}{2} = gn$ (the golden number). Thus $\hat{\beta}$ has a minimum for $\sin \mu/2 = gn$. The value of this minimum is $l/2.gn^{2.5} = 1.6651.l$.

We examine now how $\hat{\beta}$ changes when the beam momentum is increased with constant quadrupole gradients $G$. We start from the case where $\hat{\beta}$ is minimum. As $K_Q = Gc/p$ for a beam momentum $p$, $c$ being the speed of light, we can write : $K_Q(p) = K_Q(p_0)p_0/p$. From the definition of $x$, we have also : $x(p) = gn \times r$,r being the ratio $p_0/p$ which varies from one to zero when the beam momentum increases from $p_0$ to infinity. Eventually we obtain :

$$\frac{\hat{\beta}(p)}{\hat{\beta}_{min}} = \frac{1}{r}\sqrt{\frac{(1-gn)(1+r.gn)}{(1+gn)(1-r.gn)}} \tag{1}$$

This function is equal to 1 for r=1 and goes to infinity for r=0 (infinite beam momentum). i.e. there is no focusing for an infinite beam momentum.